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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS, ABSTRACTS OF PAPERS, AND

REPORTS OF THE PROCEEDINGS

OF THE SOCIETY

FROM NOVEMBER 1916 *TO NOVEMBER* 1917.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LXXVII. November 10, 1916. No. 1

Professor R. A. Sampson, M.A., D.Sc., F.R.S., President, in the Chair.

James Hartness, Springfield, Vermont, U.S.A.;
Mrs. Annie Scott Dill Maunder, 8 Maze Hill, Greenwich Park, London, S.E.; and
Reginald Lawson Waterfield, The Principal's House, Cheltenham,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

The Hon. Sir Charles Algernon Parsons, K.C.B., M.A., D.Sc., LL.D., F.R.S., 6 Windsor Terrace, Newcastle-on-Tyne (proposed by R. A. Sampson);
Rohani Thirunal Raja M. Rama Varma Raja, Prince of Mavelikara Palace, Assistant Director and Astronomer, H.H. The Maharaja's Observatory, Trivandrum, Travancore, S. India (proposed by Raja A. V. Jugga Rao); and
Manning Robertson, A.R.I.B.A., 50 Norfolk Square, London, W. (proposed by Major P. A. MacMahon).

One hundred and twelve presents were announced as having been received since the last meeting, including, amongst others:—
Royal Observatory, Cape of Good Hope, Annals, vol. xii.

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part 4: Determination of the Elements of the orbits of Jupiter's satellites from photographs by Bryan Cookson, presented by the Observatory;

Cordoba, Observatorio Nacional, Resultados, vol. xxi.: Catalogo de las Zonas de Exploración, $-52°$ á $-62°$, presented by the Observatory;

H. Knox Shaw: Photographs of the Variable Nebula in Corona Australis (24 lantern slides), presented by the Helwân Observatory;

Two photographs of the turret of the White Tower, used by Flamsteed as an Observatory, presented by Mr. Charles ffoulkes;

Thirty-nine lantern slides, various subjects, presented by Professor Hale.

On Chandler's Period in the Latitude Variation.
By Oskar Backlund.

Mr. Witting in an interesting investigation on the motion of the pole has collected all the international latitude observations from 1890 to 1914, and arranged them according to x (meridian of Greenwich) and y (perpendicular to x) for every tenth of a year. It seemed to me that this rich material might afford some indications concerning the variation of Chandler's period and the amplitude. I therefore undertook to calculate θ, the mean motion in 0·1 year, the amplitude and the phase for every tenth of a year. In order to render the calculations as simple as possible, the first task was to work out appropriate formulæ.

1. Employing the notations of Mr. Witting, we suppose the latitude may be expressed by

$$P = P_0 + \kappa \cos(\theta t + c),$$

unit of $t = 0{\cdot}1$ year.

Let n be the number of equidistant values of P covering nearly one period, and $m+1$ the number of periods, then, summing the $(m+1)$ values of P for a given value of ν, we have

$$\sum_{\mu=0}^{\mu=m} P_{\mu,\nu} = S_\nu = (m+1)P_0 + \kappa R \cos(\theta\nu + \psi),$$

where

$$R = \frac{\sin\frac{m+1}{2}n\theta}{\sin\frac{n}{2}\theta}; \quad \psi = \frac{mn}{2}\theta + c,$$

in which ν may have the values 0, 1, . . ., $(n-1)$.

If we take $n = 12$, $m = 4$, the influence of short periods, especi-

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ally of that of one year, is nearly excluded. The following formula is easily verified:

$$H_\nu = \frac{S_\nu - S_{\nu+1}}{S_{\nu+3} - S_{\nu+4}} = \frac{\sin\left(\frac{2\nu+1}{2}\theta + \psi\right)}{\sin\left(\frac{2\nu+1}{2}\theta + \psi + 3\theta\right)}.$$

As an approximate solution we assume $\theta_0 = 30°$. We have

$$\theta = \theta_0 + \Delta\theta.$$

We put

$$V_\nu = \frac{2\nu+1}{2}\theta + \psi,$$

and obtain then

$$H_\nu = \frac{\sin V_\nu}{\cos(V_\nu + 3\Delta\theta)},$$

or

$$\frac{1}{H_\nu} = \cot V_\nu \cos 3\Delta\theta - \sin 3\Delta\theta.$$

Each of these expressions furnishes the following three formulæ, which are easily verified, and in which higher powers of $\Delta\theta$ than the first are omitted. W_ν stands for arctan H_ν, and Q for

$$\frac{\sin\frac{(m+1)}{2}n\theta}{\sin\frac{n}{2}\theta} \cdot \sin\frac{\theta}{2}.$$

$$\theta = \frac{1}{36}\sum_{\nu=0}^{\nu=5}(W_{n-1-\nu} - W_\nu),$$

$$\kappa = \frac{1}{24}Q\sum_{\nu=0}^{\nu=11}\{(S_\nu - S_{\nu+1})\sin V_\nu + (S_{\nu+3} - S_{\nu+4})\cos V_\nu\},$$

$$\psi = \frac{1}{12}\sum_{\nu=0}^{\nu=5}(W_{n-1-\nu} + W_\nu) - 6\theta - 3\Delta\theta.$$

2. The procedure of the calculation is as follows: Commencing with 1890·8, the sum of the respective x and y values for 1890·8, 1892·0, 1893·2, 1894·4, 1895·6 gives S_0; S_1 is formed in the same manner, commencing with 1890·9, and so on. Every consecutive group contains, then, 11 constituents of the preceding one; only the 12th is a new one. Each such group gives a value of θ, κ, and ψ. The means (M_x, M_y) of 12 consecutive values of the x and y components respectively, thus found, are contained in the annexed table. Thus No. 1 of each table is the mean of 12 values derived from x and y corresponding to 1890·8 . . . 1891·9; No. 2 to 1892·0 . . . 1893·3, etc. It is not difficult to understand that the values corresponding to odd numbers have no common

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constituents. The same remark holds for the even numbers. No. 16 is extrapolated for the use of mechanical quadrature. No observations were available for this number. A small error in the extrapolated value is of no consequence for the result.

3. The values of θ_x in the column M_x, and of θ_y in the column M_y, are well represented by

$$\theta_x = 30^\circ{\cdot}26 - 0^\circ{\cdot}43 \cos\left[18^\circ{\cdot}74(t - 1892{\cdot}0)\right] + 0^\circ{\cdot}09 \sin\left[18^\circ{\cdot}74(t - 1892{\cdot}0)\right]$$

$$\theta_y = 30^\circ{\cdot}20 - 0^\circ{\cdot}43 \cos\left[18^\circ{\cdot}74(t - 1892{\cdot}0)\right] - 0^\circ{\cdot}01 \sin\left[18^\circ{\cdot}74(t - 1892{\cdot}0)\right]$$

t in units of a year.

These agree within the limits of their probable errors. We therefore conclude that there is no appreciable systematical difference between θ_x and θ_y. Taking the mean of M_x and M_y, we find from the mean values

$$\theta = 30^\circ{\cdot}23 - 0^\circ{\cdot}43 \cos\left\{18^\circ{\cdot}74(t - 1892{\cdot}0) + 9^\circ\right\}.$$

In the computations I have employed

$$\theta = 30^\circ{\cdot}23 - 0^\circ{\cdot}44 \cos\left\{18^\circ{\cdot}74(t - 1892{\cdot}0) + 11^\circ\right\},$$

which in regard of the probable errors may be considered as identical with the preceding formula.

From the last formula we find for the period

$$C = 434^{d}{\cdot}7\ (1 + 0{\cdot}01455 \cos\left\{18^\circ{\cdot}74(t - 1892{\cdot}0) + 11^\circ\right\}),$$

where t is expressed in units of a year, which gives

$$C_{max} = 441^{d}{\cdot}0\ ;\ C_{min} = 428^{d}{\cdot}5.$$

In order to have an idea of the bearing of the expression for C we will test it on observations made before those which have been used in its deduction. From the observations made with the vertical circle in Pulkovo, MM. Ivanoff and Bonsdorff have determined C for 1844 and 1869, and Nyrén for 1887·2.

	Bonsdorff, 1844.	Ivanoff, 1869.	Nyrén, 1887.
Period observed . .	428 days.	436 days.	433 days.
Period by formula .	428 ,,	437 ,,	434 ,,
κ observed . . .	0″·090	0″·203	0″·184
κ by formula . .	...	0″·13 (Chandler)	0″·169*
κ_x formula . . .	...	0″·142	0″·171

p'' (the value of κ in 1845) is provisionally taken = 0″·090

Our formula represents the observations pretty well 45 years backward from 1890·8. It is therefore probable that it will also represent observations with the same exactitude 45 years forwards from 1908·8.

4. The deduction of a representative formula for κ is not so

* One value 0″·40 omitted.

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easy as that for θ. The trouble lies in the fact that it is impossible to express κ by a trigonometrical series of only one argument, ωt. The Table II. evidences a gradual increase of κ, besides the fluctuations which can be expressed by a Fourier's Series of ωT_1. From the observations in Pulkovo, it seems certain that κ on an average increased from 1845, when it was very small, to 1907·6. The hypothesis that κ had a minimum 1845 and a maximum 1907·6 may serve for the deduction of an interpolation formula. According to this hypothesis, we assume

$$\kappa = F + G,$$

in which

$$F = a + b\cos(\sigma T_1)\,;\ \sigma = 2^\circ{\cdot}875\,;\ T_1 = t - 1907{\cdot}6,$$

and

$$G = \sum_i \{\alpha \cos i\omega T_1 + \beta \sin i\omega T_1\}.$$

By means of the values of κ (Table II.), given in columns M_x and M_y, I obtained

$$F_x = 0''{\cdot}080 + 0''{\cdot}080 \cos(\sigma T_1) + \Gamma\,,$$
$$F_y = 0''{\cdot}089 + 0''{\cdot}089 \cos(\sigma T_1) + \Gamma\,,$$
$$\Gamma = +0{\cdot}456p'' - 0{\cdot}534p'' \cos \sigma T_1\,,$$
$$G_x = G_y = +0''{\cdot}036 \cos \omega T_1.^*$$

p'' signifies the value of κ in 1845 expressed in seconds.

These formulæ can, in view of the hypothesis concerning the maximum and minimum of F, claim only the character of interpolation formulæ for 1845—1907·6. While θ_x and θ_y are identical, F_x and F_y are decidedly different.

5. Having computed ψ, the phase c was obtained by means of

$$c = \psi - \frac{mn}{2}\theta = \psi - 24\theta\,;$$

or, since $24\theta_0 = 4\pi$,

$$c = \psi - 24\Delta\theta.$$

As is seen from the Table III., the single values do not agree very well. The columns M_x and M_y respectively give as mean values

$$c_x = 124^\circ\,;\ c_y = 214^\circ.$$

Hence

$$c_y - c_x = 90^\circ.$$

* The more precise formulæ are:

$$G_x = +0''{\cdot}035 \cos \omega T_1 + 0''{\cdot}009 \cos 2\omega T_1 + 0''{\cdot}003 \cos 3\omega T_1 - 0''{\cdot}014 \sin \omega T_1 - 0''{\cdot}011 \sin 2\omega T_1 - 0''{\cdot}004 \sin 3\omega T_1.$$

$$G_y = +0''{\cdot}036 \cos \omega T_1 + 0''{\cdot}007 \cos 2\omega T_1 + 0''{\cdot}000 \cos 3\omega T_1 - 0''{\cdot}004 \sin \omega T_1 - 0''{\cdot}010 \sin 2\omega T_1 - 0''{\cdot}002 \sin 3\omega T_1.$$

Different values of p'' between 0''·0 and 0''·1 will not appreciably affect the coefficients of these series.

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We regard this result as an accidental agreement with the perpendicularity of x and y, as the probable error of $c_y - c_x$ is $\pm 3^\circ$.

Table I.

No.	θ_x.	No.	θ_x.		M_x.	No.	θ_y.	No.	θ_y.		M_y.
	°		°		°		°		°		°
1	30·0	2	29·7	I.	29·9	1	29·5	2	30·1	I.	29·8
3	29·7	4	30·1	II.	29·9	3	30·2	4	30·1	II.	30·1
5	30·3	6	30·6	III.	30·4	5	29·8	6	30·2	III.	30·0
7	30·6	8	30·6	IV.	30·6	7	30·3	8	30·4	IV.	30·4
9	30·6	10	30·8	V.	30·7	9	30·7	10	31·0	V.	30·8
11	30·6	12	30·6	VI.	30·6	11	30·6	12	30·4	VI.	30·5
13	30·2	14	29·9	VII.	30·1	13	30·5	14	30·0	VII.	30·2
15	30·0	16	29·9	VIII.	29·9	15	29.7	16	29·8	VIII.	29·8

Table II.

No.	κ_x.	No.	κ_x.		M_x.	No.	κ_y.	No.	κ_y.		M_y.
	″		″		″		″		″		″
1	0·146	2	0·123	I.	0·135	1	0·156	2	0·141	I.	0·148
3	·113	4	·118	II.	·116	3	·144	4	·146	II.	·145
5	·124	6	·125	III.	·125	5	·145	6	·139	III.	·142
7	·121	8	·128	IV.	·125	7	·141	8	·137	IV.	·139
9	·126	10	·138	V.	·132	9	·134	10	·145	V.	·140
11	·158	12	·186	VI.	·172	11	·162	12	·193	VI.	·177
13	·205	14	·218	VII.	·211	13	·214	14	·229	VII.	·222
15	·219	16	·205	VIII.	·212	15	·226	16	·241	VIII.	·234

Table III.

No.	c_x 1892·0.	No.	c_x 1892·0.		M_x 1892·0.	No.	c_y 1892·0.	No.	c_y 1892·0.	M_y 1892·0.
	°		°		°		°		°	°
1	81	2	92	I.	87	1	186	2	167	177
3	87	4	77	II.	82	3	165	4	174	170
5	74	6	71	III.	73	5	173	6	162	170
7	89	8	71	IV.	80	7	166	8	163	165
9	41	10	66	V.	68	9	155	10	151	153
11	43	12	72	VI.	73	11	145	12	169	156
13	53	14	88	VII.	85	13	168	14	183	176
15	53	16	75	VIII.	79	15	190	16	168	179

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The Mean Parallax of Stars of different Magnitudes.

By Sir F. W. Dyson and W. G. Thackeray.

The proper motions of nearly 12,000 stars between the limits of declination $+24^\circ{\cdot}0$ and $+32^\circ{\cdot}0$ have been determined by comparison of Greenwich observations 1906–1915 with the Astronomische Gesellschaft Catalogues (Leiden, Cambridge, and Berlin B) and the earlier observations of Bessel and Lalande.

To the Greenwich observations in right ascension corrections for magnitude equation have been applied which were derived by comparison of the observations of different observers with the positions given in Kustner's Catalogue. An independent check on the freedom of Kustner's Catalogue from magnitude equation was obtained from observations with the altazimuth in the meridian, using the travelling-wire micrometer. The accordance was very satisfactory.

To the Greenwich declinations a correction of $-0''{\cdot}4$ was applied to make them agree with the declinations of Boss's Fundamental Catalogue. To the catalogues of the Astronomische Gesellschaft the corrections given by Boss were applied both to right ascensions and declinations. With these data preliminary proper motions were obtained, and the results used to obtain systematic corrections to the separate zones of Bessel and Lalande. Proper motions were then obtained from Greenwich–Bessel and from Greenwich–Lalande. The adopted proper motions were formed by giving a double weight to the Greenwich–A.G.C. determination as compared with Greenwich–Lalande and Greenwich–Bessel.

The resulting proper motions are thus systematically Greenwich–A.G.C. The difference of epoch is generally from 30 to 40 years. When the difference was less than 20 years no determination was made. The number of stars for which a comparison with Bessel was possible is approximately 7000, with an interval of about 80 years; and with Lalande about 4500, with an interval of approximately 110 years.

The stars were divided according to magnitude into four groups—

$$6^{m}{\cdot}0–7^{m}{\cdot}8,\quad 7^{m}{\cdot}9–8^{m}{\cdot}5,\quad 8^{m}{\cdot}6–8^{m}{\cdot}9,\quad 9^{m}{\cdot}0– \quad ,$$

the magnitudes being those of the A.G. Catalogues. The means of the proper motions were taken for each hour, and the results analysed for the direction and amount of the solar motion. The following stars were omitted on account of their very large proper motions:—

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Star.	Mag.	R.A.	Dec.	Centennial Motion.	
				R.A.	Dec.
		h m	°	s	"
C 1581	8·1	3 3	26	−1·31	− 83·6
C 3695	7·3	6 58	29	+1·21	− 82·3
L 3313	8·2	7 48	31	+5·63	−184·8
C 4285	7·5	7 55	29	−1 02	−117·9
L 3472	8·3	8 13	31	−2·27	− 81·4
B 5072	9·0	14 22	24	+5·66	−110·5
L 6797	8·7	18 38	31	+0·61	− 83·5
C 14250	8·6	23 41	29	+7·01	+ 2·2
C 14413	6·9	23 57	27	+6·22	− 98·6

With these omissions the means for each hour are as follows (the units being centennial proper motions of 0s·01 and 0"·1):—

Hour.	$6^m·0–7^m·8$.			$7^m·9–8^m·5$.			$8^m·6–8^m·9$.			$9^m·0$ –		
	R.A.	Dec.	No.	R.A.	Dec.	No.	R.A.	Dec.	No.	R.A.	Dec.	No.
h h												
0–1	+22	−21	83	+13	− 8	123	+ 6	−14	142	− 2	+ 1	139
1–2	+21	−37	61	+ 6	−13	144	− 2	−13	133	− 2	− 5	102
2–3	+14	−31	72	+ 6	−12	108	0	−10	111	+ 1	− 9	99
3–4	+ 4	−30	72	+ 3	−28	128	+ 4	−21	113	− 1	−12	93
4–5	+ 2	−25	62	+ 4	−13	89	+11	−22	89	+ 1	−17	92
5–6	− 1	−23	109	− 1	−13	184	+ 2	−15	208	− 2	−11	144
6–7	− 2	−22	104	+ 3	−20	198	+ 1	−15	265	− 2	−11	163
7–8	− 4	−25	69	+ 1	−26	125	− 4	−20	196	− 6	−12	121
8–9	−15	−44	67	− 5	−24	127	− 4	−24	128	−12	−16	96
9–10	−16	−48	56	−11	−20	112	−10	−16	115	−15	−12	88
10–11	−12	−25	62	−18	−33	104	−19	−23	129	−15	−19	64
11–12	−17	−21	68	−17	−21	72	−24	−13	86	−20	−10	75
12–13	−22	−24	76	−12	−21	90	−15	−20	113	− 7	− 8	59
13–14	−29	−16	61	−20	−12	97	−23	− 3	84	−23	− 6	74
14–15	−14	−18	56	−19	− 9	99	−18	0	100	−11	− 2	84
15–16	−10	−15	44	−18	−18	115	−12	− 3	100	−16	+ 3	86
16–17	−13	−18	64	− 2	−18	105	− 5	− 8	120	−11	− 9	82
17–18	0	−24	78	− 7	− 1	148	− 6	− 9	164	− 4	−13	140
18–19	+ 6	− 2	102	+ 4	− 5	187	+ 1	− 8	196	− 1	+ 3	200
19–20	+10	− 9	165	+ 4	− 1	229	+ 2	− 2	309	+ 1	+ 1	269
20–21	+ 8	− 4	111	0	− 4	256	− 2	− 1	278	− 1	+ 8	204
21–22	+11	−11	88	+ 2	− 3	195	+ 3	+ 4	211	− 2	+ 4	169
22–23	+ 4	−11	85	− 1	− 8	151	− 2	− 1	190	− 9	+ 5	148
23–0	+20	− 8	85	+ 2	− 8	135	+ 7	− 3	154	+ 1	0	130
			1900			3321			3734			2921

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The results for each hour were taken to one more figure in the actual computations.

From the analysis of these figures giving equal weight to each hour we find :—

Analysis of the Proper Motions in Right Ascension.

m m	s	s	s
6·0–7·8	– ·017	+ ·002 sin α	+ ·182 cos α + 2″·40
7·9–8·5	– ·034	+ ·025 sin α	+ ˢ·113 cos α + 1″·49
8·6–8·9	– ·044	+ ·023 sin α	+ ˢ·100 cos α + 1″·32
9·0	– ·066	+ ·010 sin α	+ ˢ·077 cos α + 1″·01

Analysis of the Proper Motions in Declination.

m m	″	″	″
6·0–7·8	– 2·13	– 1·09 sin α	+ 0·48 cos α
7·9–8·5	– 1·41	– 0·75 sin α	+ 0·65 cos α
8·6–8·9	– 1·08	– 0·86 sin α	+ 0·49 cos α
9·0	– 0·62	– 0·75 sin α	+ 0·49 cos α

These expressions of the mean proper motions are considered as arising from—

(1) Residual error in the observations from magnitude equation.
(2) Corrections to Newcomb's precession.
(3) Solar motion.

Corrections Δm and Δn to the precession will give rise to—

$\Delta m + \Delta n \tan \delta_0 \sin \alpha$ in the proper motions in right ascension,

and

$\Delta n \cos \alpha$ in the proper motions in declination,

where $\delta_0 = 28°$, the mean declination of the zone.

Let X, Y, Z be the components of the solar motion, so that

$$X = M \cos A \sin D, \quad Y = M \sin A \sin D, \quad Z = M \cos D,$$

where M = centennial parallactic motion,
A = R.A. of solar apex,
D = Dec. of solar apex.

Then the solar motion gives rise to the terms

$X \sin \alpha - Y \cos \alpha$ in the proper motions in R.A.,

and

$X \cos \alpha \sin \delta_0 + Y \sin \alpha \sin \delta_0 - Z \cos \delta_0$ in the proper motions in Dec.

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Magnitude Equation and Precession.

The absolute term in the right ascensions may in part indicate a correction Δm to the precession, but this quantity is so mixed up with magnitude equation that it seems impossible to determine m in this way unless the correction for magnitude equation for the two catalogues is known with accuracy. The progressive change in the value of the constant term shows that it arises in all probability from erroneous magnitude equation.

If the interval between the A.G. and Greenwich Catalogues be taken roughly as 33 years, then the adopted values of magnitude equation are only in error by ^{s}·006, ^{s}·011, ^{s}·015, ^{s}·022 for the successive magnitude groups, and as the absolute term in the right ascensions does not enter into the determination of the solar motion it need not be considered further.

The correction Δn to the precession enters into the sin α terms of the right ascensions and the cos α terms of the declinations.

From the R.A.'s : $X + \Delta n \tan \delta_0 = +''\cdot03, \quad +''\cdot33, \quad +''\cdot30, \quad +''\cdot13.$
From the Decs. : $X \sin \delta_0 + \Delta n = +''\cdot48, \quad +''\cdot65, \quad +''\cdot49, \quad +''\cdot49.$

As there is no sign of progression with the different magnitudes, it is clear that X is very nearly zero, but if the mean be taken, we find

$$X = -''\cdot10 \quad \text{and} \quad \Delta n = +''\cdot58.$$

This value of X, in conjunction with $-1''\cdot60$ for the mean value of Y found in the next paragraph, gives $266\frac{1}{2}^\circ$ as the right ascension of the solar apex. If, however, the position of the solar apex be assumed to be 270°, we find

$$\Delta n \tan \delta_0 = +''\cdot20 \quad \text{or} \quad \Delta n = +''\cdot38 \text{ from R.A.}$$
$$\text{and} \quad \Delta n = +''\cdot53 \text{ from Dec.}$$

Assuming it to be at $273\frac{1}{2}^\circ$, we find

$$\Delta n \tan \delta_0 = +''\cdot10 \quad \text{or} \quad \Delta n = +''\cdot19 \text{ from R.A.}$$
$$\text{and} \quad \Delta n = +''\cdot48 \text{ from Dec.}$$

A positive correction of $+''\cdot4$ or $+''\cdot5$ to Newcomb's precession is indicated, in the direction of Struve-Peter's value.

Solar Motion. Greenwich—A.G.C.

The right ascension of the solar apex is seen from the preceding paragraph to be nearly 270°. This is in close accordance with determinations made from stars situated all over the sky, as, *e.g.*, those based on the proper motions of Boss's Catalogue.

The values of the Y co-ordinate of the solar motion are as follows :—

		$6^m\cdot0$–$7^m\cdot8$.	$7^m\cdot9$–$8^m\cdot5$.	$8^m\cdot6$–$8^m\cdot9$.	$9^m\cdot0$–	
		''	''	''	''	
	Y =	2·40	1·49	1·32	1·01	from R.A.
	$Y \sin \delta_0$ =	1·09	0·75	0·86	0·75	from Dec.
or	Y =	2·32	1·59	1·83	1·59	from Dec.

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Weighting these in the ratio 4 : 1

$$Y = 2''.42 \quad 1''.51 \quad 1''.42 \quad 1''.13$$

The values of Z found from the declinations are—

	$Z \cos \delta_0$	=	$2''.13$	$1''.41$	$1''.08$	$0''.62$
	Z	=	2·41	1·59	1·22	0·70
Thus	tan D	=	·98	1·05	·86	·62
and	D	=	45°	46°	41°	32°

The adopted values of the parallactic motion for the different magnitudes are—

Limits of Mag. (B.D.)	$6^m.0–7^m.8$.	$7^m.9–8^m.5$.	$8^m.6–8^m.9$.	$9^m.0–$
Mean Mag. . .	$7^m.25$	$8^m.25$	$8^m.75$	$9^m.0$
M	$3''.40$	$2''.19$	$1''.87$	$1''.33$

or corrected for magnitude equation in Dec. (see Addendum)—

M	$3''.28$	$2''.23$	$2''.21$	$1''.71$

The different values of the declination are not to be regarded as indicating any real difference in the direction of the solar apex for bright and faint stars. A personality in the bisection of stars of magnitude 9·0 relatively to those of magnitude 7·0 of only $0''.1$ in amount would have changed Z by $0''.3$, or from $0''.62$ to $0''.92$, thus increasing D from 32° to 39°. This point is fully discussed at the end of this paper.

We have therefore, in all what follows, adopted for the position of the solar apex A = 270°, D = 37°·5.

As the zone crosses the Milky Way, and at 12^h is about 90° distant from it, mean parallactic motions were found for each separate hour, but the results gave no clear evidence of any systematic difference in different parts of the sky.

Solar Motion. Greenwich 1890—*Groombridge.*

To obtain further material, the position of the solar apex for stars of different magnitudes was redetermined from the Groombridge observations, stars of large proper motion being included, except the following:—

Star.	Mag.	R.A.	Dec.	Centennial Motion. R.A.	Centennial Motion. Dec.
	m	h m	°	″	″
Gr. 34	8·3	0 10	43·2	+285	+ 38
μ Cass.	5·3	0 58	54·2	+337	+156
Gr. 1618	6·8	10 2	50·2	−134	+ 51
Gr. 1830	6·5	11 44	38·8	+398	−577

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The results are given in the following table:—

Group.	Mean Mag.	D.	M.	No. of Stars.
m m	m	°	"	
5·0–5·9	5·50	37	4·57	488
6·0–6·9	6·50	35	3·85	1048
7·0–7·9	7·50	34	2·84	1274
8·0–8·9	8·25	47	2·72	825

Solar Motion. Greenwich—Carrington.

Similarly, the parallactic motions were deduced for stars of different magnitudes in Carrington's Catalogue by comparison with the Greenwich Astrographic observations. The results are given in the following table:—

Group.	Mean Mag. (B.D. scale).	No. of Stars.	M.
m m	m		"
–7·9	7·42	124	3·13
8·0–8·8	8·50	521	2·56
8·9–9·1	9·00	447	2·08
9·2–9·3	9·26	598	2·14
9·4	9·40	472	1·36
9·5	9·50	1530	1·29

Solar Motion. Abbadia–Albany, A.G.C.

A comparison similar to that made between the Greenwich Catalogue for 1910 and the A.G. Catalogues of Leiden, Cambridge, and Berlin B has also been made between the Abbadia Catalogue and that of Albany. The observations of the declinations both at Abbadia and Albany were made by the use of parallel lines. At Abbadia corrections for magnitude equation in declination were applied to reduce the observations of separate observers to a uniform system. A comparison with Kustner gives the following magnitude equation to the Abbadia Catalogue places:—

$6^{m}\cdot0$–$6^{m}\cdot9$.	$7^{m}\cdot0$–$7^{m}\cdot9$.	$8^{m}\cdot0$–$8^{m}\cdot4$.	$8^{m}\cdot5$–$8^{m}\cdot9$.	$9^{m}\cdot0$–
–″·09	–″·11	–″·12	″·00	+″·15

These corrections applied to the mean proper motions derived from the comparison of the declinations of the Abbadia and Albany Catalogues arranged in order of magnitude still left outstanding differences of such a character that it seemed likely that the Albany Catalogue may be affected to some extent in the same way. It was therefore decided to use only the right ascensions in this paper.

Group.	No. of Stars.	Parallactic Motion in R.A.	M.
m m		"	"
6·0–7·8	994	2·94	3·70
7·9–8·5	1481	2·57	3·24
8·6–8·9	2570	1·65	2·08
9·0	1879	1·29	1·63

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Collected Results for Parallactic Motion.

The assumed magnitudes of the stars are on the scale of the Bonn Durchmusterung, with which the catalogues of the Astronomische Gesellschaft are in substantial agreement. These have been reduced to the Harvard scale by use of the tables given in *Harvard Annals*, vol. 54.

In the following table the magnitude on the Harvard scale is given in the first column. The source of the proper motions, number of stars, and the parallactic motion are given in succeeding columns. The mean parallax is derived from the parallactic motion by division by 410; that is, adopting 19·5 km. per second as the velocity of the solar motion, or 4·1 × the earth's distance from the sun in one year.

The discordances from the formula of the next paragraph are given in the last column.

Mag. (Harvard).	Source of Proper Motions.	No. of Stars in Group.	M.	Mean Parallax.	Difference from Formula.
m			″	″	″
5·5	Greenwich 1890—Groombridge	488	4·57	·0111	+ ·0020
6·5	,, ,,	1,048	3·85	·0094	+ 5
7·3	Greenwich 1910—A.G.C.	1,900	3·28	80	− 1
7·3	Abbadia—Albany	994	3·70	90	− 11
7·5	Greenwich 1890—Groombridge	1,274	2·84	69	+ 6
7·5	Greenwich 1900—Carrington	124	3·13	76	− 1
8·3	Greenwich 1890—Groombridge	825	2·72	66	− 6
8·4	Greenwich 1910—A.G.C.	3,321	2·23	54	+ 5
8·4	Abbadia—Albany	1,481	3·24	79	− 20
8·5	Greenwich 1900—Carrington	521	2·56	63	− 6
9·0	Greenwich 1910—A.G.C.	3,734	2·21	54	− 4
9·0	Abbadia—Albany	2,570	2·08	51	− 1
9·3	Greenwich 1900—Carrington	447	2·08	51	− 5
9·4	Greenwich 1910—A.G.C.	2,921	1·71	42	+ 2
9·4	Abbadia—Albany	1,829	1·63	40	+ 4
9·7	Greenwich 1900—Carrington	598	2·14	52	− 11
10·0	,, ,,	472	1·36	33	+ 5
10·3	,, ,,	1,530	1·29	·0031	+ 4
		26,077			

The mean parallaxes are represented fairly satisfactorily by the formula

$$\log \varpi = -1{\cdot}224 - {\cdot}120\, m,$$

the differences in the sense formula − observation being shown in the last column.

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The formula given by Kapteyn * is

$$\varpi = \ ''\cdot 0135\ (0\cdot 78)^{m-5\cdot}$$

$$\text{or} \quad \log \varpi = -\bar{1}''\cdot 275 - \cdot 108\ m.$$

Mag.	Mean Parallax.	
	Greenwich.	Kapteyn.
m	"	"
5·0	·0150	·0153
5·5	·0131	·0135
6·0	·0114	·0120
6·5	·0099	·0105
7·0	86	·0093
7·5	75	82
8·0	65	73
8·5	57	64
9·0	50	57
9·5	43	50
10·0	38	44
10·5	·0033	·0039

The agreement with Kapteyn's results lends no support to the view that the diminution of mean parallax with increasing magnitude is slower than is represented by his formula.

Addendum.

By adopting the position of the solar apex and the table of mean parallax, we are now enabled to compare the computed and observed values of Y and Z, and we get—

	$6^m\cdot 0$–$7^m\cdot 8$.	$7^m\cdot 9$–$8^m\cdot 5$.	$8^m\cdot 6$–$8^m\cdot 9$.	$9^m\cdot 0$ –	Mean.
	"	"	"	"	"
Y (C – O)	+ ·21	+ ·41	+ ·22	+ ·32	+ ·29
Z (C – O)	– ·32	– ·06	·00	+ ·31	+ ·02

Taking 33 years as the interval between the catalogues, the correction to Y will be produced by a correction to the right ascensions of $^s\cdot 010 \cos \alpha$ in a direction to increase the solar motion.

The correction to Z evidently points to a magnitude equation in declination.

The declinations of the Cambridge Catalogue were made by bisecting the star with parallel lines 11"·2 apart, the same method as is used at Albany and Abbadia, and one which appears to be especially liable to introduce magnitude equation.

The system of corrections in Boss's Preliminary Catalogue were adopted as being the result of the latest material for forming

* Using 19·5 km./sec. for the solar motion.

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such a system, and Boss has given no material for such a correction in declination, but Auwers has done so.

The quantities, less a constant given by Auwers in his tables for reducing star catalogues to his fundamental system, are as follows:—

Magnitude.	Cambridge. +	Cambridge. −	Leiden. +	Leiden. −	Berlin B. +	Berlin B. −
m	″	″	″	″	″	″
6·0		·13	·06			·02
6·5		·11	·04			·03
7·0		·09	·01			·02
7·5		·05		·01		·02
8·0		·00		·03		·01
8·5	·12			·03	·02	
9·0	·24			·03	·06	

A comparison between Kustner's Catalogue for 1900 and the Greenwich Catalogue for 1910 gives the following corrections to Greenwich declinations:—

$6^m·0$–$7^m·8$.	$7^m·9$–$8^m·5$.	$8^m·6$–$8^m·9$.	$9^m·0$–
+″·04	+″·03	−″·05	−″·06

Giving a weight of 5 to the Cambridge values, 2 to Leiden values, and 1 to Berlin B values, because Cambridge covers $\frac{5}{8}$, Leiden $\frac{2}{8}$, and Berlin B $\frac{1}{8}$ of the Catalogue zone, we get—

Limits of Mag.	Cambridge. +	Cambridge. −	Leiden. +	Leiden. −	Berlin B. +	Berlin B. −	Mean. +	Mean. −	Greenwich. +	Greenwich. −	Greenwich − A.G.C. +	Greenwich − A.G.C. −
m m	″	″	″	″	″	″	″	″	″	″	″	″
6·0–7·8		$·07_5$		$·00_2$		$·02_1$		·04	·04		·08	
7·9–8·5	$·06_5$			$·03_2$	$·00_1$		·03		·03		·00	
8·6–8·9	$·18_5$			$·03_2$	$·04_1$		·11			·05		·16
9·0–	$·24_5$			$·03_2$	$·06_1$		·15			·06		·21

In estimating the effect of these corrections on the proper motions it is to be remembered that the comparisons with Lalande and Bessel have to some extent reduced the effect of the magnitude equation of the A.G. Catalogues on the determined proper motions. This has been roughly allowed for by taking 40 years instead of 33 years as the difference of epoch.

Limits of Mag.	Correction to Centennial P.M. +	Correction to Centennial P.M. −	Z cos δ_0.	Z cos δ_0. Corrected.	Z Corrected.	Z (C − O). +	Z (C − O). −
m m	″	″	″	″	″		″
6·0–7·8		·20	2·13	1·91	2·19		·23
7·9–8·5		·00	1·41	1·41	1·59		·15
8·6–8·9	·40		1·08	1·48	1·68		·45
9·0–	·52		·62	1·14	1·27		·17
							·25

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The correction to Z has now become a constant, and may be expressed as a correction to the observed declination of ″·09 in a direction to diminish the value of Z. As the corrections to the right ascensions are in the opposite direction, these corrections to Y and Z only slightly affect M, but materially alter D. The original values of M and as corrected for (1) magnitude equation and further corrected for (2) +″29 to Y and −″25 to Z are

	$6^m·0–7^m·8.$	$7^m·9–8^m·5.$	$8^m·6–8^m·9.$	$9^m·0–$
M	3″·40	2″·19	1″·87	1″·33
(1)	3·28	2·23	2·21	1·71
(2)	3·35	2·26	2·24	1·78

On the Radiative Equilibrium of the Stars.

By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor.

1. *Outline of the Investigation.* — The theory of radiative equilibrium of a star's atmosphere was given by K. Schwarzschild in 1906.* He did not apply the theory to the interior of a star; but the necessary extension of the formulæ (taking account of the curvature of the layers of equal temperature) is not difficult. It is found that the resulting distribution of temperature and density in the interior follows a rather simple law.

Taking a star—a "giant" star of low density, so that the laws of a perfect gas are strictly applicable—and calculating from its mass and mean density the numerical values of the temperature, we find that the temperature gradient is so great that there ought to be an outward flow of heat many million times greater than observation indicates. This contradiction is not peculiar to the radiative hypothesis; a high temperature in the interior is necessary in order that the density may have a low mean value notwithstanding the enormous pressure due to the weight of the column of material above.

There is a way out of the difficulty, however, if we are ready to admit that the radiation-pressure due to the outward flow of heat may under calculable conditions of temperature, density, and absorption nearly neutralise the weight of the column, and so reduce the pressure which would otherwise exist in the interior. For the giant stars it is necessary that only a small fraction of the weight should remain uncompensated. (For the dwarf stars, on the other hand, radiation-pressure is practically negligible.)

We thus arrive at the theory that a rarefied gaseous star adjusts itself into a state of equilibrium such that the radiation-pressure very approximately balances gravity at interior points. This condition leads to a relation between mass and density on the one side and effective temperature on the other side, which seems to correspond roughly with observation. The laws arrived at differ considerably from those of Lane and Ritter.

* *Göttingen Nachrichten*, 1906, p. 41.

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The principal results are given in §§ 7–10. The theory enables us to estimate the average densities of the giant stars of different spectral types; it shows that the average luminosity will be roughly the same for the different types, and determines this luminosity as compared with the Sun; it determines the maximum effective temperature which a star can attain; and it indicates the extent to which the masses of individual stars are likely to deviate from the mean mass. It is scarcely necessary to say that the conclusions here given are tentative, being based on analysis which is only concerned with obtaining a probable approximation; but there seems to be a satisfactory accordance with observation, so far as is known. The present results also remove an objection which might be urged against the theories of Lane and Ritter, viz. that they require the heat-energy retained in the star to be much greater than that generated by contraction.

The outermost layers of the star are outside the scope of this investigation, and the formulæ here given do not apply to them. In speaking of conditions at the boundary of the star, I refer to a depth negligible compared with the radius, but deep from the point of view of the spectroscopist. Hence the theory has no bearing on the interpretation of spectroscopic results. Frequent reference is made to the effective temperature, because it affords a measure of the total outflow of radiation; we can use this measure without discussing the conditions of the layer which actually possesses the effective temperature.

It is clear that we cannot arrive at much certainty with regard to the conditions in a star's interior, except in so far as the treatment can be based on the most general laws of nature. There are some physical laws so fundamental that we need not hesitate to apply them even to the most extreme conditions; for instance, the density of radiation varies as the fourth power of the temperature, the emissive and absorbing powers of a substance are equal, the pressure of a gas of given density varies as its temperature, the radiation-pressure is determined by the conservation of momentum, —these provide a solid foundation for discussion. The weak link in the present investigation is that I have assumed without much justification that a certain product $k\epsilon$ is constant throughout a star. I have given some evidence that if it is variable the general character of the results would not be greatly altered; and, as a step towards the elucidation of the problem of stellar temperatures, I plead to be allowed provisionally one rather artificial assumption.

2. *Radiative Equilibrium in the case of Spherical Symmetry.*—Schwarzschild's treatment deals with the case when the surfaces of equal temperature are parallel planes. We have to consider the case when they are concentric spheres. The fundamental principle is that the transfer of energy takes place by radiation; convection and conduction are considered negligible.

Let ξ be the distance of a point P from the centre O of a star. At P, let the intensity of radiation travelling outwards in a direc-

tion making an angle θ with OP be expressed as a series of zonal harmonics,

$$(A + B\cos\theta + CP_2(\cos\theta) + DP_3(\cos\theta) + \ldots)dSd\omega,$$

where dS is the cross-section of the stream of radiation, and $d\omega$ is the solid angle containing directions near to θ.

Consider a small cylinder of matter of cross-section dS, and with axis ds in the direction θ. The fraction of the radiation absorbed in this cylinder will be proportional to the density ρ, and may be set equal to

$$k\rho ds,$$

where k is a coefficient of absorption and represents the absorption by a cylinder of unit *mass* and unit cross-section. Assuming that each molecule absorbs independently of the others, it will make no difference whether the cylinder is long and of low density, or short and of high density.

Hence the loss of the beam of radiation in traversing the cylinder is

$$k\rho ds(A + B\cos\theta + CP_2(\cos\theta) + \ldots)dSd\omega \quad . \quad . \quad (1)$$

The matter in the cylinder emits energy equally in all directions at a rate proportional to (1) the mass contained, (2) the fourth power of the temperature T, and (3) the specific emissive power of the substance. The last quantity may be set equal to k, since the absorbing and emissive powers of a substance are necessarily equal. Hence the energy emitted from the cylinder in directions included in $d\omega$ is

$$\mu k T^4 \rho ds dS d\omega \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where μ is an absolute constant of nature, connecting the units of energy and temperature.

The loss (1) and gain (2) together make up the change of intensity of the radiation in traversing a length ds. Hence

$$\frac{d}{ds}(A + B\cos\theta + CP_2(\cos\theta) + \ldots)dsdSd\omega$$
$$= -k\rho(A + B\cos\theta + CP_2(\cos\theta) + \ldots)dsdSd\omega + k\rho\mu T^4 dsdSd\omega \quad (3)$$

Now
$$\frac{d}{ds} = \frac{d\xi}{ds}\frac{d}{d\xi} + \frac{d\theta}{ds}\frac{d}{d\theta}$$
$$= \cos\theta\frac{d}{d\xi} - \frac{\sin\theta}{\xi}\frac{d}{d\theta} \text{ by geometry.}$$

Hence, since A, B, C are functions of ξ only, (3) becomes

$$\cos\theta\frac{dA}{d\xi} + \cos^2\theta\frac{dB}{d\xi} + \cos\theta \,.\, P_2\frac{dC}{d\xi} + \frac{\sin^2\theta}{\xi}B - \frac{\sin\theta}{\xi}\frac{dP_2}{d\theta}C + \ldots$$
$$= -k\rho(A + B\cos\theta + CP_2 + \ldots) + k\rho\mu T^4 \quad (4)$$

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Let us choose a unit of length comparable with the radius of the star. Then k is a very large quantity, since radiation must be strongly absorbed in a length small compared with the radius. It is then found that A, B, C rapidly diminish, so that B is of order k^{-1} and C of order k^{-2} compared with A. We shall therefore neglect C on the left side of (4), retaining it on the right, where it is multiplied by the large factor k.

Arrange the left side in zonal harmonics by using

$$\cos^2\theta = \tfrac{1}{3} + \tfrac{2}{3}P_2(\cos\theta),$$
$$\sin^2\theta = \tfrac{2}{3} - \tfrac{2}{3}P_2(\cos\theta),$$

and equate coefficients.

The constant terms give

$$\tfrac{1}{3}\frac{dB}{d\xi} + \tfrac{2}{3}\frac{B}{\xi} = k\rho(-A + \mu T^4) \quad . \quad . \quad . \quad (5)$$

The coefficients of $\cos\theta$ give

$$\frac{dA}{d\xi} = -k\rho B \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The coefficients of $P_2(\cos\theta)$ give

$$\tfrac{2}{3}\frac{dB}{d\xi} - \tfrac{2}{3}\frac{B}{\xi} = -k\rho C,$$

which verifies the statement made above that C is small compared with B. We shall therefore neglect C henceforth.

Equation (5) can be written

$$\frac{1}{\xi^2}\frac{d}{d\xi}(B\xi^2) = 3k\rho(-A + \mu T^4) \quad . \quad . \quad . \quad (7)$$

To interpret the quantity B, we remark that the total radiation flowing outwards across unit surface transverse to the radius is obtained by integrating $(A + B\cos\theta)\cos\theta d\omega$ over the corresponding hemisphere. The factor $\cos\theta$ is required, since the cross-section of a beam of radiation flowing obliquely through a surface dS is equal to $dS\cos\theta$. The result is

$$\pi A + \tfrac{2}{3}\pi B \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Similarly the amount flowing outwards is

$$\pi A - \tfrac{2}{3}\pi B \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

Thus the net flow outwards is $\frac{4}{3}\pi B$ per unit area, or $\frac{16}{3}\pi^2 B\xi^2$ across the sphere of radius ξ.

In a strictly steady state the total energy between two boundaries must be constant, and therefore the net outward flow across all boundaries must be the same. Thus $B\xi^2$ is constant, and by (7)

$$A = \mu T^4.$$

In an actual star the stream of energy flowing outwards is supplied by slow changes occurring within the star. The simplest theory results if we suppose that the energy is produced by radioactive processes. Let the amount thus liberated per unit mass be $4\pi\epsilon$. Then in the steady state the outward flow of heat across the outer boundary of a spherical shell will exceed that across the inner boundary by the amount of heat generated in the shell. That is,

$$d(\tfrac{16}{3}\pi^2 B\xi^2) = 4\pi\epsilon \,.\, 4\pi\rho\xi^2 d\xi.$$

Hence

$$\frac{1}{\xi^2}\frac{d}{d\xi}(B\xi^2) = 3\rho\epsilon \qquad (9)$$

Imagine the successive spherical layers to be expanded or contracted until the whole star has a uniform density τ. Let x be the radius in the uniform star, which corresponds to ξ in the original star. Then

$$\rho\xi^2 d\xi = \tau x^2 dx \qquad (10)$$

Hence from (6) and (9)

$$\frac{1}{\tau}\frac{dA}{dx} = -kB\frac{x^2}{\xi^2} \qquad (11)$$

$$\frac{1}{\tau}\frac{d}{dx}(B\xi^2) = 3\epsilon x^2 \qquad (12)$$

Integrating (12),

$$B = \tau\epsilon\frac{x^3}{\xi^2} \qquad (13)$$

and

$$\frac{1}{\tau}\frac{dA}{dx} = -k\tau\epsilon\frac{x^5}{\xi^4} \qquad (14)$$

If the outflowing energy is produced by contraction instead of by radioactivity, it is not so easy to give a precise statement, because, strictly speaking, the conditions are changing with the time. We may write down equation (13) as a definition of ϵ; and it will be seen that $4\pi\epsilon$ is then the net flow of radiation across the spherical surface divided by the mass within the surface, *i.e.* the average energy per unit mass generated by contraction. I shall generally take ϵ to be constant, representing roughly a state of affairs such that the energy of stellar radiation comes from processes going on in all parts of the star, and not from a singularity at the centre. It must be noticed that, except near the surface, ϵ is extremely small compared with μT^4, so that in any case $A = \mu T^4$ approximately.

Let g be the value of gravity at ξ

and G the constant of gravitation,

then
$$g = \tfrac{4}{3}\pi G\tau x^3/\xi^2,$$

since the numerator gives the total mass within a sphere of radius ξ.

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The pressure equation is

$$\frac{1}{\rho}\frac{dp}{d\xi} = -g,$$

whence by (10)

$$\frac{1}{\tau}\frac{dp}{dx} = -g\frac{x^2}{\xi^2}$$

$$= -\tfrac{4}{3}\pi G\tau\frac{x^5}{\xi^4}$$

$$= \tfrac{4}{3}\frac{\pi G}{k\epsilon}\frac{1}{\tau}\frac{dA}{dx} \quad \text{by (14)} \quad . \quad . \quad (15)$$

If k and ϵ are constants, integrating and setting $A = \mu T^4$,

$$p = \frac{4\pi\mu G}{3k\epsilon}T^4 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Strictly speaking, there is a small constant of integration, but it can be neglected in comparison with the large values of p in the interior of the star (see § 5 (*b*)).

Up to this point we have not used the gas-equation. For a *gaseous* star obeying Boyle's Law, $p = RT\rho$. Hence from (16), $\rho \propto T^3$ and $p \propto \rho^{\frac{4}{3}}$.

The distribution of density, temperature, and pressure for radiative equilibrium is identical with that occurring in the adiabatic equilibrium of a mass of gas for which the ratio of the specific heats is $\tfrac{4}{3}$.*

A gas for which $\gamma = \tfrac{4}{3}$ can be at the same time in radiative and adiabatic equilibrium; if γ alters, adiabatic equilibrium alters but radiative equilibrium does not.

3. *Intensity of the Outward Radiation.*—The net outward radiation has been found to be $\tfrac{16}{3}\pi^2 B\xi^2$

$$= \tfrac{16}{3}\pi^2\tau\epsilon x^3 \quad \text{by (13)}.$$

If the effective temperature of the star is T_1, the outflow is equivalent to that due to isotropic radiation of intensity $A_1 = \mu T_1{}^4$. By (8) this outflow is πA_1 per unit area. Hence

$$\pi\mu T_1{}^4 \,.\, 4\pi\xi^2 = \tfrac{16}{3}\pi^2\tau\epsilon x^3 \quad . \quad . \quad . \quad (17)$$

where x and ξ have their surface values.

Eliminate ϵ between (16) and (17), we obtain

$$kT_1{}^4 = \tfrac{4}{3}\pi G\tau\frac{x^3}{\xi^2}\frac{T^4}{p}$$

$$= \tfrac{4}{3}g_1\frac{T^4}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where g_1 is the value of gravity at the surface.

* It may be well to restate the restriction, viz. that $k\epsilon$ must be constant. This point is discussed in § 5.

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This equation gives the relation between the constant of absorption and the effective temperature. T and p are any corresponding values of the temperature and pressure. It is usually most convenient to use the central values, T_0 and p_0, so that

$$k \doteq \tfrac{4}{3}\frac{g_1}{p_0}\left(\frac{T_0}{T_1}\right)^4 \quad . \quad . \quad . \quad . \quad (19)$$

4. *Numerical Example.*—Before discussing possible modifications of the somewhat ideal conditions assumed, we shall give an example. The necessary tables and formulæ are given in Emden's *Gaskugeln* (pp. 85, 69). Emden actually works out this case (regarded as a case of adiabatic equilibrium) for the Sun, but we shall take a star in a more perfect gaseous condition. For the method of calculation reference may be made to Emden's examples (p. 96).*

Let the radius of the star be $r = 7 \times 10^{11}$ cm.
Let the mean density be $\rho_m = 0{\cdot}002$ gm. cm.$^{-3}$
Then the mass is $M = 2{\cdot}87 \times 10^{33}$ gm. (about $1{\cdot}5 \times$ Sun).

The central density in this form of equilibrium is *always* 54·2515 times the mean density. In this case

Central density $\rho_0 = 0{\cdot}1085$,

so that no serious deviation from Boyle's Law will occur.

To calculate the temperature and pressure, the only other quantity needed is the molecular weight. I have taken this to be 54 as a likely average (*e.g.* monatomic iron vapour). The corresponding value of R is $1{\cdot}536 \times 10^6$. I find at the centre

$$T_0 = 1{\cdot}52 \times 10^8 \text{ degrees Centigrade}$$
$$p_0 = 2{\cdot}535 \times 10^{13} \text{ dynes cm.}^{-2}$$

At the surface

$$g_1 = 390{\cdot}5 \text{ cm. sec.}^{-2}$$

Temperature gradient $= 6^\circ{\cdot}35$ per kilometre.

(The temperature gradient is nearly uniform for a great distance inwards; it increases slowly to a maximum of $6\frac{1}{2}$ times its surface value at a depth equal to $\frac{3}{4}$th of the radius.)

The following formulæ show how the conditions at the centre are altered by varying the mean density, mass, molecular weight (R^{-1}), or constant of gravitation:

$$\left.\begin{aligned} \rho_0 &\propto \rho_m \\ T_0 &\propto \rho_m^{\frac{1}{3}} M^{\frac{2}{3}} G R^{-1} \\ p_0 &\propto \rho_m^{\frac{4}{3}} M^{\frac{2}{3}} G \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (20)$$

To calculate k we must assume a value of the effective temperature. A star having the assumed mean density would probably be

* There is a misprint in the formula for Θ^3 on p. 97. The last bracket in the denominator should be squared.

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of type F – G, and have an effective temperature of, say, 6500° absolute.

Substituting in (19) we find

$$k = 6{\cdot}2 \times 10^6.$$

This value is quite absurd, for it indicates that the radiation would be almost wholly absorbed in a path of 10^{-6} cm. at unit density, or 10^{-3} cm. at atmospheric density. Such a high opacity can scarcely be reconciled with the known properties of matter.

Inverting the argument and taking a more reasonable value of k, say 40, the effective temperature would have to be 130,000°C., which is, of course, out of the question.

A further indication that something is wrong is given by a calculation of the total amount of radiant energy contained in the star. The radiant energy per unit volume is aT^4 ergs, where $a = 7{\cdot}1 \times 10^{-15}$ and T is reckoned in degrees. On integrating throughout the volume, we find the total imprisoned radiant energy is

$$H = \frac{1}{2}\frac{aT_0{}^4 GM^2}{p_0 r} = 5{\cdot}85 \times 10^{52} \text{ ergs},$$

whereas the whole energy so far generated by contraction is

$$\Omega = \frac{3}{2}\frac{GM^2}{r} = 1{\cdot}18 \times 10^{48} \text{ ergs}.$$

One naturally asks, Where has all the radiant energy come from?

The radiant energy probably possesses electromagnetic mass; but this does not give rise to any difficulty. The mass is found by dividing H by the square of the velocity of light

$$H/c^2 = 6{\cdot}5 \times 10^{31} \text{ grams},$$

which is less than $\frac{1}{40}$th of the whole mass of the star.

5. *Discussion of the Assumptions.*—The last paragraph shows that the theory has led to improbable results. We therefore examine the assumptions made.

(*a*) *The Radiative Hypotheses.*—It can scarcely be doubted that at the high temperatures which (on any theory) are held to prevail in the star, radiation is far more effective than any other cause in transferring heat. Its importance rapidly increases as the temperature rises, since it depends on the fourth power of the temperature. Convection currents would only arise if the transfer of heat by radiation were not sufficiently rapid to prevent the outer parts from cooling and sinking; our difficulty is that the transfer by radiation seems to be much too rapid.

(*b*) *The Boundary Condition.*—It will be found that the terms A, B, C become of the same order of magnitude near the boundary and our equations break down. The stellar atmosphere terminates in a way different from what we have supposed, and the correct equations are those given by Schwarzschild. The boundary tem-

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perature is, in fact, $T_1 / \sqrt[4]{2}$ instead of zero, and the pressure-temperature relation is more nearly

$$p \propto T^4(1 - T_1^4/2T^4);$$

that is to say, a small constant of integration is required in (16).

Until we are close to the boundary T_1^4 is very small compared with T^4, and it is easy to verify that as regards the interior of the star the correction is insignificant. Our calculation of k in § 3 is not affected by any failure of the equations at the boundary—we have not used T_1 as the temperature of a definite layer, but as measuring the total outflow of heat. The actual outflow must evidently be continuous with the outflow across a sphere contained a little within the boundary where our equations apply.

(*c*) *Variation of* k.—It has been assumed that k is constant, but it may vary with the temperature—either of the absorbing matter or of the radiation to be absorbed.

We can make calculations assuming that k varies as any power of the temperature, say T^δ. If $k = \kappa T^\delta$, then instead of (15) we have

$$\frac{dp}{dx} = \frac{4\pi G\mu}{3\kappa\epsilon}\frac{1}{T^\delta}\frac{dT^4}{dx} \quad . \quad . \quad . \quad . \quad (21)$$

whence $p \propto T^{4-\delta}$.

The equilibrium is still a form of adiabatic equilibrium, but with a different value of the ratio of the specific heats γ. Reference to the examples worked out by Emden shows that the central temperature is not very seriously altered by changing γ. We can show this best by proceeding at once to an extreme case. Let k vary as T^3, then $p \propto T$ and the star is a sphere of uniform density. I find in this case for the star discussed in § 4,

$$T_0 = 8{\cdot}9 \times 10^7 \text{ degrees Centigrade}$$
$$p_0 = 2{\cdot}7 \times 10^{11} \text{ dynes cm.}^{-2}$$

Temperature gradient near surface $= 25^\circ{\cdot}4$ per kilometre.

It is evident from the slight alteration of the central temperature that the outflow of radiation and store of radiant energy will again be excessively great. In fact, for an effective temperature of 6500° we find

$$k = 2{\cdot}7 \times 10^8 \text{ at the centre.}$$

(*d*) *Variation of* ϵ.—Similar small changes are produced if the radiated heat is not contributed by all parts equally, but comes more from the centre than the exterior layers, or vice versa. There are two cases in which we can give exact results:—

If $\epsilon \propto (1 - \xi^2/r^2)^3$, the density is constant.

If $\epsilon \propto (c^2 + \xi^2)$, where c is a certain constant, the case corresponds to adiabatic equilibrium with $\gamma = 1{\cdot}2$.*

* ϵ is defined by equation (13).

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The two cases correspond to the heat being generated mainly in the interior and mainly in the exterior parts respectively, and give some idea of the range that is possible.

(*e*) *Effect of Scattering.*—In the introduction to his theory of radiative equilibrium Schwarzschild states that he has neglected *scattering*; but it appears that scattering can be included—at least in the simple theory given by him. He considers radiation of intensity A propagated along the positive direction of ξ, and radiation of intensity B along the negative direction. The equations, taking account of absorption and emission, are then

$$\left.\begin{aligned} \frac{dA}{d\xi} &= -k\rho A + k\rho\mu T^4 \\ -\frac{dB}{d\xi} &= -k\rho B + k\rho\mu T^4 \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

If there is a coefficient of scattering s, so that, considering the radiation A, in a length $d\xi$, $\frac{1}{2}s\rho d\xi A$ is scattered forwards and the same amount backwards, the equations become

$$\left.\begin{aligned} \frac{dA}{d\xi} &= -k\rho A + k\rho\mu T^4 - \tfrac{1}{2}s\rho A + \tfrac{1}{2}s\rho B \\ -\frac{dB}{d\xi} &= -k\rho B + k\rho\mu T^4 - \tfrac{1}{2}s\rho B + \tfrac{1}{2}s\rho A \end{aligned}\right\} \quad . \quad (22a)$$

whence

$$\begin{aligned} \frac{d}{d\xi}(A - B) &= k\rho(2\mu T^4 - A - B) \\ &= 0 \text{ for the steady state} \\ \frac{d}{d\xi}(A + B) &= -(k + s)\rho(A - B). \end{aligned}$$

These differ only from the corresponding equations derived from (22) by the replacement of k by $(k + s)$. Thus scattering should be included in the absorption coefficient, and the theory then applies without modification.

6. *Radiation-Pressure.*—It appears then that no modification of the assumptions mentioned in § 5 could affect the main result, viz. that the interior temperature is so high as to cause much too great an outflow of radiation, unless we imagine the material to be almost perfectly opaque. The high temperature in the interior is inevitable if the gas—necessarily of moderate density in a giant star—is to support the weight of the enormous column of material above.

This points to a way out of the discrepancy—the weight of the column may be partly supported by radiation-pressure. In introducing radiation-pressure at this stage we do so not as a *hypothesis* to explain the discrepancy, but because in the conditions we have found radiation-pressure would be extremely powerful. Whether radiation-pressure is important or not depends on the value of k. If we had arrived at a low value

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of k, our neglect of radiation-pressure would have been justified. With the high value found radiation-pressure would far outbalance gravity, and our neglect of it is clearly illegitimate.

As there seems to be a rather widespread impression that gases are not subject to radiation-pressure, it may be advisable to state the theory briefly. The pressure is simply a consequence of absorption or scattering. A beam of radiation carries a certain forward-momentum proportional to its intensity; after passing through a sheet of absorbing medium a weaker beam emerges carrying proportionately less momentum; the difference of incident and emergent momentum is retained by the medium and constitutes the pressure. The medium, in fact, absorbs the momentum of the beam in the same proportion as it absorbs the energy. The calculations of radiation-pressure on small solid particles are simply calculations of absorption and scattering by these particles; it is not possible to apply such methods to atoms and molecules, which absorb by some internal mechanism. But the relation between absorption and pressure is a perfectly general one, depending only on the conservation of momentum.

Consider a small disc of thickness $d\xi$, and let radiation carrying momentum h fall on it, travelling at an angle θ to the normal. The length of path is $d\xi \sec\theta$. Hence the momentum absorbed is $k\rho h d\xi \sec\theta$. Resolving along the normal, the normal outward momentum absorbed is $k\rho h d\xi$, which is independent of θ.

In the present problem we have energy $\pi A + \frac{2}{3}\pi B$ flowing outwards and $\pi A - \frac{2}{3}\pi B$ inwards across unit surface; hence the net outward momentum absorbed is $c \, . \, \frac{4}{3}\pi B k\rho d\xi$, where c is the factor relating the momentum and energy of a beam. Now the pressure on a black body is numerically equal to the density of the energy (assumed isotropic). The density of the energy at temperature T is aT^4, where a is the universal constant $7{\cdot}06 \times 10^{-15}$. The outward flow of isotropic energy across unit surface is $\pi A = \pi\mu T^4$, and the outward flow of momentum is $c \, . \, \pi\mu T^4$. Hence

$$aT^4 = c\pi\mu T^4,$$

so that

$$c = a/\pi\mu.$$

It follows that the force of radiation-pressure on an element $d\xi$ is

$$\frac{4}{3}\frac{a}{\mu}Bk\rho d\xi \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$= -\frac{4}{3}\frac{a}{\mu}dA \quad \text{by (6)}$$

$$= -\frac{4}{3}ad(T^4) \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Equation (24) is quite general. Assuming now the constancy of $k\epsilon$ so that $T^4 \propto p$, the radiation-force is

$$-\frac{4}{3}a\frac{T_0^4}{p_0}dp.$$

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For the star discussed the values of T_0 and p_0 in § 4 give

$$\tfrac{4}{3}a\frac{T_0^4}{p_0} = 2\cdot 0 \times 10^5,$$

so that radiation-pressure would be 200,000 times stronger than gravitation. Of course, the result does not mean that the existing radiation-pressure is so strong as this; it shows that our conclusions contradict our premises, viz. that radiation-pressure was negligible.

We must now form the modified equations in which radiation-pressure is included. The weight of an element $g\rho d\xi$ will be partly counterbalanced by the radiation-force (23)

$$\tfrac{4}{3}\frac{a}{\mu}Bk\rho d\xi.$$

Hence in the pressure-equation we replace g by

$$g - \tfrac{4}{3}\frac{a}{\mu}Bk.$$

Using (13), this gives

$$g - \tfrac{4}{3}\tau\frac{x^3}{\xi^2}\cdot\frac{ak\epsilon}{\mu}$$

$$= g\left(1 - \frac{ak\epsilon}{\pi G\mu}\right) = \beta g, \text{ say} \quad . \quad . \quad . \quad (25)$$

We shall suppose as before that $k\epsilon$ is constant, and accordingly β is constant. Evidently the effect of radiation-pressure is exactly equivalent to an alteration of the constant of gravitation to βG. We see by (20) that the density distribution will be unaltered, but the pressures and temperatures previously calculated must be multiplied by β. Instead of (16) we shall have

$$p = \frac{4\pi\mu\beta G}{3k\epsilon}T^4 \quad . \quad . \quad . \quad . \quad (26)$$

To determine β we have from (25)

$$1 - \beta = \frac{ak\epsilon}{\pi G\mu}$$

$$= \tfrac{4}{3}a\beta\frac{T^4}{p} \quad \text{by (26),} \quad . \quad . \quad . \quad (26a)$$

so that

$$1 - \beta = \tfrac{4}{3}a\beta^4\left(\frac{T_0^4}{p_0}\right)_c \quad . \quad . \quad . \quad . \quad (27)$$

where $(T_0^4/p_0)_c$ refers to the values calculated in § 4 without taking account of radiation-pressure.

Also k is given by changing g_1 to βg_1 in (19)

$$\tfrac{3}{4}k = \frac{\beta g_1}{T_1^4}\frac{T^4}{p},$$

or

$$\tfrac{3}{4}k = \beta^4\frac{g_1}{T_1^4}\left(\frac{T_0^4}{p_0}\right)_c \quad . \quad . \quad . \quad . \quad (28)$$

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Moreover, from (27) and (28),

$$1-\beta = kaT_1^{\,4}/g_1 \quad . \quad . \quad . \quad . \quad (29)$$

7. ***Results for a Giant Star.***—Substituting in (27) the values calculated in § 4, we have

$$1-\beta = [5\cdot2980]\beta^4.$$

I have given the logarithm of the coefficient. Solving this equation

$$\beta = 0\cdot0468,$$

so that less than one-twentieth of the weight is left uncompensated. It will be seen from (20) that $(T_0^{\,4}/p_0)$, does not depend on the density; thus β is the same for all giant stars of the same mass.

Using this value of β in (29) we obtain

$$k = 29\cdot5.$$

This value should presumably be the same for all stars. It indicates that the radiation would be reduced in the ratio $1/e$ after passing through a column containing $\frac{1}{30}$ gm. per sq. cm. section (about 25 cm. of air). This seems possible, since it must be remembered that at the high temperatures concerned the bulk of the radiation is of very short wave-length.

The new values of the central temperature and pressure are

$$T_0 = 7\cdot12 \times 10^6 \text{ degrees}$$
$$p_0 = 1\cdot19 \times 10^{12} \text{ dynes cm.}^{-2}$$

Surface temperature gradient $= 0^\circ\cdot30$ per kilometre.

The total radiant energy imprisoned is

$$H = 2\cdot81 \times 10^{47} \text{ ergs}$$
$$= 0\cdot238\,\Omega,$$

so that it is no longer an excessive amount.

8. *The Relation between Density and Effective Temperature.*—It appears that a gaseous star must settle itself into a state in which radiation-pressure almost balances gravitation. If the outward flow of radiation is too strong, so that gravitation is overbalanced, the star will blow out, the temperature will fall, and radiation will be reduced. On the other hand, if radiation-pressure falls much below gravitation the star will contract and the outflow of radiation will increase. We can make use of the approximate relation radiation-pressure = gravitation (or the more rigorous equation (29)) to determine the variation of effective temperature with mean density and mass.

Since g_1 (gravity at the surface) is proportional to $M^{\frac{1}{3}}\rho^{\frac{2}{3}}$, we have from (29)

$$T_1^{\,4} \propto M^{\frac{1}{3}}\rho^{\frac{2}{3}}(1-\beta),$$

whence, approximately

$$T_1 \propto M^{\frac{1}{12}}\rho^{\frac{1}{6}} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

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The relation is rigorous as regards ρ; and as regards M the correction due to the dependence of β on M is usually insignificant. For example, if M is increased eightfold, formula (30) gives an increase of T_1 in the ratio 1·189, whereas the exact ratio is 1·198.

The range of mass in stars is believed to be small, and will affect T_1 only slightly. The following table shows how T_1 varies with the density for a star of mass $1·5 \times$ Sun.

Effective Temperature.	Mean Density (Water = 1).
13 000°	0·128
10 000	·026
8 000	·007
6 000	·0012
4 500	·00022
3 000	·00002

This seems to correspond fairly well with what we know of the densities and temperatures of the giant stars of different types.*

At 10 000° the central density should be $54·25 \times ·026 = 1·4$, so that deviations from Boyle's Law will occur near the centre. We may, however, anticipate the results of § 10, and state that it is not until temperatures above 13 000° are reached that the deviations from the gas-law begin to influence the effective temperature.

The total radiation varies as $r^2 T_1^4$, which by (29) varies as $g_1 r^2(1-\beta)$, that is, as $M(1-\beta)$, and is independent of the density. We thus obtain the result that

The bolometric magnitude of a gaseous star is independent of its stage of evolution, and depends only on its mass.

This is in accordance with H. N. Russell's conclusion that the absolute magnitudes of giant stars of different types are nearly the same. Differences of one or two magnitudes may occur, owing partly to differences of mass and partly to differences of the ratio of the luminous to the total radiation.

From the table the density of a giant star having the same effective temperature as the Sun (6000°) is ·0012. The Sun's density is 1·38. Hence by comparing their superficies I find that a giant star is $5^m·4$ brighter than the Sun, which agrees well with observation. (According to Russell, the average magnitude of a giant star at 10 parsecs distance is probably about zero; that of the Sun is usually given as $5^m·5$, but some recent determinations make it about $0^m·5$ brighter.)

* Russell has given the following estimates of the average densities of giant stars of different types in terms of the Sun's density (*Nature*, vol. xciii. pp. 282–3):—Type A, 1/10; G, 1/350; K, 1/2800; M, 1/25000. Assuming that type G has the same effective temperature as the Sun (6000°), the law $T_1 \propto \sqrt[6]{\rho}$ gives the following effective temperatures:—Type A, 10800°; G, 6000°; K, 4250°; M, 2950°. This agrees almost exactly with the temperatures usually assigned.

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Comparison with the Lane-Ritter Theory.—According to Ritter * the effective temperature of a star varies as the square-root of the mass. Our result is the twelfth root, which seems to accord better with the comparatively small range of surface temperatures observed in stars of similar density. Ritter further states that the temperature is nearly independent of the density, whereas according to the present view density is the main determining factor.

Had Ritter taken $\gamma = \frac{4}{3}$, his formula would have led to $T_1 \propto \rho^{\frac{1}{6}}$, agreeing in this respect with (30).

Lane's law, strictly speaking, is not concerned with the effective temperature, but with the temperature at any definite point in the interior; this is proportional to the cube root of the density on Lane's theory as on the present theory. It has often been recognised that the effective temperature will follow a different law; but I do not think the sixth root has been suggested before.

Range of Stellar Masses.—We have found that the total radiation of a giant star varies as $M(1-\beta)$; or, since β is small and varies only slightly, the radiation is practically proportional to the mass. On this view we must attribute the differences in luminosity of individual giant stars of the same spectral type to differences in the masses; and if we knew the range of variation of luminosity we should be able to deduce the range of variation of the masses.

H. N. Russell has given provisionally the necessary data for giant stars of type M.† He has found that these are distributed about a mean absolute magnitude approximately according to the law of errors, the "probable error" being $\pm 0^m{\cdot}6$. By the law of errors more than four-fifths of the stars will be included within a total range of $2^m{\cdot}4$ (*i.e.* twice the probable error on either side). The corresponding range of luminosity is 1 : 9; and the range of the masses must be the same.

We conclude that the masses of the stars are dispersed about a median mass M, so that four-fifths of the whole number are between the limits $\frac{1}{3}$M and 3M. This result applies primarily to the giant M stars; but Russell's investigation indicates that the range is very similar in the other types. This remarkably small range of mass is in general agreement with our knowledge derived from observation of binary stars.

9. *Energy of a Star.*—Former investigators have found a difficulty that the heat energy of a star amounts to a large proportion of the energy acquired by contraction, leaving only a small fraction to account for the radiation during past history. Thus Perry showed that if $\gamma < \frac{4}{3}$ the heat (molecular) energy alone is greater than could be acquired by contraction; he neglected the imprisoned radiant energy, which would have made the deficit very much worse. On the present theory this difficulty is avoided, and only about a quarter of the energy of contraction is retained within the star.

* *Astrophysical Journal*, vol. viii. p. 307.

† *Observatory*, vol. xxxvii. p. 170; *Nature*, vol. xciii. p. 255.

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Let c_v be the specific heat at constant volume,
γ the ratio of specific heats,
dV an element of volume,
K the molecular energy of the star,
H the imprisoned radiant energy,
Ω the energy acquired by contraction from infinite diffusion.

Then $$K = \int c_v \rho T dV = \frac{c_v}{R}\int p dV = \frac{1}{\gamma - 1}\int p dV,$$

since $$c_v + R = \gamma c_v.$$

Also $$H = \int a T^4 dV = a\frac{T_0^4}{p_0}\int p dV = \frac{3(1-\beta)}{4\beta}\int p dV \quad \text{by } (26a).$$

Now if there were no radiation-pressure we should have

$$\int p dV = 4\pi\int p\xi^2 d\xi = -\tfrac{4}{3}\pi\int \xi^3 dp = \tfrac{1}{3}\Omega. \quad (\textit{Gaskugeln}, \text{ p. } 124).$$

But radiation-pressure multiplies the values of p in the ratio β, so that

$$\int p dV = \tfrac{1}{3}\beta\Omega.$$

Thus $$K = \frac{\beta}{3(\gamma - 1)}\Omega \qquad H = \frac{1-\beta}{4}\Omega.$$

We cannot estimate K accurately, since we do not know γ. The material is probably monatomic at the high temperature, but it seems unlikely that the usual value $\gamma = \frac{5}{3}$ is valid for radiating atoms. The factor β in the numerator will make this energy small in almost any case.

Taking Perry's case, $\gamma = \frac{4}{3}$, we have (for mass $= 1{\cdot}5 \times \odot$)

$$K = {\cdot}047\Omega \qquad H = {\cdot}238\Omega.$$

Altogether 0·285 of the energy of contraction is retained within the star as heat, leaving plenty of margin for radiation or for the formation of radioactive elements.

It is interesting to note that the greater part (H) of the energy resides in the æther. Only a small part of the star's store of heat is represented by the molecular movements; the bulk of it consists of æther-waves travelling in all directions but unable to escape, except very slowly, through the meshes of matter which imprison them. This condition applies to the gaseous stars only, and not to dwarf stars such as the Sun.

As the star contracts Ω varies as $\rho^{\frac{1}{3}}$, or as T_1^2. Hence, since the total rate of radiation remains constant, we ought to have—

The age of a gaseous star increases as the square of its effective temperature.

I do not place much reliance on this, because it is well known that a star cannot have a reasonably long life if its supply of energy is due solely to contraction.

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10. *Dwarf Stars.*—If we were to extrapolate the law $T_1 \propto \rho^{\frac{1}{6}}$ for a star having the density of the Sun, 1·38, we should find an effective temperature of 19 300,° whereas the Sun's effective temperature is about 6000°. The difference is due to the failure of the gas-law at this high density, which renders the calculated values of $(T_0^4/p_0)_c$ erroneous. It should be noted that we have not used the gas-law in deriving equations (27), (28), (29), or indeed in any of the numbered equations except (20). Thus the whole theory holds for the dwarf stars, except that the calculations of § 4 break down.

The effect of a failure of the gas-law is to alter the value of $(T_0^4/p_0)_c$, which by (27) alters β. β begins to increase. By (29) T_1 varies as $\sqrt[4]{(1-\beta)}$. Since β is at first about 0·05, the increase of β two- or three-fold makes at first but little difference in this factor. It is not until long after the internal distribution of temperature and density has deviated strongly from that given by the gas-law that T_1 begins to be affected.

It is of great interest to determine the turning-point at which the effective temperature ceases to increase with the density.

For high densities and pressures the necessary modification of Boyle's Law is given approximately by Van der Waal's equation, and the gas-equation becomes

$$p = R\rho T\left(1 - \frac{\rho}{\rho_0}\right)^{-1} \quad . \quad . \quad . \quad . \quad (31)$$

where ρ_0 is the maximum possible density of the material. This equation must be combined with (26*a*)

$$p = \tfrac{1}{3}a\frac{\beta}{1-\beta}T^4 \quad . \quad . \quad . \quad . \quad (32)$$

and

$$\frac{1}{\rho}\frac{dp}{d\xi} = -\beta g \quad . \quad . \quad . \quad . \quad . \quad (33)$$

to determine the interior conditions. An exact solution is, of course, impossible; but we can without much difficulty attain our object by quadratures.

Since it is clear that on this theory the Sun must be compressed to something near the maximum density throughout most of its mass, I take ρ_0 to be 1·5.* Our object is—given the mass and mean density, to find β. We have to proceed by guessing β and then working from the outside inwards with equations (31), (32), and (33) to test whether the value fits.

For a star of the same mass as before, and density 0·7, I find that β must be 0·18. The following table gives the radii of the surfaces of equal density, the mass contained within them, the temperature, and the acceleration of gravity there.

* N. Ekholm gives 1·486. His neglect of radiation-pressure and other divergences will make no serious difference in the case of the Sun. See *Gaskugeln*, p. 467.

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Dense Star.

(Mass = 1·5 ☉ ; Density = 0·7.)

Density.	Radius.	Interior Mass.	Temperature.	Gravity.
0	$9·92 \times 10^{10}$	$2·87 \times 10^{33}$	...	$1·94 \times 10^{4}$
0·05	9·32	2·87	$3·4 \times 10^{6}$	2·20
0·15	9·05	2·85	5·0	2·31
0·25	8·87	2·81	6·1	2·38
0·35	8·70	2·76	7·0	2·43
0·45	8·54	2·70	7·8	2·46
0·55	8·37	2·62	8·6	2·49
0·65	8·18	2·52	9·5	2·51
0·75	7·95	2·39	10·4	2·52
0·85	7·67	2·22	11·3	2·51
0·95	7·29	1·98	12·5	2·48
1·05	6·76	1·65	13·8	2·41
1·15	5·86	1·16	15·4	2·25

The mass remaining within the last sphere must correspond to a mean density greater than 1·15, and less than the limiting density 1·5. The actual value deduced from the table is 1·37, and this constitutes our test that the value of β is correct—we have stripped off just the right amount. Had we chosen $\beta = 0·20$, the interior density would have been less than 1·15; similarly a smaller β would have given a density greater than 1·5.

According to formula (30) this star would have an effective temperature of 17 300°; the factor $(1-\beta)^{\frac{1}{4}}$ reduces this to 16 600°. It is remarkable that such an extreme deviation from the conditions of a perfect gas should lead to so small a correction. It is clear that 16 600° must be about the turning-point of the temperature, because when once the term in β has begun to take effect it will soon counterbalance any increase of density.

By (30) the maximum effective temperature will be proportional approximately to the twelfth root of the mass. The correction depending on β can be determined from (27), since *for the same mean density* $(T_0{}^4/p_0)_c$ varies as M^2, whether the star be gaseous or otherwise (*cf.* (20)). Thus for the same density

$$\frac{1-\beta}{\beta^4} \propto M^2.$$

Introducing this correction, I find the following results:—

Mass = 4·5 × Sun.	Maximum temperature =	18 600°
„ 1·5 „	„ „	16 600°
„ 0·5 „	„ „	14 600°

These values are perhaps too high. The observational evidence of Nordmann and of Wilsing and Scheiner seems to point to a maximum of about 14 000°. But the agreement seems sufficiently

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close to be of interest. Lower maxima, in the ratio five-sixths, would have been obtained if Russell's densities (*footnote*, p. 29) had been used instead of the datum density 0·002 corresponding to temperature 6500°.

The dwarf-stars must be comprised within the range of density 0·7 to 1·5. The importance of radiation-pressure compared with gravitation decreases rapidly as we descend along the dwarf series. For the Sun, radiation-pressure is practically negligible. The differences of luminosity of the dwarf types must be due mainly to differences of effective temperature, since there is not much scope for variation of superficial area.

11. *Molecular Weight.*—The calculations have been based throughout on a molecular weight 54. To illustrate the effect of modifying this, I give some results for a molecular weight 18. The gas-constant R becomes multiplied by 3. The central pressure becomes multiplied by 3 very approximately, the central temperature being almost unaltered. The new value of β is ·137, so that about $\frac{6}{7}$ of gravity is compensated by radiation-pressure, instead of $\frac{19}{20}$. The constant of absorption k is slightly reduced to 26·7. The results of § 8 are based on the general relation $T_1 \propto M^{\frac{1}{12}}\rho^{\frac{1}{6}}$, and are not altered. The calculations of the internal energy of the star in § 9 involve β, and the numerical results are altered, but they are of the same general character; the molecular energy is, however, increased and the radiant energy slightly diminished.

The most important effect of diminishing the molecular weight to 18 is to reduce the maximum possible effective temperature and to cause it to occur at a lower mean density. It is not possible to give accurate figures without a great deal of labour, but it appears that the maximum effective temperature would be decreased by about 1200°.

The importance of radiation-pressure in determining stellar equilibrium would be reduced if the molecular weight were still smaller; but even for a star composed of helium, it would still be intense enough to counterbalance half the gravitation.

12. *The Constant of Absorption.*—We have found that k has a value about 30 in C.G.S. units, and this should be the constant of absorption for stellar material at temperatures chiefly between 10^6 and 10^7 degrees prevailing within stars. Unfortunately, there seems no other evidence to indicate whether this value is at all near the truth. At temperatures of 10^6 and 10^7 degrees the radiation of maximum intensity is of wave-length 30 and 3 tenth-metres respectively—shorter than ordinary spectral radiation, but longer than X-rays and γ-rays. It has one merit for our purpose—it cannot very well experience selective absorption, since no spectrum (X-ray or ordinary) contains lines within this range.

It happens that the experimental values of k for hard X-rays absorbed by solid material are usually about equal to the value here found.* In the absence of a theory to guide us, we cannot

* Bragg, *X-Rays and Crystal Structure*, p. 177. Table A gives k, defined as in this paper.

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infer that the same values would hold at a temperature of 10^6 degrees and for considerably longer waves. The agreement is probably accidental. The work which seems to bear most nearly on the question is J. J. Thomson's theory of the scattering of X-rays.* For very short waves the scattering depends only on the number of electrons present—for a gas, practically on the mass. The theory, confirmed by the experiments of Barkla, gives a scattering of ·00025 by a cubic centimetre of air. This is equivalent to $k = 0{\cdot}2$ only; but this is only the part of the coefficient due to scattering, and the full absorption coefficient must be larger.

Experimental results for the extreme ultra-violet have, I think, no application to our problem, for the absorption may be molecular and is not likely to persist at high temperatures.

The Stellar Magnitude Scales of the Astrographic Catalogue. Tenth Note. The Melbourne Magnitudes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. During the visit of the British Association to Australia in 1914 we were glad to learn that the measurement and reduction of the Melbourne zones ($-65°$ to $-90°$, the portion corresponding to that undertaken by Greenwich in the northern hemisphere) were nearing completion, and that funds had been provided for the printing. It is earnestly to be hoped that neither the war nor the retirement of Mr. Baracchi from the Directorship will affect the availability of these funds, and that the printing may go forward rapidly. The Council of the British Association, after consultation with the Council of the Royal Astronomical Society, drew the attention of the Victorian Government to the importance of this printing, expressing great satisfaction that the means had been promised.

2. Meanwhile Mr. Baracchi, in kind response to a request, sent me the counts of the plates in zone $-65°$; material of great value, seeing that nothing similar has yet been published south of $-42°$ (Cape). Experience of the Vatican plates in similar latitudes and of the Oxford plates indicates that the results of a single zone are liable to be sensibly affected by accidental errors (*Mon. Not.*, lxxv. pp. 468 and 603). Hence the following results may possibly be modified when other zones are available.

3. We will depart a little from previous procedure by considering first the magnitude scale as a whole. In Table I. are given, first, d, the diameter as measured at Melbourne in units of $0''{\cdot}25$; next, N′, the total number of stars on all the plates in the zone with a diameter greater than d; and next, log N, the average *per plate*, *i.e.* log N′ − log 80, there being 80 plates in the zone.

* Richardson, *Electron Theory of Matter*, p. 485.

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Using then the scale of Chapman and Melotte (as expanded in *Mon. Not.*, lxxv. p. 145), we get the magnitudes under the heading C. & M. Then follows the $\sqrt{d}$: and on adding this column to

TABLE I.

General Magnitude Scale for Zone − 65° (Melbourne).

d>.	N'.	log N.	C. & M.	$\sqrt{d}$.	Sum.	Calc. d.	O − C.
55	84	0·02	7·10	7·40	14·50	54·3	+0·7
50	117	0·17	7·40	7·05	14·45	50·0	0·0
45	189	0·38	7·80	6·70	14·50	44·5	+0·5
40	265	0·52	8·10	6·35	14·45	40·6	−0·6
35	470	0·77	8·60	5·90	14·50	34·5	+0·5
30	735	0·97	9·00	5·50	14·50	29·9	+0·1
25	1169	1·17	9·45	5·00	14·45	25·2	−0·2
20	1864	1·37	9·90	4·50	14·40	20·9	−0·9
15	4863	1·79	10·90	3·85	14·75	12·7	+2·3
10	15,675	2·30	12·25	3·15	15·40	4·9	+5·1
All	33,813	2·63	13·20	...	...	1·6	...

the former we see that the results are nearly constant until we come to the faintest stars, so that magnitudes may be obtained from the Melbourne magnitudes by the formula

$$m = 14{\cdot}47 - 1{\cdot}00\sqrt{d}$$

with considerable accuracy down to $d = 20$. For diameters smaller than this, however, the law ceases to be applicable. The numbers in the last column but one show the actual diameter corresponding to the value of log N on the scale of C. & M.: and the differences O − C between this and the first column are satisfactorily small.

4. But now these diameters were measured on three different machines, two of Repsold's and one "slide" machine. Mr. Baracchi sends the following note:—

"On the slide machine all diameters over 16 were measured, on Repsold I. all diameters over 24, and on Repsold II. all diameters over 50. Repsold I. has two parallel wires and Repsold II. four parallel wires in each co-ordinate which served as standards for the estimation of diameter. The unit used in the table is $\frac{1}{4}$ second of arc, the original estimates being made to quarter seconds. In the tables the results for the northern and southern halves of each plate are given separately. The measured area is a square whose side is 26 réseau intervals, the value of one réseau interval being $5' \times (1 - 1/87)$."

5. It is therefore important to see whether the diameters found by the three machines accord. The plates measured by the three machines are fairly well scattered through the 24^h, and in the first instance we may adopt the plan of comparing simply the totals for

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each instrument, or rather the mean totals per plate, and it will suffice to give the log totals, which occupy the 2nd, 3rd, and 4th columns of Table II.

TABLE II.

Log N for the Three Different Machines R_1, R_2, S.

d>.	R_1.	R_2.	S.	R_2-R_1.	$\frac{1}{2}(R_1+R_2)$.	$S-\frac{1}{2}(R_2+R_1)$.	−0·44.
55	9·93	9·76	0·32	−0·17	9·84	+0·48	+0·04
50	9·97	0·01	0·47	+0·04	9·99	+0·48	+0·04
45	0·19	0·24	0·65	+0·05	0·22	+0·43	−0·01
40	0·35	0·32	0·83	−0·03	0·34	+0·49	+0·05
35	0·58	0·67	1·03	+0·09	0·62	+0·41	−0·03
30	0·80	0·86	1·22	+0·06	0·83	+0·39	−0·05
25	1·04	1·02	1·43	−0·02	1·03	+0·40	−0·04
20	1·24	1·28	1·59	+0·04	1·26	+0·33	−0·11
15	1·77	1·76	1·84	−0·01	1·76	+0·08	−0·36
10	2·31	2·18	2·41	−0·13	2·25	+0·16	−0·28
All	2·64	2·59	2·66	−0·05	2·62	+0·04	−0·40

6. In the 5th column are shown the differences $R_2 - R_1$. There is a trace of systematic error near the bottom: but the accordance is good on the whole, and we may fairly take the mean of the two results, as in the 6th column, to represent the Repsold machines.

7. But when we compare this with S as in the 7th column there are striking differences, which average 0·44 at the top and fall to nearly zero at the bottom. Now it is a fair presumption that the average number of stars per plate is the same for the plates measured by each machine, in which case the small differences at the bottom are presumably correct. But what are we then to make of the very considerable differences at the top? They are the more remarkable because, according to Mr. Baracchi's note, the diameters of the brighter stars were deliberately *measured*, while those of the fainter were estimated only. Hence the first possibility considered was that, after all, the bottom differences must be wrong, and that we should subtract 0·44 from all the numbers in the 7th column (as shown in the 8th column), representing the difference between the average number of stars on plates measured in the S machine and those measured in the R machines.

8. Since log $3 = 0·48$, it follows that the S plates contain nearly three times as many stars as the R plates. But this is so large a discrepancy that it should be easily discernible. To test its reality the number of stars on every S plate was compared with the mean of the R plates immediately preceding and following it. The totals were—

$$S = 8509 \qquad R = 7041$$
$$\log = 3·93 \qquad \log = 3·84 \qquad \text{Diff.} = +0·09.$$

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The difference 0·09 accords fairly well with the run of the numbers near the bottom of column 7, Table II., and is quite inconsistent with any such systematic difference as 0·44. We are thus led to consider these differences for bright stars as real, and their remarkable character is shown by the consideration that they involve calling a diameter $d=42$ on one machine and $d=55$ on another.

9. It was natural to write to Melbourne to see what light could be thrown on the discrepancy, and at my request Mr. Baldwin very kindly had three plates which had been measured on the S machine remeasured (as regards diameters only) on R_2. But it may fairly be said that these remeasures only increase the mystery. There is some evidence of personality between observer P, who made all the remeasures on machine R_2, and observer S, who measured one plate on machine S. But it does not occur in the right place in the scale, and the personality between P and the four other measurers (T, MH, L, and P) who first measured the other two plates is insensible.

The following are the values of log N :—

TABLE III.

$d>$.	Plates $1^h\ 39^m$ and $5^h\ 33^m$. S.	R_2.	$S-R_2$.	Plate $14^h\ 33^m$. S.	R_2.	$S-R_2$.
50	0·84	0·70	+0·14	0·30	0·48	−0·18
45	1·00	1·00	·00	0·84	0·84	·00
40	1·18	1·11	+ ·07	1·04	0·95	+ ·09
35	1·34	1·36	− ·02	1·18	1·15	+ ·03
30	1·38	1·40	− ·02	1·42	1·23	+ ·19
25	1·52	1·45	+ ·07	1·73	1·49	+ ·24
20	1·66	1·59	+ ·07	2·00	1·63	+ ·37
15	2·03	1·96	+ ·07	2·27	1·89	+ ·38
10	2·53	2·48	+ ·05	2·58	2·53	+ ·05
All	2·74	2·74	·00	2·71	2·71	·00

The numbers of bright stars concerned are of course small, and this evidence may not be conclusive. But the systematic difference is so large that the origin of it ought to be traceable with little difficulty.*

10. Although the existence of this systematic error may somewhat diminish the value of our conclusions, we will now see what this single Melbourne zone (the only part of the *Astrographic Catalogue* yet to hand, even in MS., from −52° to the South Pole) tells us about the stellar distribution in R.A.

11. In Table IV. are given the values of the counts for each

* Mr. Baldwin's letter is dated 1916 January 13, and I have delayed this note a little in the hope that further information may be forthcoming: but nothing more is yet to hand.

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hour of R.A. at certain definite limits of diameter. Under $d>40$ are given the total numbers of images with diameters of 41 or more: and so for the other columns. The ninth column, headed "All," gives the total number of stars on the plates in each hour. Two-thirds of the "hours" contain three plates each, while hours 1^h, 4^h, 7^h, etc., contain four plates: but as in what follows we are chiefly concerned with log differences for each hour, from which a factor 3 or 4 disappears, the simple counts have been left as they stand, to avoid the use of fractions.

Table IV.

Cumulative Totals (N') for Melbourne − 65° for each Hour of R.A.

$d>$.	40.	35.	30.	25.	20.	15.	10.	All.	(α)	(β)
0^h	2	7	9	18	32	77	265	361	− 9	+ 1
1	9	19	26	29	44	107	345	554	− 4	− 29
2	7	9	12	16	28	73	242	314	− 9	− 16
3	3	5	8	18	31	94	299	391	− 11	+ 10
4	2	9	21	36	73	182	545	745	− 9	+ 4
5	12	16	19	36	51	146	448	685	− 6	− 6
6	6	17	25	40	61	189	615	1058	− 5	− 3
7	10	21	32	43	63	223	813	1796	+ 1	+ 4
8	6	14	23	37	74	200	430	1251	+ 12	+ 2
9	23	35	54	84	131	266	855	1779	+ 2	− 15
10	41	66	105	139	219	457	1568	3532	+ 3	− 16
11	3	4	10	24	37	103	386	1494	+ 16	+ 38
12	31	57	99	169	238	420	1448	3040	+ 2	− 19
13	15	25	35	70	119	422	1619	3356	− 2	+ 28
14	21	33	51	88	168	363	990	1443	− 6	− 14
15	9	18	29	42	80	357	1140	3074	+ 6	+ 29
16	27	48	66	103	152	403	1059	3035	+ 10	− 6
17	5	10	13	16	26	114	472	1029	0	+ 19
18	7	11	20	37	56	203	607	1765	+ 9	+ 20
19	9	17	29	45	56	126	449	841	− 1	− 19
20	4	7	12	20	30	83	290	654	+ 2	+ 3
21	5	8	13	24	35	92	302	668	+ 2	+ 2
22	5	8	15	23	40	115	354	704	0	0
23	3	6	9	12	20	48	134	244	− 1	− 20
Totals	265	470	735	1169	1864	4863	15675	33813		
Mag. C. & M.	8·10	8·60	9·00	9·45	9·90	10·90	12·25	13·20		

12. In the last two columns are given—

under heading (α), log "All" − log "10" − 0·56,
and under heading (β), log "All" − log "30" − 1·93,

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log "10" denoting log N′ for the hourly total in the column $d > 10$, and the constants 0·56, 1·93 being the mean values of the log differences for the 24 hours. From the magnitude scale given at the foot of the table we see that the column (α) compares mags. 13·20 and 12·25, differing by 0·95 mag.: and column (β) compares 13·20 and 9·00, a difference of 4·20 mags.

13. For harmonic analysis of these columns it will suffice to add together 23^h and 0^h, giving a total applicable to the midpoint $0^h{\cdot}0$: with similar totals for the points $2^h{\cdot}0$, $4^h{\cdot}0$, etc., a set of 12 terms, which is quickly dealt with. The first harmonics are (applicable to hourly results)—

$$\text{for } (\alpha),\ -3{\cdot}5 \sin\theta - 4{\cdot}9 \cos\theta = -\ 6{\cdot}0 \cos(\theta - 36^\circ),$$
$$\text{for } (\beta),\ -7{\cdot}5 \sin\theta - 6{\cdot}7 \cos\theta = -10{\cdot}0 \cos(\theta - 49^\circ).$$

We may perhaps add an intermediate determination, comparing mags. 13·20 and 9·90 ($d > 20$), giving

$$-8{\cdot}7 \sin\theta - 4{\cdot}8 \cos\theta = -10{\cdot}0 \cos(\theta - 61^\circ).$$

14. Now the phases 36°, 61°, and 49° are fairly accordant: the mean value is 48°, meaning that the "obscuration" is greatest at about $3^h{\cdot}0$. To represent the effect as an obscuration of faint stars the coefficients −6·0, −10·0, and −10·0 have been given with negative signs (as proposed on p. 154 of *M.N.*, lxxvi.). But the coefficients do not increase as fast as the difference of magnitude, there being no change at all between mags. 9·00 and 9·90, a change of 4·0 between mags. 9·90 and 12·25, and a change of 6·0 in the short interval 12·25 to 13·20. This, however, is in good accordance with the Cape counts, which do not show any sensible "obscuration" effect until we reach faint stars (see Table XVI. on p. 480 of *M.N.*, lxxv.).

15. We may now see how these results fit the "spiral" suggested (*M.N.*, lxxvi. p. 154 or 157).

We find

$$\psi_0 = A + 3{\cdot}66\delta - 247^\circ,$$
$$= 48^\circ - 3{\cdot}66 \times 65^\circ - 247^\circ,$$
$$= -77^\circ.$$

The Cape zones −49°, −50°, and −51° give $\psi_0 = +39^\circ$, −15° and −93°, so that there is some hint of a negative value. But, in the absence of intermediate zones, we cannot yet say anything definite. It is much to be regretted that we have as yet so little material in this part of the sky: a single zone from the Sydney Observatory would be of great value. To supply the deficiency counts have been made of the *visual* magnitudes observed at Cordoba, and will be given in another note. But an appeal is hereby made to the Directors of the Melbourne and Sydney observatories for more material as soon as it can be furnished, either in print or in MS., but preferably, of course, in print.

16. Meanwhile another point may be noticed about the Melbourne results which seems to strengthen the evidence for the

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"spiral." It was shown on p. 153 of *M.N.*, lxxvi., that when the circuits of R.A. are started with ψ_0 in each case, one edge of the spiral is much sharper than the other, the peak occurring at 15^h from ψ_0. Now the same character attaches to the Melbourne results. There are, of course, large accidental irregularities which mask it to some extent; but they can be partly nullified by grouping. Let us, for instance, take two hours of R.A. together, and also combine columns (α) and (β) of Table IV. We will start with the group $3^h + 4^h$, the mean of the four numbers $-11-9+10-4$ in columns (α) and (β) being -4; and we then follow round with similar groups of 4, as below:—

	(0^h).	(2^h).	(4^h).	(6^h).	(8^h).	(10^h).	(12^h).	(14^h).	(16^h).	(18^h).	(20^h).	(22^h).
M	−4	−5	+5	−6	+9	+2	+10	+12	−4	+1	−7	−14
C	−4	−3	−3	0	+4	+8	+11	+16	−1	−9	−10	−10

The Melbourne numbers are given in the row M: in the row C are given the simple sums, for every two hours, of the last column of Table IV. on p. 153 of *M.N.*, lxxvi.

It will be seen that the numbers run together very fairly. If we again add every consecutive pair to get six groups, we find—

M	−9	−1	+11	+22	−3	−21
C	−7	−3	+12	+26	−10	−20

17. The unit of magnitude for Melbourne has been arbitrarily chosen to fit, so that the similarity of the scales has no significance: it is the *arrangement* of the numbers in circuit to which attention is called. We must, however, justify the starting-point chosen for Melbourne: the Cape zones have been started with the epoch of greatest obscuration, which for Melbourne we have found to be 48° or close to $3^h{\cdot}0$: hence to correspond as nearly as possible with the Cape arrangement we must begin with 3^h, as in fact we did.

18. Finally, one word more may be said as to the systematic difference between the machines. We might perhaps have tried the effect of correcting S to accord with R_1 and R_2, but Table II. shows that if the distribution of S measures round the circuit had any sensible effect on the phenomenon under consideration, it would be shown more markedly in the case of $d > 30$ and $d > 20$ than in that of $d > 10$. Since all three phases are in good accord, we may fairly conclude that to make this correction would be an unnecessary refinement at the present stage.

Summary.

(α) Counts of the Melbourne zone −65° show that the recorded diameters d of the star images can be converted into magnitude (C. and M. scale) by the formula

$$\text{mag.} = 14{\cdot}47 - 1{\cdot}00\sqrt{d},$$

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the constant 14·47 being a mean value only, but the coefficient of $\sqrt{d}$ being probably applicable to the plates generally (§§ 1–3).

(β) There is, however, a systematic difference between measures made on the S machine and those made on the two R machines, which calls for further investigation (§§ 4–9).

(γ) Analysis of the R.A. circuit shows that the "greatest obscuration" is at $3^h{\cdot}2$, giving

$$\psi_0 = 48^\circ - 3{\cdot}66 \times 65^\circ - 247^\circ = -77^\circ.$$

(δ) The "obscuration" only begins among the faint stars —say after mag. 10 or 11,—as was found for the Cape zones (§ 14).

(ϵ) The following edge of the *unobscured* spiral is the sharper, the distribution in R.A. or ψ being similar to that shown by the Cape zones (§§ 16, 17).

An Observation by Lamont of Professor Barnard's Proper Motion Star. By A. C. D. Crommelin, B.A., D.Sc.

The discovery by Professor Barnard of a faint star with a proper motion of more than 10″ per annum is of such great interest that it seems worth while to point out that the star was almost certainly observed by Lamont at Munich on 1842 July 24. (It is No. 15040 in Munich (1) 1880.) The only reason for the smallest doubt in the matter is that there is another faint star within some 14″ of the position recorded by Lamont. The establishment of identity involves (1) the proof from independent sources that Barnard's star was within a few seconds of arc of Lamont's position, (2) the proof that Barnard's star is at least half a magnitude brighter visually than the other star.

Lamont's observations were made in zones of declination about $1\frac{1}{2}^\circ$ in width, a graduated sector being used whose position relatively to the declination circle could be changed from night to night. As his readings on the night in question show that the sector shifted 13″ between the beginning and the end of the zone, it was necessary to re-reduce the observations to find when the shift occurred. The zone consists of 80 stars, the Barnard star being the eighth. The method adopted was to carry back the places of the Albany A.G. Catalogue to the epoch 1842 with the aid of proper motions supplied to me by Mr. Thackeray, which were deduced from comparison of Albany with Abbadia. I have used the seven stars preceding the Barnard star and seven following. They indicate corrections to Lamont as follows:—

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Star.	B.D. Mag.	Corr. to Lamont. R.A.	Corr. to Lamont. Decl.	Star.	B.D. Mag.	Corr. to Lamont. R.A.	Corr. to Lamont. Decl.
		s	″			s	″
B.D. +3°3493	6·9	−·04	−0·7	B.D. +3°3552	9·2	−·38	−2·7
+4·3517	7·7	−·39	+1·3	3·3562	8·7	−·30	+1·4
+4·3530	9·0	−·30	+1·2	4·3573	8·5	+·46	−0·6
+4·3541	7·0	−·19	+1·2	4·3574	9·2	−·46	+2·8
+4·3542	8·6	+·34	+0·7	4·3602	9·0	−·22	−0·5
+4·3545	8·7	+·17	+2·7	3·3597	8·7	−·24	−1·7
+3·3533	8·1	−·42	−0·7	3·3611	7·9	−·19	−1·0

The mean correction is $-^{s}\!\cdot\!15$, $+0''\!\cdot\!2$; the average discordances are $0^{s}\!\cdot\!20$, $1''\!\cdot\!4$, which are not unexpectedly large considering that transits were observed over only one wire. It is clear that the shift in the sector did not occur till after these stars had been observed. This is confirmed by the fact that Lamont himself bracketed the declinations of all the stars at the end of the zone; apparently it occurred between the 67th and 68th star.

Lamont's place of the Barnard star for 1842·0 is

$$17^{h}\ 50^{m}\ 7^{s}\!\cdot\!09 + 4^{\circ}\ 16'\ 4''\!\cdot\!7.$$

Applying the corrections $-^{s}\!\cdot\!15$, $+0''\!\cdot\!2$ and reducing to the equinox of 1916·0, we obtain

$$17^{h}\ 53^{m}\ 46^{s}\!\cdot\!72 + 4^{\circ}\ 15'\ 12''\!\cdot\!8,\ \text{epoch } 1842\cdot56.$$

Comparing this with the position from Greenwich meridian observations, reduced without proper motion to the equinox of 1916·0, viz. $17^{h}\ 53^{m}\ 43^{s}\!\cdot\!49 + 4^{\circ}\ 27'\ 49''\!\cdot\!8$, epoch 1916·59, we obtain the proper motion $-^{s}\!\cdot\!0436$, $+10''\!\cdot\!225$, or in great circle $10''\!\cdot\!25$ in P.A. $356^{\circ}\!\cdot\!35$. Professor Barnard's value as printed in *Astr. Journ.*, 695, was $10''\!\cdot\!30$ in P.A. $359^{\circ}\!\cdot\!7$, but the latter angle appears to be affected by a clerical error, and should read $356^{\circ}\!\cdot\!7$. This will be seen if the P.M. be re-determined from Professor Barnard's data. As the error is repeated in *Popular Astronomy* for October, it is as well to call attention to it.

As a further check I measured, with the aid of a microscope, the Toulouse Astrographic Chart of 1898 July 25. (The catalogue of this region is not yet to hand.) The distance of the Barnard star from the star that Professor Barnard denotes by *n* was found to be $15''\!\cdot\!9$ in position-angle $104^{\circ}\!\cdot\!5$. I assumed for *n* the position $17^{h}\ 53^{m}\ 43^{s}\!\cdot\!31 + 4^{\circ}\ 24'\ 48''\!\cdot\!4$ (eq. 1916·0), derived from Professor Barnard's micrometer measures, and the Abbadia position of B.D. $+4^{\circ}$, 3560. This gives for the Barnard star on 1898 July 25 $17^{h}\ 53^{m}\ 44^{s}\!\cdot\!34 + 4^{\circ}\ 24'\ 44''\!\cdot\!4$ (eq. 1916·0).

From Lamont, Toulouse, we obtain P.M. $10''\!\cdot\!23$ in P.A. $356^{\circ}\!\cdot\!43$.
From Toulouse, Greenwich, „ $10''\!\cdot\!31$ „ $356^{\circ}\!\cdot\!03$.

Since the spectroscopic results of Professors Adams, Campbell, and Slipher indicate a speed of approach of the order of 100 kms.

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per second, we may reasonably expect an appreciable secular increase of the P.M. owing to diminished distance.*

Having now proved that the Barnard star was within a few seconds of arc of the object observed by Lamont, it remains to consider the question of magnitude. Lamont assigns the magnitude 10 to the Barnard star, but his magnitudes require a correction to reduce to the B.D. scale. I compared his magnitudes of all stars observed on 1842 July 24 with the B.D. magnitudes, with the following result:—

Mag. given by Lamont .	7–8	8	8–9	9	9–10	10
Mean of B.D. mags. of stars	5·7	...	7·3	7·7	8·2	8·7
Number of stars . . .	1	0	2	11	11	54

There is a reasonable presumption that the object observed by Lamont was not fainter than 9·4 or 9·5 on the B.D. scale. He observed scarcely any stars that are not in the B.D.

Professor Bauschinger revised Lamont's zones at Munich about 1886, using the same instrument (aperture $4\frac{1}{3}$ inches), but without the zone sector. In seeking for the star 15040 he naturally did not find it, as it was over 7′ distant from its former place, but he observed a fainter star in the same region on two nights, 1886 July 18 and August 12. (It is No. 6966 in Munich (2) 1880.) I re-reduced his observations of each night in the same way as those of Lamont, and obtained (eq. of 1880·0)—

	Observed R.A.	No. of Wires observed.	Observed Decl.
	h m s		° ′ ″
1886 July 18	17 51 59·50	1	+4 15 52·1
1886 Aug. 12	17 51 59·96	2	4 15 46·0

Giving the second R.A. double weight, we obtain for the equinox of 1916·0, $17^h\ 53^m\ 46^s{\cdot}83 + 4°\ 15'\ 26''{\cdot}7$. Assuming that this star has no P.M. (I have ascertained from the Toulouse chart and from recent measures that its P.M. is small), in 1842 July it was 14″ north of the Barnard star. The two stars must have been very near each other at the end of 1843.

Professor Bauschinger gave the magnitude of 6966 as 11·0 and 10·0 respectively on the two nights. Comparison of his magnitudes of faint stars with those in the B.D. indicate a correction of $-0^m{\cdot}6$ on each night, so that the mean magnitude of 6966 on the B.D. scale is 9·9, exactly the same as the value independently assigned to it by Mr. W. H. Steavenson last July.

Professor Bauschinger noted on each night that 6966 was too faint to observe accurately, although the observing conditions were described as good. It is therefore quite unlikely that Lamont would have selected such a faint star for observation.

* Taking the parallax 0″·50 given by Schlesinger, the annual P.M. would increase 0″·05 per century.

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The Barnard star could be observed comfortably with the Greenwich Transit Circle with illuminated field, so that its magnitude on the B.D. scale can scarcely be fainter than 9·4 or 9·5. Mr. W. H. Steavenson, who kindly observed the region at my request, made it 9·4, and 6966 just half a magnitude fainter. My own experience of magnitude-estimation is small, but so far as it goes it fully confirms these figures.

The photographic magnitude of the Barnard star on the Toulouse chart is given as 10·5 by Mr. Innes, 10·8 by Mr. Melotte, 11·0 by myself. Being of spectral type Mb, its colour index would be nearly 2 magnitudes, so that its visual magnitude cannot be much fainter than 9. Star 6966 appears about $10^m·9$ on the chart, and has probably a smaller colour index than the Barnard star.

The question remains, whether the Barnard star or 6966 is identical with B.D. + 4°·3561. I cannot ascertain exactly when this region was surveyed at Bonn, but it appears to have been either in 1852 or 1853.

In the summer of 1852 the position of the Barnard star (eq. of 1855·0) was

$$17^h\ 50^m\ 45^s·11 + 4°\ 17'\ 36''·3,$$

that of star 6966 was

$$17^h\ 50^m\ 45^s·49 + 4°\ 16'\ \ 8''·0,$$

the B.D. place of + 4°·3561 is

$$17^h\ 50^m\ 42^s·9 + 4°\ 16'·9.$$

It will be seen that the Barnard star is the nearest to the B.D. place; owing to this, and its greater brightness, it would seem to have the greater claim to be considered identical with 4°·3561. But, considering the low power employed in the Durchmusterung, the position may possibly refer to the mean of the two stars, considered as a single entity.

If one desires an old designation for the Barnard star, I submit that the name Munich (1) 15040 may be adopted with tolerable certainty, or B.D. + 4°·3561 with considerable probability.

55 *Ulundi Road, Blackheath, S.E.:*
1916 *November* 6.

[*Note added* 1916 *Dec.* 9.—A further confirmation of the identity of Lamont's star is afforded by the Proper Motion published by M. Gonnessiat in *Comptes Rendus* for 1916 Nov. 27, viz. 10″·286, in P.A. 356°·07, deduced from Astrographic plates taken at Algiers 1897 Aug. 4, 1910 June 15, 1916 Oct. 7. Owing to the large scale of the plates, and the smaller star images, this is more trustworthy than any of the other photographic determinations.]

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Micrometrical Measures of 250 *Double Stars.*
By W. S. Franks.

The following measures of Σ stars were made with the Cooke position micrometer on the 6⅛-inch equatorial at Mr. Frederick J. Hanbury's observatory, Brockhurst, East Grinstead. They form a continuation of the list published in vol. lxxvi., part 1, of the *Monthly Notices*—and the same remarks as there given will also apply. A few objects from OΣ's catalogue, left over from a previous list, are included, their numbers being placed within brackets.

Measures of Double Stars.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
26 *rej.*	0 15·8	+66 27	7·2, 9·5	114·4	13·58	1915·678	2	wh, prpsh.
[OΣ 11 *rej.*]	,, 25·4	31 35	7·8, 7·9	317·4	197·11	·678	2	wh, wh.
46	,, 34·6	20 53	5·0, 8·2	193·1	6·49	·688	2	55 Pisc. ; orge, bl.
47	,, 35·1	23 30	6·7, 8·6	205·0	16·54	·687	2	AB. wh, bl.
60	,, 43·1	57 17	4·0, 7·6	253·3	6·44	·687	3	η Cass. ; p. yel, ruddy purp.
79	,, 54·5	44 13	6·0, 7·0	192·4	7·74	·688	2	wh, grnsh.
127	1 26·8	78 37	8·0, 9·0	186·9	24·20	·715	2	
180	,, 48·0	18 48	4·2, 4·4	359·4	8·04	·728	2	γ Ari. ; v. wh, blsh wh.
188 *rej.*	,, 52·2	62 26	Fr.8·5, 9·5	236·8	33·18	·715	2	p. orge, bl (not in Lewis).
205	,, 57·7	41 51	2·5, 5·5	63·2	10·11	·728	2	γ And. ; gold yel, grnsh bl.
219	2 4·4	32 53	8·2, 9·0	182·6	11·63	1916·014	2	
237	,, 10·4	10 19	8·4, 8·7	238·1	14·35	·029	2	
238	,, 11·2	37 1	8·3, 9·0	357·3	10·92	·029	2	
270	,, 23·8	55 7	7·2, 9·0	302·7	21·18	·029	2	wh, blsh.
281	,, 30·6	5 9	5·0, 9·6	83·1	7·62	·050	2	ν Ceti ; yel, bl.
296	,, 37·3	48 48	4·2, 9·5	302·1	18·00	·050	2	AB. p. yel, two blsh. θ Persei. } Triple.
,,	,, ,,	,, ,,	,, 10·0	227·8	75·44	·050	2	AC. A and D < in both.
318	,, 47·6	37 56	5·2, 9·2	237·6	14·02	·020	2	20 Pers. ; wh, bl.
332	,, 52·7	0 1	8·5, 8·5	53·3	12·24	·029	2	
336	,, 55·4	32 1	6·5, 8·0	8·2	8·53	·011	2	yel, v. bl.
378	3 16·6	58 4	8·2, 9·5	313·0	18·67	·041	2	
392	,, 22·9	52 33	7·5, 9·7	347·2	25·85	·041	2	yel, o.
427	,, 34·5	28 27	6·6, 7·4	208·5	6·70	·085	2	ylsh wh, blsh wh.
430	,, 35·1	4 48	6·0, 9·0	55·4	26·24	·050	2	AB. p. orge, two bl. } Triple
,,	,, ,,	,, ,,	,, 9·8	301·5	37·30	·050	2	AC.

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Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
470	3 49·3	− 3 15	4·0, 6·0	347·8	6·75	1916·085	2	32 Erid.; yel, blsh grn.
471	,, 51·1	+39 43	3·1, 8·3	9·6	8·88	·085	2	ε Pers.; grnsh wh, lilac.
485	,, 59·1	62 5	6·1, 6·2	303·1	17·80	·085	2	ylsh wh, ylsh wh.
523	4 13·8	23 30	7·2, 9·2	162·1	10·36	·097	2	wh, bl.
545	,, 21·3	17 59	7·5, 9·3	57·6	18·99	·099	2	v. wh, bl.
552	,, 24·5	39 46	6·3, 6·5	114·0	8·80	·099	2	wh, ylsh wh.
[OΣ 84]	,, 25·7	6 35	6·8, 7·7	255·2	9·56	·087	2	yel, bl.
[,, 87]	,, 30·8	8 1	7·5, 9·2	234·4	6·04	·097	2	wh, blsh.
570	,, 30·5	− 9 56	7·0, 8·0	259·5	12·85	·102	2	ylsh wh, p. bl.
576	,, 33·4	−13 13	7·0, 8·0	172·2	12·27	·100	2	Σ mgs. 6·7, 7·2; Lewis decl. 13° 4′.
590	,, 38·8	− 8 59	6·2, 6·7	316·9	8·99	·102	2	55 Erid.; yel, blsh wh.
612	,, 48·9	+ 7 13	7·7, 7·8	197·4	16·44	·099	2	wh, wh.
616	,, 52·5	37 44	5·1, 7·7	354·2	6·09	·099	2	ω Aur.; wh, purp.; Σ mgs. 4·0, 7·9.
645	5 3·5	27 54	6·2, 8·2	26·8	11·85	·134	2	wh, lilac.
646	,, 4·1	39 9	8·2, 9·0	74·0	15·76	·134	2	
653	,, 8·9	32 33	5·0, 7·2	225·6	14·64	·134	2	14 Aur.; p. yel, lilac. AB.
668	,, 9·7	− 8 19	1·0, 8·0	201·6	9·66	·102	2	β Ori.; v. wh, bl.
688	,, 14·7	−10 51	7·0, 7·4	274·3	10·43	·102	2	Lewis decl. − 19° 51′ p. yel, blsh wh.
692	,, 15·6	− 8 8	7·8, 8·8	3·8	35·12	·102	2	ylsh wh, blsh.
721	,, 24·3	+ 3 4	7·0, 9·0	149·9	24·70	·134	2	A–BC; wh, prpsh.
[OΣ 109]	,, 30·2	71 35	7·7, 9·0	127·8	10·78	·216	2	ylsh, blsh.
785	,, 39·7	25 53	6·7, 7·7	348·1	14·07	·134	2	wh, bl.
792	,, 41·8	− 3 18	8·2, 8·7	132·6	25·00	·134	2	
817	,, 49·4	+ 7 1	8·2, 8·3	72·7	18·42	·134	2	
869	6 5·9	− 9 50	7·5, 8·5	278·5	24·28	·232	2	wh, blsh.
896	,, 17·8	+51 56	8·3, 8·7	82·1	18·70	·232	2	D>.
897	,, 16·2	26 43	8·2, 8·5	348·0	17·83	·232	2	
914	,, 21·9	− 7 25	6·7, 9·0	297·6	20·92	·232	2	blsh wh, bl.
[OΣ 143]	,, 25·3	+17 1	6·8, 9·9	101·6	8·03	·216	2	ylsh, o.
924	,, 26·0	17 51	6·0, 6·9	210·0	20·03	·216	2	20 Gem.; ylsh wh, grnsh wh.
960	,, 41·6	53 9	7·3, 9·2	66·8	21·75	·243	2	P. VI 215; wh, prpsh.
1010	,, 56·4	− 2 58	7·8, 8·8	5·6	23·11	·232	2	wh, blsh.
1069	7 13·6	−13 31	8·3, 8·3	14·9	25·26	·247	2	A<s?
1082	,, 18·3	+10 54	8·5, 9·0	326·7	19·92	·247	2	Σ 8·0, 8·7—but mgs. as Lewis.
1111	,, 27·0	− 8 28	8·2, 8·7	219·8	20·00	·247	2	

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
[OΣ 173 *rej.*]	7 27·9	+33 20	Fr. 7·0, 9·5	348·5	43·67	1916·247	2	AC. OΣ 7·5, 9·0 wh, bl.
1138	„ 40·9	−14 27	6·2, 7·0	339·5	16·62	·247	2	2 Argûs; wh, lilac.
1141	„ 41·9	+ 0 15	8·0, 8·7	8·9	17·73	·247	2	
1188	8 3·2	30 38	8·0, 8·5	201·5	16·42	·254	2	D<s?
1193	„ 10·0	72 44	6·0, 9·0	85·7	43·47	·254	2	yel, bl. D>s?
1232	„ 26·3	66 38	8·0, 8·2	350·6	31·50	·254	2	
1254	„ 34·6	+20 2	Fr. 6·5, 9·5	53·5	20·10	·254	2	p. or, bl; Lewis 5·7 9·8. P. VIII 29. In *Præsepe.*
1266	„ 38·3	28 49	8·0, 9·2	65·1	23·60	·254	2	A and D<?
[OΣ 195]	„ 48·6	8 48	7·4, 7·9	138·6	9·50	·254	2	P. VIII 200; ylsh wh, blsh wh.
1311	9 1·7	23 21	6·7, 7·0	200·1	7·17	·265	2	wh, wh.
1327	„ 9·6	28 20	8·0, 9·2	73·4	10·84	·265	2	AB. A and D >. } Triple.
„	„ „	„ „	„ 9·1	21·7	26·75	·265	2	AC. A >. D <. } Triple.
1364	„ 26·1	20 28	7·7, 9·2	155·6	16·30	·265	2	wh, blsh.
1416	10 7·5	−15 35	7·4, 8·6	277·4	11·43	·265	2	Σ has 6·7 mg; but overrated.
1427	„ 16·0	+44 25	7·2, 7·7	214·3	9·54	·278	2	wh, blsh wh.
1440	„ 24·4	− 3 23	8·0, 9·5	347·6	15·29	·294	2	
1473	„ 42·7	−15 6	8·0, 8·9	9·9	30·73	·268	2	P. X 159.
1474	„ 42·7	−14 44	6·9, 8·0	23·1	69·55	·265	2	AB. D >; A is moving. All white. } Triple.
„	„ „	„ „	8·0, 8·0	197·5	6·65	·265	2	CB. (not BC, as in Lewis). } Triple.
1509	11 1·5	−12 52	7·2, 9·0	16·1	32·93	·294	2	p. yel, bl.
1529	„ 14·3	− 1 6	7·0, 8·0	251·3	9·89	·278	2	ylsh wh, blsh wh.
[OΣ 239 *rej.*]	„ 39·0	+25 46	5·5, 9·8	23·3	38·07	·294	2	yel, blsh; A<s.
1570	„ 40·3	46 10	8·3, 8·8	48·8	10·70	·308	2	
1595	„ 59·2	7 57	8·3, 9·0	329·7	27·63	·308	2	
1605	12 5·3	− 1 41	8·0, 8·5	278·4	23·44	·308	2	
1608	„ 6·5	+53 59	7·5, 7·7	223·1	11·35	·315	2	wh, wh.
1635	„ 16·0	−10 55	7·7, 8·7	173·4	13·33	·313	2	wh, wh.
1645	„ 23·2	+45 21	7·0, 7·5	160·2	10·27	·315	2	ylsh wh, ylsh wh.
1649	„ 26·5	−10 31	7·5, 7·8	194·3	15·43	·313	2	ylsh wh, blsh wh.
1670	„ 36·6	− 0 54	3·0, 3·0	324·0	6·00	·315	3	γ Virg; both yelsh wh.
1675	„ 39·6	+34 58	8·3, 9·0	7·4	30·52	·321	2	A>s?
1691	„ 50·7	58 42	8·2, 9·0	276·5	18·83	·321	2	
1701	„ 54·2	7 2	7·5, 9·5	305·9	21·53	·321	2	yel, blsh.

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Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
1719	13 2·2	+ 1 8	7·3, 7·8	360·2	7·16	1916·335	2	A>s; ylsh wh, ylsh wh.
1724	,, 4·8	− 5 0	4·0, 9·0	343·2	7·09	·335	2	AB. θ Virg.; wh, prp, o. } Triple.
,,	,, ,,	,, ,,	,, 10·0	299·2	72·36	·335	2	AC. A and D<. } Triple.
1741	,, 19·1	− 1 35	8·2, 9·7	261·9	25·58	·343	2	D<s.
1775	,, 38·3	− 3 46	7·0, 9·7	335·6	27·75	·343	2	P. XIII 171; p. orge, o.
1791	,, 52·0	+14 56	8·7, 9·5	159·9	21·02	·343	2	
1803	14 2·3	38 54	7·7 9·5	42·7	17·88	·370	2	ylsh wh, blsh.
1827	,, 11·1	59 42	8·5, 9·0	209·3	11·28	·370	2	A>s.
1835	,, 18·5	8 53	5·5, 6·8	191·0	6·39	·371	3	P. XIV 69; wh, lilac; A<.
1864	,, 36·0	16 50	4·9, 6·0	103·7	5·91	·371	3	π Boöt.; grnsh wh, prpsh wh; A<.
1899	,, 56·4	− 2 43	7·2, 9·7	67·3	28·25	·370	2	yel, blsh.
1914	15 6·5	− 5 6	8·0, 8·7	335·5	31·12	1915·492	2	
1920	,, 7·5	+47 13	8·5, 8·5	290·8	18·96	1916·376	2	
[OΣ 292 *rej.*]	,, 10·0	32 10	6·5, 8·0	155·6	124·75	1915·492	2	yel, bl; D>?
1936	,, 16·9	27 22	8·5, 9·0	231·9	20·19	1916·373	2	
1939	,, 22·1	−10 37	8·0, 9·0	131·2	8·69	·373	2	D>s?
1948	,, 24·0	+55 14	8·0, 8·7	48·8	12·90	1915·495	2	A>s, D<s.
1977	,, 45·3	25 46	7·7, 9·7	357·2	13·74	·622	2	ylsh, o.
1978	,, 46·3	14 58	8·5, 9·0	235·0	15·27	1916·373	2	
1996	,, 54·4	57 33	8·7, 9·0	109·2	19·70	·376	2	D < s.
[OΣ 305]	16 7·8	33 36	5·8, 9·8	263·4	5·45	·386	2	p. yel, o.
2019 *rej.*	,, 8·8	−10 10	Fr. 8·0, 9·8	154·1	22·33	1915·503	3	ylsh, o.
2033	,, 13·0	− 2 2	8·5, 8·7	174·8	10·64	1916·373	2	
[OΣ 312]	,, 22·6	+61 44	2·1, 8·1	142·1	5·12	·386	2	η Drac.; yel, bl.
2071	,, 34·2	13 52	8·7, 9·0	311·1	24·93	·373	2	
2083	,, 38·1	13 48	8·3, 8·8	334·3	12·56	·373	2	
[OΣ 317]	,, 49·9	44 34	7·2, 8·0	317·5	118·90	1915·492	2	AC. ylsh wh, blsh wh; D<.
2124	,, 58·9	65 21	8·5, 9·2	88·1	15·02	1916·386	2	
2128	17 1·9	59 43	8·0, 9·2	49·6	12·36	1915·505	2	A > D<.
2140	,, 10·1	14 30	3·0, 6·1	114·5	4·86	·608	3	α Herc.; orge, blsh green.
2144 *rej.*	,, 11·9	− 7 45	Fr. 8·0, 9·5	183·2	25·38	·508	2	Lewis decl. 7° 39′; yel, o.
2155	,, 14·9	+60 48	6·2, 9·5	111·1	9·96	1916·386	2	ylsh wh, blsh; A > D<s.
2158	,, 19·1	3 10	8·0, 9·7	78·2	23·31	1915·622	2	
2190	,, 31·7	21 3	6·0, 9·5	23·2	10·10	·606	2	P. XVII 163; wh, prpsh.
2211	,, 41·5	− 1 10	8·2, 9·2	115·8	10·55	·508	2	D<s.

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Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
2216 *rej.*	17 42·1	+ 5 43	Fr.8·0, 9·5	31·3	28·16	1916·508	2	wh, bl (not in Lewis).
2217	,, 42·1	14 49	8·4, 7·8	284·1	6·78	1915·608	3	wh, wh.
[OΣ 336 *rej.*]	,, 44·2	34 19	6·3, 10·3	163·7	42·40	·518	2	p. yel, o.; Hussey decl. 35° 19′.
2227	,, 44·6	5 23	8·8, 8·8	110·7	17·88	1916·508	2	A and D>.
2232	,, 46·2	25 18	7·0, 8·5	142·1	6·50	1915·633	2	wh, blsh.
2234	,, 47·3	− 7 55	8·6, 9·3	198·9	17·23	1916·538	2	D<s.
2247	,, 55·6	+29 30	8·5, 9·0	189·7	11·52	·532	2	
2250	,, 53·9	− 6 51	8·0, 9·0	346·4	7·85	·609	2	
261	,, 55·8	+52 14	7·5, 9·5	261·5	9·48	·532	2	ylsh wh, blsh.
2263	,, 56·9	26 33	8·2, 9·2	160·6	7·47	·609	2	
2265	,, 59·2	6 28	8·4, 9·4	282·2	24·36	1915·508	2	
2291	18 6·6	34 1	8·5, 9·0	159·3	27·20	1916·538	2	D<.
[OΣ 346]	,, 11·1	19 44	7·5, 8·3	328·5	5·60	1915·518	2	ylsh wh, blsh wh.
2301	,, 11·6	23 58	8·5, 9·0	123·6	21·66	1916·544	2	D>.
2313	,, 19·3	− 6 39	7·2, 8·5	198·5	6·09	1915·634	2	wh, blsh.
2333	,, 27·4	+32 10	7·5, 8·1	333·9	6·50	·610	2	wh, blsh wh.
2337	,, 29·2	−14 48	7·8, 8·8	297·3	16·29	·527	2	P. XVIII 116; ylsh wh, bl.
2351	,, 33·0	+41 12	7·4, 7·4	339·9	5·06	·634	2	D>s? wh, wh.
2361	,, 35·6	3 1	8·3, 8·8	211·8	24·97	·537	2	
2371	,, 38·3	27 32	8·5, 8·5	56·2	9·61	1916·609	2	
2378	,, 39·8	35 28	8·3, 8·8	190·0	11·13	1915·541	2	Lewis decl. 35° 7′.
2411	,, 47·7	14 25	7·0, 9·8	95·8	13·36	·622	2	ylsh wh, o.
2413	,, 48·4	3 15	8·2, 8·7	200·3	10·01	1916·544	2	Lewis R.A. $43^m·4$.
2421	,, 52·3	33 39	8·0, 8·7	62·3	23·02	1915·537	2	
2427	,, 54·6	38 5	8·5, 9·0	61·0	49·03	1916·547	2	AB. } Triple.
,,	,, ,,	,, ,,	9·0, 9·2	80·2	7·20	·547	2	BC. } Triple.
2452	,, 56·9	75 39	6·7, 7·5	218·6	5·76	·532	2	wh, lilac.
2459	19 3·2	25 49	8·4, 9·1	232·9	13·67	1915·568	2	
2489	,, 11·9	14 22	6·5, 9·5	347·1	8·23	1916·547	2	wh, bl.
2494 *rej.*	,, 14·6	− 6 49	Fr.7·0, 10·0	82·2	25·68	1915·562	2	=Krüger 1544 (OR). ruddy orge, bl (fine cols.).
2505	,, 16·2	+35 22	8·0, 8·7	315·2	10·20	1916·547	2	
2506 *rej.*	,, 17·2	14 10	8·7, 9·1	351·2	16·74	·570	2	wh, wh.
2508	,, 14·4	67 44	8·7, 9·0	116·7	17·70	·570	2	
2512	,, 18·8	31 32	7·5, 9·8	310·6	22·08	1915·622	2	ylsh wh, o.
2523	,, 22·5	20 57	7·3, 7·4	148·7	6·18	·610	2	wh, wh.
2531	,, 24·5	2 53	7·8, 9·7	30·4	31·82	·601	2	wh, bl.
2548	,, 32·3	24 46	8·0, 9·0	100·2	9·20	1916·570	2	

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Measures of Double Stars—continued.

Σ	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
2585	19 44·5	18 54	+5·7, 8·8	311·7	8·71	1916·578	2	AB–C. ζ Sagittæ; wh, bl.
[OΣ 388]	,, 48·1	25 36	7·6, 8·8	134·7	27·14	1915·518	2	BC. (AB 7·6, 7·6: 3″·97). } Triple.
,,	,, ,,	,, ,,	7·6, ,,	135·8	31·11	·518	2	AC. 2 ylsh wh, 1 bl. } Triple.
2621	,, 59·7	8 57	7·7, 7·9	223·1	5·65	·633	2	wh, wh.
2648	20 7·4	49 31	7·9, 9·2	117·1	6·33	1916·578	2	Lewis 8·2 mag. (B); ylsh wh, blsh.
2655	,, 9·7	21 55	7·5, 7·5	2·8	6·10	1915·610	2	wh, wh.
2667	,, 14·2	45 19	8·2, 8·5	224·3	7·99	1916·578	2	
2669	,, 17·0	55 47	8·7, 9·1	260·2	23·43	·578	2	
2675	,, 12·3	77 25	4·5, 8·0	123·3	7·38	1915·555	3	κ Ceph.; wh, bl.
2683	,, 22·9	−13 30	8·0, 8·5	67·1	22·82	·568	2	
2699	,, 31·4	−13 5	8·0, 9·0	195·1	9·58	1916·601	2	
[OΣ 409]	,, 35·3	+3 5	6·8, 10·3	84·0	17·13	1915·568	2	AB. wh, o. } Triple.
,,	,, ,,	,, ,,	,, 8·0	333·9	65·30	·568	2	AC. ,, bl. } Triple.
2718	,, 37·8	12 22	7·4, 7·6	86·6	8·33	·572	2	wh, wh.
2722	,, 39·1	19 22	8·2, 8·7	306·9	7·18	1916·578	2	
2726	,, 41·5	30 21	4·0, 9·2	65·6	6·04	1915·625	2	52 Cyg.; or yel, bl; A<D>.
[OΣ 413]	,, 43·5	36 7	4·6, 9·0	105·6	84·75	·552	2	λ Cyg. AB–C; ylsh wh, bl; A<D>s.
[,, 416]	,, 48·4	43 22	7·8, 8·1	130·9	7·67	·572	2	wh, wh. A>D<s.
2759	21 2·3	32 3	8·5, 9·5	326·6	17·05	1916·585	2	A and D<.
2760	,, 2·7	33 44	7·3, 8·1	228·7	5·20	1915·634	2	A<D>; wh, blsh.
2763	,, 4·8	16 56	8·5, 9·7	292·9	17·50	1916·585	2	D<.
2789	,, 16·8	52 33	7·1, 7·1	116·3	6·00	1915·625	2	wh, wh.
3112	,, 29·6	9 3	7·6, 9·4	239·2	7·14	·612	2	=OΣ 528; ylsh wh, o.
2813	,, 32·9	57 2	8·5, 9·0	272·9	10·35	1916·585	2	
[OΣ 443]	,, 32·6	6 16	8·0, 8·3	349·1	8·16	1915·612	2	wh, wh.
[,, 447]	,, 35·6	41 16	7·0, 7·9	45·2	29·01	·616	2	yel, ylsh.
2816	,, 35·8	57 2	6·3, 7·9	120·4	11·66	·616	2	AC. P. XXI 248. } Triple.
,,	,, ,,	,, ,,	,, 8·0	339·5	19·95	·616	2	AD. ylsh wh, 2 blsh. } Triple.
2817	,, 37·3	−0 2	8·2, 8·5	156·3	25·84	·616	2	
2828	,, 44·5	+2 56	8·0, 9·0	142·3	28·24	·616	2	AB. } Triple.
,,	,, ,,	,, ,,	,, 9·2	133·3	27·85	·616	2	AC. } Triple.
[OΣ 454]	,, 51·5	23 52	7·0, 9·0	279·2	7·13	·616	2	wh, blsh.
[,, 455]	,, 51·9	15 39	7·5, 9·0	269·3	9·88	·616	2	ylsh, blsh.
2851	,, 56·3	−12 28	8·0, 8·3	121·3	18·32	·628	2	
2855	22 0·2	−1 55	7·9, 9·5	299·9	26·93	·628	2	wh, o.
2860	,, 0·0	+60 21	7·7, 9·3	256·0	7·56	1916·585	2	D<; yel, bl.

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Measures of Double Stars—continued.

Σ.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nts.	Remarks.
[OΣ 461]	22 1·0	+59 23	5·9, 10·6	298·3	11·18	1916·585	2	AB. 15 Ceph. } Triple.
,,	,, ,,	,, ,,	,, 9·5	39·3	89·51	·585	2	AC. yel, o, o.
2861	,, 1·4	20 18	7·7, 8·2	220·1	6·99	1915·631	2	wh, wh.
2863	,, 0·9	64 8	4·7, 6·5	281·1	7·18	·620	2	ξ Ceph.; ylsh wh, ruddy lilac.
2877	,, 9·5	16 42	6·4, 9·6	7·2	12·97	·620	2	P. XXII 33; p. or, grnsh.
2891	,, 12·4	47 29	8·2, 9·2	309·2	13·00	·620	2	
2902	,, 19·3	44 51	7·1, 8·0	89·2	6·37	·625	2	yel, grnsh.
2908	,, 23·3	16 45	7·0, 8·7	115·1	8·88	·625	2	p. yel, blsh.
2913	,, 25·2	− 8 38	Fr. 7·7, 8·7	329·9	8·16	1916·777	2	Σ mags. 7·0, 8·0.
2922	,, 31·4	+39 7	6·0, 6·5	185·7	22·38	·590	2	AB. 8 Lacertæ; ylsh wh, blsh wh. } Quad.
,,	,, ,,	,, ,,	6·5, 10·2	155·3	28·06	·590	2	BC. blsh wh, o.
,,	,, ,,	,, ,,	,, 8·5	131·4	66·72	·590	2	BD. ,, bl.
2923	,, 30·4	69 51	6·9, 9·2	46·7	9·33	1915·620	2	creamy wh, prpsh.
2926	,, 33·0	38 23	8·5, 8·5	335·8	21·11	1916·590	2	
2930	,, 34·4	6 38	8·3, 9·3	77·2	21·42	·595	2	
2932	,, 36·8	29 30	8·7, 9·2	281·3	20·44	·595	2	D<s.
2938	,, 39·9	− 3 11	8·2, 8·2	162·8	19·47	·777	2	
[OΣ 529]	,, 41·7	+67 36	8·5, 9·2	203·5	3·58	·590	2	AB. ylsh wh, blsh. } Triple.
,,	,, ,,	,, ,,	,, 9·0	219·2	20·68	·590	2	AC. ylsh wh. blsh.
2952 *rej.*	,, 49·4	27 29	7·5, 10·0	135·7	18·24	1915·624	2	ylsh wh, o.
2953	,, 48·7	60 24	6·9, 9·4	137·9	8·40	1916·590	2	yel, blue.
2954	,, 49·8	14 40	9·0, 9·0	23·8	39·65	·601	2	A>D<.
[OΣ 536]	,, 53·5	8 50	7·0, 10·0 (Fr.)	84·1	238·31	1915·665	2	AB–C; yel, o.; A and D>.
2978	23 2·7	32 17	6·8, 8·0	145·3	8·43	·625	2	P. XXII 306; wh, p. bl.
2991 *rej.*	,, 8·4	10 31	6·0, 10·0	358·2	33·03	·631	2	yel, o. (not in Lewis).
2993	,, 8·8	− 9 28	7·0, 7·8	177·2	25·57	·665	2	AB. ylsh wh, wh. } Triple.
,,	,, ,,	,, ,,	,, (Fr.) 8·5	116·6	120·96	·665	2	AC. ylsh wh, blsh wh.
2997	,, 12·1	+20 53	8·5, 9·0	222·4	24·39	1916·794	2	
3004	,, 16·0	43 35	6·5, 10·0	178·1	13·07	·794	2	wh, o.
3006	,, 16·4	34 54	8·5, 9·0	161·0	5·67	·801	2	A>.
3007	,, 17·8	20 1	6·5, 9·5	82·8	6·28	·801	2	wh, o.
3009	,, 19·2	3 11	6·8, 8·8	228·1	7·02	1915·647	2	orge yel, bl.
3010	,, 18·7	45 15	8·0, 8·7	132·1	25·76	·640	2	
3021	,, 26·3	15 40	7·7, 8·9	306·3	8·45	·647	2	ylsh wh, blsh.

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Measures of Double Stars—continued.

Σ.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nts.	Remarks.
[OΣ 501]	23 35·1	+37 6	6·8, 10·2	164·3	14·99	1916·794	2	p. yel, o.
[,, 504]	,, 37·4	18 7	7·2, 10·0	175·3	7·70	·794	2	yel, blsh.
3039	,, 41·8	27 52	7·3, 9·7	32·4	33·80	1915·647	2	yel, o.
[OΣ 509]	,, 45·5	42 52	7·6, 9·5	106·1	5·50	1916·801	2	wh, o.
3051	,, 57·6	79 43	7·5, 9·4	23·4	16·72	1915·640	2	ylsh wh, o.
[OΣ 514]	,, 59·5	41 32	6·9, 9·5	168·5	5·31	1916·801	2	wh, o.
[OΣ 547]	0 0·4	45 16	Fr.8·3, 8·5	141·2	4·70	·801	2	both p. orge; A<; mgs. underrated.
3061	,, 0·6	17 17	8·0, 8·0	147·5	7·66	·801	2	
Hh 503	16 14·3	−19 19	7·5, 8·0	332·6	47·05	·501	2	P. XVI 44–5; ylsh wh, blsh wh.
,, 504	,, 18·8	−19 53	8·0, 9·0	22·3	12·72	·501	2	P. XVI 48–9; wh, greyish.
,, 505	,, 15·1	−25 21	3·1, 8·0	271·6	20·34	·501	2	σ Scorpii; grnsh wh, prpsh.
,, 507	,, 18·4	−29 29	5·9, 6·6	355·6	7·00	·501	2	p. yel, blue (very pretty).
,, 522	,, 38·7	+ 6 48	7·5, 8·0	291·1	54·00	·480	2	ylsh wh, blue.
,, 523	,, 41·0	8 46	5·4, 8·0	230·3	81·34	·480	2	43 Herc.; p. orge, blue.
,, 562	18 9·4	28 14	7·7, 7·9	136·7	56·50	·508	2	wh, wh; D<(β 54·9: 1880).
Sh 263	,, 11·1	−18 49	7·0, 8·5	12·8	54·39	·598	2	wh, blue.
,, 264	,, 12·8	−18 40	7·0, 7·7	52·3	17·18	·598	2	ylsh wh, lilac.
Hh 605	19 10·0	+38 52	8·0, 8·3	56·7	39·34	·566	2	wh, wh.
,, 608	,, 12·9	37 57	4·5, 9·0	71·2	100·24	·516	2	θ Lyræ; yel, blue.
,, 611	,, 15·8	−19 25	6·4, 9·0	309·2	90·53	·516	2	P XIX 67; ylsh wh, o.
"Cel. Obj."	,, 33·5	−17 8	7·0, 7·5	237·5	10·60	·516	2	wh, wh; *sp* 54 Sagittarii.
Hh 630	,, 35·0	+16 23	7·0, 9·0	303·0	28·18	·550	2	orge, blue (v. fine cols).
"Cel. Obj."	,, 38·2	1 0	8·5, 9·0	18·4	27·95	·556	2	wh, blsh.
S 738	20 6·1	33 20	7·1, 7·7	109·7	41·60	·582	2	wh, wh.
,, 752	,, 25·7	19 5	7·0, 7·2	288·5	106·10	·582	2	ylsh wh, blsh wh.
"Cel. Obj."	,, 29·0	28 31	8·0, 9·0	339·7	33·96	·578	2	AB. wh, blsh. } Triple.
,,	,, ,,	,, ,,	,, 9·5	223·6	36·90	·578	2	AC. ,, blsh. } Triple.
Hh 706	,, 42·6	−18 34	7·0, 7·1	293·6	15·85	·570	2	wh, wh.
,, 753	22 8·8	−21 35	5·5, 8·0	113·6	5·27	·777	2	AB. A> D<; orge yel, blue. 41 Aquarii. } Triple.
,, ,,	,, ,,	,, ,,	,, 9·0	43·5	211·81	·777	2	AC. orge yel, blue. } Triple.
,, 762	,, 21·2	−17 15	6·4, 6·6	312·3	6·96	·777	2	53 Aquarii; both ylsh wh; A<D>.
,, 807	23 40·8	−19 14	5·5, 6·5	137·8	6·18	·787	2	107 Aquarii; ylsh wh, lilac; A>D<.

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Notes on some doubtful Hh Objects.

Hh 491. Is pretty certainly identical with Σ 1999 (Hh 492), with an error of 14′ in decl.

550. Not found; so Innes (*Transvaal Obs. Circular*, No. 1). But it bears a suspicious resemblance to Hh 552, which is 1° 14′ to the south. Query identical!

612. There is little doubt that this is Σ 2531, with 2° error in R.A. It is the *f* of two stars (not the *p*, as J. H. has it) about 20′ apart, forming a right angle with a wider pair about 25′ *n*.

615. Probably identical with Σ 2533; no other corresponding object near the place.

659. Not 64 Sagittarii. But it may be identical with Σ 2625 (P. XIX 396) which is *sf* 65 Sagittarii? A similar mistake occurred with 64 Draconis (Hh 664), afterwards corrected to 65.

670. Not found. In a rich part of the Milky Way; the vague description might apply to several stars.

672. Strong probability that this is identical with Hh 671 (P. XX 12), 42′ *n* of this place.

715. Burnham identifies this with β 269; which seems unlikely, as it is much too unequal and difficult for one of H's close pairs. It is more probably identical with Σ 2742 (λ Equulei), which fits it better, and is quite as near to the place given.

Mr. Frederick J. Hanbury's Observatory:
Brockhurst, East Grinstead.

Sir W. Herschel's Double Star Measures. By W. S. Franks.

When going through the Synoptic Catalogue of H's double stars (vol. xxxv., *Memoirs*) recently, in connection with my working list, I was struck with the pretty constant difference between H's distance measures and my own (the latter being almost invariably in excess) of certain *wide* double stars. I then consulted Burnham's measures of some of these objects, and was astonished to find the same peculiarity—his average deviation being even greater than mine. Pursuing the matter further, I made a comparison between Herschel and Struve, taking all measured objects common to both from the *Mensuræ Micrometricæ* (Lewis's revision and Lord Lindsay's summary)—except those of H's Class I., which are more like estimates than measures. The result was a confirmation of the previously found discrepancy, as will be seen by the following epitome:—

I. *Comparison between* H *and* Σ.

H's Class.	Limiting Distance.	No. of Stars compared.	Mean Distance.	Mean Difference.	Percentage.
	″ ″		″	″	
II.	4 to 8	18	5·6	−0·27	−4·79
III.	8 ,, 16	52	11·2	−0·20	−1·79
IV.	16 ,, 32	60	21·1	−1·07	−5·08
V.	32 ,, 60	15	41·7	−3·56	−8·54

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II. *Comparison between H and β.*

H's Class.	Limiting Distance.	No. of Stars compared.	Mean Distance.	Mean Difference.	Percentage.
	" "		"	"	
IV.	16 to 32	11	21·8	−1·11	−5·09
V.	32 ,, 60	33	44·7	−3·78	−8·46
VI.	above 60	19	106·8	−9·01	−8·43

III. *Comparison between H and* Fr.

IV.	16 to 32	6	22·8	−0·95	−4·17
V.	32 ,, 60	27	42·0	−3·35	−7·93
VI.	above 60	21	78·3	−4·85	−6·20

IV. *Combined Results.*

						H *minus*
IV.	16 to 32	77	21·9	−1·04	−4·75	Σ, β, and Fr.
V.	32 ,, 60	75	42·8	−3·56	−8·31	do.
VI.	above 60	40	92·5	−6·93	−7·31	β and Fr.

No duplicates are included in the three lists, which are composed of entirely different stars. Any objects showing definite evidence of motion, as in Σ's Appendix II., were rejected. It may be mentioned that there are complete measures for only about half the stars in the Hh list; or, excluding duplicates in different classes, about 370 out of 812.

It will be noticed that H's distance measures show the smallest error for Class III., and increase in nearly equal ratio from that on either side. Probably this was because those doubles were easier and more certain to measure than either the closer or wider ones. Considering, however, the very rough and crude appliances at Herschel's command, and the absence of a driving-clock, the error is no more than might have been expected. On the whole, his distance measures require a *plus* correction of 5 to 8½ per cent., from Class IV. outward, to make them comparable with those by other observers. It seems necessary to emphasise this point, as I have come across some instances in which H's measures of wide double stars were quoted in conjunction with quite modern ones, with about a century's interval, and the difference deduced as proper motion—an assumption which is not justified by any means. Despite the temptation to utilise what, in many cases, are the earliest recorded measures, it is certainly not safe to depend upon them as evidence of motion, unless corroborated by subsequent observations.

Mr. Frederick J. Hanbury's Observatory:
Brockhurst, East Grinstead.

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B.D. Stars observed as Missing or Faint. By Rev. T. E. Espin, M.A.

In the course of the systematic examination of the stars of the B.D. for new pairs, some have been found to be altogether missing, others were so faint that it seems almost impossible that they could have been observed by Argelander unless they were at that time brighter than they are now. Where a star is missing, this may arise from (1) an error in the original observation, (2) variation, (3) proper motion. As far as I am aware, none of the stars in this list are known variables.

Column 1 is a current number; columns 2, 3, 4, 5 give the number, place, and magnitude in the B.D.; column 6, the observed magnitude, if seen; the symbol < is employed to indicate the faintest star that would be seen at the time; column 7, the date of observation; column 8, the apparent magnitude on the Harvard map of the sky (section, Decl. +45°). In the notes will be found additional observations, and any stars near. Lists of other missing B.D. stars are given in Chandler's "Third Catalogue of Variable Stars," and in Harvard Circular No. 128.

No.	B.D.	R.A. 1855.	Decl. 1855.	Mag.	Obs.	Date.	H.M.
		h m s	° ′				
1	+45° 92	0 18 49·6	+45 23·8	9·5	12	14 Jan. 6	11
2	+43° 120	29 32·0	43 59·3	9·5	...	15 Jan. 16	
3	+47° 192	38 2·4	47 11·6	9·2	...	13 Nov. 17	
4	+49° 327	1 5 1·1	49 34·2	9·5	12	12 Jan. 5	
5	+48° 441	19 18·0	48 26·5	9·5	<13·5	13 Nov. 5	10
6	+47° 439	22 52·4	47 24·2	9·2	9·5	13 Dec. 1	10·5
7	+49° 525	52 7·1	49 33·5	9·5	...	12 Jan. 27	
8	+44° 542	2 27 52·8	44 51·7	9·5	<14	14 Nov. 26	
9	+45° 639	28 20·3	45 54·3	9·4	13	13 Dec. 23	?
10	+44° 551	30 20·6	44 57·1	9·4	...	14 Jan. 6	
11	+43° 632	57 9·6	43 9·3	9·5	12·5	15 Dec. 8	
12	+48° 869	3 4 40·4	48 30·5	9·5	<12	12 Jan. 27	?
13	+44° 799	38 50·1	44 47·9	9·3	...	15 Jan. 16	
14	+49° 1052	43 2·3	49 45·5	9·5	<13	11 Dec. 20	
15	+50° 968	4 6 59·0	50 56·6	9·5	...	11 Jan. 20	
16	+43° 1105	42 58·4	43 45·3	9·5	13·5	15 Feb. 29	
17	+44° 1216	5 19 22·8	44 11·6	9·5	<13·5	16 Feb. 7	
18	+50° 1344	6 31 58·7	50 42·2	9·5	13	1 Jan. 13	?
19	+42° 1656	55 48·1	42 6·0	9·5	...	16 Mar. 31	
20	+42° 1683	7 4 54·2	42 5·8	9·4	...	15 Apr. 7	
21	+46° 1267	16 24·6	46 28·2	9·5	...	13 Apr. 4	
22	+42° 1776	43 36·6	42 49·8	9·5	...	15 Mar. 30	?
23	+41° 1778	53 58·0	41 20·6	9·5	12	16 Apr. 2	10
24	+46° 1365	8 1 14·3	46 49·5	9·3	...	12 Mar. 28	

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No.	B.D.	R.A. 1855. h m s	Decl. 1855. ° ′	Mag.	Obs.	Date.	H.M.
25	+44° 1727	8 5 22·5	+44 44·2	9·5	...	13 Apr. 4	11
26	+46° 1408	20 14·5	46 53·8	9·3	. 11·8	12 Mar. 28	11
27	+43° 1850	35 28·1	43 59·9	9·5	13	13 Apr. 5	11
28	+41° 1894	49 35·1	41 44·9	9·5	13·5	16 Apr. 4	
29	+46° 1465	52 45·6	46 16·9	9·3	<12	12 Mar. 28	
30	+42° 1966	57 14·3	42 11·2	9·5	12·5	16 Apr. 1	
31	+41° 1939	9 6 51·5	41 33·9	9·5	...	16 Apr. 2	
32	+43° 1939	29 12·9	43 21·4	9·5	...	13 Mar. 25	
33	+43° 1946	30 44·3	43 6·8	9·5	<14	13 Apr. 5	
34	+42° 2037	32 26·9	42 14·6	9·4	...	16 Apr. 1	
35	+46° 1553	40 50·5	46 28·2	9·3	...	12 Mar. 20	
36	+53° 1379	52 24·3	53 34·1	9·5	...	09 Apr. 7	?
37	+46° 1585	58 38·9	46 50·6	9·5	11·5	12 Apr. 10	11
38	+46° 1597	10 2 54·8	46 52·5	9·5	...	12 Apr. 2	
39	+40° 2302	4 19·8	40 45·2	9·5	...	16 Apr. 4	
40	+43° 2010	13 39·5	43 3·0	9·5	...	15 Mar. 31	
41	+41° 2083	15 50·6	41 34·4	9·5	...	15 May 2	
42	+45° 1858	35 19·3	45 37·1	9·5	<13	12 Apr. 10	
43	+41° 2149	52 7·4	41 16·0	9·5	...	15 Apr. 29	
44	+45° 1893	58 31·4	45 9·8	9·5	<13·5	12 Apr. 11	
45	+43° 2120	11 21 32·4	43 3·0	9·5	...	13 Apr. 19	
46	+40° 2522	12 11 23·7	40 50·0	9·5	...	15 May 12	
47	+43° 2268	45 55·7	43 8·8	9·5	13	13 May 2	
48	+43° 2305	13 4 39·2	43 20·5	9·5	...	13 May 18	
49	+44° 2267	14 51·9	44 43·0	9·3	...	12 May 13	
50	+42° 2404	23 27·5	42 48·7	9·5	12	13 May 24	
51	+40° 2707	49 46·7	40 12·9	9·5	...	16 May 3	?
52	+42° 2460	56 24·9	42 10·8	9·5	...	13 May 25	
53	+43° 2546	15 53 31·2	43 26·7	9·5	...	16 May 26	?
54	+41° 2736	16 35 48·7	41 21·5	9·5	...	16 July 27	
55	+42° 2861	17 25 11·1	42 7·4	9·5	...	16 July 27	
56	+43° 2745	26 12·5	43 49·2	9·5	<13·5	15 Sept. 10	
57	+41° 2862	31 57·0	41 55·0	9·5	...	16 July 28	
58	+40° 3183	33 1·5	40 40·7	9·5	...	16 Aug. 8	
59	+43° 2845	51 46·5	43 9·1	9·1	...	15 Sept. 10	
60	+41° 3166	18 47 23·6	41 54·5	9·5	...	16 Aug. 2	
61	+42° 3228	57 26·9	42 6·2	9·5	11·5	16 July 28	
62	+41° 3230	19 1 4·0	41 54·5	9·5	...	16 Sept. 24	
63	+43° 3180	7 54·3	43 44·3	9·5	13	15 Aug. 26	10
64	+43° 3218	15 52·6	43 18·6	9·5	...	15 Sept. 10	
65	+41° 3348	22 4·9	41 34·0	9·5	<12	16 Oct. 26	

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No.	B.D.	R.A. 1855. h m s	Decl. 1855. ° ′	Mag.	Obs.	Date.	H.M.
66	+43° 3493	20 2 35·3	+43 40·9	9·5	...	15 Aug. 25	?
67	+48° 3072	11 38·3	48 37·7	9·5	...	11 Sept. 16	
68	+42° 3742	20 52·0	42 27·3	9·5	...	16 Sept. 1	
69	+45° 3251	38 45·2	45 3·4	9·5	...	12 Oct. 14	
70	+42° 3991	21 4 49·3	42 55·1	9·5	<14	15 Sept. 9	
71	+45° 3707	46 25·6	45 43·4	9·3	13	13 Nov. 5	11
72	+47° 3606	49 29·0	47 1·2	9·5	...	12 Oct. 14	
73	+41° 4340	53 52·2	41 4·5	9·5	...	16 Sept. 23	
74	+42° 4378	22 17 47·0	42 1·7	9·5	11	16 Sept. 5	11
75	+46° 3811	38 33·5	46 4·8	9·5	13	13 Nov. 3	
76	+46° 3867	47 12·5	46 18·3	9·5	...	13 Oct. 21	
77	+49° 4032	58 27·3	49 18·8	9·5	13	12 Oct. 4	?
78	+45° 4193	23 12 24·0	45 58·2	9·1	9·5	13 Oct. 9	9·5
79	+45° 4202	14 48·5	45 59·6	9·5	12	13 Oct. 9	11
80	+46° 4067	24 8·9	46 2·3	9·5	13·5	13 Oct. 9	
81	+45° 4266	27 26·9	45 56·8	9·5	13·5	13 Oct. 21	?

Notes.

2. A star 2′ N.
3. Dec. 31. No star here.
8. 15 Jan. 16. No star.
10. A star 2′ N.
12. H.C. No. 128 missing. The star is apparently visible on H.M.
13. A star $1^m f$. (This is corrected in Argelander, vol. vi. p. 379.)
16. 16 Feb. 2, 13·5.
20. A star at +42° 8′.
21. A star 2′ S.
23. 16 April 4, 10·8. Var. ?
24. A star 1′–2′ N.
25. 13 April 5, 12.
27. 14 March 12, 13.
31. 16 April 4. Not seen.
32. 13 April 5. Not seen.
34. A star about 20 sec. *p*.
36. 10 Feb. 11. Faint star seen. 11 March 9, 10 mag. Var.
38. A star 2′ N.
43. A star 2′ S.
44. Misplaced in chart where R.A. is 1^m too great.
45. 13 May 2, no star above 13·5. 15 March 31, not seen. 16 April 4, not seen.
49. A star 1′ N, faintly seen in H.M.
50. Several small stars near this place.
52. 15 April 29. Not seen.
54. In H.M. a star some 2′ N.
55. B.D. +42°·2856, 42·4 sec. *p*, 1′·3 S. is a wide pair 9·3, 9·3.
57. The double star Espin 636 is 4·4 sec. *f*, 7′ S. of this place.
58. A star 4′ N.
59. A star 2′ S.
60. A star 30 sec. *p*.
63. 15 Sept. 2, 12·5.
70. A star 5′ S.
73. A star 2′ N.
76. A star 2′ N.
78. 13 Nov. 7, 9·6 Argelander's scale.
79. 13 Nov. 7, 12.

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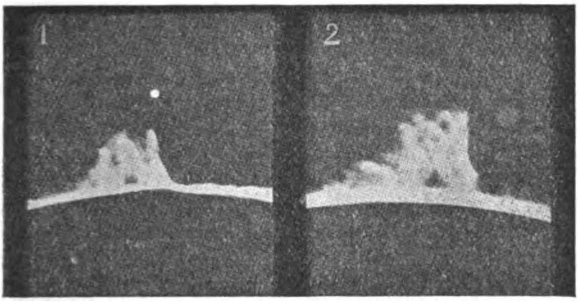
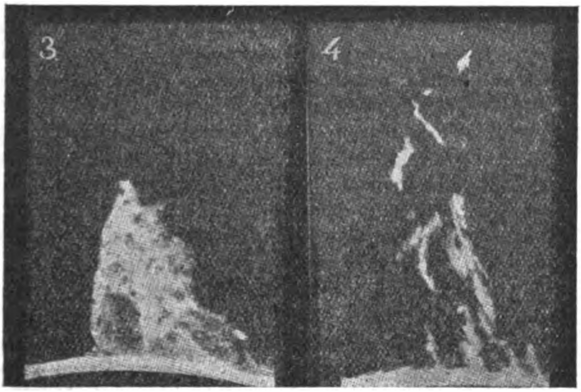
Note on the Uprush of Gases in the Solar Prominences. By the Hon. R. J. Strutt, F.R.S.

(Communicated by the Secretaries.)

The great velocities with which the gases shoot up radially from the level of the chromosphere have long attracted attention.



$14^h\ 4^m$ $16^h\ 27^m$



$17^h\ 0^m$ $17^h\ 40^m$

Prominence Photographs by Deslandres, 1894 May 31. The little white circle in the first figure represents the dimensions of the earth on the scale employed.

These uprushes of gas have apparently been considered analogous to the similar though infinitely less rapid movements in our own atmosphere due to convection. It has perhaps been somewhat vaguely supposed that the enormous velocity is quite in keeping with the huge scale and high temperature of solar phenomena generally.

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The object of the present note is to point out what seems to be an insuperable difficulty in this view.

For purposes of discussion I take as a concrete example the photographs published by Deslandres (*Annuaire du Bureau des Longitudes*, 1907, p. 87) of a polar prominence taken on 1894 May 31. They are here reproduced for convenience of reference.

It appears from these photographs that during the interval from $14^h\ 04^m$ to $16^h\ 27^m$ the mean upward velocity of the highest part of the prominence was $4{\cdot}5 \times 10^6$ cm. per sec. But during the succeeding interval from $16^h\ 27^m$ to $17^h\ 0^m$ the velocity of the highest part had increased to $8{\cdot}34 \times 10^7$. Thus practically the whole of the latter velocity was acquired during this interval. It is important to notice that before this velocity was acquired the top of the prominence was already at a height above the general level of the chromosphere equal to seven times the earth's diameter. It appears certain therefore that this velocity was acquired by the gas not within the sun, but *after emerging into the sun's atmosphere.* This circumstance materially simplifies the discussion. If the velocity were acquired mechanically inside the sun's mass, as for instance by a huge bubble of hydrogen rising through denser material, it would be very difficult to reason quantitatively about what value the velocity might be expected to have.

I wish now to discuss whether such velocities as are observed can be attributed to convection currents, including in this term any motion of the gas under a pressure gradient, whether of thermal origin or not.

Two cases may be distinguished—convection in the form of continuous circulation, and convection in one defined upward direction, starting from rest.

First, as regards continuous circulation: this occurs under conditions analogous to those of a domestic hot-water circuit, an upward current of hot gas, and a descending current of cold gas. A circuit of this kind is analogous to a heat engine. Heat is taken in at a high temperature, and a less amount of heat is rejected at a lower temperature, the difference representing the mechanical work done in setting the gas in motion. There is no easily calculable limit to the velocity which may be acquired in a circulation of this kind. Such a limit is only set by frictional resistance encountered by the moving fluid against stationary surroundings. Apart from this, there is no reason why the velocity should not increase indefinitely, something being added to it each time the gas ascends.

Whatever the importance of motion of this kind may be at the lower levels of the sun's atmosphere, the evidence of the photographs clearly justifies us in assuming that it is not concerned in the case of the prominences. What we have to do with is *one* upward ascent, the gas starting from rest, without any kinetic energy carried on from a previous horizontal motion.

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Although convection may occur in the open (as in the earth's atmosphere), yet it acts to greater advantage in setting a gas in motion if the latter is confined in a wide pipe or chimney, open at the bottom. If therefore we imagine a chimney to be placed over the prominence so as to prevent horizontal movements of the gas, we shall not make the conditions less favourable for a convective uprush than they would be in the absence of a chimney, but more so. At the same time the conditions are made definite and calculable.

Let us suppose such a chimney to be in position. Then the upward motion of the hydrogen requires a greater pressure at the bottom than at the top. As regards the top, let the most favourable supposition be made, namely, that the pressure is zero. As regards the bottom, let the pressure be p_0 dynes per sq. cm.

It might appear at first sight that the greater the value of p_0 the more rapid would be the ultimate motion; but, as is well known, for adiabatic expansion this notion is illusory.

We suppose the gas at the bottom to be at a temperature of 6600° centigrade or 6900° absolute, and to rush up into a vacuum above. What velocity will it attain in doing so?

The question may be approached conveniently by considering how much energy is available to set the gas in motion.

Suppose that 1 gram of hydrogen enters the bottom of the chimney. The work done upon it by the external atmosphere in pushing it in will be equal to

$$p_0 v_0,$$

the product of the initial pressure and volume, under the conditions prevailing outside the base of the chimney.

But we have

$$p_0 v_0 = \mathrm{RT}_0,$$

when T_0 is the absolute temperature outside, and R is the gas constant.

During this initial part of the process the gas may be regarded as behaving like an incompressible fluid such as water, and it acquires velocity under the action of the superincumbent pressure, just as water does in issuing out of a small side-hole in the bottom of a vessel.

Having entered the chimney, which has a vacuum maintained at the top, the gas will rush upwards with increasing velocity. Energy of motion is derived from the expansion of the gas.

Let us suppose this expansion in the first instance to be adiabatic. Then, if p and v are the pressure and corresponding volume at any stage, and γ the ratio of specific heats,

$$pv^{\gamma} = p_0 v_0^{\gamma} = p_0 v_0 \,.\, v_0^{\gamma-1} = \mathrm{RT}_0 \,.\, v_0^{\gamma-1}.$$

Thus

$$p = \mathrm{RT}_0 v_0^{\gamma-1} \,.\, v^{-\gamma}$$

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and the work to be got by unlimited expansion from the initial volume v_0

$$= \int_{v_0}^{\infty} p dv = \int_{v_0}^{\infty} RT_0 v_0^{\gamma-1} \cdot v^{-\gamma} dv = \frac{RT_0}{\gamma - 1}.$$

To this must be added the work RT_0 done on the gas in entering the chimney, and we get as the total energy available to produce motion

$$RT_0\left(1 + \frac{1}{\gamma - 1}\right)$$

or

$$\frac{RT_0\gamma}{\gamma - 1}.$$

If V is the velocity generated,

$$\tfrac{1}{2}V^2 = \frac{RT_0\gamma}{\gamma - 1}$$

$$V = \sqrt{\frac{2RT_0\gamma}{\gamma - 1}}.$$

Taking

R as 4.158×10^7
γ „ 1.408
T_0 „ 6.900

V comes out as 1.4×10^6 cm. per sec.

This then is the greatest velocity that could be generated by adiabatic expansion of hydrogen initially at the temperature of the sun, even if the process were carried out to an infinite volume. Yet it is only $\frac{1}{80}$th part of the velocity of the prominence. Although molecular considerations are in no way essential to the argument, it is worth while to remark on what is occurring from a molecular point of view in the case supposed. The gas enters the chimney with a certain velocity, derived from the work done upon it by the external atmosphere. It then expands, and the energy of the molecular motions, both of translation and of rotation, is converted into velocity of the gas as a whole in one direction. If the whole energy were translational as in a monatomic gas like helium, then the ultimate velocity of the gas would be equal to the molecular velocity of mean square at the initial temperature. If, as has been supposed above,* hydrogen remains diatomic at the temperature of the sun, it will acquire a somewhat larger velocity, for the initial energy of rotation is added.

It appears therefore that the greatest velocity which can possibly result from adiabatic expansion of the gas is very small compared with the velocities observed.

It may, however, be objected with reason that the expansion will not be adiabatic. Heat may be supposed to be conducted

* In taking the value $\gamma = 1.408$.

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through the walls of the imaginary chimney, and in this case the energy of molecular agitation which the gas gives up in the process of expansion will be restored to it. Let us take the case when this restoration is complete, so as to make the expansion isothermal, the gas remaining at solar temperatures throughout instead of being cooled by expansion. As before, the work done in expansion is

$$\int_{v_0}^{v_1} pdv,$$

where v_0 and v_1 are the initial and final volumes.

In the present case pv is constant, and equal to RT_0, when R is the gas constant and T_0 the absolute temperature.

Thus the work done in expansion is

$$RT_0 \int_{v_0}^{v_1} \frac{1}{v} dv$$

or

$$RT_0 \log \frac{v_2}{v_1}.$$

To this must be added as before the work RT_0 done upon the gas by the external atmosphere.

Thus if V is the ultimate velocity

$$\tfrac{1}{2}V^2 = RT_0 + RT_0 \log \frac{v_2}{v_1}$$

$$= RT_0\left(1 + \log \frac{v_2}{v_1}\right)$$

$$V = \sqrt{\left\{ 2RT_0\left(1 + \log \frac{v_2}{v_1}\right) \right\}}.$$

This expression can be made to exceed any assignable value if the expansion $\frac{v_2}{v_1}$ is made sufficiently great, and it might appear from this circumstance that there could be no difficulty in accounting for the observed velocities of the uprising gas by assuming a large enough expansion. But when the actual figures are considered the matter wears a very different aspect.

First, the photographs enable us to set a close limit on the expansion that can have actually occurred. (Compare stage 3 with stage 2.) The mean heights of the prominence in these two stages are nearly in the same ratio as the maximum heights—about 2 . 3 : 1 : The base remains about the same in each. Thus the volumes are also in the ratio 2 . 3 to 1, and if we suppose that no fresh gas has come up, this is the mean expansion.*

By assuming that all the expansion has occurred in the higher

* If fresh gas *has* come up, the mean expansion of the original gas will of course be still less.

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portions of the prominence, we may get a rather larger figure. But I do not think that the most violent straining of the probabilities will allow us to assume that more than a tenfold expansion has occurred in these upper portions, particularly if the evidence of the subsequent photograph, No. 4, is taken into account.

Calculating the velocity obtainable by a tenfold isothermal expansion, we find

$$V = 1{\cdot}37 \times 10^6 \text{ cm. per sec.},$$

which is only $\frac{1}{60}$th part of the observed upward velocity of the top of the prominence.

Again, it is instructive to consider what isothermal expansion would be required if the observed velocity were to be reached in this way.

In this case we get, for a velocity of $8{\cdot}34 \times 10^7$,

$$\log_e \frac{v_1}{v_0} = 1{\cdot}21 \times 10^4,$$

which gives

$$\frac{v_1}{v_0} = 10^{5250} \text{ roughly.}$$

Such an expansion of the whole prominence, or of any appreciable fraction of it, would make it fill a space far in excess of the estimated volume of the stellar universe.

It seems therefore quite impossible to attribute the observed velocities of the hydrogen to the same kind of forces which produce the movements in our own atmosphere.

What view, then, is to be taken of these movements?

Can it be supposed that it is not the gas itself, but the *luminous condition*, which spreads upwards?

This suggestion is decisively negatived by the observations which have been made on the Doppler effect, when the motion has a component towards the observer. This effect can only occur when the material emitting centre is moving towards the observer.

Again, can we find any escape in supposing that the moving centres of emission are not hydrogen atoms, but something much lighter?

Such a suggestion would be utterly at variance with our experimental knowledge of the circumstances under which the hydrogen spectrum is emitted, and with all our theoretical notions of the origin of line spectra.

It seems much more feasible to suppose—and the notion has been contemplated before,* though I do not know that it has ever been strongly pressed—that the masses of glowing hydrogen

* See, for instance, Deslandres, *C.R.*, clv. p. 1579, 1912, "Les protubérances sont alors simplement un amas des ions lumineux qui sont rejetées au dehors par les forces *aérodynamiques ou électriques.*"

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are urged upwards by electrostatic forces. There seems to be little extravagance in such an hypothesis, for Hale's observations on the Zeeman effect in sun-spots may be said to compel the assumption of strongly electrified masses of gas in rotation.

Summary.

The motion of gas in the solar prominence is discussed from a theoretical point of view, and it is concluded to be impossible to regard this motion as due to convection. The reasons for this view are—

1. The photographs show clearly that no continuous circulation of the gas is in question. We cannot therefore regard the velocity as gradually built up in successive ascents.

2. The velocities are enormously greater than can be accounted for by any adiabatic expansion at solar temperature, however great.

3. If the expansion is regarded as isothermal, the observed velocities could only be accounted for by assuming a ratio of expansion enormously exceeding what observation will admit.

Note.—Since writing the above I have found that the impossibility of explaining the motion as resulting from adiabatic expansion has previously been insisted upon by Pringsheim, who has also suggested the alternative of electrostatic force (*Physik der Sonne*, p. 225, Teubner, Leipzig, 1910). I think, however, from remarks in the subsequent literature, and from what I have been able to learn in conversation, that the above discussion may not be without usefulness in directing attention to the subject.

The Total Solar Eclipse of 1916 *February* 3.

By C. D. Perrine, Assoc. R.A.S.

This eclipse was observed at Tucacas, Venezuela, by the expedition from this observatory in charge of Third Astronomer Chaudet.

Owing largely to the financial difficulties resulting from the great European war it was not possible to send the 40-foot camera nor the 11-foot spectrographs.

Following was the programme of observations :—

A. Photographs of the corona with cameras of 11 feet and 6 feet focal length.

B. Photographs of the flash spectrum with prismatic cameras of 6 feet and 27 inches focal length.

C. Photographs of the general spectrum of the corona with a one-prism slit spectrograph.

D. Photographic determination of the total amount of light from the corona.

E. Visual observations of the times of contacts.

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It was impossible to obtain fresh orthochromatic plates as well as some moderately slow plates for the extreme inner corona and photometric work.

Standard intensity squares were impressed on the long exposure for the 11-foot camera and upon the four plates for the photometer, before leaving Cordoba. They were then sealed up in tin along with the other plates for use on eclipse day, and not opened until the night before. The plates, which had been standardised before leaving Cordoba, were somewhat mottled, due, it is believed, to the exposure to the atmosphere two months before development.

The expedition left Cordoba on 1915 December 2 and arrived in Tucacas on January 14.

Although the weather for the first week was unfavourable, Mr. Chaudet succeeded in getting all of the six instruments mounted and adjusted. Assistance for taking the photographs was freely offered from among the officials and residents of Tucacas, and an efficient corps of observers was trained for a week before the eclipse.

The morning of the eclipse was unpromising, with heavy rain. Conditions improved, however, to such an extent that, during totality, the sky was covered only by haze. This interfered slightly with the coronal extensions, the general spectrum of the corona, and the photometric observations.

The entire programme was carried through, and 20 plates with 28 exposures obtained.

The foci of the 11-foot camera and of the large prismatic camera were excellent. There were some very small vibrations to the 11-foot camera which disturbed the short exposures, particularly the images of the prominences, to some extent. They were not sufficient, however, to interfere greatly with the longer exposures.

The images with the 6-foot camera and the small prismatic camera are not sharp. The latter appears not to have been in good focus, owing to lack of opportunity to focus properly the instrument before the eclipse—clouds preventing the completion of a focus plate on the stars. This instrument had not been focussed before leaving Cordoba.

Diffusion by the clouds may have caused a part of this appearance in the case of the 6-foot camera, but it is feared that some disturbance of the focus took place, although no such disturbance was noted after the eclipse, nor can any be accounted for. The focal image was examined visually shortly before totality, and was then excellent. The general features of the corona and the streamers are well enough shown for some purposes, as are also the prominences.

In the small prismatic camera negatives the general forms of the images are easily distinguishable, and many of the lines well enough separated for measurement on their sharp edges.

The negatives of the corona with both cameras show streamers to a distance of one and a half solar diameters.

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The corona is of the intermediate type, resembling that of 1898 more closely than any other recent one.

Five groups of prominences are shown at the bases of the four principal wings of the corona. A series of well-marked hoods is shown over the prominences in the S.W. quadrant. One of comparatively rare, long, very slender and nearly radial streamers is shown in the N.E. quadrant. This streamer is composed of two distinct branches near the limb.

The chromospheric layer was of considerable depth.

Both flashes were obtained with the prismatic cameras; and three exposures on the corona were made (instead of a single one during all of totality) on account of the great light-gathering power of these instruments.

No trace of gaseous radiation in the corona proper has been found. The principal hydrogen lines can be seen in the prominences in the exposures at mid-totality.

The exposures with the large prismatic camera were made on flat plates. The focus from the extreme red end (λ 7300) to about H is good. Above this it is not sharp, although many lines are visible, and as the edges are generally sharp it may be possible to measure them with some accuracy owing to the large scale. The spectrum is recorded to λ 3300, where it is cut off by the edge of the field.

Bright lines are well shown to about λ 5300. Below this the sensitiveness of the plate (Lumière Violet Label) falls off considerably, and at the extreme lower end only the D_3 and H_α lines are visible. These are both strong, and the forms of some prominences are plainly discernible in them. Continuous spectrum extends to λ 7300 or below.

About 40 lines can be distinguished in the flash spectrum above λ 3600.

Probably the greatest value these observations will have is the heights of the different vapours in comparison with similar results from other eclipses.

The spectrum with the slit spectrograph is faint, and does not extend beyond the extreme inner corona, 3′ from the moon's limb on the east and not more than 1′ on the west.* The slit was placed east and west across the sun's diameter. The spectrum appears to be perfectly continuous on both sides, without a trace of either dark or bright lines, except for small bright beads on the linear continuous spectrum on the west at the H and K positions.

The photometric plates present some interesting results—chief of which appear to be that the prominences and chromospheric layer at the beginning of totality emitted more light than the corona proper. It may be possible to determine the total amount of light from some of the groups of prominences separately, as distinct rings can with some difficulty be traced on the negative taken near the end of totality.

* The exposure was stopped intentionally 20 seconds before the end of totality.

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The second and third contacts were well observed by Mr. Chaudet as follows:—

1916 February 3, Tucacas Mean Time.

	Observed.			Computed.		
	h	m	s	h	m	s
Beginning of totality	11	23	21 a.m.	11	23	21 a.m.
End " "	11	25	51 "	11	25	53 "
Duration . .		2	30		2	32

These agree very closely with the times obtained by the party from the Caracas Observatory. The computed times are given for comparison, the data being taken from the American Ephemeris and the correction of -10^s given by Ross in *Astronomical Journal*, No. 681, having been applied.

It will be seen that the beginning of totality agreed within the second, and the end differed but 2^s from these predictions.

The position of Tucacas obtained by a commission from the Caracas Observatory is as follows:—

Latitude . . .	10° 47′·5 North.
Longitude . .	68 19·2 West of Greenwich.

Observatorio Nacional Argentino,
Cordoba:
1916 *June* 26.

The Rotation Period of the Great S. Tropical Disturbance on Jupiter. By Frank Sargent.

The Great S. Tropical Disturbance on the planet Jupiter has shown a curious variation in its periods of rotation during recent years.

In a general way these have been lengthening. The object being subject to considerable changes in length, it naturally follows that the periods of its ends will likewise be subject to corresponding fluctuations. The purpose of this note is to direct attention to a peculiar oscillation in the values for these ends.

A very usual method of tabulating the various periods of Jovian features is to evaluate them as from "opposition" to "opposition." But observations are only possible for a portion of the time, and during any such observing season the rotation periods derived may, and frequently do, vary from those derived from "opposition" to "opposition."

By using positions observed at the close of an "opposition," and others obtained at the beginning of the next, it is also possible to derive values for the time of the planet's invisibility.

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Recently it has been noted that the rotation periods derived for the ends of the S. Tropical Disturbance have been longer during the time of its visibility than during the time when Jupiter has been lost in the Sun's rays.

This has been consistently so during the last three years (beyond which my personal records do not go with sufficient amplitude to justify their use respecting this matter), and recent observations indicate that the peculiarity is being maintained during the present opposition.

During this time the S. Tropical Disturbance has varied considerably in length, but the changes have not succeeded in breaking down the sequence of these up-and-down movements in the rotation periods of its ends.

In support of the foregoing the following data are submitted, consisting of certain dates, with corresponding positions of longitude for the ends of the S. Tropical Disturbance believed to be well established, and the rotation periods derived for the intervals between succeeding dates.

The terms "opposition" and "conjunction" are used to cover the intervals during which these events take place.

S. T. D. Rotation Periods.

Preceding End:—

From	To	Period. h m s	♃ in
1913 June 14 λ_2=210	1913 Sept. 26 λ_2=184	9 55 29·3	Opposition.
1913 Sept. 26 ,, 184	1914 July 11 ,, 80·7	,, 25·9	Conjunction.
1914 July 11 ,, 80·7	1914 Nov. 7 ,, 47·3	,, 29·1	Opposition.
1914 Nov. 7 ,, 47·3	1915 June 8 ,, 339·6	,, 27·6	Conjunction.
1915 June 8 ,, 339·6	1916 Feb. 5 ,, 268·8	,, 28·6	Opposition.
1916 Feb. 5 ,, 268·8	1916 July 19 ,, 206·4	,, 25·2	Conjunction.
1916 July 19 ,, 206·4	1916 Oct. 16 ,, 178·5	,, 27·7	Opposition.

Following End:—

From	To	Period. h m s	♃ in
1913 June 27 λ_2=346	1913 Sept. 15 λ_2=320	9 55 27·6	Opposition.
1913 Sept. 15 ,, 320			*
1914 Aug. 5 ,, 185·1	1914 Nov. 17 ,, 153·5	,, 28·1	Opposition.
1914 Nov. 17 ,, 153·5	1915 Aug. 13 ,, 52·7	,, 25·4	Conjunction.
1915 Aug. 13 ,, 52·7	1916 Jan. 24 ,, 13·7	,, 30·9	Opposition.
1916 Jan. 24 ,, 13·7	1916 July 22 ,, 317·6	,, 27·8	Conjunction.
1916 July 22 ,, 317·6	1916 Oct. 15 ,, 298·3	,, 31·3	Opposition.

It will be noted that the variations are of such magnitude that observational errors sufficient to remove them seem impossible.

* (When Jupiter came under observation in May 1914 the F.E. of the S. Tropical Disturbance was involved with the Red Spot Hollow, with a result that it was not possible to observe its position accurately, therefore a rotation period for the preceding months cannot be given. On Aug. 5 an excellent observation re-established the position.)

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Respecting this I desire to point out that observations of the "Red Spot Hollow" during the same time show no such oscillation, and a prominent object on the S. Temperate Belt recently re-observed from last "opposition" yields a rotation period for the interval precisely the same as that derived during 1915.

These consistent results from features contiguous to the zone in which the S. Tropical Disturbance is situated seem to negative the possibility of observational error sufficient to account for the curious fluctuations recorded.

The search for a physical explanation has not been successful. The relative positions of the Sun, Earth, Jupiter system seem clearly involved.

Gravitational effects may be immediately dismissed, in the first place, because of their minuteness as far as the Earth is concerned; and secondly, because in view of their proximity the

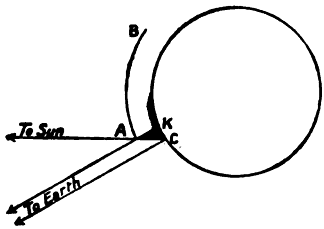

Fig. 1.

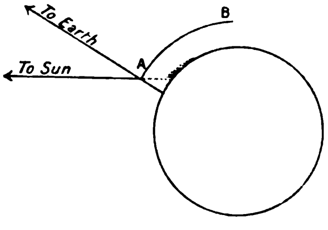

Fig. 2.

satellites of Jupiter are in this direction a much more potent factor.

The phenomenon has been considered on the assumption that the S. Tropical Disturbance is of a cloud formation, capable of casting a shadow on to the light zone which it overlays.

During the months previous to the date of "opposition" the shadow cast by the P.E. of the S. Tropical Disturbance will be in view of an observer on the earth, whilst after "opposition" the shadow of the F.E. will be in a position to be seen. This being so, it is highly probable that the S. Tropical Disturbance being itself a dark feature, will merge itself in its shadow as seen from the Earth, and when the shadow is in a position to be seen, it may be timed by the observer, instead of the actual material of the S. Tropical Disturbance.

The effect of this may be inferred after consideration of figs. 1 and 2.

In fig. 1 the positions shown are as at "quadrature" preceding "opposition."

Jupiter is represented by a circle, the S. Tropical Disturbance by the arc AB: A being the P.E. The shadow CK will be seen from the Earth, resulting in the time being taken early, the actual amount being taken from the "conjunction" rotation periods, and eventually distributed over the "opposition" periods,

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or more correctly, over the periods from the time of the observation until the time of true "opposition," when the shadow will cease to be visible from the Earth.

From the date of "opposition" onwards the shadow will not be visible from the Earth, being overlaid by the S. Tropical Disturbance itself, as may be seen from fig. 2.

For the F.E. the process will be reversed, the shadow being invisible until after the date of opposition, when it will appear as an extension of the S. Tropical Disturbance, resulting in late timing of transits, and a corresponding lengthening of the rotation periods.

As a question of theory this may be accepted, but as a matter of actual observation the magnitude of the shadow effect is very small indeed, probably not large enough to influence observations at all, unless we assume a height for the alleged "cloud" for which we have no warrant. The suggestion is of interest only because it agrees in kind and direction with the motion necessary to explain the observations.

Further investigation seems desirable, is in fact proceeding, and, aided by earlier observations kindly supplied by the Rev. T. E. R. Phillips, M.A., and Messrs. W. F. Denning and Stanley Williams, it is hoped to carry the inquiry back into the earlier history of the object under consideration.

16 *Cottrell Road, Eastville,*
Bristol:
1916 *November* 1.

The Meteor System of Pons-Winnecke's Comet.

By Charles P. Olivier, M.A., Ph.D.

Before discussing the question of the connection between the 1916 May-June meteors and the Pons-Winnecke Comet, it is of interest to review briefly the history of this body.

It was first discovered by Pons on 1819 June 12, and though its period was computed at that time to be only 5·62 years, it escaped observation until 1858 March 8, when it was rediscovered by Winnecke at Bonn. It has since reappeared in 1869, 1875, 1886, 1892, 1898, 1909, and 1915.

This comet belongs to Jupiter's family, and at different times in its history has approached very near that planet. The strong resulting perturbations have greatly changed its orbit. Fortunately, the progression of the changes is very easy to follow, thanks to several papers on the subject. It will be sufficient to mention an article by Winnecke in *A.N.*, No. 1160, which gives the 1858 orbit; one by von Haerdtl in *A.N.*, No. 2873, which gives orbits for 1858, 1869, 1875, and 1886; and, lastly, one by Hillebrand in *A.N.*, No. 4330, for the 1909 return. Unfortunately, no orbit

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seems to have been computed from the few observations made in 1915. On this last return it was discovered in April, when of the 16th magnitude, and at its closest approach to the earth was more than a unit distant. At its brightest it was of the 12th magnitude and 20″ in diameter, as observed at the Union Observatory, South Africa.

The three orbits for 1819, 1858, and 1909 follow, so that the changes in the elements may be readily seen:

Year.	log *a*.	log *e*.	log *q*.	*i*.	π.	Ω.	U.
				° ′	° ′	° ′	y
1819	0·4997	9·8781	9·8885	10 43	274 41	113 11	5·62
1858	0·4965	9·8778	9·8859	10 48	275 39	113 32	5·56
1909	0·5135	9·8462	9·9879	18 17	271 37	99 21	5·892

If all the orbits between 1858 and 1909 were to be included in the table, they would show progressive changes in all the elements from 1858 on. However, as these two mentioned show their directions clearly, it is thought unnecessary to copy in the others.

The element of greatest interest to us is the perihelion distance. It will be seen at once that this has increased from 0·77 in 1858 to 0·97 in 1909, and doubtless has continued to increase since. This increase in distance has made possible our meeting with the meteors which belong to the comet's system.

Some rough calculations show that on June 28·5 the comet's (1909) orbit was about three million miles from the earth, while on May 27 it was nearly five times as distant. That we should meet outlying meteors even at this great distance from the parent comet's orbit has already been observed in the cases of the Perseids and the Eta Aquarids (the Halley Comet meteors).

The observations of these meteors for testing their connection with the Pons-Winnecke Comet are quite numerous, thanks to the energy of some of the observers who are members of the American Meteor Society. Altogether, in the interval 1916 May 20 to July 10, which seems about the extreme limits advisable to use in the discussion, five observers, working in all 47 nights, sent in 1219 observations of meteors. Also in *Nature* of July 10 two English observers, working on June 28, reported, one seeing a large number of meteors, the other finding a radiant between ζ and η Ursæ Majoris, with a rate of 100 ± meteors per hour. Later, in *The Observatory* for August, Mr. W. F. Denning gives his own observations, with a rate of 32 per hour and the radiant being diffused, having two principal centres. In *Nature* of July 17 Mr. Denning says that the English observations show some resemblance in their elements to those of Pons-Winnecke's Comet. Absolutely no mention of this is made in his more complete article in *The Observatory* of August. In the September number of that journal, however, he says that there is a significant resemblance. All English observers reported that meteors were visible in extraordinary numbers on June 28. Unfortunately, only one of the

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American observers, Mr. Donald Brooks of Washington, D.C., worked on June 28. His rate was only 5·5 per hour. This need, however, excite no surprise, because it has long been known that the richest portions of meteor streams, even the Leonids, are traversed in a comparatively few hours, hence the six hours which elapsed between the English and American observations were quite sufficient to allow the earth to pass out of the dense portion.

From our data seven radiants are clearly shown on the maps of Mr. John Koep and Philip Trudelle, of Chippewa Falls, Wis., between May 21 and June 4 inclusive, which certainly belong to this system, and one radiant observed by Mr. Raymond Lambert of Newark, N.J., on July 3, which almost certainly does also.

Using the eight radiants determined in America, the radiant at $\frac{1}{2}(\zeta + \eta)$ Ursæ Majoris by the observer at Birmingham, and Mr. W. F. Denning's two principal centres, orbits have been computed for all eleven positions. They were first computed on the usual assumption of parabolic velocity. When this had proved that the meteors followed orbits similar to that of the comet, elliptical orbits were next computed for each, assuming that their major axes were equal to that of the comet. This produced so marked an improvement in the agreement of the elements with those of the comet that no reasonable doubt could remain, and the connection may be considered fully proved.

The two tables of elements follow:—

Parabolic Elements.

Date. G.M.T. 1916.	No.	α.	δ.	ι.	log q.	ω.	π.	Ω	Observer.	Nos.	Notes.
		°	°			°	°	°			
May 21·6	1	224·5	+25·3	20·2	9·954	219·0	279·7	60·7	J.K.	8	4 on May 19, 4 on May 23.
May 26·68	2	230·3	+27·4	22·4	9·958	217·9	283·4	65·5	P.T.	7+	
May 27·25	3	231·0	+27·5	22·5	9·958	218·1	284·2	66·1	J.K.	7	4 on May 26, 3 on May 27.
May 27·67	4	232·1	+26·8	22·8	9·954	218·9	285·4	66·5	P.T.	4+	
May 30·14	5	232·7	+28	22·1	9·978	209·0	277·9	68·8	J.K.	15	8 on May 29, 7 on May 30.
June 3·7	6	234·4	+27·5	22·2	9·965	215·1	288·4	73·2	J.K.	16	June 2, 3, 4.
June 4·68	7	235·8	+25·6	22·0	9·961	216·9	291·0	74·2	P.T.	9	
June 28·5	8	203	+53	20·9	0·005	171·4	268·3	96·9	B.	100∓	
June 28·5	9	231	+54	27·3	0·007	182·9	279·8	96·9	W.F.D.	?	} 69 seen.
June 28·5	10	223	+41	19·7	0·006	186·8	283·7	96·9	W.F.D.	?	
July 3·67	11	206·7	+61·2	26·0	0·001	165·8	267·6	101·8	R.L.	5	

Note.—The observers were :—J.K. = John Koep.
P.T. = Philip Trudelle.
B. = Observer at Birmingham.
W.F.D. = W. F. Denning.
R.L. = Raymond Lambert.

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Elliptical Elements.

No.	log *a*.	log *e*.	log *q*.	ι.	π.	ω.	☊.
1	0·514	9·857	9·962	16°·7	280°·1	219°·4	60°·7
2	,,	9·865	9·948	18·6	287·6	222·1	65·5
3	,,	9·854	9·970	17·6	282·6	216·1	66·1
4	,,	9·856	9·964	19·0	286·6	220·1	66·5
5	,,	9·848	9·983	16·6	278·0	209·2	68·8
6	,,	9·852	9·974	17·6	287·6	214·3	73·2
7	,,	9·855	9·967	18·2	291·6	217·4	74·2
8	,,	9·839	0·005	17·1	2·683	171·4	96·9
9	,,	9·840	0·002	22·9	303·0	206·3	96·9
10	,,	9·839	0·005	16·1	285·2	188·3	96·9
11	,,	9·857	9·962	22·6	263·7	161·8	101·8
Comet (1909)	0·514	9·846	9·988	18·3	271·6	172·3	99·3
Comet (1858)	0·496	9·878	9·886	10·8	275·6	162·1	113·5

It will be seen that, contrary to our rule, the mean positions from more than one night's work have been combined in Nos. 1, 3, 5, and 6. This was due to the fact that Mr. Koep used four maps in all, and it was impossible to detect appreciable deviations in radiant points for the meteors shown on each map when taken for each night separately, and then as combined into the adopted means. It goes without saying that the remarkably exact progression shown in the first seven positions is partly due to good fortune. Nevertheless, all the radiants were well determined and quite sharply defined on the maps.

If it be objected that the node positions differ considerably for the earlier dates, it must be remembered that the ☊ for a meteor stream is simply the longitude of the earth on that date, and such differences only inform us that the meteors are met at a considerable distance from the parent comet's orbit. Hence rather considerable disagreements in ☊ may mean little or nothing provided the *a*, *e*, *q*, and *π* are accordant. Again, we must remember the comet passed perihelion in 1915 September, and therefore was an enormous distance ahead of these meteors. Therefore the perturbations to which it and these groups had been of late years subjected were by no means the same, and some differences of elements are to be fully anticipated.

If the perihelion distance of the comet's orbit, and hence inferentially those of the meteors, continues to increase as rapidly as in recent years, it seems that this interesting system of meteors may be carried beyond the earth's orbit, and hence can be seen for only a few years more at most. It was never observed before 1916, so far as I have been able to find, and, as has been explained,

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unless the changes in the elements of the orbits shortly reverse their direction, it cannot long be visible to us. It is therefore of very real importance to make every effort to secure observations in 1917 May and June, and probably for the next few years. It is believed that this exact cycle of changes just described offers a new case in meteoric-cometary astronomy, paralleled partially only by that of Biela's Comet.

The way in which the connection was detected here was that partial calculations, in June, showed that there was strong probability of the meteors belonging to the Pons-Winnecke Comet. Not until further data came in was I willing to publish anything on the subject, however, though the results were shown to the members of our observatory staff at once. Other data coming later in June made the connection certain, but not until the report from England was seen in July, giving a new and distant radiant by the observer at Birmingham, for which an orbit was calculated at once, did I think it necessary to publish the conclusion immediately, since I desired first to read a paper on the subject before the American Astronomical Society at its meeting late in August. The preliminary results were then sent to Professor E. C. Pickering on August 3, and kindly announced by him in Harvard Circular No. 614. About four days after sending this on a later copy of *Nature* arrived here from England, stating that Mr. Denning had arrived at possibly similar conclusions, though no orbits were given.

While of course the working up of the American observations and all the calculations are my own, yet I wish to give all possible credit to the two young men mentioned, through whose observations, made with the greatest energy and enthusiasm, this proof was first made possible. It is of interest to say that during the first seven months of 1916 Mr. Koep sent in 1429 observations, and Mr. Trudelle 908. This in itself is quite sufficient guarantee of their ability.

Leander McCormick Observatory,
University of Virginia,
Virginia, U.S.A.:
1916 *Sept.* 14.

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Observations of the Perseids on 1916 *August* 10 *and* 11, *made at the temporary University Observatory, Rostow-on-Don.* By Professor S. D. Tscherny.

August 10.

No.	Rostow-on-Don Mean Time of Appearance.			The Point of Appearance.		The Point of Disappearance.		Remarks.
				$\alpha_{1855\cdot0}$	$\delta_{1855\cdot0}$	$\alpha_{1855\cdot0}$	$\delta_{1855\cdot0}$	
	h	m	s	°	°	°	°	
1	13	26	2	44·0	+53·0	73·0	+57·0	White.
2	13	38	7	44·7	+40·3	32·1	+37·6	White; streak.
3	13	40	32	28·7	+41·5	11·0	+42·7	White; streak.
4	13	47	37	81·1	+43·8	80·1	+39·0	White.
5	13	52		72·9	+43·7	77·3	+40·3	White.
6	13	59	13	91·6	+59·0	107·7	+55·4	White.
7	14	10	57	87·2	+44·8	100·0	+53·6	White; trained.
8	14	18	44	57·0	+39·5	60·0	+39·8	}
9	14	24	45	87·4	+37·2	91·6	+33·0	}
10	14	33	44	12·2	+59·9	341·4	+65·4	} White.
11	14	38	13	58·9	+49·9	62·0	+47·0	}
12	14	50	22	72·6	+60·2	91·0	+57·4	}
13	14	56	39	60·0	+62·0	68·2	+65·8	}
14	15	8	46	80·8	+32·1	80·0	+28·5	White; streak.
15	15	30	18	36·4	+56·1	39·2	+49·7	White.
16	15	34	59	68·4	+42·8	74·2	+41·5	Meteor of second mag.; white.

August 11.

No.	h	m	s	α	δ	α	δ	Remarks.
17	12	52	46	0·6	+58·4	347·2	+53·8	Meteor of second mag.; white; streak.
18	12	55	7	5·9	+62·5	357·2	+67·1	Meteor of second mag.; white; streak.
19	13	3	55	226·8	+83·3	222·5	+74·8	Meteor of first mag.; white; streak.
20	13	22	43	15·1	+54·0	5·8	+46·4	
21	13	48	57	44·8	+49·0	57·6	+56·0	Meteor of third mag.; white; streak.
22	13	52	18	41·0	+52·2	38·9	+42·9	Meteor of fourth mag.; white; streak.
23	13	57	11	43·8	+52·9	44·0	+51·6	}
24	14	13	49	53·5	+21·6	50·9	+16·5	} White.
25	14	44	53	48·5	+49·3	48·7	+46·1	}
26	14	50	49	49·7	+50·3	67·0	+58·3	White; streak.
27	15	2	10	51·0	+46·9	52·4	+43·5	}
28	15	21	38	43·8	+56·1	46·0	+52·9	} White.
29	15	27	12	54·0	+41·5	54·0	+36·1	}

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I have drawn the apparent paths of preceding meteors on the meteor-chart in gnomonical projection, and by the method of least squares I have obtained the three following apparent radiants:—

(1) The radiant of the meteors Nos. 1, 5, 6, 9, 12, 13, 15:

$$\alpha_{1855 \cdot 0} = 42^\circ \cdot 2, \quad \delta_{1855 \cdot 0} = +55^\circ \cdot 6,$$
$$\alpha_{1916 \cdot 0} = 43^\circ \cdot 3, \quad \delta_{1916 \cdot 0} = +55^\circ \cdot 9.$$

(2) The radiant of the meteors Nos. 2, 3, 8, 10, 11, 16:

$$\alpha_{1855 \cdot 0} = 61^\circ \cdot 8, \quad \delta_{1855 \cdot 0} = +44^\circ \cdot 3,$$
$$\alpha_{1916 \cdot 0} = 62^\circ \cdot 9, \quad \delta_{1916 \cdot 0} = +44^\circ \cdot 5.$$

(3) The radiant of the meteors Nos. 21, 22, 23, 25, 26, 27, 28, 29:

$$\alpha_{1855 \cdot 0} = 49^\circ \cdot 0, \quad \delta_{1855 \cdot 0} = +50^\circ \cdot 3,$$
$$\alpha_{1916 \cdot 0} = 50^\circ \cdot 1, \quad \delta_{1916 \cdot 0} = +50^\circ \cdot 5.$$

The meteors Nos. 4, 7, 14, 17, 18, 19, 20, 24 have not a definite radiant.

Near the constellation of *Perseus* were seen on August 10 forty-three meteors from $11^h\ 33^m$ till $15^h\ 43^m$, and on August 11 seventy-one meteors from $12^h\ 43^m$ till $15^h\ 47^m$ Rostow-on-Don mean time. The moon was near full, and set on August 10 about 13^h and on August 11 about $14^h\ 30^m$ Rostow-on-Don mean time.

Position of the temporary University Observatory (the centre of the platform on sixth floor of the University building): longitude, $2^h\ 38^m\ 49^s \cdot 52$ E. (Greenwich); latitude, $47^\circ\ 13'\ 9'' \cdot 4$ N.

University Observatory, Rostow-on-Don:
1916 *August* 27.

On a Graphical Construction for obtaining the Wave-lengths in Prismatic Spectra. By F. Henroteau, D. ès Sc.

It has been established that in the case of a prismatic dispersion the Hartmann formula

$$\lambda = \lambda_0 + \frac{c}{R_0 - R} \quad . \quad . \quad . \quad . \quad (1)$$

(where λ is the wave-length, R the micrometer reading, and λ_0, c, and R_0 computed constants) gives good results for the values of λ. As a rule, then, a small correction only has to be used for obtaining the real value of λ. In astrophysical observatories this correction is obtained from a curve in which the abscissæ are the wave-lengths and the ordinates are the corrections, several corrections being deduced from known lines. The computation of λ by the

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Hartmann formula, although simple, is rather a laborious operation, and a graphical method might be advantageous.

The graphical method given below is very simple, each wave-length being obtained by the tracing of only one straight line.

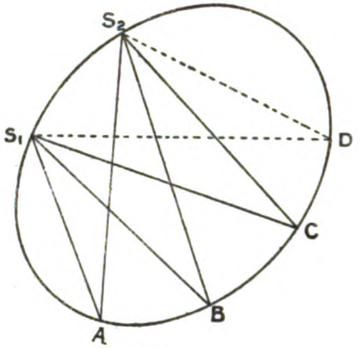

Fig. 1.

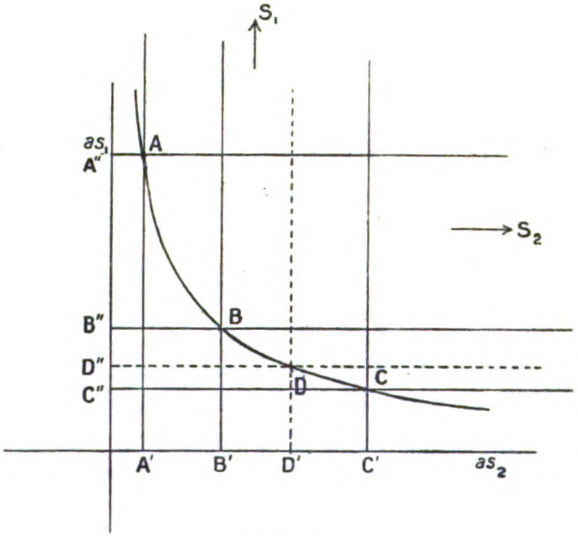

Fig. 2.

I shall first explain the theory of this construction. The Hartmann formula may be written

$$(\lambda - \lambda_0)(R - R_0) = c \quad . \quad . \quad . \quad . \quad (2)$$

This is the equation of a rectangular hyperbola, the axes of co-ordinates being parallel to its asymptotes. But if on any conic section (fig. 1) we take five fixed points S_1 S_2 A B C, and a

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moving point D, we know that the pencil S_1ABC is projective with the pencil S_2ABC. In other words, the cross ratio or double ratio of those four rays is the same for each pencil.

Let us now consider a branch of the above rectangular hyperbola and its two asymptotes as_1 and as_2. Let us take ABC anywhere on this branch, and let S_1 and S_2 be the points at infinity on the same branch (fig. 2).

S_1ABC is then a pencil of rays parallel to as_1, and S_2ABC a pencil parallel to as_2.

These two pencils are projective; hence the range of points A″ B″ C″ D″ is projective with the range of points A′ B′ C′ D′.

But A′B′ and A′C′ represent certain increments of the micrometer

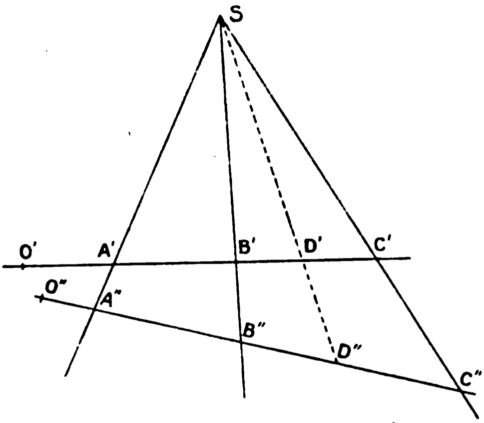

Fig. 3.

reading, and A″B″, A″C″ the corresponding known increments for the wave-length. We may then, knowing any increment A′D′ of the micrometer reading, obtain the corresponding increment of the wave-length; we have simply to put these two ranges of points in perspective. Hence we shall have the following simple construction (fig. 3).

Suppose we know three micrometer readings R_1 R_2 R_3 and the three corresponding wave-lengths λ_1 λ_2 λ_3.

Let us take a graduated scale (or piece of paper) where

$$O'A' = R_1$$
$$O'B' = R_2$$
$$O'C' = R_3$$

and let us draw that part of the scale in A′B′C′. Having taken a point S outside A′B′, we join SA′, SB′, SC′. Taking then another graduated scale, where $O''A'' = \lambda_1$, $O''B'' = \lambda_2$, $O''C'' = \lambda_3$, we fit it so that A″, B″, and C″ be each on the corresponding rays joining S to A′, B′, and C′.

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O″D″ will be the wave-length corresponding to the micrometer reading O′D′ = R.

The construction becomes easier still if we have to deduce a

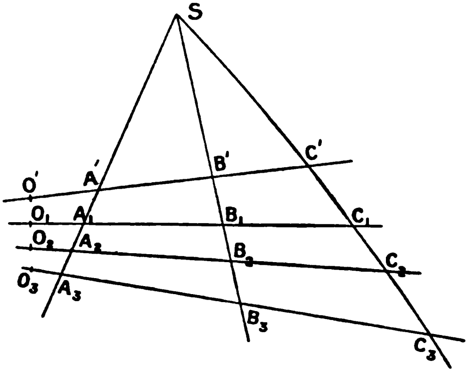

FIG. 4.

table of wave-lengths corresponding to micrometer readings increasing systematically (fig. 4).

For instance, if we take

$$O'A' = R$$
$$O'B' = R + 10$$
$$O'C' = R + 20,$$

we have

$$O_1A_1 = f(R)$$
$$O_1B_1 = f(R + 10)$$
$$O_1C_1 = f(R + 20).$$

But we may take also

$$O'A' = R + 30$$
$$O'B' = R + 40$$
$$O'C' = R + 50$$

and have the corresponding O_2A_2, and so on.

So that, if A′C′ is divided, say in twenty parts, each of the twenty-one rays drawn from S will serve more than once in dividing the series of lines A_1C_1, A_2C_2, A_3C_3. . . .

This construction has given good results. It has the advantage of being mathematically exact (as far as the Hartmann formula is concerned) and also of occupying a very small space. This space may be reduced proportionately to the number of lines A_1C_1, A_2C_2 . . . to be considered.

To give an idea of the accuracy of the construction, I have applied this method to the reduction of part of a photograph taken at Ann Arbor.

In the Hartmann formula the constants are

$$\lambda_0 = 2213{\cdot}640, \qquad c = 223802{\cdot}23, \qquad R_0 = 185{\cdot}336.$$

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Having the following micrometer readings with their corresponding wave-lengths

R.	λ.
40·755	3761·577
50·668	3875·521
59·963	3998·731

I desired the wave-lengths corresponding to each integral value of R, from 40 to 60.

I took then (see fig. 3) on A′C′, each value = $\frac{1}{2}$ inch (A′C′ = 10 inches), and the point S at a distance of 7 inches from A′C′ on a perpendicular to A′C′ at the point marked 50.

I took A″C″ so that each tenth of an inch would correspond to an Ångström, and made the construction as indicated above.

In the following table the wave-lengths corresponding to the values of R, 40, 41, 48, 52 will give an idea of the accuracy of the results :—

R.	Measured Diff. in Angströms.	Wave-lengths from the Drawing.	Computed Wave-lengths.
40	− 8·00	3753·577	3753·535
41	+ 2·65	3764·227	3764·204
48	+ 81·65	3843·227	3843·236
52	+ 130·60	3892·127	3892·123

We thus obtain an accuracy of about $\frac{1}{20}$ of an Ångström. A yet greater accuracy (of $\frac{1}{100}$ or $\frac{1}{1000}$ Å.) may be obtained by taking more values of R and λ, and applying the fig. 4.

The construction is identical for finding R, knowing λ.

P.S.—The construction has been tried at the Yerkes Observatory. An accuracy greater than that mentioned above has been obtained by Mr. Hubble. When applied to the reduction of spectra obtained with the three-prism spectrograph, it gives perfect results up to the thousandth of an Ångström.

Detroit Observatory,
Ann Arbor, Mich., U.S.A.:
1916 *October* 8.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVII. DECEMBER 8, 1916. NO. 2.

Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

Edward Lindsay Ince, M.A., B.Sc., Trinity College, Cambridge, and
John Cromwell Prior, Rosedene, South Hayling, Hants,
were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Mrs. Louisa Petitot Freeman, 67 High Street, Oxford (proposed by A. A. Rambaut);
William Henry Pick, B.Sc. (Lond.), F.C.S., Queen Mary's School, Basingstoke, Hampshire (proposed by Alfred W. Porter); and
J. G. E. G. Voûte, Royal Observatory, Cape of Good Hope (proposed by S. S. Hough).

Mr. J. Jackson, Mr. G. J. Newbegin, and Mr. W. Shackleton were appointed Auditors of the Treasurer's Accounts for 1916.

Forty presents were announced as having been received since the last meeting, including, amongst others:—

Vols. 36 to 107 inclusive of the *Astronomische Nachrichten*, presented by Joseph and Walter Baxendell.

A special vote of thanks was passed to the Messrs. Baxendell for their valuable gift.

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On Certain Possible Distributions of Meteoric Bodies in the Solar System. By Harold Jeffreys, B.A., M.Sc.

This investigation deals with problems that arise when a swarm of small solid bodies, of size comparable with a meteor, or even smaller, is supposed to move under gravity about a large nucleus. In all cases the united mass of the particles is supposed sufficiently small for their mutual attractions to be neglected in a first approximation, and the object of the work is to find out in what way the systems will degenerate on account of their mutual impacts.

The first part of the paper deals with cases in which the motion is steady, and the swarm is symmetrically distributed about an axis through the central mass. Two examples that are considered are the rings of Saturn and the zodiacal light. It is shown that in the case of Saturn's rings the frequency of collision is very great, and consequently, on account of the loss of relative motion at every collision, the rings must long ago have reached a state in which all the particles are moving in very accurate circles, all in the same plane. The zodiacal light being assumed to be caused by meteors, it is shown that it must be of later origin than the planets; no information is obtained, however, when the particles causing it are assumed to be small compared with the wave-length of light.

The second part treats of the motion of meteors in the neighbourhood of the earth's orbit. The motion is not assumed to be steady, and the frequency of collision is small, the life of the system being comparable with the age of the earth itself. From the ratio of the numbers of meteors picked up by the earth in front and behind as it moves, it is shown to be highly probable that a large majority of these bodies move in direct orbits.

The third part of the paper considers the Planetesimal Hypothesis of Chamberlin and Moulton. The objections to the Nebular Hypothesis are first summarised, and it is then shown that while the Planetesimal Hypothesis avoids most of them, it is nevertheless open to a new one, namely, that collisions between the small bodies would be so frequent, and would occur with so high a velocity, that the particles would be fused and volatilised long before accretion could produce any important effect on bodies as large as the planets. If the planetesimals be supposed to have moved initially in nearly circular orbits, this objection is avoided, but the planets are required to have always been very large. An alternative hypothesis is, however, suggested, which may combine the advantages of the Nebular and Planetesimal Hypotheses. The original nebula is in this case assumed to be gaseous, with a strong condensation to the centre, and perhaps highly heterogeneous in other respects, but moving in such a way that the period of revolution about the centre depends on the distance from the

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centre, so that the criterion of the constancy of angular momentum is automatically satisfied.

I. Problems of Steady Motion.

When a large number of particles are moving in a field of force in such a way that collisions are infrequent and the distribution of mass and velocities with respect to space remains constant, the motion is said to be steady. The condition that collisions are rare makes the problem totally different from that considered by Lockyer, Darwin, and du Ligondès. Mathematically the nearest analogue is the motion of stars in clusters, as treated by Professor Eddington * and Mr. J. H. Jeans.

If N denotes the number of particles within the element of volume $dx\,dy\,dz$ whose velocity components lie between u and $u+du$, v and $v+dv$, w and $w+dw$, we have

$$N = f\,du\,dv\,dw\,dx\,dy\,dz \quad . \quad . \quad . \quad (1)$$

where f is some function of (u, v, w, x, y, z).

Then it has been shown by Jeans that the condition for steady motion is that $f=$ constant is a first integral, independent of the time, of the equations of motion of a particle.† If, however, this is so, f must be a function of the existing first integrals, and conversely any function of these will satisfy the conditions for steady motion.

Now in the case considered the mutual attraction of the particles is neglected in comparison with the attraction of the central body, and consequently the potential is μ/r. There are then four first integrals of the equations of motion, namely, the energy equation and the three momental equations.

Taking spherical polar co-ordinates, we have

$$x = r\sin\theta\cos\phi\,;\; y = r\sin\theta\sin\phi\,;\; z = r\cos\theta.$$

The angular momenta are found to be given by

$$h_1 \equiv y\dot{z} - z\dot{y} \equiv -r^2\dot{\theta}\sin\phi - r^2\dot{\phi}\sin\theta\cos\theta\cos\phi \quad . \quad (2)$$

$$h_2 \equiv z\dot{x} - x\dot{z} \equiv r^2\dot{\theta}\cos\phi - r^2\dot{\phi}\sin\theta\cos\theta\sin\phi\,. \quad . \quad (3)$$

$$h_3 \equiv x\dot{y} - \dot{x}y \equiv r^2\sin^2\theta\dot{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

All these must be constant in the motion of any particle.

Also the equation of energy gives

$$E \equiv \frac{2\mu}{r} - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 \equiv \frac{2\mu}{r} - \dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2 \quad . \quad (5)$$

Then the most general steady motion is given by making

$$f = \chi(E, h_1, h_2, h_3) \quad . \quad . \quad . \quad (6)$$

where χ is an arbitrary function.

* *Monthly Notices*, vol. lxxvi., 1915, p. 37.
† Jeans, *Monthly Notices*, vol. lxxvi., 1915, p. 78.

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Now we intend to consider only distributions that are symmetrical about an axis, so that f must not involve ϕ explicitly. The functions h_1 and h_2 can therefore only be involved through the sum of their squares, which is independent of ϕ. Then

$$\left.\begin{aligned} f &= \chi(\mathrm{E},\ h_1^2+h_2^2,\ h_3) \\ &= \chi\left(\frac{2\mu}{r} - \dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2,\ r^4\dot{\theta}^2 + r^4\sin^2\theta\cos^2\theta\dot{\phi}^2,\ r^2\sin^2\theta\dot{\phi}\right) \end{aligned}\right\} \quad (7)$$

It follows that in any symmetrical distribution the mean values of $\dot{r}$ and $\dot{\theta}$ among the particles at every point must be zero.

(*a*) A very natural type of distribution to try would be one in which f is of the form

$$\chi\{\mathrm{E} + 2ah_3 - \beta h_3^2 - \gamma(h_1^2+h_2^2)\} = \chi(\nu), \text{ say,}$$

where a, β, and γ are constants. Then

$$\begin{aligned} \nu = 2\mathrm{K} - \dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2 + 2ar^2\sin^2\theta\dot{\phi} \\ - \beta r^4\sin^4\theta\dot{\phi}^2 - \gamma r^4\dot{\theta}^2 - \gamma r^4\sin^2\theta\cos^2\theta\dot{\phi}^2 \quad (8) \end{aligned}$$

where K at present denotes the gravitational potential.

Put

$$\dot{r} = u,\ r\dot{\theta} = v,\ r\sin\theta\dot{\phi} = \frac{ar\sin\theta}{1+\beta r^2\sin^2\theta + \gamma r^2\cos^2\theta} + w \quad . \quad (9)$$

It is seen at once that

$$\begin{aligned} \nu = \frac{2\mu}{r} - u^2 - (1+\gamma r^2)v^2 - (1+\beta r^2\sin^2\theta + \gamma r^2\cos^2\theta)w^2 \\ + \frac{a^2r^2\sin^2\theta}{1+\beta r^2\sin^2\theta+\gamma r^2\cos^2\theta} \quad (10) \end{aligned}$$

Thus the velocity distribution at any point has ellipsoidal symmetry about the mean velocity that makes $u = v = w = 0$. This hypothesis is, however, not satisfactory, for a reason that will be given.

The numerical density at any point is $\iiint f\,du\,dv\,dw$, all the integrations being from $-\infty$ to $+\infty$.

Let

$$2\mathrm{K} + \frac{a^2r^2\sin^2\theta}{1+\beta r^2\sin^2\theta+\gamma r^2\cos^2\theta} = \mathrm{V}^2 \quad . \quad . \quad (11)$$

Now substitute

$$u = \mathrm{V}\xi;\ v = \mathrm{V}\eta/(1+\gamma r^2)^{\frac{1}{2}};\ w = \mathrm{V}\zeta/(1+\beta r^2\sin^2\theta+\gamma r^2\cos^2\theta)^{\frac{1}{2}} \quad (12)$$

Then the density becomes

$$\begin{aligned} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\chi\{\mathrm{V}^2(1-\xi^2-\eta^2-\zeta^2)\}\mathrm{V}^3 d\xi\,d\eta\,d\zeta \\ \div (1+\gamma r^2)^{\frac{1}{2}}(1+\beta r^2\sin^2\theta+\gamma r^2\cos^2\theta)^{\frac{1}{2}} \quad (13) \end{aligned}$$

provided β and γ are not negative.

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Now as long as the form of χ is such that the triple integral has any meaning, it is evidently a function of V only.

So far the solution is exact. When r is great, provided the mass is finite, a is not zero, and β and γ are not both zero, V^2 tends to $a^2/(\beta+\gamma\cot^2\theta)$, and is therefore constant for any particular colatitude. The density at a great distance is then of the form $F(\beta+\gamma\cot^2\theta)/r^2\sin\theta$. The total mass between two spheres of radii r and $r+dr$ is then $dr\iint F d\theta d\phi$, and is therefore equal to $M dr$ where M is a constant: M cannot be zero, as it is physically obvious that F is essentially positive. Thus the condition that the total mass of the system is finite cannot be satisfied.

If for $\chi(\nu)$ we write $e^{h^2\nu}$, where h is a constant, the distribution reduces to the Schwarzschild rotational distribution found by Professor Eddington in the paper already mentioned.

If β and γ are both zero and a is not zero, the mean velocity at any point is $ar\sin\theta$. If then this is greater than $(2\mu/r)^{\frac{1}{2}}$, the particles moving with velocities greater than the mean will leave the system. Hence the system is confined to a region surrounding the axis and bounded by the surface $r^3\sin^2\theta = 2\mu/a^2$.

If either β or γ is negative, a region will exist such that at the margin of it infinite velocities are possible without making ν different from V^2.

The argument does not apply if a is 0, corresponding to a state without any general rotation. In that case $V^2 = 2K$, and forms of χ can be found so that the total mass is finite. This has been done in the case of globular star clusters by Professor Eddington * and Mr. Jeans,† who find that with a law of the type $\nu = E$ it is possible to make the density proportional to r^{-5} at a great distance.

(*b*) Consider next a distribution in which χ is always zero unless Eh_3^2 lies between μ^2 and $\mu^2(1-a)$, where a is a small positive quantity that may be a function of E, $h_1^2+h_2^2$, and h_3.

Put $w = r\sin\theta\dot\phi$, so that $h_3 = r\sin\theta w$ . . . . . (14)

Also put $w = \operatorname{cosec}\theta\sqrt{(\mu/r)} + w'$ . . . . . . (15)

Then

$$\frac{\mu^2}{h_3^2} - E = \dot r^2 + r^2\dot\theta^2 + \frac{h_3^2}{r^2}\operatorname{cosec}^2\theta - \frac{2\mu}{r} + \frac{\mu^2}{h_3^2}$$

$$= \dot r^2 + r^2\dot\theta^2 + \left(\frac{\mu}{h_3} - \frac{h_3}{r}\right)^2 + \frac{h_3^2}{r^2}\cot^2\theta \quad . \quad . \quad (16)$$

But $\frac{\mu^2}{h_3^2} - E$ lies between 0 and $\frac{\mu^2 a}{h_3^2}$. Hence each of the four squares of which it is the sum must separately be small. If for simplicity we regard a as a small quantity of the *second* order, then it follows that $\dot r$, $\dot\theta$, $h_3^2 - \mu r$, and $\cot\theta$ are all small quantities of the first order; thus all the particles must be moving in very nearly circular orbits close to the invariable plane. This condition alone

* *Monthly Notices*, vol. lxxvi., 1916, p. 579. † *Ibid.*, p. 570.

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is therefore enough to define a system in which the orbits are of this type, and may thus give a fair representation of the movement of Saturn's rings and the zodiacal light, if the composition of this is meteoric. Such systems as these are meant when the term "a flat system" is used later in this paper.

If, for instance, we make χ equal to $H\left(a - 1 + \frac{Eh_3^2}{\mu^2}\right)^k$, when the quantity inside the bracket is positive and less than $\mu^2 a$, and zero in all other circumstances, the distribution is of the type considered. H and a may be functions of E and h_3.

Put $\dot{r} = u$, $r\dot{\theta} = v$, and substitute for h_3 in terms of w'.

Then

$$\frac{\mu^2}{h_3^2} - E = u^2 + v^2 + \left(\frac{2\sqrt{(\mu/r)}\sin\theta w' + \sin^2\theta w'^2}{\sqrt{(\mu/r)} + \sin\theta w'}\right)^2 + \frac{\mu\cot^2\theta}{r}\left(1 + \sqrt{\frac{r}{\mu}}\sin\theta w'\right)^2.$$

When we neglect small quantities of order above the second, this reduces to

$$u^2 + v^2 + 4w'^2 + (\mu/r)\cot^2\theta \quad . \quad . \quad . \quad (17)$$

Hence

$$\chi = H\left\{a - \frac{r}{\mu}(u^2 + v^2 + 4w'^2) - \cot^2\theta\right\}^k \quad . \quad . \quad (18)$$

Now H and a are functions of E and h_3. For any particular value of r we have therefore

$$H = H_0 + w'\left(\frac{\partial H}{\partial w'}\right)_0 + \tfrac{1}{2}w'^2\left(\frac{\partial^2 H}{\partial w'^2}\right)_0 + u^2\left(\frac{\partial H}{\partial u^2}\right)_0 + v^2\left(\frac{\partial H}{\partial v^2}\right)_0 + \cot^2\theta\left(\frac{\partial H}{\partial \cot^2\theta}\right)_0,$$

where the suffixes indicate that the quantities are evaluated for those particles for which u, v, w' and $\cot\theta$ are zero. All terms after the first are evidently of at least the first order of small quantities, and hence in (18) H_0 can be written for H. Similarly

$$a = a_0 + w'\left(\frac{\partial a}{\partial w'}\right)_0 + \text{similar terms}$$

$$= a_0 + r\sin\theta w'\left(\frac{\partial a}{\partial h_3}\right)_0 + \text{etc.},$$

so that every term after the first is of at least the third order, provided a does not vary too rapidly with h_3.

Hence H and a for any particle can be given their values for a particle at the same distance from the centre, moving in a circular orbit in the equatorial plane. In other words, H and a may be regarded as functions of r only, and not of u, v, w', and θ.

To the first order the mean velocity at any point is equal to the circular velocity, and consequently a swarm of this type cannot affect the mean distance of a planet to any important extent.

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The density is

$$\iiint \chi \, du\, dv\, dw = 2^{2k+2}\pi \frac{k!\,(k+1)!}{(2k+3)!}\left(\frac{\mu}{r}\right)^{\frac{3}{2}} \frac{\mathrm{H}(a-\cot^2\theta)^{\frac{2k+3}{2}}}{\sin\theta} \qquad (19)$$

provided $\cot^2\theta$ is less than a.

If $\cot^2\theta$ is greater than a, the density is zero.

If a and H were constants, the total mass would evidently be infinite. If, however, H is constant and a is a function of h_3 so that it tends to zero at infinity more rapidly than $h_3^{-(3+2\epsilon)/(k+2)}$ where ϵ is a small positive quantity, the mass is finite. There will therefore be no difficulty in ensuring finiteness of mass.

If $\mathrm{H}a^{k+2}$ is proportional to h_3 between definite limits for h_3, and otherwise zero, the number of particles in a small cross-section of the system parallel to the axis of symmetry is constant between two definite radii, and zero everywhere else. Such a system must have a strong resemblance to Saturn's rings.

The Effect of Collisions in a Flat System.

On the hypothesis that collisions are rare, it is possible to find steady motions that a given type of system is capable of describing. These collisions must, however, occur occasionally, so that the steady state can only be regarded as a first approximation. In investigating the permanence of a system it is therefore necessary to estimate the frequency of collisions and the character of their effect on the system.

Let ν be the number of particles per unit volume at any point, and let the number of particles per unit volume whose component velocities lie respectively between u and $u+du$, v and $v+dv$, w and $w+dw$, be $\nu f(u, v, w)du\,dv\,dw$. Further, if all the particles in any system are supposed to be equal spheres of diameter σ, the probability that a particle with velocities (u_1, v_1, w_1) will have a collision in time dt is

$$dt\iiiint \nu\sigma^2 f(u, v, w)\{l(u_1-u)+m(v_1-v)+n(w_1-w)\}du\,dv\,dw\,d\omega$$

where the range of integration for u, v, and w is through all values from $-\infty$ to $+\infty$, subject to the condition that $l(u_1-u)+m(v_1-v)+n(w_1-w)$ is positive. The last two integrations are with regard to a surface element $d\omega$ whose co-ordinates are (l, m, n) and extend over a unit sphere.

In an average element of volume the velocity distribution is given by $\nu f=(r/\mu)^k\mathrm{H}\{(\mu/r)(a-\cot^2\theta)-(u^2+v^2+4w^2)\}^k$ when the quantity in the bracket is positive and is otherwise zero; in this w is now written for w'.

If now we put

$$(\mu/r)(a-\cot^2\theta)=\delta, \qquad . \quad . \quad . \quad (20)$$

the first three integrations give, by the theorem proved in the appendix,

$$(r/\mu)^k\frac{\mathrm{H}\sigma^2\pi\delta^{k+2}}{2(k+1)}(l^2+m^2+\tfrac{1}{4}n^2)^{\frac{1}{2}}\left[\frac{(1-\mathrm{L}_0{}^2)^{k+2}}{2(k+2)}-\mathrm{L}_0\int_0^{\theta_0}\sin^{2k+3}\theta\,d\theta\right] \quad (21)$$

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where

$$L_0 = (lu_1 + mv_1 + nw_1)/\{\delta(l^2 + m^2 + \tfrac{1}{4}n^2)\}^{\frac{1}{2}}$$

and $\cos\theta_0 = L_0$, subject to the condition that θ_0 lies between 0 and π.

Calling the quantity inside the square brackets $\lambda(L_0)$, and the factor outside them A, then the probability of collision within the elementary solid angle $d\omega$, whose direction cosines are (l, m, n), is $A\lambda(L_0)d\omega\,dt$. Again, the probability of a collision within an equal angle, the direction cosines being $(-l, -m, -n)$, is $A\lambda(-L_0)d\omega\,dt$. The two together give $A\,d\omega\,dt\{\lambda(L_0)+\lambda(-L_0)\}$.

Now

$$\frac{d}{dL_0}\left\{\lambda(L_0)+\lambda(-L_0)\right\} = \frac{d}{dL_0}\left\{\frac{(1-L_0{}^2)^{k+2}}{k+2} + L_0\int_{\theta_0}^{\pi-\theta_0}\sin^{2k+3}\theta\,d\theta\right\}$$

$$= -2L_0(1-L_0{}^2)^{k+1} + \int_{\theta_0}^{\pi-\theta_0}\sin^{2k+3}\theta\,d\theta$$

$$+ L_0\sin^{2k+3}(\pi-\theta_0)\frac{d(\pi-\theta_0)}{dL_0} - L_0\sin^{2k+3}\theta_0\frac{d\theta_0}{dL_0}.$$

But

$$\frac{d\theta_0}{dL_0} = -\frac{1}{\sin\theta_0}, \quad \text{and} \quad \sin^{2k+2}\theta_0 = (1-L_0{}^2)^{k+1}.$$

Hence the last two terms together give $+2L_0(1-L_0{}^2)^{k+1}$, and therefore

$$\frac{d}{dL_0}\{\lambda(L_0)+\lambda(-L_0)\} = \int_{\theta_0}^{\pi-\theta_0}\sin^{2k+3}\theta\,d\theta \quad . \quad . \quad (22)$$

which has the same sign as L_0 and vanishes when $L_0 = 0$. Hence $L_0 = 0$ is the only stationary value of the frequency of collision within a complete cone, and it gives a minimum, namely, $\frac{1}{k+2}$. Thus particles moving with the mean velocity at a point undergo fewer collisions than any others. We shall therefore under-estimate the number of collisions if we put $L_0 = 0$. This gives for the probability of collision for a particular particle

$$dt\iint\frac{(r\mu)^k H\sigma^2\pi\delta^{k+2}(l^2+m^2+\tfrac{1}{4}n^2)^{\frac{1}{2}}}{4(k+1)(k+2)}d\omega \quad . \quad . \quad (23)$$

Also

$$\iint(l^2+m^2+\tfrac{1}{4}n^2)^{\frac{1}{2}}d\omega = 4\pi E \quad . \quad . \quad . \quad (24)$$

where E denotes the second complete elliptic integral with modulus $\frac{1}{2}\sqrt{3}$. Its value is given by Greenhill to be 1·212. Hence the probability of collision is greater than

$$1{\cdot}212\,dt(r\mu)^k\pi^2 H\sigma^2\delta^{k+2}/(k+1)(k+2) \quad . \quad . \quad (25)$$

On the other hand, it is easy to see that when $L_0 = 1$, which is the greatest value it can have, $\lambda(L_0)+\lambda(-L_0)$ is $2\int_0^{\pi/2}\sin^{2k+3}\theta\,d\theta$,

which is not overwhelmingly greater than $1/(k+2)$, so that this estimate of the probability of collision will be of the correct order of magnitude for particles for which L_0 is not zero.

Next, it is evident that in a given short interval a particle is more likely to collide with a particle moving in the opposite direction than with one moving in the same direction. The average effect of collisions will therefore be greater than if each particle, instead of colliding with another moving particle, actually collided with a particle moving with the average velocity at the point. The relative velocity of collision is supposed not great enough to cause fusion.

If the coefficient of restitution is λ, the reduction in the velocity relative to the mean in a collision is on the average greater than is expressed by the factor $\frac{1}{2}(1-\lambda)$.

Thus if the average absolute velocity relative to the mean is U initially, then after a time dt it will be reduced to less than

$$U\{1 - \cdot 606(1-\lambda)dt(r\mu)^k\pi^2 H\sigma^2\delta^{k+2}/(k+1)(k+2)\}.$$

Thus

$$\frac{dU}{Udt} = -\frac{\cdot 606\pi^2(1-\lambda)(r\mu)^k H\sigma^2\delta^{k+2}}{(k+1)(k+2)} \quad . \quad . \quad (26)$$

gives a minimum estimate of the rate of degeneration. The actual rate is not likely to be more than five times this in any case.

Application to Saturn's Rings.

No data are available with regard to the actual size of the small bodies believed to compose Saturn's rings, but the product $\nu\sigma^2$ can be estimated from considerations of the luminosity and opacity of these rings. Observations of the satellite Iapetus * have shown that it is quite invisible when it passes into the shadow of either of the main rings, but visible behind Cassini's division and the crape ring. This suggests that the number of particles is such that very little light can get through the rings, owing to reflection, absorption, and scattering. This is confirmed by the high albedo of the rings, which is comparable with that of the body of the planet. The outer ring, however, is not so bright as the inner one.

If we consider a pencil of light from the sun striking the plane of the ring at an angle i, and if the cross-section of the pencil be dS and the thickness T, the number of particles in the part of the ring within this pencil is $\operatorname{cosec} i dS\int\nu dz$, the integral being taken right through the ring. Each particle stops the light over an area equal to that of its own diametral plane, and if nearly all the light is stopped it follows that $\frac{\pi\sigma^2}{4}\operatorname{cosec} i\int\nu dz$ must be comparable with unity. On the other hand, if this quantity is much greater than unity, light will be stopped and reflected very little more than if it

* Barnard, *Monthly Notices*, vol. l. pp. 107–110.

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is only equal to unity, as most of the stoppage will be confined to the first layers. The fact that the inner and outer rings differ in luminosity suggests, then, that the number is not very different from unity for the outer ring, and somewhat greater for the inner one. Young gives* 27° as the value of the inclination of the plane of the rings to the plane of Saturn's orbit, and i must always be equal to or less than this. If ν_0 is the numerical density in the diametral plane,

$$\int \nu dz = 2r\nu_0 \sqrt{a} \int_0^{\pi/2} \cos^{2k+4}\psi \, d\psi.$$

Hence

$$\nu_0 \sigma^2 = 2 \sin i \Big/ \pi r a^{\frac{1}{2}} \int_0^{\pi/2} \cos^{2k+4}\psi \, d\psi$$

for the outer ring. For the inner ring its value must be greater.

Then

$$-\frac{1}{U}\frac{dU}{dt} = \frac{1\cdot212(1-\lambda)\sin i}{a^k \int \cos^{2k+4}\psi \, d\psi} \frac{(2k+3)!}{(k+1)!\,(k+2)!} \frac{1}{2^{2k+2}} \left(1 - \frac{\cot^2\theta}{a}\right)^{k+2} \left(\frac{\mu}{r^3}\right)^{\frac{1}{2}} \quad (27)$$

k may have any value, but it is not likely to be negative, on account of the fact that this would prevent the velocities at any point from being concentrated around a mean value. It may, however, be zero. The rest of the numerical coefficient is of order unity. It follows that the time of degeneration at any point is comparable with the time of revolution of the particles at that point, namely, a few hours.

The ring was known to be thin in Galileo's time. Taking its thickness at that date to be less than 10,000 miles, then, since the time of revolution at all points of the ring is less than a day, its present thickness would, from the above estimate, be less than 10,000 exp. (− 100,000) miles, which would be absolutely inappreciable. The hypothesis that the motion is steady would break down before this stage, on account of the frequency of collisions, but it is nevertheless evident that the ring must now be very thin, and practically confined to one plane. The constituent particles must then be distributed in narrow circular strips, each strip being practically a broken solid ring, and perhaps even undergoing something not very different from solid friction from the two on each side of it, which must be moving with slightly different velocities. All estimates of the thickness that have been made visually up to the present are of the nature of upper limits; and from the above it appears very unlikely that any measurable thickness will ever be found. The last visual estimate was made by H. N. Russell,† who states that the thickness must be much less than 13 km.

* *General Astronomy*, p. 392.
† *Astrophysical Journal*, vol. xxvii. p. 230.

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The Zodiacal Light.

This consists of a broad luminous band, surrounding the sun, and apparently nearly in the plane of the ecliptic. It extends to more than 90° from the sun, and is quite conspicuous between the orbits of Mercury and Venus. Its cause is not yet quite certain. It is believed to be due to light diffusely reflected or scattered by small particles revolving round the sun, but whether these particles are large or small in comparison with the wave-length of light is not known. Newcomb * states that they are considered to be of meteoric character, but that the final judgment is quite likely to be that the structure is a tenuous gas. If it is gaseous, or composed of particles of dust small compared with the wave-length of light, no progress can be made by the methods here employed, as the amount of light scattered is proportional to $\nu\sigma^6$, which cannot be found without a knowledge of the size of the particles. On the other hand, if it is meteoric, $\nu\sigma^2$ can be at once found from the luminosity. This case will now be considered.

Consider a particle, large compared with the wave-length of light, moving within the earth's orbit and near maximum elongation as seen from the earth. Its distance from the sun is r, and from the earth is ρ. The intensity of solar illumination at the earth being I, and the albedo of the particles p, the amount of light reflected by any particle is $\mathrm{I}(r_0/r)^2\frac{1}{2}\pi\sigma^2 p$, and hence the contribution to the intensity of the light at the earth by the particles in unit volume is $\frac{1}{4}\pi\nu\sigma^2 p\mathrm{I}r_0^2/r^2\rho^2$ where r_0 is the earth's mean distance. The further factor $\frac{1}{2}$ has been introduced because only half the visible surface is illuminated. If then we consider a solid angle $d\omega$ as seen from the earth, the intensity within it is $d\omega\int d\rho\frac{1}{4}\pi\nu\sigma^2 p\mathrm{I}r_0^2/r^2$.

In all, then, the radiation per unit solid angle of the sky is roughly equal to $\frac{1}{4}\pi\nu\sigma^2 p\mathrm{I}r_0^2\mathrm{P}/r^2$, where P denotes the depth of the structure in a definite line of observation, and ν and r refer to the point on that line that is nearest to the sun.

If then ω denote the solid angle subtended by a circle of diameter 1°, the above estimate of the intensity gives

$$\frac{\text{total light from an element } \omega \text{ of the sky}}{\text{the solar radiation}} = \frac{\frac{1}{4}\pi\nu\sigma^2 p\mathrm{I}r_0^2\mathrm{P}\omega}{\mathrm{I}r^2} \qquad (28)$$

Now Newcomb states † that the luminosity of the "dark" background of the heavens is equivalent to 0·9 stars of magnitude 5·0 per circle of diameter 1°, near the galactic pole, whereas in the Milky Way itself this estimate must be increased by 1·0 to 1·5 for the same area. This excess is to be attributed to the light of the galaxy, and that of the zodiacal light is of about equal intensity. Thus the above fraction is equal to the ratio of the radiation of

* Article "Zodiacal Light," *Ency. Brit.*, 11th edition.
† *Astrophysical Journal*, xiv., 1901, p. 307.

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1·5 stars of magnitude 5·0 to the radiation of 1 star of magnitude $-26{\cdot}5$, or $1{\cdot}5 \times 10^{-12{\cdot}6}$.

Then $\omega = \pi^3/129600$; p is taken as $\frac{1}{8}$, equal to the albedo of the moon; $r_0/r = 1{\cdot}4$; $P = 2{\cdot}2 \times 10^{13}$ cm. This estimate of **P** is obtained by taking the equivalent depth as equal to the diameter of the orbit of Venus.

Then we find

$$\nu\sigma^2 = 3{\cdot}5 \times 10^{-22} \div 1 \text{ cm.} \qquad (29)$$

Adopting this value, determining H from it, and substituting in dU/Udt, we find

$$\frac{1}{U}\frac{dU}{dt} = -\frac{{\cdot}606\pi(1-\lambda)(2k+3)!}{(k+1)!\,(k+2)!\,2^{2k+2}}\left(\frac{\mu a}{r}\right)^{\frac{1}{2}}\frac{3{\cdot}5 \times 10^{-22}}{1 \text{ cm.}} \qquad (30)$$

The numerical coefficient is not much different from unity, and a is of order $\frac{1}{4}$ for particles at the distance of Venus.*

The order of the time of degeneration is then found to be 10^7 years, which is short compared with the estimates of the age of the earth derived from radioactivity. The zodiacal light may then be of more recent origin than the planets, and possibly its matter may have been derived from meteors originally moving in more eccentric orbits, or even from the detritus of periodic comets, as has been suggested by Fessenkoff.†

II. The General Distribution of Meteors in the Solar System.

According to Professor Newton,‡ the average distance between the meteors picked up by the earth is about 250 miles. Thus the numerical density ν is $1/6{\cdot}4 \times 10^{22}$ (cm.)^3. Further, the average mass appears to be about 1 grain. Taking the density to be 3, this gives $\sigma = \frac{1}{3}$ cm. roughly. Hence $\nu\sigma^2 = 1{\cdot}7 \times 10^{-24}/1$ cm. This is only $\frac{1}{200}$ of the value of $\nu\sigma^2$ already deduced for the zodiacal light near the orbit of Venus. Thus either the density must increase with great rapidity as we approach the sun, or meteors can account for only a small part of the zodiacal light. Smaller bodies, such as might be produced by the collisions and consequent volatilisation of meteors, would have a larger value of $\nu\sigma^2$ for the same mass.

The rate of degeneration near the earth is of order $\nu\sigma^2 V$, where V is the average velocity with which meteors enter the earth's atmosphere. This is nearly the parabolic velocity of 40 kms./1 sec. Then the time of degeneration is found to be about 5×10^9 years. Nearer the sun it must be much shorter, as both $\nu\sigma^2$ and V are greater. This indicates that in a time that is probably less than the ages of the planets most of the meteors within the orbit of

* Fessenkoff, *Ast. Nach.*, 4693, 1913, vol. cxcvi. p. 229.

† "Sur l'origine de la lumière zodiacale," *Ast. Nach.*, 4752.

‡ Young, *General Astronomy*, art. 771.

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Venus would have been volatilised by collision. It appears unlikely, however, that the amount of gas that could thus be produced would scatter enough light to account for the zodiacal light. Thus while the meteoric theory of the zodiacal light is not impossible, it appears to offer no advantage over Fessenkoff's theory that the matter is derived from the detritus of short-period comets, and probably molecular in character.

The Sweeping Action of the Earth.

Consider an element dS of the surface of the earth, which is supposed spherical. Let U, V, and W be the excesses of the component velocities of the centre of the earth over the mean velocities of the meteors around it, and let the velocity of any individual meteor relative to this mean velocity be u, v, w. Let (l, m, n) be the direction cosines of the outward normal to the surface at the centre of dS.

Then in time dt the element dS picks up a number

$$dS\,dt\,\nu f(u, v, w)du\,dv\,dw\{l(U-u)+m(V-v)+n(W-w)\}$$

of the meteors whose velocities before impact lay within the element $du\,dv\,dw$. This being integrated gives

$$\nu\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int f(u, v, w)\{l(U-u)+m(V-v)+n(W-w)\}du\,dv\,dw\,dS\,dt \quad (1)$$

wherein u, v, and w range from $-\infty$ to $+\infty$, subject to the condition that the quantity within the bracket is always positive.

The directions of the axes have not yet been assigned. Suppose them chosen so that $V=W=0$. If the distribution of velocities is assumed to be Maxwellian, we have

$$f(u, v, w)=\frac{a^3}{\pi^{\frac{3}{2}}}\exp\{-a^2(u^2+v^2+w^2)\} \quad . \quad . \quad (2)$$

Integrating with regard to u, v, and w, we find that the number of meteors falling per unit time on any area is

$$\nu\int\!\!\int\left\{\frac{\xi_1}{2}(1+\text{Erf}\,a\xi_1)+\frac{1}{2a\pi^{\frac{1}{2}}}e^{-a^2\xi_1^2}\right\}dS \quad . \quad . \quad (3)$$

where $\xi_1 = lU$.

The integrand is evidently greater when ξ_1 is positive than when it is negative, so that the number picked up is greater in front than behind.

Let the angular distance from the direction of U be θ. Then $\cos\theta = l$, and $dS = R^2\sin\theta\,d\theta\,d\phi = -R^2 dl\,d\phi$, where R is the radius of the earth. The number falling on the area for which l lies between l_1 and l_2 is then

$$R^2\nu\int_{l_1}^{l_2}\int_0^{2\pi}\left\{\tfrac{1}{2}Ul(1+\text{Erf}\,aUl)+\frac{1}{2a\pi^{\frac{1}{2}}}e^{-a^2l^2U^2}\right\}dl\,d\phi.$$

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Putting $alU = p$ and integrating with regard to ϕ, this becomes

$$\frac{\pi R^2 \nu}{a^2 U}\int_{p_1}^{p_2}\left\{p(1+\operatorname{Erf}p)+\frac{1}{\pi^{\frac{1}{2}}}e^{-p^2}\right\}dp$$

$$=\frac{\pi R^2 \nu}{a^2 U}\left\{\tfrac{1}{2}p^2+\tfrac{1}{4}\operatorname{Erf}p+\tfrac{1}{2}p^2\operatorname{Erf}p+\frac{1}{2\pi^{\frac{1}{2}}}pe^{-p^2}\right\}_{p_1}^{p_2} \quad (4)$$

(1) To get the ratio between the numbers falling in front and behind, we see that the limits in the two cases are 0 to aU and $-aU$ to 0 respectively. Then

Number falling in front hemisphere ÷ number falling in back hemisphere

$$=\frac{a^2U^2+\frac{1}{2}\operatorname{Erf}aU+a^2U^2\operatorname{Erf}aU+\pi^{-\frac{1}{2}}aU\exp(-a^2U^2)}{-a^2U^2+\frac{1}{2}\operatorname{Erf}aU+a^2U^2\operatorname{Erf}aU+\pi^{-\frac{1}{2}}aU\exp(-a^2U^2)} \quad . \quad (5)$$

For first magnitude shooting stars the value of this ratio is given by Denning * to be $\frac{64{\cdot}5}{35{\cdot}5}$. This corresponds to the case of $aU = {\cdot}3394$.

It is evident from the positive sign of this quantity that the mean transverse velocity of the meteors near the earth is less than the earth's velocity. On the other hand, if it were zero, and a were determined by the above condition, the number of meteors with velocities greater than $U\sqrt{2}$, the parabolic velocity, would be

$$\nu\{1+2^{\frac{3}{2}}a\pi^{-\frac{1}{2}}U\exp(-2a^2U^2)-\operatorname{Erf}aU\sqrt{2}\}={\cdot}926\nu \quad . \quad (6)$$

Thus the majority of the meteors would be moving in parabolic or hyperbolic orbits, and therefore could not be true members of the system. The same would hold even if the law of velocity-distribution was considerably altered. It is evident, then, that the mean transverse velocity of the meteors must be in the direction in which the earth moves, and therefore the great majority of the meteors must move in direct orbits.

(2) As the velocity ellipsoid is symmetrical about the axis of U, we can see that the amounts of momentum in the directions of V and W communicated to the earth are zero. The amount carried by the particles picked up by the element dS in time dt is evidently

$$\nu\, dS\, dt\iiint ma^3\pi^{-\frac{3}{2}}\exp\{-a^2(u^2+v^2+w^2)\}$$
$$\{l(U-u)+m(V-v)+n(W-w)\}u\,du\,dv\,dw \quad (7)$$

Turn the axes so that the axis of ξ is parallel to the direction (l, m, n). Then

$$\left.\begin{aligned}\xi &= lu+mv+nw\\ \eta &= l'u+m'v+n'w\\ \zeta &= l''u+m''v+n''w\end{aligned}\right\}\text{with the relation }\begin{vmatrix} l & m & n\\ l' & m' & n'\\ l'' & m'' & n''\end{vmatrix}=1.$$

* *Monthly Notices*, vol. liv. p. 540.

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Then the integral transforms into

$$\int_{-\infty}^{\xi_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ma^3\pi^{-\frac{3}{2}} \exp\{-a^2(\xi^2+\eta^2+\zeta^2)\}(\xi_1-\xi)(l\xi+l'\eta+l''\zeta)d\xi\, d\eta\, d\zeta \quad (8)$$

Now

$$\int_{-\infty}^{\infty} e^{-a^2\eta^2}\eta\, d\eta = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} e^{-a^2\zeta^2}\zeta\, d\zeta = 0.$$

Hence the integral becomes

$$mal\pi^{-\frac{1}{2}}\int_{-\infty}^{\xi_1} e^{-a^2\xi^2}(\xi_1\xi - \xi^2)d\xi = -\frac{ml}{4a^2}[1+\text{Erf}\,a\xi_1].$$

The total momentum acquired by the sphere in time dt is then

$$-\frac{vm}{4a^2}R^2dt\int_{-1}^{1}\int_{0}^{2\pi} l(1+\text{Erf}\,alU)dl\, d\phi \quad . \quad . \quad (9)$$

$$=\frac{\pi vmR^2dt}{2a^4U^2}[(a^2U^2-\tfrac{1}{2})\text{Erf}\,aU+\pi^{-\frac{1}{2}}aU\exp(-a^2U^2)] \quad . \quad (10)$$

Hence in time dt the mass of the earth changes from

$$M \text{ to } M+\frac{\pi R^2vm}{a^2U}\{\tfrac{1}{2}\text{Erf}\,aU+a^2U^2\text{Erf}\,aU+\pi^{-\frac{1}{2}}aUe^{-a^2U^2}\}dt = M+\delta M, \text{ say,}$$

and the momentum from

$$MU \text{ to } MU-\frac{\pi R^2vm}{a^4U^2}\{(a^2U^2-\tfrac{1}{2})\text{Erf}\,aU+\pi^{-\frac{1}{2}}aUe^{-a^2U^2}\}dt = MU+\delta(MU), \text{ say.}$$

Hence the velocity changes from

$$U \text{ to } \{MU+\delta(MU)\}/(M+\delta M) = U+\frac{1}{M}\delta(MU)-\frac{\delta M}{M}U.$$

The effect of the meteors is to modify the orbit in the same way as a tangential retardation of amount

$$\frac{\pi R^2vm}{4a^4U^2M}\left\{(4a^4U^4+4a^2U^2-1)\text{Erf}\,aU+\frac{2}{\sqrt{\pi}}aU(1+2a^2U^2)e^{-a^2U^2}\right\} (11)$$

It is interesting to note that in a case where U is small the expansion of this in powers of U starts with $\frac{8}{3}\sqrt{\pi}\cdot\frac{R^2vmU}{aM}$, so that it has not a singularity at $U=0$. In this case the portion of the retardation due to increase of mass is only $\frac{1}{4}$ of the whole, the rest being due to diminution of momentum.

Increase of Sweeping Power of a Planet due to Gravity.

Owing to the attraction of the planet on a meteor, the orbit of this in the neighbourhood of a planet becomes curved towards it, and consequently more meteors will strike the planet than would be the case if gravity did not act. In calculating the

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effect, it will be legitimate to treat the planet as at rest, and the meteors as initially moving in straight lines relative to it. Before a meteor comes within the influence of the planet, let its velocity be V, and the perpendicular from the centre of the planet to its path be p. Then, referred to the centre of the planet as origin, the motion of the particle is given by

$$\dot{r}^2 + r^2\dot{\theta}^2 = \frac{2\mu}{r} + V^2 \quad . \quad . \quad . \quad . \quad (12)$$

$$r^2\dot{\theta} = pV \quad . \quad . \quad . \quad . \quad (13)$$

Within the sphere of influence of the planet the solar attraction is supposed the same for both bodies, and hence does not appear in the equations of motion. If the least distance between the centres is λ, the condition that $\dot{r} = 0$ when $r = \lambda$ gives

$$p^2 = \frac{2\mu\lambda}{V^2} + \lambda^2 \quad . \quad . \quad . \quad . \quad (14)$$

The condition for a collision is that $\lambda < R$, the radius of the planet. Hence if $p^2 < \frac{2\mu R}{V^2} + R^2$, there will be a collision. The maximum value of p for a collision to be possible is thus determined as a function of the initial relative velocity. Let it be denoted by P.

Now suppose the distribution of meteoric velocities to be a Maxwellian one about a mean, the velocity of the planet relative to this mean being U; we have

$$f(u, v, w) = a^3\pi^{-\frac{3}{2}} \exp\{-a^2(u^2+v^2+w^2)\}, \quad .$$

and the mean value of P^2 is given by

$$[P^2]_{\text{mean}} = a^3\pi^{-\frac{3}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{R^2 + \frac{2\mu R}{V^2}\right\}e^{-a^2(u^2+v^2+w^2)}du\,dv\,dw \quad (15)$$

where V^2 is now $(U-u)^2 + v^2 + w^2$.

Put

$$\left.\begin{aligned} U - u &= V\cos\chi \\ v &= V\sin\chi\sin\psi \\ w &= V\sin\chi\cos\psi \end{aligned}\right\} \quad . \quad . \quad . \quad (16)$$

and $\cos\chi = l$.

Then this becomes

$$a^3\pi^{-\frac{3}{2}}\int_0^{\infty}\int_{-1}^{1}\int_0^{2\pi}\left(R^2 + \frac{2\mu R}{V^2}\right)\exp\{-a^2(V^2 - 2UVl + U^2)\}V^2\,dV\,dl\,d\psi$$

$$= a\pi^{-\frac{1}{2}}\int_0^{\infty}\frac{(R^2V^2 + 2\mu R)}{UV}[\exp\{-a^2(V-U)^2\} - \exp\{-a^2(V+U)^2\}]dV$$

$$= \frac{\pi^{-\frac{1}{2}}R}{c}\int_0^{\infty}\left(Rz + \frac{2\mu a^2}{z}\right)\{e^{-(z-c)^2} - e^{-(z+c)^2}\}dz$$

where $aV = z$, $aU = c$.

$$\int_0^\infty Rz\{e^{-(z-c)^2} - e^{-(z+c)^2}\}dz = cR\pi^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (17)$$

$$\int_0^\infty \frac{dz}{z}\{e^{-(z-c)^2} - e^{-(z+c)^2}\}dz = 2e^{-c^2}\int_0^\infty \frac{dz}{z}\sinh(2cz)e^{-z^2}$$

Expanding sinh $2cz$, we get *

$$2e^{-c^2}\left[2c\int_0^\infty e^{-z^2}dz + \frac{(2c)^3}{3\,!}\int_0^\infty z^2e^{-z^2}dz + \quad . \quad . \quad . \quad . \right]$$

$$= 2\pi^{\frac{1}{2}}e^{-c^2}\left[c + \frac{c^3}{3} + \frac{c^5}{2\,!\,5} + \frac{c^7}{3\,!\,7} + \quad . \quad . \quad . \quad . \right]$$

$$= 2\pi^{\frac{1}{2}}e^{-c^2}\int_0^c e^{x^2}dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Then

$$[P^2]_{\text{mean}} = R^2 + 4\mu a^2 R \cdot \frac{1}{c}e^{-c^2}\int_0^c e^{-x^2}dx \quad . \quad . \quad (19)$$

Putting for c its value ·339, found already, this gives

$$[P^2]_{\text{mean}} \div R^2 = 1 + 3{\cdot}62ga^2R \quad . \quad . \quad . \quad (20)$$

Now $2g$R is the square of the velocity of a body at the earth's surface that would make it leave the earth. This is known to be about 11 kms./1 sec. The information has already been obtained that aU = ·339, while U is a considerable fraction of the earth's orbital velocity, as may be seen from the great radial velocities of meteors. Hence $1/a$ is not likely to be less than 40 kms./1 sec. Thus the number of meteors picked up by the earth is increased by less than a tenth of its amount when gravity is taken into account, so that the result already stated, that the majority of the meteors in the solar system must move in direct orbits, is not affected by the fact that gravity was omitted in calculating the value of aU.

III. The Planetesimal Hypothesis.

The Nebular Hypothesis of Laplace.

This hypothesis supposed the solar system to have formerly consisted of a mass of gas, extending beyond the orbit of Neptune, and rotating like a rigid body. In the process of condensation the mass would contract, its angular velocity would correspondingly increase, and ultimately become too great for the equatorial portion to be retained by gravity. This would then be left behind in the form of a ring, which, it was believed, would subsequently collect into a planet. By the repetition of the process the planets were supposed to have been formed; in a

* The method is the same as is employed by Bromwich, *Infinite Series*, p. 473, ex. 33.

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similar way the satellites might have been formed from the planets. This hypothesis has been criticised on several grounds, notably by Professor F. R. Moulton.

(*a*) In the first place, the angular momentum of the system, when the nebula extended to the orbit of a particular planet, should correspond to an angular velocity just sufficient to make the centrifugal force balance gravity at the equatorial boundary, in order that any matter should be detached. Now, as angular momentum is a constant, it is possible to calculate what the speed of rotation must have been when the gaseous mass extended to any planetary orbit, and it is actually found to be very much too small to cause any detachment. This criterion has been stated in various forms, first by Babinet, and later by Moulton * and See.† Poincaré considered ‡ that this objection could be met by supposing that very much the greater part of the mass was condensed to a central point before the annulation commenced. The total mass of the surrounding nebula could only be about $\frac{1}{800}$ of the mass of the sun, so that practically the whole of it must have condensed into planets.

(*b*) As contraction would be a continuous process, it follows that instead of the detached matter forming definite rings, it would produce a continuous system of molecules, moving in elliptical orbits, apart from collisions.

(*c*) A ring of matter of the type described by Laplace could not condense into a planet. If it condensed at all, it would produce several planets of about the same size and with the same mean motion, and these could not unite into one.

(*d*) On Laplace's hypothesis every planet must rotate more rapidly than any of its satellites revolve. This condition is violated by Phobos, the inner satellite of Mars. Sir G. H. Darwin's work on tidal friction explained this anomaly on the hypothesis that the solar tides in Mars have slackened Mars's rotation so much as to make its period longer than the period of revolution of Phobos. A similar objection, however, has been again raised by Moulton, who points out that the same condition is violated by the particles that compose the inner edge of Saturn's ring, where tidal friction can scarcely be of sufficient importance to account for the discrepancy.

(*e*) All satellites ought to revolve in the same direction as their primaries rotate. This condition is not satisfied by the outermost satellite of Saturn, and the two outermost satellites of Jupiter. If planets originally had a retrograde rotation, and this was subsequently reversed by tidal friction, these facts might be accounted for; so likewise might the retrograde revolutions of the satellites of Uranus and Neptune, but it remains doubtful whether tidal friction is of sufficient importance at the distances of Jupiter and Saturn to have reversed their rotations. Mr.

* *Astrophysical Journal*, vol. xi., 1900, p. 126.
† *Ast. Nach.*, vol. clxxx., No. 4308, p. 185, 1909.
‡ *Leçons sur les Hypothèses Cosmogoniques*, p. 18.

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J. Jackson* has pointed out that at a great distance from a planet a retrograde satellite orbit would be less perturbed by the sun than a direct one would be. This gives an adequate reason for the persistence of these orbits, when once they had commenced, but how they were produced in the first place remains unexplained.

The Planetesimal Hypothesis.

Lockyer, in his work *The Meteoritic Hypothesis*, suggested that nebulæ were composed of swarms of meteors, moving irregularly, and liable to frequent collisions. A similar hypothesis has been advanced by M. du Ligondès. It seems, however, that such a swarm of meteors would behave very like a gas,† and that its method of evolution would also not be very different. It indeed seems likely that it would actually turn into a gaseous nebula very soon.

A hypothesis advanced more recently by Chamberlin and Moulton,‡ called the "Planetesimal Hypothesis," seems open to fewer objections than the older theories. The original nebula is supposed to be of spiral form, and composed of solid or liquid particles called planetesimals moving freely in elliptic orbits about a large nucleus, the two arms of the spiral coinciding roughly with the locus of the aphelia. Several large "knots," or parts of excessive density, were supposed to lie on the arms. In their revolution these are considered to have swept up most of the small particles, and in the process to have developed a direct rotation, for the most part, and to have had the eccentricities of their orbits reduced. The first four objections offered against the theory of Laplace do not arise against this hypothesis. The satellites, according to this theory, grew from smaller nuclei close to the large planetary nuclei. It is stated by Moulton on p. 481 of his *Astronomy* (*infra*) that those with retrograde revolutions would be more resisted by the scattered material than would the direct moving ones, that their orbits would consequently shrink more rapidly, and that they would ultimately be for the most part precipitated on their primaries. From this point of view it seems remarkable that all the known retrograde satellites are on the *outside* of their systems. If the eccentricities were small, it seems more likely that in the neighbourhood of a planet most of the planetesimals nearer the sun would be moving faster than the planet, and those further away would be moving more slowly. The medium as a whole, then, in the neighbourhood of a planet would have a retrograde motion relative to that planet. The retrograde satellites would then approach the primaries less rapidly than would the direct ones, and might even recede. Their present positions can thus be explained on the planetesimal hypothesis

* *Monthly Notices*, vol. lxxiv., 1913, p. 69.

† Sir G. H. Darwin, *Sci. Papers*, vol. iv., or *Phil. Trans.*, 1888.

‡ Chamberlin and Salisbury, *Textbook of Geology*, vol. ii.; Moulton, *Introduction to Astronomy*.

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more easily than by Moulton's method, which assumes the eccentricities of the planetesimal orbits to have been fairly great throughout the process, and gives exactly the opposite result.

The difficulty in the way of accepting the hypothesis in its present form arises from the statement that the planetesimals were solid or liquid particles. The spiral form of the nebula obviously could not have been retained for more than a few years at the outside, unless the orbits were highly eccentric. Now let us consider the probable course of events when two solid bodies impinge. When the relative velocity along the normal at the point of incidence is of the order of magnitude that we ordinarily produce in the laboratory, the two bodies rebound, with a smaller relative velocity than before impact. A greater velocity, about that of a rifle bullet, may cause partial fusion, and in this case the bodies may unite into one. With still higher velocities, enough energy may be transformed into heat to convert the bodies into vapour.

Let each of the two bodies be of mass m, and let the relative velocity along the normal be $2v$. Then the energy liberated if they unite is mv^2, so that each unit of mass will receive an amount of heat equal to $v^2/2J$, where J is the mechanical equivalent of heat. Now 400 calories per gram would be sufficient to volatilise most solids. Thus the value of v necessary to vaporise may be taken to be 6 kilometres per second, a comparatively small planetary velocity.

Next, let M and (UVW) be the mass and components of velocity of a large body, and m the mass of a smaller body whose component velocities are (u, v, w), the axis of U being along the normal at the point of contact. If they impinge and unite, the velocity afterwards has components $\left(\frac{MU+mu}{M+m}, \frac{MV+mv}{M+m}, \frac{MW+mw}{M+m}\right)$.

It is then easy to verify that if the bodies were moving before impact in anything like the same direction, which is a condition of the planetesimal hypothesis, the resultant velocity after impact would be intermediate between those before impact, and consequently the major axis of the orbit of the combined body would be between those of the components before their union. This seems to be necessarily true in the great majority of cases. Thus if the planet Mercury was formed according to the planetesimal manner, a large number of planetesimals must have formerly existed whose orbits had major axes less than that of Mercury. Similarly many must have existed whose mean distances were greater than that of Neptune. Further, the orbital eccentricities of the planets, and also those of the planetesimals, must have been large, probably greater than 0·5 originally.

Under these conditions the relative velocity of impact between planetesimals near the orbit of Mercury must have been of the same order of magnitude as half that planet's orbital velocity, or 20 kilometres per second, even in the case postulated by Chamberlin and Moulton, that the collisions were all overtakes. This is above

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the velocity required for vaporisation, by what has already been shown. Indeed, observations of meteors at the surface of the earth are by themselves adequate to show that impact at such a velocity would volatilise the bodies concerned. The condition that all the collisions are overtakes seems itself to be very uncertain. On account of the attractions of the planetary nuclei, the perihelia and nodes of the planetesimals must have advanced or retrograded, and it is highly improbable that all would move at the same rate. Thus after a time comparable with 100,000 years the original spiral state would be converted into a state where the motions of the particles near any point would be in widely different directions. Now, as the probability of a collision between two particles is greater the greater is their relative velocity, within certain limits, it follows that in most impacts taking place after that interval the relative velocity must have been comparable with the velocity of a particle in a circular orbit at the same point. Thus in nearly all impacts occurring within the orbit of Jupiter the planetesimals must have turned to gas.

A possible objection to the above argument is that collisions between planetesimals must have been very rare, and consequently that the planets would have time to go through their evolution before the swarm was destroyed. This, however, does not appear to be the case. Let a body of class A be one whose component velocities lie between u and $u+du$, v and $v+dv$, w and $w+dw$ respectively, and one of class B be one whose velocities are between u' and $u'+du'$, v' and $v'+dv'$, w' and $w'+dw'$ respectively, and the number of bodies of class A be $\nu f(uvw)du\,dv\,dw$, where ν is the number of bodies per unit volume. Further, let both bodies be spherical, for greater definiteness, and let (l, m, n) be the direction cosines of the normal at the point of contact, supposed to lie within a small solid angle $d\omega$ at the centre of the first. Then the number of collisions occurring in unit time in unit volume, one body belonging to class A and the other to class B, and the normal lying inside $d\omega$, is

$$\nu^2 f(uvw) f(u'v'w') \mathrm{F}\sigma^2\, du\, dv\, dw\, du'\, dv'\, dw'\, d\omega$$

where σ is the distance between the centres of the bodies and

$$\mathrm{F} = l(u-u') + m(v-v') + n(w-w'),$$

and is thus the normal component of the relative velocity.

Now, in the case of a collision between planetesimals, σ is practically the mean of their diameters, but when a planet picks up a planetesimal, σ is the radius of the planet. Then suppose we consider any particle of class A, its expectation of a collision in unit time with another planetesimal is

$$\iiiint \nu f(u'v'w') \mathrm{F}\sigma^2 du'\,dv'\,dw'\,d\omega.$$

As $\iiint f(uvw)du\,dv\,dw = 1$, this is of the same order of magnitude as $4\pi\nu\sigma^2\sqrt{u^2+v^2+w^2}$. Again, the expectation of

collision with a planet in unit time is similarly of the order of $4\pi Na^2\sqrt{u^2+v^2+w^2}$, velocities being measured relatively to the movement in a circular orbit at the point considered, and $1/N$ denoting the volume of the region that the hypothesis requires the planet to sweep out.

If then each planetesimal has a diameter 10^{-4} of that of the planet which is likely to absorb it, and the increase of the mass of the planet by accretion is to be as little as 10^{-3}, both these estimates being very conservative, we see on averaging through the region just mentioned that $\nu/N = 10^{-3}a^3/\sigma^3$, each side being the number of particles within it, so that the number of collisions between planetesimals is about ten times the number of planetesimals picked up by the planet. If the planetesimals are to be smaller and the amount of growth greater, the importance of collisions between planetesimals must be greater still, so that there seems to be little doubt that if eccentricities were at all high at the commencement the planetesimal system must have turned to gas long before the planets could grow appreciably by accretion.

It may be remarked that as in impact the relative velocity of the bodies is converted into molecular motion, and the mass is the same before and after collision, the average molecular velocity in the gas produced by impact must be less than a planetary velocity, and consequently very few molecules could leave the system. The system would then presumably degenerate to something not unlike a Laplacian nebula with a very strong central condensation and some pronounced secondary nuclei. This nebula would act on the planetary nuclei as a resisting medium would, and the eccentricities of their orbits would be diminished secularly in the well-known manner.

An alternative to the hypothesis of complete vaporisation is that the colliding particles may be broken up into solid fragments, which would then fly off with velocities comparable with the relative velocity before impact. As, however, the fragments would together have a surface greater than the original particles, and likewise similar velocities relative to each other, it follows that in the degenerate nebula collisions would be much more frequent than in the original one; thus the process would be repeated more and more rapidly until the conversion to vapour was complete. This alternative, therefore, only postpones the difficulty without avoiding it.

The Case of Initially Small Eccentricities.

If all the particles were initially moving in nearly circular orbits in one plane, the relative velocity at impact would be small and they would not be volatilised. In such a case as this it would apparently be impossible to regard existing meteors with their high eccentricities as a survival of the planetesimals, and they must be supposed to be of later origin. As before, collisions between the small bodies would cause their orbits to tend rapidly

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to the circular form, whether their collisions took the form of imperfectly elastic rebounds or accretions. The spiral structure is in this case impossible; the eccentricities of the planetary orbits, though small, are still supposed to exist, but their squares are neglected. This hypothesis has the advantage over the standard form of the planetesimal hypothesis that retrograde satellite orbits are more easily accounted for; the most evident defect is that it resembles no known form of system, and its inception would be difficult to account for, unless it arose from the degeneration of a Laplacian nebula.

Consider a planet J moving in such a system of planetesimals. Let the major axis of its orbit be a, and its mean motion n. Further, let μ be the product of the gravitational constant and the mass of the sun.

Let S be the sun. From S draw a line SO whose length is a, moving about S with constant angular velocity n, and nearly coinciding initially with SJ. Let its production OX be the axis of X, and take the axis of Y in the plane of the system. If x and y be the co-ordinates of J relative to these moving axes, its real velocities are $\dot{x} - ny$, $na + \dot{y} + nx$.

The equations of motion are then

$$\left.\begin{aligned} \frac{d}{dt}(\dot{x} - ny) - (na + \dot{y} + nx)n &= \frac{\partial}{\partial x}\left(\frac{\mu}{r}\right) + \mathrm{X} \\ \frac{d}{dt}(\dot{y} + nx + na) + (\dot{x} - ny)n &= \frac{\partial}{\partial y}\left(\frac{\mu}{r}\right) + \mathrm{Y} \end{aligned}\right\} \quad . \quad . \quad (1)$$

where $r = \mathrm{SJ}$, and X and Y are the components of the frictional acceleration. They are proportional to the velocities of J relative to the planetesimals in its immediate neighbourhood.

Now if in a short time dt J picks up a mass dm, and if the mean velocity of the particles constituting dm was $(\bar{u}, \bar{v})$ before impact, then the gain of momentum is $(\bar{u}\,dm, \bar{v}\,dm)$. The gain of mass is dm. Let (U, V) be the velocity of J. Then the change of velocity that friction would produce if acting alone has for its x-component the value

$$\frac{m(\mathrm{U}) + \bar{u}dm}{m + dm} - \mathrm{U} = -(\mathrm{U} - \bar{u})\frac{dm}{m} \text{ to the first order.}$$

Hence the accelerations due to accretion are

$$\left\{-\frac{(\mathrm{U} - \bar{u})}{m}\frac{dm}{dt}, \ -\frac{(\mathrm{V} - \bar{v})}{m}\frac{dm}{dt}\right\} \quad . \quad . \quad (2)$$

If all the particles picked up were moving in exactly circular orbits, then the velocities of all of them would be the same, namely, the circular velocity. Actually, however, their velocities are distributed about this value as a mean. If the planet is moving more rapidly than the mean of the particles, then it tends to pick up more of those particles moving less rapidly than the mean than of those moving more rapidly than the mean; so that

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the accelerations will be underestimated if we put $(\bar{u}, \bar{v})$ equal to the circular velocities. It has been shown that if the velocity distribution were Maxwellian about the mean, and the planet's velocity relative to the mean were small, the actual accelerations would be four times those given on the hypothesis here adopted.

Now the mean velocity of the particles at J is $\sqrt{\mu/r}$ in the direction at right angles to SJ, whose direction cosines are $\left(-\frac{y}{r}, +\frac{a+x}{r}\right)$.

Its x-component is then

$$-\frac{\mu^{\frac{1}{2}}y}{\{(a+x)^2+y^2\}^{\frac{3}{4}}} = -\frac{\mu^{\frac{1}{2}}y}{a^{\frac{3}{2}}} = -ny \text{ to the first order,}$$

and its y-component is

$$\frac{\mu^{\frac{1}{2}}(a+x)}{\{(a+x)^2+y^2\}^{\frac{3}{4}}} = na - \tfrac{1}{2}nx \quad . \quad . \quad . \quad (3)$$

We can then write for the accelerations X and Y the values

$$-k\dot{x}, \ -k(\dot{y}+\tfrac{3}{2}nx), \text{ where } k \text{ is greater than } \frac{1}{m}\frac{dm}{dt} \quad . \quad (4)$$

Again,

$$\frac{1}{r} = \frac{1}{\{(a+x)^2+y^2\}^{\frac{1}{2}}} = \frac{1}{a}\left(1 - \frac{x}{a} + \frac{x^2}{a^2} - \frac{1}{2}\frac{y^2}{a^2}\right) \text{ to the second order,}$$

and thus the equations of motion become

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} - n^2a - n^2x &= -\frac{\mu}{a^3}(a-2x) - k\dot{x} \\ \ddot{y} + 2n\dot{x} - n^2y &= -\frac{\mu y}{a^3} - k(\dot{y}+\tfrac{3}{2}nx) \end{aligned}\right\} \quad . \quad . \quad (5)$$

These reduce to

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} - 3n^2x + k\dot{x} &= 0 \\ \ddot{y} + 2n\dot{x} + k(\dot{y}+\tfrac{3}{2}nx) &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

These are the equations of motion of a dissipative system in which

$$\left.\begin{aligned} 2\text{R} &= \dot{x}^2 + \dot{y}^2 + 2nx\dot{y} - 2ny\dot{x} + 3n^2x^2 \\ 2\text{F} &= k\dot{x}^2 + k\dot{y}^2 + 3nkx\dot{y} \end{aligned}\right\} \quad . \quad . \quad (7)$$

In accordance with a theorem already proved,* if these equations have a solution in which x and y are proportional to $e^{\gamma t}$ when k is treated as constant, then they will have a solution proportional to exp $\int\gamma dt$ when k is variable. The period equation is then

$$\begin{vmatrix} \gamma - 3n^2 + k\gamma & -2n\gamma \\ 2n\gamma + \tfrac{3}{2}nk & \gamma^2 + k\gamma \end{vmatrix} = 0,$$

* *Monthly Notices*, vol. lxxv., 1915, p. 577.

which reduces to

$$\gamma^2(\gamma^2 + n^2 + 2k\gamma) = 0 \qquad . \quad . \quad . \quad (8)$$

The first factor, giving $\gamma = 0$, represents the motion in an orbit of slightly different major axis and epoch, and does not concern us. The other gives

$$\gamma = \pm \iota n - k \text{ to the first order} \qquad . \quad . \quad . \quad (9)$$

Hence the amplitude of the oscillation about a circular orbit is proportional to $\exp \int -k\,dt$. Also there is no secular change in the mean distance to this order.

We thus see that the eccentricity of a planetary orbit is proportional to $\exp \int -k\,dt$, and it therefore diminishes with the time more rapidly than

$$\exp \int -\frac{1}{m}\frac{dm}{dt}\,dt, \text{ or } \frac{1}{m} \qquad . \quad . \quad . \quad . \quad (10)$$

Hence if a planet picks up sufficient material to double its mass, its eccentricity is reduced to less than half its former amount.

Now the eccentricity of Jupiter's orbit at present is 0·048. Hence if its mass had ever been so small as $\frac{1}{20}$ of its present amount, its eccentricity must have been so large that the present approximation would break down. In other words, on this hypothesis of initially nearly circular orbits Jupiter can never have been smaller than Uranus is now. Planetary perturbations cannot account for the present eccentricity of Jupiter's orbit, as it was shown by Laplace that $\Sigma m a^{\frac{1}{2}} e^2$ for all the planets cannot be changed by their mutual perturbations, and most of this quantity is due to Jupiter.

This form of the planetesimal hypothesis might then be satisfactory if we could explain how the large planetary nuclei required by it originally came to exist, moving in very eccentric orbits, and with mean motions possibly not very different from their present values.

Further Note on Lagrange's Three Particles.

In my former paper* "On the Effect of a Resisting Medium on Lagrange's Three Particles" it was shown that the motion of the third particle is secularly stable, on the hypothesis that the velocity of the medium itself is constant through space. This hypothesis appears to agree with that made by Dr. See in the Capture Theory, as he considers the major axes of the orbits to be secularly diminished by the resistance, which seems to imply that he considers the medium to be at rest. In the case just considered, however, the velocity is a function of the position, so that if we take the same axes as were used in the previous paper, and (u, v, w) are the velocities of the medium at (x, y, z), we have $u = -ny$, $v = n\mathrm{R} - \frac{1}{2}nx$, $w = 0$. Further, n is in this case a constant.

* *Monthly Notices*, vol. lxxv., 1915, p. 580.

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The velocities of the particle relative to fixed axes instantaneously coinciding with the movable ones are $\dot{x}-ny$, $\dot{y}+nx$, $\dot{z}$. The equations of motion in a non-dissipative system being

$$\left.\begin{aligned} \ddot{x}-2n\dot{y} &= ax+hy \\ \ddot{y}+2n\dot{x} &= bx+hy \\ \ddot{z} &= -n^2z \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (11)$$

we see that in this case the equations must be

$$\left.\begin{aligned} \ddot{x}-2n\dot{y} &= ax+hy-\kappa\dot{x} \\ \ddot{y}+2n\dot{x} &= hx+by-\kappa(\dot{y}+\tfrac{3}{2}nx) \\ \ddot{z} &= -n^2z-\kappa\dot{z} \end{aligned}\right\} \quad . \quad . \quad . \quad (12)$$

The last equation is independent of the other two, and hence the vibration in latitude dies down in such a way that its amplitude is proportional to $\exp -\frac{1}{2}\int\kappa\,dt$.

In the other two, suppose x and y proportional to $\exp\int\gamma\,dt$. Then γ is given by the equation

$$(\gamma^2-a)(\gamma^2-b)+4n^2\gamma^2-h^2+\kappa\{2\gamma^3-(a+b)\gamma+\tfrac{3}{2}n(2n\gamma+h)\}=0.$$

Now $a+b=3n^2$, and $ab-h^2=27n^4\mathrm{SJ}/8(\mathrm{S}+\mathrm{J})^2$.

Substituting these values we find,

$$\gamma^4+n^2\gamma^2+\frac{27n^4\mathrm{SJ}}{8(\mathrm{S}+\mathrm{J})^2}+\tfrac{3}{2}nh\kappa+2\kappa\gamma^3=0 \quad . \quad . \quad (13)$$

If κ were zero, the solutions would be $\gamma=\pm\iota\alpha$ and $\pm\iota\beta$, say.

Suppose then $\gamma-\iota\alpha$ to be small. The second approximation is found to be given by

$$\gamma-\iota\alpha=\frac{2\kappa\iota\alpha^3-\frac{3}{2}nh\kappa}{2\iota\alpha(\beta^2-\alpha^2)}.$$

The real part of γ is thus $\kappa\alpha^2/(\beta^2-\alpha^2)$ . . . . (14)

Now let α correspond to the more rapid vibration about the steady motion. Then α is not very different from n, while β is much smaller. We see then that as the amplitude of the more rapid vibration is proportional to $\exp -\alpha^2\int\kappa dt/(\alpha^2-\beta^2)$ and that of the slower one is by symmetry proportional to $\exp\beta^2\int\kappa dt/(\alpha^2-\beta^2)$, it follows that the amplitude of the more rapid vibration decreases steadily, while that of the slower one increases with the time. The motion is therefore secularly unstable when the resisting medium is supposed to revolve.

The criticisms of the Planetesimal Hypothesis that have here been made rest entirely on the supposition that the planetesimals were solid bodies. If, instead of being solid particles, they were discrete molecules, then the nebula was indistinguishable from a gaseous one, possibly at a low temperatnre, and not rotating like a rigid body. Such a nebula would thus partake of features of

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both the planetesimal nebula and the Laplacian nebula, and may combine advantages from both. A system of nuclei revolving in eccentric orbits within this would evolve to something not very different from the present solar system; it remains to be examined, however, whether these nuclei could have been small originally. If the relation that *em* decreases holds for this type of nebula as well as for the planetesimal one with nearly circular orbits, then the eccentricities of the present planetary orbits will be difficult to explain on this hypothesis, as they appear to be on almost any other. The further question of whether gravitational instability could occur in such a nebula as this requires to be discussed.

It scarcely seems likely that a system of this type would bear any resemblance to any known form of nebula. Viewed from α Centauri it would appear as a first-magnitude star, with a very faint, or even invisible, nebulosity round it; it would be enormously smaller than any nebula that is known to exist, and if any clue to the past history of the solar system is to be obtained from observations of bodies outside it, this hypothesis can be tested only by observations of nebulous stars, and not of nebulæ.

Appendix.

Note on a Type of Definite Integral.

In certain problems that have arisen earlier in this paper it has been found necessary to evaluate integrals of the type $I \equiv \iiint \phi(S)(lx+my+nz+p)dx\,dy\,dz$, where S is the general quadratic function of x, y, and z, denoted by $ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d$. The limits are defined by the condition that $lx+my+nz+p$ shall be positive; otherwise the integration is through all space. In all cases that have arisen the equation $S=0$ represents an ellipsoid. The axes are supposed rectangular.

In the first place, the centre of this ellipsoid is given by the equations

$$ax'+hy'+gz'+u=0.$$
$$hx'+by'+fz'+v=0.$$
$$gx'+fy'+cz'+w=0.$$

Putting $x-x'=X$, $y-y'=Y$, $z-z'=Z$, we have

$$S \equiv aX^2+bY^2+cZ^2+2fYZ+2gZX+2hXY+\Delta/D,$$

where Δ denotes the discriminant of S, and capital letters (ABCFGHD) denote the minors of the corresponding small letters in Δ. Write $\Delta/D=\delta$. There is no loss of generality if S is taken such that δ is always positive, so that S is positive inside the ellipsoid and negative outside.

Now let ξ denote the length of the perpendicular from the

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point (x, y, z) to the plane through the centre parallel to the plane $lx+my+nz+p=0$. Then $\xi = lX + mY + nZ$.

If $-\xi_0 = lx' + my' + nz' + p$, then

$$lx+my+nz+p = \xi - \xi_0.$$

ξ_0 is then given by the equation

$$\begin{vmatrix} l & m & n & p+\xi_0 \\ a & h & g & u \\ h & b & f & v \\ g & f & c & w \end{vmatrix} = 0$$

Now take as a new set of axes any pair of conjugate diameters in the plane $\xi = 0$ and the diameter conjugate to this plane. Let η and ζ be the distances measured along the axes in the plane $\xi = 0$. The co-ordinate ξ is, however, the perpendicular from the point to the plane $\xi = 0$, and not the distance measured along the ξ-axis. If we transform to co-ordinates (ξ, η, ζ), S becomes $\delta - \alpha\xi^2 - \beta\eta^2 - \gamma\zeta^2$, where α, β, and γ are constants. As the quadric is an ellipsoid, all have the same sign as δ, and there is no loss of generality in taking them all to be positive. Then the integral becomes

$$\int_{\xi_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(\delta - \alpha\xi^2 - \beta\eta^2 - \gamma\zeta^2)(\xi - \xi_0)d\xi\, d\eta\, d\zeta \frac{\partial(x, y, z)}{\partial(\xi, \eta, \zeta)}.$$

Putting $\xi = L\sqrt{(\delta/\alpha)}$, $\eta = M\sqrt{(\delta/\beta)}$, $\zeta = N\sqrt{(\delta/\gamma)}$, we have

$$I = \int_{L_0}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi\{\delta(1 - L^2 - M^2 - N^2)\}(L - L_0)dL\,dM\,dN\frac{\delta^2}{\alpha(\beta\gamma)^{\frac{1}{2}}}\frac{\partial(x,y,z)}{\partial(\xi,\eta,\zeta)}$$

where $\xi_0 = L_0\sqrt{(\delta/\alpha)}$.

The numerical factor must now be evaluated. To do this, consider the plane $lX + mY + nZ + q = 0$. This touches the quadric $S = 0$ if $(bc - f^2)l^2 + (ca - g^2)m^2 + (ab - h^2)n^2 + 2(gh - af)mn + 2(hf - bg)nl + 2(fg - ch)lm + Dq^2/\delta = 0$.

Now the appropriate values of q must be $\pm(\delta/\alpha)^{\frac{1}{2}}$. Hence

$$-\frac{D}{\alpha} = (bc - f^2)l^2 + (ca - h^2)m^2 + (ab - h^2)n^2 + 2(gh - af)mn + 2(hf - bg)nl + 2(fg - ch)lm.$$

This determines α.

Now an element of volume $= dx\, dy\, dz$

and also $= d\xi\, d\eta\, d\zeta \sin\psi$,

where ψ is the angle between the conjugate diameters chosen for axes of η and ζ. Hence

$$\frac{\delta(x, y, z)}{\delta(\xi, \eta, \zeta)} = \sin\psi.$$

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Now $\dfrac{\delta}{\sqrt{(\beta\gamma)}}\sin\psi$ = area of the parallelogram with the semi-conjugate diameters as adjacent sides,
$=\frac{1}{8}$ of the volume of a parallelopiped circumscribing the ellipsoid $\div\frac{1}{2}$ the thickness of this parallelopiped perpendicular to the plane $\xi=0$.

But $\frac{1}{8}$ of the volume of this parallelopiped $=\alpha_1\beta_1\gamma_1$, where α_1, β_1, and γ_1 are the principal semi-axes of the ellipsoid. We find at once that $\alpha_1\beta_1\gamma_1=(-\delta^3/D)^{\frac{1}{2}}$. Also the divisor $=(\delta/\alpha)^{\frac{1}{2}}$.

Hence we find on substitution that

$$\frac{\delta^2}{\alpha(\beta\gamma)^{\frac{1}{2}}}\frac{\delta(x,y,z)}{\delta(\xi,\eta,\zeta)}=\frac{\delta^2}{(-\alpha D)^{\frac{1}{2}}}.$$

Finally, by a transformation of the variables M and N, the integral is reduced to the form

$$I=\frac{2\pi\delta^2}{(-\alpha D)^{\frac{1}{2}}}\int_{L_0}^{\infty}\int_0^{\infty}\phi\{\delta(1-L^2-P^2)\}(L-L_0)P\,dL\,dP.$$

By putting $P^2=R^2-L^2$, it is possible to reduce this to a single integral, equal when L_0 is positive to

$$\frac{\pi\delta^2}{(-\alpha D)^{\frac{1}{2}}}\int_{L_0}^{\infty}(R-L_0)^2\phi\{\delta(1-R^2)\}R\,dR.$$

When L_0 is negative, equal to $-H_0$, the limits become more complicated, and the integral is

$$\frac{\pi\delta^2}{(-\alpha D)^{\frac{1}{2}}}\left[\int_{H_0}^{\infty}(R+H_0)^2\phi\{\delta(1-R^2)\}R\,dR-4\int_0^{H_0}L_0R^2\phi\{\delta(1-R^2)\}dR\right].$$

In practice, however, it appears to be often simpler to evaluate the double integral directly by integrating with regard to P than to use this single integral.

In the general case it is not possible to make any further progress without specifying the form of ϕ, which at present is quite arbitrary except that it must be such that no logical difficulties arise in the transformations of the infinite integrals. In the two particular cases that have arisen the integration can be completed.

(1) Let $\phi(S)=\rho\exp S$.

Then it is evident that, on account of the rapidity with which $\exp S$ tends to zero when any of X, Y, and Z becomes large, all the previous transformations are justified in this case. Hence

$$I=\frac{\rho\pi\delta}{\sqrt{(-\alpha D)}}\left\{\frac{1}{2\delta}e^{-\delta L_0^2}-\frac{L_0}{\sqrt{\delta}}\frac{\pi^{\frac{1}{2}}}{2}(1-\mathrm{Erf}L_0\sqrt{\delta})\right\}.$$

(2) Let $S=\delta-aX^2-bY^2-cZ^2$, and $\phi(S)=\rho S^k$ when S is positive, and zero when S is negative.

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Then
$$\frac{1}{a}=\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{e}.$$

$$I=\frac{\pi\rho\delta^{k+2}}{(k+1)\sqrt{(abca)}}\left[\frac{(1-L_0^2)^{k+2}}{2(k+2)}-L_0\int_0^{\theta_0}\sin^{2k+3}\theta\,d\theta\right]$$

where $\cos\theta_0=L_0$ and θ_0 is between 0 and π.

The Secular Perturbations of the Four Inner Planets. By Harold Jeffreys, B.A., M.Sc.

It was shown by Newcomb* that the observed secular variations in the orbits of the inner planets differ slightly from those calculated from the law of gravitation, the best available values of the masses being adopted. The principal discrepancies are in the movements of the perihelion of Mercury and the node of Venus, but certain others also have notable values. Two important attempts have been made to account for the differences. The first was by Seeliger,† who assumed that about the sun are two superposed oblate homogeneous ellipsoids of gravitating matter. In addition he assumed that the axes of reference, which are fixed relative to the mean of the stars observed, are actually rotating in the plane of the ecliptic. By means of a method of least squares he found the best values of the densities of the ellipsoids and the rotation; all the residuals, with the exception of those in the eccentricity of Mercury and Mars, the explanation of which does not seem to have been attempted by anyone, were less than their mean errors. It thus appeared very satisfactory. Nevertheless, it seems to require serious modification, on account of the doubtful character of some of the physical hypotheses involved.

(*a*) The rotation found appears too great to be possible. De Sitter‡ found, however, that it is not necessary to the theory, for on rejecting it he obtained a satisfactory representation by means of the ellipsoids alone.

(*b*) The density of the outer ellipsoid, which extends outside of the orbit of the earth, was found by De Sitter to be $3{\cdot}7\times10^{-15}$, and by Seeliger to be $3{\cdot}1\times10^{-15}$ of that of the sun. It will be shown later that this density, small as it seems, is greater than can actually be allowed.

On the other hand, Einstein§ has recently, by means of a form of the Principle of Relativity, given an excellent representation of the perihelion of Mercury and a fair one of that of Mars; but

* *Astronomical Constants*, p. 109.

† *Sitzungsber. K. Bayer. Akad.*, *Math.-Phys. Klasse*, 1906, p. 595.

‡ *Observatory*, vol. xxxvi., 1913, p. 296. This also gives an account of the earlier work on the subject.

§ *Annalen der Physik*, Bd. xlix., 1916, p. 769.

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naturally no hypothesis of this type can affect the position of the plane of an orbit, so that the empirical part of the motion of the node of Venus remains unexplained. For this reason it appears desirable to try to find the distribution of gravitating matter that will best account for the changes in the inclinations and nodes, for it seems impossible to account for them otherwise; this can then be used to determine the motions of the perihelia, to ascertain how much of these can be accounted for by such means. The disturbing matter must be assumed to be close to the sun, as the density near the orbit of Mercury and outside it will be shown to be so small that it can have no perceptible effect. When this is done, the distribution that gives the best approximation to the movements of the planes of the orbits is capable of causing an advance of the perihelion of Mercury $1\frac{1}{2}$ times as great as the observed excess.

The Density of the Matter causing the Zodiacal Light.—Seeliger adopted the special dimensions that he chose for his outer ellipsoid because the observed phenomenon of the zodiacal light indicates the presence of matter as far out as he indicated, and in particular between the orbits of Mercury and the earth. Within the orbit of Mercury the observations are not so decisive, as they must be made so soon after sunset or before sunrise that matter there is practically invisible. Presumably, however, the density increases rapidly towards the sun, as Seeliger supposed. The nature of the substance causing the light is uncertain, but it may be supposed either to be composed of particles of meteoric type, large compared with the wave-length of light, or of very fine dust or gaseous matter, the particles being small compared with the wave-length. The treatment of the problem is different in the two cases.

In the first case, let the number of particles per unit volume be ν, and their mean diameter σ. Then it was shown * from the observed luminosity of the zodiacal light that the value of $\nu\sigma^2$ is about $3.5 \times 10^{-22} \div 1$ cm. for the neighbourhood of the orbit of Venus. The determination was not an accurate one, but the true value is not likely to be more than 10 times this or less than $\frac{1}{10}$ of it. The density is $\frac{1}{6}\pi\rho\nu\sigma^3$, where ρ is the density of a single particle. Now, putting this equal to De Sitter's density of 3.7×10^{-15} of that of the sun, we see that the above value of $\nu\sigma^2$ would require the diameters of the particles to be about 10^7 cm. This value is so great that the particles would appear as small separate planets instead of as a continuous sheet. Any size that would be acceptable, taken in conjunction with the above value of $\nu\sigma^2$, would make the density so small that it is quite evident that the large particles that may help to cause the zodiacal light must be so few that their effect on the orbits of the planets must be hundreds of times less than was required of the outer ellipsoid.

If the particles are small compared with the wave-length, the

* P. 94.

problem is considerably altered. The light is no longer diffusely reflected from the surface, but is scattered instead.* The intensity is now proportional to $\nu\sigma^6$ instead of $\nu\sigma^2$, and a comparison of the theory with observation may be derived from the scattering of light by the earth's atmosphere. If the diameters of the particles were the same in the two cases, the amounts of light received from equal small angles, first, of the blue sky in daytime, and second, of the zodiacal light, would be proportional to the masses of columns of equal cross-section through the two structures considered. This ratio, on Seeliger's hypothesis, would be $1 \cdot 2 \times 10^4$. The observed ratio of the luminosity of the sky to that of an equal area of the sun is given by L. V. King † to be 80×10^{-8} at the Mount Wilson level, which was considered sufficiently high above sea level for dust to be of no importance. The intensity of the light from the sky is therefore 8×10^6 of that of the zodiacal light, so that the latter is of the order of 600 times fainter than would be expected on the hypothesis that both it and the light from the sky are due to molecular scattering. Lord Kelvin's estimate ‡ gives values ranging from 4×10^{-7} to $2 \cdot 6 \times 10^{-7}$ for the ratio of sky to sun, which would reduce the above ratio to about 200. If the zodiacal light were caused by particles much larger than the molecules in the air, the discrepancy would be still greater, as $\nu\sigma^6$ would be greater for the same value of $\nu\sigma^3$. It seems very improbable then that the matter causing the zodiacal light can have a density in any way comparable with that of the outer ellipsoid, and its effect on the motions of the planets may accordingly be neglected.

The Secular Motions of a Planet consequent on an Oblate Distribution close to the Sun.—The hypothesis here adopted to give the distribution of the disturbing matter is a modification of that of Seeliger, so as to make it consistent with photometric observations. Any important perturbing mass must then be far inside the orbit of Mercury. The potential can then be expressed in terms of a series of spherical solid harmonics of the form

$$V = f\frac{M}{r} + \frac{S_2}{r^3} + \frac{S_3}{r^4} + \frac{S_4}{r^5} + \ldots,$$

where S_2, S_3, S_4, etc., are surface harmonics of the second, third, fourth, and higher orders respectively. Further, it can be seen that as it is highly improbable that a mass of this type is not rotating about an axis through the centre of the sun, the harmonics involved must be zonal. It can also be shown without much difficulty that odd harmonics can give rise to no secular terms in the motion of a planet that moves near the equatorial plane of the distribution. S_3 may therefore be omitted. In this investigation S_4 will not be considered, as its effect will be small on every planet except Mercury; if later observations show this to be

* R. W. Wood, *Physical Optics*, pp. 478–480.

† *Phil. Trans.*, 212A, 1912–13, p. 411.

‡ *Baltimore Lectures*, p. 314.

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desirable, however, this term will have to be included and determined with the rest.

Let now the sum of the masses of the sun and the perturbing matter be M,

the moments of inertia of the perturbing mass about its polar and equatorial axes be C and A,

the colatitude of any point, measured from the polar axis, be θ,

and put $\alpha = \frac{1}{2}(C - A)$.

α is positive when the distribution is oblate.

Then

$$V = f\frac{M}{r} + f\frac{\alpha(1 - 3\cos^2\theta)}{r^3}.$$

If now x, y, z be the Cartesian co-ordinates of a point, and (l, m, n) the direction cosines of the polar axis, the disturbing function is

$$R = f\alpha r^{-5}\{r^2 - 3(lx + my + nz)^2\}.$$

Now, in the usual notation of the planetary theory,

$$r = a(1 - e^2)/(1 + e\cos w),$$
$$x = r\{\cos\Omega\cos(v - \Omega) - \sin\Omega\sin(v - \Omega)\cos i\},$$
$$y = r\{\sin\Omega\cos(v - \Omega) + \cos\Omega\sin(v - \Omega)\cos i\},$$
$$z = r\sin(v - \Omega)\sin i.$$

If ω be the mean motion of the planet, so that $\omega = (\mu/a^3)^{\frac{1}{2}}$, the disturbing function is periodic in a time $2\pi/\omega$. The secular perturbations depend only on the secular part of R, which is the average of R taken with regard to the time. If this be S, we have

$$S = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} R\,dt,$$
$$= \frac{f\alpha}{a^3(1 - e^2)^{\frac{3}{2}}}\left[1 - \tfrac{3}{2}(l\cos\Omega + m\sin\Omega)^2 - \tfrac{3}{2}(-l\sin\Omega\cos i + m\cos\Omega\cos i + n\sin i)^2\right].$$

The perturbations are now given by the equations *

$$\frac{da}{dt} = \frac{2}{\omega a}\frac{\partial S}{\partial \epsilon},$$
$$\sin i\frac{d\Omega}{dt} = \frac{1}{\omega a^2\sqrt{(1 - e^2)}}\frac{\partial S}{\partial i},$$
$$\frac{d\varpi}{dt} = \frac{\sqrt{(1 - e^2)}}{\omega a^2}\frac{\partial S}{e\partial e} + \frac{\tan\frac{1}{2}i}{\omega a^2\sqrt{(1 - e^2)}}\frac{\partial S}{\partial i},$$
$$\frac{de}{dt} = -\frac{\sqrt{(1 - e^2)}}{\omega a^2 e}\frac{\partial S}{\partial \varpi} - \sqrt{(1 - e^2)}\frac{1 - \sqrt{(1 - e^2)}}{\omega a^2 e}\frac{\partial S}{\partial e},$$
$$\frac{di}{dt} = -\frac{1}{\omega a^2\sqrt{(1 - e^2)}\sin i}\frac{\partial S}{\partial \Omega} - \frac{\tan\frac{1}{2}i}{\omega a^2\sqrt{(1 - e^2)}}\left(\frac{\partial S}{\partial \varpi} + \frac{\partial S}{\partial \epsilon}\right),$$
$$\frac{d\epsilon}{dt} = -\frac{2}{\omega a}\frac{\partial S}{\partial a} + \frac{\tan\frac{1}{2}i}{\omega a^2\sqrt{(1 - e^2)}}\frac{\partial S}{\partial i} + \sqrt{(1 - e^2)}\frac{1 - \sqrt{(1 - e^2)}}{\omega a^2 e}\frac{\partial S}{\partial e}.$$

* Tisserand, *Mécanique Céleste*, t. 1, p. 369.

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We see at once that $\frac{da}{dt}$ and $\frac{de}{dt}$ are zero. In the other variations, the parts depending on the squares of the small quantities i, l, m can be omitted, except in the case of the perihelion of Mercury. In this case squares will be retained, but no higher powers.

$$\sin i \frac{d\Omega}{dt} = -\frac{3fa}{\omega a^5(1-e^2)^2}(\sin i - l \sin \Omega + m \cos \Omega),$$

$$\frac{di}{dt} = -\frac{3fa}{\omega a^5(1-e^2)^2}(l \cos \Omega + m \sin \Omega),$$

$$\frac{d\varpi}{dt} = \frac{3fa}{\omega a^5(1-e^2)^2}\left[1 - \tfrac{3}{2}(l - \sin i \sin \Omega)^2 - \tfrac{3}{2}(m + \sin i \cos \Omega)^2 - \tfrac{1}{2}\sin i(\sin i - l \sin \Omega + m \cos \Omega)\right].$$

Now choose the units so that the unit of length is the earth's mean distance from the sun, that of time is such that the earth's mean motion is unity, and the sum of the masses of the sun and earth is the unit of mass. Then $f = 1$, and for any planet $\omega = a^{-\frac{3}{2}}$.

The variations found from the last equations will then be expressed in radians per unit of time, whereas they are recorded by Newcomb in seconds of arc per century, the former unit being $1\cdot296 \times 10^8$ times the latter. It will therefore be found convenient to introduce a new quantity $\beta = 1\cdot296 \times 10^8 a$.

The following table gives the elements of the orbits of the four inner planets, with their unexplained secular variations in a century and their mean errors:—

Planet.	a.	e.	$\frac{3}{\omega a^5(1-e^2)^2}$.	Ω.	i.	$\sin i\delta\Omega$.	δi.	δe.	$e\delta\varpi$.
Mercury	·3871	·206	72·01	47° 8′ 45″	7° 0′ 10″	0″·61 ± 0″·51	0″·38 ± 0″·80	−0″·88 ± 0″·50	8″·48 ± 0″·43
Venus	·7233	·007	10·46	75 46 47	3 23 27	0·60 ± 0·17	0·38 ± 0·33	0·21 ± 0·31	−0·05 ± 0·25
Earth	1·0000	·017	3·002	...	0 0 0	...	−0·22* ± 0·27	0·02 ± 0·10	0·10 ± 0·13
Mars	1·5237	·093	·5557	48 47 9	1 15 1	0·03 ± 0·22	−0·01 ± 0·20	0·29 ± 0·27	0·75 ± 0·34

Let us first consider the motions of the planes of the orbits separately.

The secular changes in i and Ω give then the following conditional equations to determine the unknowns β, l, m:—

$$\left.\begin{aligned}(-49\cdot9l - 52\cdot7m)\beta &= 0\cdot38 \pm 0\cdot80\\ (+52\cdot7l - 49\cdot9m - 8\cdot79)\beta &= 0\cdot61 \pm 0\cdot51\end{aligned}\right\}\text{ from Mercury.}$$

$$\left.\begin{aligned}(-2\cdot57l - 10\cdot14m)\beta &= 0\cdot38 \pm 0\cdot33\\ (10\cdot14l - 2\cdot57m - 0\cdot62)\beta &= 0\cdot60 \pm 0\cdot17\end{aligned}\right\}\text{ from Venus.}$$

$$\left.\begin{aligned}(-\cdot363l - \cdot417m)\beta &= -0\cdot01 \pm 0\cdot20\\ (\cdot417l - \cdot363m - \cdot0121)\beta &= 0\cdot03 \pm 0\cdot22\end{aligned}\right\}\text{ from Mars.}$$

* This entry is not δi, but $\delta\epsilon$.

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The normal equations derived from these, weights being assigned according to the mean errors, are—

$$(36{\cdot}22l - 13{\cdot}17m - 4{\cdot}095)\beta - 0{\cdot}594 = 0.$$
$$(-13{\cdot}17l + 29{\cdot}30m + 3{\cdot}473)\beta + 0{\cdot}466 = 0.$$
$$(-4{\cdot}095l + 3{\cdot}473m + 0{\cdot}640)\beta + 0{\cdot}069 = 0.$$

From these we find $l = +0{\cdot}09503$; $m = -0{\cdot}09005$; $\beta = {\cdot}915$. Thus the equatorial plane of the disturbing matter is inclined at $7^\circ\ 31'$ to the ecliptic, and the longitude of its ascending node is $46^\circ\ 33'$. It therefore nearly coincides with the orbit of Mercury.

The values of the perturbations caused in a century by the distribution thus found are as follows, expressed in seconds of arc:—

Planet.	$\sin i\delta\Omega$.	δi.	$e\delta\varpi$.
Mercury	0·64	0·00	13·6
Venus	0·52	0·61	0·06
Earth	...	...	0·05
Mars	0·05	0·00	0·05

The distribution found thus accounts for all the unexplained secular motions with the exceptions of those of the perihelia of Mercury and Mars and the variations in the eccentricities of the same planets. The residuals in all other cases are less than the mean errors. On the other hand, it is seen that the motions of the orbital planes of the planets actually imply a secular movement of the perihelion of Mercury half as large again as the unexplained part already known to exist.

Next, consider the motions of the orbital planes and of the perihelia together in order to determine the best values of l, m, and β. In this case the equations for $ed\varpi/dt$ will give four conditional equations in addition to those already used.

$$(14{\cdot}72 + {\cdot}660l - {\cdot}613)\beta = 8''{\cdot}48 \pm 0{\cdot}43$$

(squares of $l - \sin i \sin \Omega$ and $m + \sin i \cos \Omega$ being omitted)

$${\cdot}07\beta = -0''{\cdot}05 \pm 0{\cdot}25$$
$${\cdot}05\beta = 0''{\cdot}10 \pm 0{\cdot}13$$
$${\cdot}05\beta = 0''{\cdot}75 \pm 0{\cdot}34$$

The normal equations now found are—

$$(36{\cdot}22l - 13{\cdot}17m - 3{\cdot}99)\beta - 0{\cdot}654 = 0,$$
$$(-13{\cdot}17l + 29{\cdot}30m + 3{\cdot}375)\beta + 0{\cdot}522 = 0,$$
$$(-3{\cdot}990l + 3{\cdot}375m + 2{\cdot}984)\beta - 1{\cdot}282 = 0,$$

whence

$$l = +0{\cdot}1057;\ m = -0{\cdot}0986;\ \beta = 0{\cdot}575.$$

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The perturbations caused by this distribution, with the same units as before, are given in the following table:—

Planet.	$\sin i\delta\Omega$.	δi.	$e\delta\varpi$.
Mercury	0·94	0·00	8·53
Venus	0·41	0·42	0·04
Earth	...	...	0·03
Mars	0·04	0·00	0·03

The secular variations are thus all accounted for within their mean errors except the motions of the perihelion of Mars and the node of Venus. In the latter case, however, the residual is only just greater than the mean error. The equatorial plane of the perturbing matter is inclined at 8° 19′ to the ecliptic, and the longitude of its ascending node is 47° 0′. Thus it is sufficiently near to the plane of the orbit of Mercury for the quantity $(l-\sin i \sin \Omega)^2+(m+\sin i \cos \Omega)^2$ to be neglected. The value found for β shows that $a=4\cdot43\times10^{-9}$. If the disturbing potential was to be attributed to an ellipticity of the sun, the requisite value of this would be $9\cdot93\times10^{-3}$ (the sun being assumed homogeneous), which is much too great to be possible. The matter must therefore be outside of the sun, though its diameter must be much less than that of the orbit of Mercury. A somewhat better representation could be obtained by including a fourth harmonic, but in view of the size of the other unexplained discrepancies this scarcely seems worth doing at present.

If then Einstein's hypothesis be accepted, any perturbing matter that may exist must be such as to produce considerable effects on the positions of the planes of the orbits, without affecting the much more sensitive perihelion of Mercury. This can only occur if a fourth harmonic is included in the potential, and so accurately adjusted that its effect on the perihelion of Mercury just balances that of the second harmonic. In the figure of the disturbing mass the fourth harmonic must then be very much more prominent than the second harmonic, which appears unlikely. It appears therefore that on the hypothesis of Einstein it will be found very difficult to find a plausible explanation of the motions of the planes of the orbits, and in particular of the node of Venus.

The Irregular Variable RS Camelopardalis.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The valuable paper by C. Martin and H. C. Plummer in the May number of the *Mon. Not.* (p. 612) gives not only a fine series of observations of this star extending over more than four years, but a careful analysis of them resulting in an apparent solution of the problem of irregular variability, possibly extending to other

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cases. The solution is, moreover, put into elegant shape; and it is an ungracious task to challenge the interpretation of it (nothing here said challenges the analysis). Indeed, the following criticism did not shape itself until the paper had been studied with much pleasure and approval, and it represents a considerable disappointment to the writer; for it seemed on first reading the paper as though a satisfactory case had at last been found of a phenomenon which has hitherto proved very elusive.

2. It is well known that Argelander gave a formula for the oscillations of the maxima of variable stars of the form

$$\text{Julian Date} + 255\text{E} + 15 \sin (9^\circ\text{E} + 230^\circ)$$

where 255 days is the mean period, but the maxima are presumed to oscillate 15 days either way from their mean position in a long cycle completed in $360^\circ/9^\circ = 40$ periods of 255 days, say 30 years. Now although formulæ of this kind have been fitted by Chandler and others to the observations of variables extending over a limited period, *no case* has (so far as I am aware) yet been found in which the periodic term has satisfied a complete cycle of its own changes, and brought back the maxima to their original sequence. On the other hand, there have been various cases of breakdown when such a formula *might* have come round again, but has failed to do so: so that a quite different hypothesis of several abrupt changes of period has been adopted for more than one such case, after the Argelander-Chandler formula had clearly broken down.

3. Now Messrs. Martin and Plummer seemed at last to have found a satisfactory case with a complete oscillation of the maximum: they do not express it in that form, but it can be so expressed. Their mode of expression contemplates "a simple oscillation in the period 88·7 days, with slowly changing amplitude and phase." We shall omit consideration at present of the amplitude, and consider only the phase ψ for which they give the equation (p. 627):

$$\tan \psi = \frac{\sin (0^\circ{\cdot}32t' - 63^\circ)}{3 + 6 \cos (0^\circ{\cdot}32t' - 63^\circ)} = \frac{\sin (\theta - \phi)}{3 + 6 \cos (\theta - \phi)}$$

the second form being virtually used on the previous page where the variation in 88·7 days is represented by the angle $(\theta + \phi)$.

4. In this form the phase ψ consists partly of a secular and partly of a periodic term: for when

$$\begin{array}{rcccccc} (\theta - \phi) & = & 0^\circ & 120^\circ & 180^\circ & 240^\circ & 360^\circ \text{ etc.} \\ \psi & = & 0^\circ & 90^\circ & 180^\circ & 270^\circ & 360^\circ \text{ etc.} \end{array}$$

so that the secular part of ψ is clearly $(\theta - \phi)$ or χ say.

To get the periodic part ω write

$$\psi = \chi - \omega.$$

Then

$$\tan \omega = \tan (\chi - \psi) = \frac{\sin \chi (3 + 5 \cos \chi)}{1 + 3 \cos \chi + 5 \cos^2 \chi},$$

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and since the denominator can never vanish, ω oscillates between limits less than $\pm 90°$. If, then, we add this secular variation of the phase to the argument, we are left with an oscillating phase such as could be represented by an Argelander-Chandler formula. Moreover, the oscillation of the phase takes place in the period of χ or $\theta - \phi$ or $(0°\cdot 32t' - 63°)$, which is 1125 days, well covered by the extent of the observations. The beautiful accordance of these with the suggested formula is shown by the details of Table VIII. in the paper. Hence at last we seem to have an oscillation of the maximum, satisfactorily confirmed: it is really heart-breaking to have to raise doubts as to the reality of this long-delayed success. But previous personal experience of a noteworthy pitfall leaves no option.

5. Let us first recall the main point here criticised. On p. 626 the authors obtain the elegant expression

$$2a \sin 2\theta + (a+b) \sin (\theta + \phi) + 2b \sin 2\phi$$

for the variation, in which both the coefficients $2a$, $a+b$, $2b$, and the arguments 2θ, $\theta + \phi$, 2ϕ are in A.P. Attention is here entirely directed to the significance of the latter A.P. The relation between the coefficients may very possibly be of valuable significance: this point is not at present under examination: we confine ourselves to the inquiry whether the fact that a great part of the variation can be represented by three periodic terms with arguments in A.P. gives us any important information—for instance, any information which can be used for prediction—and the point of view here taken is that it does *not*; that the variation *may* be quite irregular (though with an approximate average period) and still be expressible in this form.

6. It will be convenient to give some notion of what is meant by an *irregular* variable, with an approximate average period.

The calendar months in our year have either 28, 29, 30, or 31 days; but their lengths are assigned in a definite way, and this is a case of regular, though multiple, periodicity. But suppose that when any month was starting, a die were to be thrown to determine its length. Let the usual numbers 1 to 6 mean that the length shall be 28, 29, . . . 33 days. Then the *average* length would remain much as at present but the month would become an irregular variable; i.e. *its actual length could not be predicted.*

7. Now let a series of observations of the lengths of such months be made for a year. The average period of something like $30\frac{1}{2}$ days would be easily apparent: but it would also be clear that this was not the whole story. Periodogram analysis would give us a maximum "intensity" at about $30\frac{1}{2}$ days, and the idea might occur to us, "Let us remove this term and see what is left." Call the term so removed $B \sin (\theta + \phi)$. Then when we removed it, we should find a term $A \sin 2\theta$ suggesting itself: it would be natural to remove that, and we should find then a term $C \sin 2\phi$ very clearly marked: and we should find the greater

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part of the variation represented by these three terms. We should be tempted (the present writer has easily yielded to the temptation in more than one instance) to think that we had found out something definite about the law of variation. But the neat mathematical expression is applicable to all cases: it arises naturally just as much in the case of absolutely irregular variability above sketched as in the case of sun-spots, where several actual and determinable periods are superposed. So far this is a mere assertion: it remains to justify it.

8. When, in the instance before us, we determine the term B sin $(\theta+\phi)$ we are not finding an actual periodicity of $30\frac{1}{2}$ days: indeed, there is not any such: the nearest actual periodicities are 30 and 31 days. We shall include in B sin $(\theta+\phi)$ *contributions* from these two periodicities, but not the whole of them. We shall also include contributions from the 29-day and 32-day months, though not so large, owing to the greater deviation from the adopted period of $30\frac{1}{2}$ days. But as we leave the adopted period further we come to periodicities which are scarcely included at all, because at one time of the year-of-observation they are in accord, at another time in discord, with the average $30\frac{1}{2}$-day term. For 12 periods of 30·5 days make 366 days, and

$$366/11 = 33{\cdot}3, \quad 366/12 = 30{\cdot}5, \quad 366/13 = 28{\cdot}2.$$

Hence a 33-day period "interferes" with a $30\frac{1}{2}$-day in something like a year; and so does a 28-day. When the term B sin $(\theta+\phi)$ is determined and removed, two periodicities on opposite sides of it will be *untouched*, and will consequently become prominent. One or other of these will probably catch the eye more than the other—in § 7 we have assumed that it was the A sin 2θ term: and up to this point the notation used is quite general, for neither ϕ nor θ has been defined separately: but once we have determined A sin 2θ + B sin $(\theta+\phi)$, then the chief term left will be C sin 2ϕ, so that the arguments are in A.P.

9. It is worth going over the selection of θ and ϕ more explicitly. When we deliberately analyse the observations into a Fourier sequence of exact submultiples of the whole time T (in this case one year), then the relations of θ and ϕ are quite definite. But the existence of three terms with arguments in A.P. is a little more general than this. We may start as above with an argument $(\theta+\phi)$ selected by inspection, which gives us two arbitrary variables, θ and ϕ: then we may select, also by inspection perhaps, another argument 2θ. But the difference $(\theta+\phi-2\theta)=(\phi-\theta)$ is not really arbitrary: it is such that the $(\phi+\theta)$ term runs through its cycle *just once more* than the 2θ term during T the time available for discussion: and a term which similarly runs through its cycle just once less must therefore have argument $(\theta+\phi)+(\phi-\theta)=2\phi$.

10. Besides these terms in 2θ and 2ϕ there are others which run through their cycle exactly twice, three times, etc., more or less than the adopted $(\theta+\phi)$ term with which we started, and which will therefore be quite untouched when we remove $(\theta+\phi)$.

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The arguments for the next pair will be

$$(\theta+\phi) \pm 2(\phi-\theta) = 3\phi-\theta \text{ and } 3\theta - \phi,$$

and so on. Whether these will have sensible coefficients depends upon how near they lie to the periods which actually occur. In the above example the actual period of 33 days lies *between* $(\theta+\phi)$ which has been adopted as of 30·5 days, and 2θ which has been assumed to be 33·3 days. Hence it can be divided into components the chief of which has 33·3-day period, the next has 30·5, and but little is left for anything outside these. But 28 days lies *outside* the corresponding space between 30·5 and 28·2; so that it will have a sensible component on the small side of 28·2 days. In this actual example we should probably find that three terms would not be sufficient to express the main features of the observations: but if we either shorten the whole period of observation, or suppose the die thrown has only four numbered faces, restricting the possible periods to 29, 30, 31, or 32 days, we shall find three terms enough. The mode of distribution of the actual periodicities among the constituents of a Fourier sequence is given in *Monthly Notices, R.A.S.*, lxxiii. p. 718. and it seems probable (though the point has not yet been examined) that it is applicable also in the case when we start (as above) with an arbitrary term instead of an exact submultiple of the time of observation.

11. To sum up: the point here made is that *any* case of variation with an approximate average period, however irregularly the variation may be distributed about that period, can be represented by three terms with *arguments in A.P.*, provided the whole period of observation is short enough. How short it should be depends on the limits of departure from average period. Whatever these limits may be (provided they remain the same) then at first three terms in A.P. will suffice: but as the time of observation is extended the common difference of the A.P. will diminish and five terms will ultimately be required, still in A.P.: then seven, and so on. Hence there is no *necessary* significance in the relation of the arguments of the three terms given by Martin and Plummer on p. 626, *loc. cit.* The relation of the *coefficients* may be significant: that point is not here raised.

12. In the course of meditation on the particular problem here presented another point of a more general kind was suggested, which seems worthy of notice. The variation of RS Camelopardalis is represented by Martin and Plummer as a "simple oscillation in a period 88·7 days with a slowly changing amplitude and phase" (p. 626). The variations of the phase ψ are, as already quoted in § 3 of this paper, given by

$$\tan\psi = -\frac{\sin(\theta-\phi)}{3+6\cos(\theta-\phi)},$$

and it is there shown that we may write

$$\psi = (\theta-\phi) - \omega$$

where ω is periodic. Hence the secular part of ψ is $(\theta-\phi)$. But ψ is the variable phase of the adopted period $(\theta+\phi)$. Should we not therefore include the secular part of ψ and say that the true period is $(\theta+\phi)+(\theta-\phi)=2\theta$? If so, then we just undo the work which Martin and Plummer go through in *combining* the three terms

$$0{\cdot}172 \sin 2\theta + \quad 0{\cdot}147 \sin(\theta+\phi) + \quad 0{\cdot}122 \sin 2\phi.$$

They combine them, by a pretty piece of analysis, to form a single term in $\theta+\phi$ with varying phase ψ; and we insist on restoring the secular change in ψ, thus obtaining a term in 2θ.

13. On full consideration, it is here urged that the latter is the better and truer view: that 2θ is the best measure of the periodicity, because the term in 2θ has the largest coefficient: and in general that *when several periodic terms co-exist, the terms with the largest coefficient settles the period*, no analytical combination of terms being really effective.

Thus the expression

$$(1+n^2)\cos 2pt + (1-n^2)\cos 2qt$$

can be written analytically

$$2\cos(p-q)t\cos(p+q)t + 2n^2\sin(p-q)t\sin(p+q)t$$

which may be regarded (following Martin and Plummer) as having an argument $(p+q)t$ with slowly varying amplitude and phase, viz.

$$\text{amplitude} = \{4\cos^2(p-q)t + 4n^2\sin^2(p-q)t\}^{\frac{1}{2}}$$

and phase ψ given by

$$\tan\psi = n^2\tan(p-q)t.$$

But it is clear that ψ contains the secular term $(p-q)t$, and if we remove this by adding it to the argument we get as before $2pt$, which is the term with the largest coefficient $(1+n^2)$

14. The impulse to combine two terms of different periods comes without doubt from the theory of "interference" in physical optics, where one coefficient is in general *equal* to the other, *i.e.* where $n^2=0$. In this particular case we cannot pick out one term as having a larger coefficient: and the sum of the terms is usually represented, from considerations of symmetry, as a periodic term $(p+q)t$, and a coefficient which is alternately positive and negative. But this is, after all, not a satisfactory way of regarding a periodicity. It would be more satisfactory to keep the coefficient always of one sign, and this we can do by choosing *either* of the arguments $2pt$ or $2qt$ instead of the symmetrical $(p+q)t$. We cannot prevent the coefficient vanishing, but we can prevent its changing sign.

15. A safer guide to the combination of two periodicities is afforded by the circulation of two planets round the Sun. In this

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case it is always the outer planet, *i.e.* the one of greater radius or coefficient, which settles the period. The period of Jupiter seen from the Earth is that of Jupiter, with a little oscillation due to the Earth. The period of Venus seen from the Earth is that of the Earth, with an oscillation due to Venus. The case of equal coefficients (and different periods) would correspond to that of two planets circulating at the same distance from the Sun in different times. Regarding this as the limiting case of an outer planet seen from the Earth, we must make its orbit go just *outside* the Earth; if as the limiting case of an inner planet, it is just *inside*. Consider the former case: on leaving the Earth (E) the planet (P) makes the whole circuit of its orbit round the Sun, but the line EP only describes 180° until the Earth is nearly reached again, when the second 180° is described instantaneously in *the same* direction. Hence the mean motion of the planet, relatively to the Earth, is the same as its mean motion round the Sun. (We deal of course, with mean motion in space, *not* with synodic motion.) The revolution round the Earth is far from uniform: but it can be regarded as uniform motion with an impressed oscillation, the uniform motion being that of P.

If, however, as in the second case, we think of P as passing just *inside* E, then the instantaneous description of 180° is in the *opposite* direction to that which has just been laboriously made in travelling round the orbit, so that the work of this latter is completely undone. The planet P therefore on the average remains stationary within the orbit of E just as the Sun does, and the mean period of the relative motion is therefore that of the revolution of E round the Sun; on which, of course, is impressed an oscillation due to P.

16. The point to which attention is here drawn concerns the use of words rather than the interpretation of phenomena: but it is submitted with some seriousness that to speak of a periodicity of which the coefficient may regularly change sign is a misuse of words. The coefficient may die down, but it should keep the same sign: zero should be its limit: if it reverses in sign persistently (and not merely accidentally as an alternative to zero or a small positive value) then we should alter the adopted value of the period so as to avoid this reversal. In particular, protest is entered against the nomination of 88·7 days by Martin and Plummer as that of RS Camelopardalis: the adopted period should be that of the term with largest coefficient

$$\cdot 172 \sin (4^\circ{\cdot}382 t' - 25^\circ)$$

which is

$$360^\circ / 4^\circ{\cdot}382 = 82{\cdot}2 \text{ days.}$$

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Baxendell's Observations of Variable Stars. Edited by H. H. Turner and Mary A. Blagg. Eighth Instalment. No. 20, T Tauri; No. 21, R Ursæ Majoris; No. 22, S Ursæ Majoris.

No. 20, *T Tauri* (1537 *Chandler*).

$4^h\ 13^m\ 33^s\ +19°\ 11'{\cdot}3$ (1855·0)
4 16 10 +19 17·8 (1900·0)

Identification of Comparison Stars.—Three charts and one small sketch have been found, in which the comparison stars have been identified as shown in Table I. In most cases the identification is fairly easy, but as none of the charts are dated, it can be decided only by internal evidence of the observations themselves which chart was in use at any particular time. There are a few cases of anomalous observations in which a slip seems to be suggested.

TABLE I.

Baxendell's Comparison Stars.

Baxendell. Large Charts. (1)	(2)	Small Chart.	Sketch, 1862 Nov. 15.	A.S.V. No.	Grade.	H.A. Letter.	Mag.	Adopted Mag.
a	*a*	*a*	*b*	2	13	*b*	8·27	...
b	*r*	*r*	...	3	33	*d*	9·30	93
c	*s*	*s*	...	7	43	*f*	10·10	101
d	*t*	*t*	...	9	48	...	...	105
e	*γ*	*γ*	...	10	51	*g*	10·76	108
f	*w*	*w*	...	11	55	...	...	109
g	*y*	*y*	*y*	...	...	...	...	110
h	*ζ*	...	...	...	...	...	...	116
k	*f*	...	...	...	...	...	...	125
l	...	...	*c*	13	76	*l*	12·37	124
m	Z	Z	*v*	16	93	...	...	128
n	...	...	*w*	...	...	...	...	130
p	*p*	*p*	...	8	44	...	...	102
...	...	*o*	...	...	...	...	...	122

TABLE II.

Baxendell's Differences between his Comparison Stars.

Years.	105–101.	109–108.	110–109.	116–109.	124–116.	128–124.	130–124.	102–101.	108–102.	109–102.	124–109.
1862–4	$1{\cdot}5_1$	$+0{\cdot}6_{14}$	$-0{\cdot}4_{15}$	$6{\cdot}3_3$	$12{\cdot}7_3$	$5{\cdot}0_2$	...	...	...	...	$19{\cdot}0_2$
1865–9	...	$+0{\cdot}3_{11}$	$+4{\cdot}2_3$	$10{\cdot}0_7$	$9{\cdot}1_6$	$4{\cdot}2_3$	...	...	...	...	$18{\cdot}8_2$
1877–81	$4{\cdot}5_2$	$-1{\cdot}5_1$	...	...	...	...	...	$1{\cdot}8_3$	$6{\cdot}2_3$	$7{\cdot}8_6$	...
1884–7	...	...	$+2{\cdot}0_1$	$14{\cdot}0_1$	$9{\cdot}5_2$	...	$7{\cdot}6_4$	...	...	...	...
Mean.	3·5	+0·4	+0·5	9·1	10·1	4·5	7·6	1·8	6·2	7·8	18·9
No. of Obsns.	3	26	19	11	11	5	4	3	3	6	4
Reduced Mean.	3·2	+0·4	+0·3	8·0	8·6	3·9	6·5	1·7	5·0	6·6	15·1

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The unit is Baxendell's "step," reduced in the lowest line approximately to a tenth of a magnitude by the usual formula applied to each comparison. The telescopes used were A 68 during the earlier series, except on the following occasions :—1862 Nov. 15 and 16 (2nd observation) and 1864 Nov. 5; R (13) 81 was used: 1864 Oct. 6 and 7; R (13) 196: 1863 Aug. 19 and Sept. 19 and 1866 Sept. 15; C 39: 1863 Feb. 17, $7\frac{1}{2}$ in. A 88. In 1870 and 1871 Mr. Gladstone's $7\frac{1}{4}$ in. was used, with powers 60 and 69. From 1877, E 80 was almost certainly used, though this is not noted.

Table III.

Baxendell's Observations of T Tauri.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1862	240+		
Mar. 19	1219	110 − 4	106
Oct. 18	1432	Invisible <125	
Nov. 15	1460	124 − 2, 128 − 7	122
16	1461	124 − 2 (2 obsns.)	122
Dec. 26	1501	124 + 0	124
1863			
Jan. 23	1529	124 − 5	119
29	1535	Invisible <102	
31	1537	Glimpsed, "☽ and v. hazy."	
Feb. 3	1540	Not seen <103	
8	1545	124 − 3	121
15	1552	124 − 4	120
16	1553	Glimpsed	
17	1554	124 − 4	120
21	1558	124 − 4	120
Mar. 3	1568	Invisible <105 "v. strong ☽."	
15	1580	124 + 0, 128 − 3	124
21	1586	124 + 0	124
22	1587	124 − 2	122
Aug. 19	1737	110 + 2, 109 + 2	112
24	1742	109 + 3, 116 − 3, 124 − 12	112
Sept. 19	1768	109 + 6	115
22	1771	109 + 6, 124 − 8, 116 + 2	116
1863	240+		
Oct. 9	1788	109 + 6, 116 + 0	116
Nov. 2	1812	116 + 2, 124 − 8	117
29	1839	116 + 2	118
Dec. 17	1857	109 − 1, 108 − 1	108
1864			
Jan. 4	1875	108 + 0, 109 + 0	108
Feb. 5	1907	108 + 1, 109 − 1	108
8	1910	108 + 1, 109 − 1	108
Mar. 28	1959	108 + 1, 109 + 0	109
Sept. 24	2139	109 + 2, 108 + 0	110
Oct. 6	2151	109 + 3, 108 + 3	112
7	2152	109 + 2, 108 + 3	111
Nov. 5	2181	108 + 1, 109 − 1	108
Dec. 21	2227	108 + 1, 109 + 0	109
1865			
Feb. 14	2282	109 − 2, 108 − 2	106
Oct. 19	2529	108 + 1	109
Nov. 12	2553	108 − 1	107
1866			
Jan. 5	2607	108 + 11, 109 + 11	120
15	2617	124 + 3, 128 − 3	126
23	2625	Not seen <125	
Feb. 3	2636	124 + 2, 128 − 2	126
4	2637	124 + 3	127
12	2645	124 + 3	127
17	2650	124 + 3	127

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TABLE III.—*Baxendell's Observations of T Tauri*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1866	240+		
Mar. 13	2674	124+4,* 128+2	? 129
Sept. 15	2860	Not seen <110	
Oct. 6	2881	,, <125	
8	2883	Glimpsed.	
10	2885	124+2	126
16	2891	124+2	126
28	2903	About 124+0. "Low and seen with difficulty."	
30	2905	124+3	127
Nov. 11	2917	Not seen <120	
16	2922	124+3	127
30	2936	Not seen <127	
Dec. 7	2943	124+3	127
30	2966	124−3	121
31	2967	116+3, 124−5	119
1867			
Jan. 2	2969	116+2, 124−7	118
4	2971	116+1	115
8	2975	116+0, 124−8	116
11	2978	116+0, 124−8	116
21	2988	Not seen <105	
28	2995	109+3, 116−2	113
Feb. 2	3000	109+5, 116−3	114
20	3018	109+8, 116−2	116
28	3026	116−2, 124−7, 109+7	116
Mar. 16	3042	109+7, 116−2	115
20	3046	109+7, 116−2	115
26	3052	109+7, 116−3, 124−9	115
Nov. 17	3288	109+2, 108+0	110
Dec. 6	3307	108+4, 109+4	112
18	3319	108+3, 109+2	111
22	3323	108+3, 109+3	112
1868			
Jan. 26	3358	108−2, 109−2	106
1868	240+		
Feb. 11	3374	108+2, 109+1	110
15	3378	109+0	109
1869			
Feb. 5	3734	109+0, 108+1	109
Mar. 2	3759	108+2, 109+3	111
24	3781	108+4, 109+3	112
Dec. 21	4053	Not seen <120	
1870			
Mar. 25	4147	124+6	130
1871			
Mar. 3	4490	Not sure if glimpsed. "☽ and hazy."	
Mar. 13	4500	124+6	130
1877			
Dec. 6	6960	102+0, 108−3	104
1878			
Jan. 31	7016	101+2, 105−3, 102−2, 108−5	102
Nov. 25	7314	102+2, 109+0	106
Dec. 5	7324	102+2, 109−5	104
23	7342	102+1	103
1879			
Sept. 13	7606	102+1, 109−5, 101+2, 105−2	103
Oct. 20	7643	102−1	101
Nov. 2	7656	102−2	100
1880			
Feb. 5	7751	93+5, 102−2, 101+1	99
Aug. 17	7945	102−3	99
Oct. 2	7991	102+0	102
1881			
Oct. 29	8383	109−2	107
Nov. 14	8399	? 109−2† (or 93−2)	? 107 (or 91)
Dec. 19	8434	102+6, 109−3	107

* "Has *m* changed? or is it a star not before laid down that I have taken for *m*?"
† The letter is *b*, but it seems probable that it is a slip for *f*.

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TABLE III.—*Baxendell's Observations of T Tauri*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1884	240+		
Jan. 20	9196	116+6, 124−7	120
Nov. 20	9501	125+0	125
Dec. 14	9525	About 124+2	126
15	9526	124+4, 130−2	128
20	9531	124+4, 130+0	129
1885			
Jan. 6	9548	124+3, 130−3	127
13	9555	124−3, 130−11	120
Feb. 2	9575	124−6, 122−4, 125−6	118
Mar. 17	9618	124−6, 125−7	118
Dec. 1	9877	116−2	114
1886	240+		
Jan. 6	9913	109+6, 116−6	112
Feb. 1	9939	125−3	122
14	9952	About 124+0. "☽ too near."	?124
Mar. 3	9969	109+3, 110+1	112
	241+		
Nov. 2	0213	124+0. "v. low."	?124
30	0241	124−6, 116−3	116
Dec. 4	0245	116−3	113
29	0270	116−2	114
1887			
Jan. 29	0301	108+6, 116−4	113
Feb. 18	0321	116+0	116

The observations of 1886 appear to be inconsistent and puzzling. The variations of this star would appear to be irregular, but the observations are so scattered and the gaps so long and so frequent, that it is impossible to deduce even that conclusion with absolute certainty. Five minima seem to be indicated; one somewhere about 240 1500 to 1600 (or it might be earlier), one between 2670 and 2930, another ending about 9530, and two others, one near 4150 and the other near 4500. But in the first case there seems to be a rise of half a magnitude, lasting for about 40 days, between two separate lowest points if we suppose the actual minimum to be observed, and this might very well be the case in the other minima also. No maximum seems to be actually observed, but one probably occurred between 7800 and 7900.

No. 21, *R Ursæ Majoris* (3825 *Chandler*; 103769 *Harvard*).

$10^h\ 34^m\ 19^s \quad +69^\circ\ 32'{\cdot}1 \quad (1855{\cdot}0)$

$10\ \ 37\ \ 34 \quad +69\ \ 18{\cdot}0 \quad (1900{\cdot}0)$

Identification of Comparison Stars.—There are four small sketches of the comparison stars, all in agreement with one another; and from 1863 Pogson's chart was used, some additional letters being inserted by Baxendell.

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Table IV.

Comparison Stars.

Chart (Pogson's.)	Baxendell Sketches. 1857. Apr. 19.	1857. July 25.	1859. Jan. 6.	1862. Apr. 15.	A.S.V. No.	A.S.V. Grade.	Harvard. Mag.	Adopted Mag.	Bax.'s and Pogson's est.
a	*a*	*a*	...	*a*	2	...	5·90	63	6·7
c	*b*	*b*	*b*	*b*	5	14	8·25	81	8·1
d	*d*	*d*	...	...	7	31	9·19	93	9·8
e	...	...	..	...	12	52	10·28	101	...
f	...	...	...	...	10	43	9·83	100	10·4
e'	...	...	...	...	9	42	9·78	102	...
f'	...	...	...	...	...	...	...	106	...
g	...	...	...	...	13	65	10·89	110	11·2
h	...	...	...	...	14	72	11·20	114	11·5
k	...	...	...	...	18	91	11·94	119	12·5
l	...	...	...	...	23	114	12·61	126	12·6
...	*c*	*c*	...	...	...	...	...	...	...

The Harvard magnitudes are obtained by interpolation, and are very uncertain. The adopted magnitudes are a compromise.

A star called *m* is used once, in 1885; there is no clue to its identification.

Table V.

Baxendell's Differences between his Comparison Stars.

Years.	81-63.	93-81.	101-93.	100-101.	110-100.	114-110.	119-114.	126-119.	106-100.
1857-8	$12{\cdot}9_{21}$	$14{\cdot}7_{5}$	...	...	...	...	...	...	...
1861-7	$15{\cdot}5_{2}$	$20{\cdot}0_{2}$	...	$+3{\cdot}0_{1}$	...	...	..	...	$4{\cdot}0_{1}$
1879	...	$18{\cdot}0_{1}$	...	...	...	...	...	...	...
1884-6	$22{\cdot}4_{7}$	$16{\cdot}7_{6}$	$8{\cdot}8_{2}$	$-1{\cdot}2_{2}$	$11{\cdot}5_{2}$	$5{\cdot}5_{2}$	$4{\cdot}3_{3}$	$8{\cdot}8_{2}$	$12{\cdot}0_{1}$
Mean.	15·3	16·5	8·8	+0·2	11·5	5·5	4·3	8·8	8·0
No. of Obsns.	30	14	2	3	2	2	3	2	2
Reduced Mean.	12·0	13·5	7·2	+0·3	10·0	4·2	3·7	7·0	6·5

In this case there are so few comparisons for most of the stars that we give even those which occur only twice; but practically no weight attaches to any of the columns as regards the *exact* intervals, though something may be inferred as to the approximate relative differences.

The telescopes used were:—A (30) in 1857 Apr. to July, 1858 Mar. 11, Apr. 18, and June; A 40, 1858 Mar. 16 and 25; A (22) 21, 1858 Aug. to Dec., and 1859 (except Jan. 18); A 68, 1861 May 17 and June 25, 1865 June 18, and 1867 Mar. 16; Tully A 35, 1857 Dec., 1858 Feb. and Mar. 3; ditto 40, 1858 Jan. 25;

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R (7) 40, 1858 Apr. 1 to 12 and July 18 and 1859 Jan. 18; R (13) 81, 1858 Mar. 23; C 39, 1861 Jan. 1, Feb. 28, May 10 and 30, June 17, Aug. 6, Sept. 1, 1862 to 1864, 1865 Nov. 12, and 1867 May 11; C 21, 1861 July 5. From 1879 E 80 was most probably used throughout.

Table VI.

Baxendell's Observations of R Ursæ Majoris.

Date.		J.D.	Corrected Comparisons.		Deduced Mag.
1857		239+			
Apr.	19	9424	81 − 1		80
	20	9425	81 − 2		79
	23	9428	81 − 2		79
	25	9430	81 − 3		78
	26	9431	81 − 3		78
	27	9432	81 − 3		78
May	4	9439	81 − 5,	$\frac{63+81}{2}$	74
	5	9440	63 + 9,	81 − 5	74
	7	9442	63 + 7,	81 − 5	73
	15	9450	63 + 5,	81 − 5	72
	19	9454	63 + 4,	81 − 6	71
	21	9456	63 + 5,	81 − 5	72
	24	9459	63 + 7,	81 − 4	74
	,,	,,	$\frac{63+81}{2}$		72
	26	9461	63 + 6,	81 − 4	73
	28	8463	63 + 5,	81 − 4	72
	30	9465	*63 + 6½,	81 − 4	73
	,,	,,	63 + 6,	*81 − 4½	73
June	4	9470	63 + 7,	81 − 4	74
	12	9478	63 + 8,	81 − 3	74
	14	9480	63 + 7,	81 − 3	74
	15	9481	63 + 8,	81 − 3	74
	17	9483	63 + 8,	81 − 2	75
	19	9485	63 + 8,	81 − 3	74
	20	9486	63 + 8,	81 − 2	75
	21	9487	63 + 9,	81 − 2	76
	22	9488	81 − 2		79

Date.		J.D.	Corrected Comparisons.		Deduced Mag.
1857		239+			
June	24	9490	63 + 9,	81 − 1	76
	25	9491	63 + 9,	81 − 1	76
	26	9492	63 + 9,	81 − 1	76
July	1	9497	81 − 1		80
	11	9507	81 + 1		82
	12	9508	81 + 2		83
	19	9515	81 + 5		86
	25	9521	81 + 7,	93 −	89
Dec.	7	9656	Glimpsed.		
	20	9669	,,		
1858					
Jan.	25	9705	Invisible ☽		
Feb.	6	9717	93 + 1		94
	7	9718 ?	†93 + 0		? 93
	15	9726	93 − 3		90
	22	9733	81 + 2		83
	26	9737	81 − 1		80
Mar.	3	9742	81 − 2		79
	11	9750	81 − 0		81
	16	9755	81 + 0		81
	20	9759	81 − 1		80
	23	9762	81 − 1		80
	25	9764	81 + 0		81
Apr.	1	9771	81 + 1		82
	4	9774	81 + 1		82
	12	9782	81 + 4,	93 − 7	86
	18	9788	81 + 6		87
	20	9790	81 + 7,	93 − 7	87
	23	9793	81 + 6,	93 − 7	86

* The half-tenths are here given to show the difference between the two observations.

† Bax. wrote δ.

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TABLE VI.—*Baxendell's Observations of R Ursæ Majoris*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1858	239+		
June 6	9837	Invisible, "not v. clear."	
7	9838	,,	
8	9839	,,	
July 18	9879	101+3, min. vis. −4	104
Aug. 10	9902	Occas. glimpsed.	
Sept. 5	9928	Barely ,,	
11	9934	Invisible.	
Nov. 5	9989	,,	
	240+		
28	0012	93−3	90
Dec. 2	0016	93−5	88
1859			
Jan. 6	0051	81+0	81
18	0063	81−1	80
23	0068	81+1	82
28	0073	81+1	82
Feb. 6	0082	81+1	82
11	0087	81+1	82
23	0099	81+4	85
May 29	0194	Invisible.	
1861			
Jan. 1	0777	,,	
Feb. 28	0835	Est. 110?	
May 10	0906	81+11, 93−7	89
17	0913	81−2	79
30	0926	63+2, 81−9	68
June 17	0944	63+3	66
25	0952	63+2	65
July 5	0962	63+3	66
Aug. 6	0994	63+11, 81−2	76
Sept. 1	1020	81+6	87
1862			
Mar. 7	1207	Not seen <93	
Apr. 15	1246	81−5	76
27	1258	81−4	77
May 19	1280	81−2	79
1862	240+		
July 4	1326	Not seen <100 "v. hazy."	
Sept. 20	1404	Not seen <115	
1863			
Apr. 30	1626	,, <100	
Dec. 17	1857	81−3	78
1864			
Jan. 2	1873	81+0	81
1865			
June 18	2406	114−1	113
Nov. 12	2553	81+6, 93−8	86
1867			
Mar. 16	3042	101+6, 100+3, 106+0	105
May 11	3098	81−5	76
1879			
Aug. 28	7590	81+11, 93−3	91
Sept. 29	7622	81−3	78
1884			
May 26	9323	110+6	116
July 19	9377	100+4, 114−6	106
25	9383	93−2, 100−6, 101−9	92
Aug. 1	9390	81+4, 93−9	84
5	9394	81+0	81
11	9400	81−3	78
15	9404	81−6	75
23	9412	81−6	75
Sept. 1	9421	63+9, 81−8	72
4	9424	63+12, 81−8	74
11	9431	63+11, 81−7	74
13	9433	81−6	75
17	9437	81−4	77
20	9440	63+11, 81−6	74
26	9446	63+11, 81−7	74
29	9449	81−7	74
Oct. 1	9451	63+11, 81−7	74

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TABLE VI.—*Baxendell's Observations of R Ursæ Majoris*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.	Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1884	240+			1885	240+		
Oct. 4	9454	63+11, 81−7	74	Jan. 6	9548	110+0	110
5	9455	81−7	74	12	9554	114−6, 110−1	109
9	9459	81−6	75	13	9555	114−2	112
12	9462	81−6	75	Feb. 19	9592	126+5	131
Nov. 2	9483	81+0	81	Apr. 2	9634	126+3, ?−3	? 129
7	9488	81+1	82	June 1	9694	81+8, 93−5	88
9	9490	81+1	82	12	9705	81−5	76
13	9494	81+6	87	July 4	9727	81−8	73
17	9498	81+5	86	9	9732	81−6	75
20	9501	81+6, 93−7	86	25	9748	81−4	77
24	9505	93−7	86	29	9752	81−2	79
27	9508	93−3	90	Aug. 1	9755	81−2	79
Dec. 3	9514	93+1	94	28	9782	81+7, 93−7	87
6	9517	93+3, 101−4, 100−6	96	Sept. 13	9798	93+1	94
20	9531	100+4, 110−5, 102+3, 106−6	104	1886			
				Jan. 31	9938	126+2	128
25	9536	100+6, 110−4, 114−8	106		241+		
				Apr. 26	0023	81−5	76

A period of 300 days was at first assumed, but it became clear that this was a little too short, so the observations were arranged in the usual manner with an assumed period of 302 days. The means for different groups of periods are as follows, the unit being 0·1 magnitude:—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A	89_6	79_{13}	$76\frac{1}{2}{}_{18}$	79_{19}	85_4	...	...	104_1	...	...	...	...
B	84_3	72_5	75_4	76_1	87_1	86_1	...	...	...	...	...	89_1
C	94_3	80_7	75_{10}	75_{10}	83_4	89_7	102_3	110_3	...	131_1	124_3	113_1
General Mean	89	78	76	78	84	89	102	108	...	131	124	10
No. of Obsns.	12	25	25	30	9	8	3	4	0	1	3	2

Group A contains Periods I. to IV.; group B Periods V. to XIII.; group C Periods XXVIII. to XXXVI.

There is some appearance of a retrograde movement of the maximum between A and B, and a forward movement between B and C, but the observations in B are too few to trust with any certainty. The observations in the second half of the curve are so few throughout that we cannot expect to obtain from them a very smooth curve, but the portion from 1 to 6 is fairly satisfactory. The only maxima which can be considered as being in any real

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sense observed are those of Periods I., II., and XXXIV., and even of these the dates of the second and last cannot be fixed with any real certainty. The following table gives a series of epochs at fixed intervals of 301·9 days, compared with Baxendell, and with later observations of the B.A.A. Those of periods LVI. to LXIV. are as published, the *most* doubtful ones being omitted (though all are really doubtful to 8 or 10 days), and the later ones are calculated by the usual method from observations kindly lent to us in MS. by Mr. Brook. Comparisons with Chandler's formula $2397949 + 302{\cdot}1E + 11 \sin(8^\circ E + 238)$ are also given.

TABLE VII.

Comparison of Epochs of Maximum.

Period.	E.	Mean Epoch.	Observed ditto.	O – M.	Chandler.	O – Ch.
I.	5	9452	9454	+ 2	9449	+ 5
II.	6	9754	9745	– 9	9751	– 6
III.	7	0056	0060	+ 4	0054	+ 6
VI.	10	0962	0952	– 10	0963	– 11
XXXIV.	38	9415	9423	+ 8	9428	– 5
XXXV.	39	9717	9723	+ 6	9729	– 6
LVI.	60	6056	6059	+ 3	6075	– 16
LXI.	65	7566	7571	+ 5	7592	– 21
LXII.	66	7868	7879	+ 11	7896	– 17
LXIII.	67	8170	8177	+ 7	8199	– 22
LXIV.	68	8472	8475	+ 3	8502	– 27
LXV.	69	8774	8774	0	8804	– 30
LXVI.	70	9075	9078	+ 3	9107	– 29
LXVII.	71	9377	9373	– 4	9409	– 36
LXVIII.	72	9679	9676	– 3	9711	– 35
LXIX.	73	9981	9976	– 5	0013	– 37
LXX.	74	0283	0275	– 8	0315	– 40

The figures of the O – M column give no trustworthy evidence of any change of period either continuous or discontinuous, and Chandler's formula has clearly broken down. It could not be brought into agreement simply by reducing the length of the mean period while retaining the curve of variation.

No. 22, *S Ursæ Majoris* (4557 *Chandler*; 123961 *Harvard*).

$12^h\ 37^m\ 35^s \quad +61^\circ\ 53{\cdot}3' \quad (1855{\cdot}0)$

$12\ \ 39\ \ 34 \quad +61\ \ 38{\cdot}5 \quad (1900{\cdot}0)$

Identification of Comparison Stars.—In this case Pogson's chart was used from 1864 Feb. 17. Before that, four small

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sketches and a chart are given, the earliest one containing a considerable number of letters, and the others very few. The identifications have been made as follows. The two queries are used in the sketches and in the observations as if they were letters.

TABLE VIII.

Baxendell's Comparison Stars.

1857 May 24.	1859 Feb. 6.	1859 May 29.	1861 Apr. 9.	Provisional Chart. 1863 Nov. 1.	Pogson's Chart.	A.S.V. III. No.	A.S.V. III. Grade.	A.S.V. IV. No.	A.S.V. IV. Grades.	Harvard. Letter.	Harvard. Mag.	Adopted Mag.
a	...	...	...	...	*a*	1	0	2	20, 11	*a*	6·65	...
e	...	...	...	...	*b*	2	13	3	31	*b*	7·02	73
f	...	...	...	...	*c*	5	29	4	37	*c*	7·30	74
(?) *x*	...	...	...	...	*d*	3	24	7	40, 43	*d*	7·52	77
g	...	...	...	*g*	*e*	6	51	11	70, 81	*f*	8·50	85
h	*h*	...	...	*h*	*f*	8	66	13	78, 92	*g*	8·83	88
...	...	...	...	...	*g*	14	122	23	117, 121	...	...	100
...	...	...	...	...	*h*	19	162	27	127, 129	...	...	105
l	...	...	*l*	*x*	*k*	16	139	24	122, 129	*m*	10·65	106
m	...	?	...	...	*l*	22	182	29	131, 133	*n*	11·08	108
...	...	...	...	...	*m*	26	217	30	143, 140	*o*	11·58	116
c	...	...	...	...	...	...	...	9	64, 66	...	...	...
k	...	...	?	...	..	13	118	21		*l*	10·09	101
...	...	...	...	...	(*n*)	28	230	31	148, 145	*p*	11·94	120

The letter *n* is used only once, with nothing to identify it, but it seems highly probable that it is A.S.V. (IV.) No. 31, as it is used in conjunction with the neighbouring star, No. 30. The magnitudes adopted are those of Harvard (xxxvii. p. 7), with slight modifications.

TABLE IX.

Baxendell's Differences between his Comparison Stars.

Years.	77–74.	85–77.	85–74.	88–85.	101–88.	106–101.	108–106.	108–101.	116–108.	100–88.	105–100.
1857–8	...	? $5{\cdot}0_1$	$3{\cdot}0_1$	$2{\cdot}3_6$	$7{\cdot}3_2$	$4{\cdot}2_5$	$1{\cdot}8_3$	$6{\cdot}5_3$	...	...	...
1861–7	...	?$17{\cdot}0_1$	$14{\cdot}0_3$	$3{\cdot}6_7$	$9{\cdot}0_1$	$4{\cdot}7_3$	...	...	...	...	...
1879–85	$3{\cdot}3_7$	$9{\cdot}8_{17}$	$12{\cdot}3_5$	$3{\cdot}8_7$	...	...	$3{\cdot}0_1$	...	$10{\cdot}1_4$	$18{\cdot}0_6$	$6{\cdot}3_3$
Mean.	3·3	9·9	11·8	3·3	7·8	4·4	2·1	6·5	10·1	18·0	6·3
No. of Obsns.	7	19	9	20	3	8	4	3	4	6	3
Reduced Mean.	3·0	8·3	10·0	2·9	6·8	3·8	1·9	5·5	8·8	14·2	5·5

In most cases the number of comparisons is too small to give trustworthy results.

The telescopes used were:—Tully 35: 1857 Dec. 20; 1858

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Feb. 22, 24 (2nd obs.), 26, Mar. 3; A (30): 1857 Dec. 7, 1858 June 7 to 15, June 20, 26, 27, Aug. 5 and 8; A 68: 1858 Mar. 11, 1861 Mar. 21, Apr. 6, May 17, June 25, 1863 Nov. 1, 1864 from Jan. 4, 1866 June 26, and Dec. 16 to 1867 Mar. 16; A 40: 1858 Mar. 16 to 25; R (13): 1858 Feb. 24 (1st obs.); R (7) 40: 1858 Apr. 1 to June 4, June 17, 21, July 18; A (22) 21: 1858 July 1 to 17, 19, Aug. 10 to 1859 May 29; C 39: 1861 Jan. 1 to Mar. 8, Mar. 29, 31, Apr. 9 to May 10, May 30, June 17, Aug. 6 to 1863 Apr. 30, 1866 Aug. 22; E 80 (probably) from 1879.

The variable was first ostensibly observed 1857 April 19, and further observations followed on six dates, ending with May 15. On the first two occasions the comparisons are with 76 Urs. and 75 Urs.; on the next two, with these and also with Gr. 1918 (later called *a*): and on the last three with the latter only. The resulting magnitudes for the supposed variable range from 6·2 to 6·3; several of the observations are marked with a query; they are all written in pencil only (all others of the same dates being in ink); and, on the last of these dates, a note is added "? is this the real S Ursæ." It seems clear that it could not have been so. On May 24 the sketch is given, and comparisons are made between certain of the comparison stars on that date, on May 26 and 30, and on June 4. These comparisons are made with A (30), and the intervals come out very small compared with those obtained with the instruments in use later on. The real S Ursæ is first observed 1857 Dec. 7.

Table X.

Baxendell's Observations of S Ursæ.

Date.		J.D.	Corrected Comparisons.		Deduced Mag.
1857		239+			
Dec.	7	9656	85+0,	? 77+4	? 83
	20	9669	85+0		85
1858					
Feb.	22	9733	85+4		89
	24	9735	85+3		88
	,,	,,	85+4,	88+3	90
	26	9737	88+3		91
Mar.	3	9742	88+6,	101+1	98
	11	9750	101+2,	106−1	104
	16	9755	101+2,	106−2	104
	20	9759	106+0		106
	25	9764	106+1		107
Apr.	1	9771	108+0		108
	4	9774	108+0		108
	9	9779	108+3		111
	12	9782	108+4		112

Date.		J.D.	Corrected Comparisons.		Deduced Mag.
1858		239+			
Apr.	20	9790	Not seen, moonlight.		
	23	9793	,,	,,	
May	21	9821	106+1		107
June	4	9835	88+3,	101−4	94
	7	9838	88+3		91
	8	9839	88+3		91
	12	9843	88+2		90
	15	9846	88+0		88
	17	9848	88+1		89
	20	9851	85+1		86
	21	9852	85+0		85
	26	9857	85+0		85
	27	9858	85+0		85
July	1	9862	85−1		84
	6	9867	85−1		84
	10	9871	85−2		83

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TABLE X.—*Baxendell's Observations of S Ursæ*—continued.

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1858		239+		
July	17	9878	85 − 3	82
	18	9879	85 − 3	82
	19	9880	85 − 2	83
Aug.	5	9897	85 − 2	83
	8	9900	85 − 1	84
	10	9902	85 − 1	84
	12	9904	85 + 0	85
Sept.	5	9928	85 − 1	84
	11	9934	85 − 1	84
1859		240+		
Jan.	6	0051	88 + 7	95
Feb.	6	0082	88 + 0	88
	11	0087	88 + 0	88
Mar.	31	0135	85 + 0	85
May	29	0194	108 + 0	108
1861				
Jan.	1	0777	85 + 2	87
	3	0779	85 + 2, 88 − 2	86
	29	0805	85 − 3	82
Feb.	6	0813	85 − 3	82
	12	0819	85 − 2	83
	24	0831	85 − 2, 88 − 4	84
	28	0835	85 + 2, 88 − 1	87
Mar.	8	0843	88 + 0	88
	21	0856	88 + 3	91
	29	0864	88 + 9	97
	31	0866	88 + 11, 106 − 2	102
Apr.	6	0872	88 + 12, 106 − 2	102
	9	0875	106 − 3, 101 + 0	102
	16	0882	101 + 3, 106 + 0	105
May	10	0906	Invisible < 106 + 3	
	17	0913	106 + 6	112
	30	0926	Invisible < 106 + 3	
June	17	0944	,, < 88, "hazy, ☽, and twilight."	
	25	0952	88 + 6, 101 − 2, 106 − 8	97

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1861		240+		
Aug.	6	0994	85 + 1, 88 − 3	86
Sept.	1	1020	? 77 + 8, 85 − 6	? 82
Oct.	5	1054	85 − 1	84
	16	1065	85 + 3	88
Nov.	7	1087	88 + 4	92
1862				
Mar.	7	1207	88 + 0	88
Apr.	15	1246	85 − 6	79
	27	1258	85 − 4	81
May	19	1280	85 − 1	84
July	4	1326	Not seen < 100	
Sept.	20	1404	88 + 7	95
1863				
Apr.	30	1626	Not seen < 100	
Nov.	1	1811	106 + 11	117
Dec.	2	1842	88 + 11	99
	17	1857	88 + 3	91
1864				
Jan.	2	1873	88 + 0	88
	4	1875	85 + 1, 88 − 1	86
Feb.	17	1919	74 + 7, 85 − 6	80
Mar.	15	1946	74 + 7, 85 − 4	81
July	8	2061	106 + 6	112
1865				
Nov.	12	2553	85 + 3, 88 − 3	86
1866				
June	26	2779	85 − 2	83
Aug.	22	2836	85 − 2	83
Dec.	16	2952	88 + 6, ?106* + 0	?100
1867				
Jan.	28	2995	85 + 1, 88 − 1	86
Mar.	16	3042	74 + 7, 85 − 4	81
1879				
Sept.	23	7616	77 + 5, 85 − 4	82
	28	7621	77 + 4, 85 − 4	81

* The query is Baxendell's.

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TABLE X.—*Baxendell's Observations of S Ursæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1879	240+		
Oct. 15	7638	88+1	89
Nov. 1	7655	88+10, 100−2	98
23	7677	106+0	106
Dec. 24	7708	116+0	116
1884			
May 27	9324	106+14	120
Aug. 6	9395	85−2	83
11	9400	85−1, 88−3	84
15	9404	85−3	82
23	9412	88−2	86
Sept. 1	9421	77+6, 85−3	82
4	9424	77+4, 85−4	81
11	9431	77+1, 85−9	77
12	9432	77+1, 85−6, 88−8	79
17	9437	85−8, 77−3	76
20	9440	77+0, 85−8, 88−9	78
26	9446	77−2, 85−5, 74+5	78
Oct. 1	9451	77+1, 85−8, 74+1	77
5	9455	77+3, 74+4, 85−9	78
9	9459	77+3, 85−5, 74+4	79
12	9462	77+3, 85−5, 74+6	80
Nov. 2	9483	88+4, 100−8	92
7	9488	88+11, 100−5	97
9	9490	100−2	98
13	9494	100−2	98
17	9498	100−4	96
20	9501	100−5	95
24	9505	100−2	98
27	9508	100−2	98
Dec. 3	9514	100−1, 105−5	100
6	9517	100+4, 105−3	103
14	9525	106+2, *108+0	? 108
20	9531	106+5, 108+3	111
1885			
Jan. 5	9547	108+1, 116−6	110
11	9553	108+4, 116−3	112
Feb. 19	9592	88+7, 100−10	92
22	9595	88+5	93
Mar. 27	9628	77+4, 85−5	80
Apr. 2	9634	77+2, 85−5	80
May 11	9673	73+2, 74+1, 77−3	75
June 9	9702	88+0, 100−11	88
July 9	9732	100+0	100
25	9748	106+3, 108+1	109
Aug. 1	9755	108+7, 116−6	112
20	9774	116+2, 120−4	117
Sept. 13	9798	108+6, 116−2	114
27	9812	105+1, 106−1	106
Oct. 29	9844	88+7, 100−9	93
Nov. 28	9874	77+5, 85−3	82
Dec. 26	9902	77+3, 85−4	80

There are, again, very long gaps in this series, but a period of 226½ days was assumed, and the observations were arranged and tabulated in the usual way. The following are the results for different groups of periods:—

* The letter is *h*, but it seems probable it should be *l*, and this correction has been made accordingly.

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	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
A	85	82	83	84	86	92	103	(107)	112	...	98½	89
B	(81)	(80)	82	(81)	(89)	(98)	(106)	(117)	(116)	105½	89	86
C	(93)	85	82	(78)	78	84	(96)	98	108	115	(114)	(105½)
Weighted Mean with C shifted.	83	81	81	83	89	96	106	114	114	105½	94	87

Group A contains Periods II. to IX., group B Periods XI. to XVII. and XXXVII., group C Periods XLIV. to XLVII.

The brackets show that one period only is observed in that portion. The maximum is in approximately the same position in the first two groups, but in the third group it has advanced about two portions.

The mean has therefore been obtained from these figures with those of the C group shifted back two portions. The result is a good, smooth curve, nearly equal in the rise and fall, but with the maximum much flatter than the minimum.

The individual observations have been compared with the curve, omitting those which are very scattered, and Table XI. shows the results for each portion of the different periods. The shift of 38 days has been applied to the observations in the C group, before comparing them with the curve.

TABLE XI.

Comparison of the Observations with a Mean Light Curve.

Period.	Initial Date.	Reference Numbers and Mean Magnitudes of Portions.											
		(1.) 83.	(2.) 81.	(3.) 81.	(4.) 83.	(5.) 89.	(6.) 96.	(7.) 106.	(8.) 114.	(9.) 114.	(10.) 106.	(11.) 94.	(12.) 87.
II.	9626·5	...	[+2]	[+4]	...	...	−9	−4	−6	[−4]	...	+11	+2
III.	9853	+1	[+1]	[+3]	...	−2	...	...	...	...	...	+1	...
IV.	0079·5	+4	...	+3	...	...	...	+6	...	...	...	...	...
VII.	0759	+5	[+5]	[+1]	0	0	0	−1	...	[−4]	...	+1	...
VIII.	0985·5	+2	[+1]	...	0	+1	−3	...	...	...	...	...	+2
IX.	1212	...	[−2]	[0]	+1	...	...	...	...	...	...	−2	...
XI.	1665	...	...	...	...	...	...	...	+2	...	−8	−6	−2
XII.	1891·5	...	[−1]	[+1]	...	...	...	...	...	...	+2	...	...
XXXVII.	7554	...	...	..	−2	0	+3	0	...	[0]	...	...	...
XLIV.	9139·5	...	...	...	...	...	...	...	[+5]	...	...	...	−2
XLV.	9366	+1	[−3]	[−3]	−2	+7	+2	+2	[−4]	...	...	−6	−5
XLVI.	9592·5	−5	...	[−6]	+4	...	+5	+4	+2	+3	+2	...	+5
XLVII.	9819	0	...	[0]	...	...	...	...	...	...	...	...	...
p−f		+6	+2	−2	−8	−13	−17	−18	−8	+8	+20	+19	+11

Very few periods are sufficiently observed to give anything like trustworthy results when the observations are dealt with in the usual manner for corrections to epoch. This has been done for Periods IV., VII., VIII., IX., XLV., and XLVI., taking the observations so as to have the date of maximum near the middle

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in each case. The observations in brackets have not been used, being too close to maximum or minimum.

These results being so scanty, it seemed well to compare them with those of later observations, and use has been made of the B.A.A. results published in 1913 and of later ones not yet published which have been kindly lent to us in MS. by Mr. C. L. Brook. The Baxendell dates in Table XII. are got by calculation, by the method used throughout this series, and so are the last six of the B.A.A. The others are as estimated in the B.A.A. Memoirs, such epochs as seemed very uncertain being omitted. It is quite evident that no continuous and unchanging length of period will suit all the dates, and this is still true when all reasonable allowance is made for possible errors of observation and flatness of maxima. A solution by the discontinuity method * suits very well, and so does one by a new formula

$$2400589{\cdot}4 + 226{\cdot}2E + 28 \sin (3^\circ{\cdot}6E - 43^\circ).$$

Chandler's formula for this star was

$$2400571 + 226{\cdot}5E + 35 \sin (5^\circ{\cdot}4E + 194^\circ).$$

TABLE XII.

Comparison of Epochs of Maximum.

E.	Observed Epoch. (Bax. or B.A.A.)	Epoch by Disc.	Epoch by New Formula.	Chandler's Epoch.	O – D.	O – F.	O – Ch.
	240 +						
−2	0115	0116	0116	0116	−1	−1	−1
+1	0799	0798·5	0798	0786	+0·5	+1	+13
2	? 1027	1026	1026	1009	+1	+1	(+18)
3	1254	1253·5	1253	1233	+0·5	+1	+21
39	9437	9436	9439	9429	+1	−2	+ 8
40	9664	9661	9665	9658	+3	−1	+ 6
	241 +						
66	5516	5511	5511	5514	+5	+5	+ 2
71	6636	6636	6634	6631	0	+2	+ 5
74	7310	7311	7309	7304	−1	+1	+ 6
75	7540	7536	7534	7529	+4	+6	+11
76	7756	7761	7759	7753	−5	−3	+ 3
77	7988	7986	7984	7979	+2	+4	+ 9
79	8430	8436	8434	8430	−6	−4	0
81	8880	8886	8885	8883	−6	−5	− 3
82	9108	9111	9111	9109	−3	−3	− 1
83	9339	9336	9337	9336	+3	+2	+ 3
84	9556	9561	9562	9564	−5	−6	− 8
85	9786	9786	9788	9791	0	−2	− 5
86	? 0011	0011	0014	0019	+0	−3	− 8

* Periods IV. to XLII. (E = 36) at 227·5 days; the rest at 225 days.

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Some of the residuals in the last column are large, and several of them can hardly be reconciled with the observations, but one is put in brackets because the "observed" epoch in that case is extremely doubtful, and Chandler's date is not impossible in face of the observations.

Analysis of the mean light curve gives the following expression:—

$$1{\cdot}666\cos(\theta-223^\circ)+0{\cdot}347\cos(2\theta-90^\circ)+0{\cdot}085\cos(3\theta-349^\circ),$$

which becomes, when divided by the range and multiplied by 6,

$$2{\cdot}87\cos(\theta-223^\circ)+0{\cdot}59\cos(2\theta-90^\circ)+0{\cdot}15\cos(3\theta-349^\circ).$$

The separate groups of periods are not sufficiently well observed to be worth analysing.

Analysis of the B.A.A. mean light curve, from the observations in MS., gives the expression

$$1{\cdot}654\cos(\phi-149^\circ)+0{\cdot}383\cos(2\phi-294^\circ)+0{\cdot}118\cos(3\phi-172^\circ),$$

which becomes, when divided by the range and multiplied by 6,

$$2{\cdot}92\cos(\phi-149^\circ)+0{\cdot}68\cos(2\phi-294^\circ)+0{\cdot}21\cos(3\phi-172^\circ).$$

If now we write $\phi=\theta-74^\circ$, the expression becomes—

$$2{\cdot}92\cos(\theta-223^\circ)+0{\cdot}68\cos(2\theta-82^\circ)+0{\cdot}21\cos(3\theta-394^\circ).$$

This is in good agreement with that of Baxendell, and the difference in phase agrees pretty well with the corrections to epoch. The magnitude at maximum and minimum is almost the same in the two curves, but the range is about 0·1 magnitude less in that of the B.A.A.

Daylight Photography of Stars as a means of testing the Equivalence Postulate in the Theory of Relativity. By A. F. and F. A. Lindemann.

One of the most important questions in physics is whether there is any interaction between gravitation and electromagnetic phenomena, *e.g.* whether light is affected by a gravitational field.

The principle of relativity leads to the conclusion that the energy E possesses inertia equal to $\frac{E}{C^2}$ if C is the velocity of light. Experiments with a pendulum consisting in large part of uranium, and which therefore contained energy at least of the order $2\times(10)^{17}\,\frac{\text{ergs}}{\text{gramme}}$, showed no deviation from the rule that weight is proportional to inertia, though the inertia is obviously increased by some 0·02 per cent. It seemed to follow that energy must be acted upon by gravity, and it was in order to treat this question

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quantitatively that Einstein formulated his equivalance postulate, namely, that there is no physical means of distinguishing between a uniform gravitational field and a field of uniform acceleration. This postulate may be shown to imply that a ray of light passing a mass M at distance r will be bent out of its course by the angle $\alpha = \frac{4kM}{c^2r}$, k being the gravitational constant.* As this angle is $\frac{0''.46}{\beta}$ in the case of a star $\beta°$ from the sun, experimental verification seems to be within the bounds of possibility.

Einstein himself suggested the possibility of determining the apparent position of fixed stars near the sun during a total eclipse. The rarity of total eclipses, however, and the large number of other problems, which can only be investigated during the few minutes of totality, seemed to render it desirable to try to work out a method of more universal applicability.

Two possibilities suggest themselves: observation and photography. Though it will be seen that the observation of stars near the sun is not impossible in certain cases, if suitable precautions are taken, the photographic method seems far superior. If it is possible to photograph stars near the sun, there are various ways of determining whether there is any apparent shift or not. Firstly, it might be possible to photograph the sun on the same plate as the star, and determine the distance from the limb. Apart from the difficulty of making an accurate measurement, this method requires elaborate apparatus for keeping the sunlight off the plate for most of the time and exposing the plate to the sun for a very short time. Another method would be to photograph a number of stars on the same plate and determine their relative positions. This relies upon there being a cluster of sufficiently bright stars near the ecliptic. A third possibility would be to photograph the star near the sun in daytime and photograph some other stars, whose positions are accurately known, on the same plate at a given time at night. A fourth method, which has many advantages, would be to photograph the same star every day as it approaches and recedes from the sun with the same setting of the telescope, shifting the plate, say, 0·5 cm. each time. If there is no interaction between gravity and light, the images should lie upon a regular curve approximating to a straight line. If Einstein's theory is correct, however, the curve will not be regular when the light from the star passes close to the sun.

The instrument at our disposal, a six-inch equatorial of 223 cm. focus, is of course not sufficient to carry out the experiments, as $1''$ only corresponds to $\frac{1}{90}$th of a millimetre. It is to be hoped, however, that the question will be investigated by an observatory more fortunately situated, since, as the following will show, daylight photography of stars is by no means difficult.

* A. Einstein, "Grundlage der allgemeinen Relativitätstheorie." In his first paper on the subject Professor Einstein gave half these values.

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Whether faint stars can be photographed or not is largely a question of the steadiness of the air. According to Pickering, a 0th magnitude star gives about as much light as a candle at 526 metres, *i.e.* it is equivalent to about $3{\cdot}6\ (10)^{-6}$ candle-power.

The diffraction disc of a star is $2{\cdot}4\ \frac{\lambda L}{D}$ in diameter, λ being the wave-length, L the focal length, and D the diameter of the objective. If the light from the star were concentrated upon a disc of this size, the surface brightness of a first-magnitude star would be

$$\frac{D^4}{5{\cdot}76\lambda^2L^2} \cdot 3{\cdot}6 \cdot 10^{-6} = 2{\cdot}5 \cdot 10^2 \frac{D^4}{L^2}$$

if λ is equal to $5\ (10)^{-5}$ cm. This is of the order $8{\cdot}8\ (10)^4$ for a 150 cm. mirror of 1200 cm. focal length. An ordinary photographic plate of comparatively high sensitiveness requires $0{\cdot}033\ \frac{\text{candle-power second}}{\text{cm.}}$ to render it developable. Thus a 16th magnitude star should be photographed in one second with the 60″ mirror at Mount Wilson, or a 27th magnitude star in seven hours. In reality the limit is about the 22nd magnitude.

It is difficult to give any very satisfactory explanation of this discrepancy. It seems that in practice the smallest stars taken into account are those whose image is about $\frac{1}{10}$th of a millimetre. It may be that this is the limit of instrumental accuracy, or it may be that the grain of the plates is too coarse to permit accurate work with smaller discs; again, it may be, and in certain cases certainly is, that the atmosphere is too disturbed to allow a smaller image than one $(10)^{-2}$ cm. in diameter to be formed. If one calls d the diameter of the disc on the plate, the surface brightness of a star of nth magnitude would be

$$\frac{D^2}{d^2} \cdot 2{\cdot}512^{-n} \cdot 3{\cdot}6 \cdot 10^{-6}.$$

An exposure of t seconds would give rise to a developable image of diameter d if

$$t \cdot \frac{D^2}{d^2} = 9{\cdot}3 \cdot 10^3 \cdot 2{\cdot}512^n.$$

In other words, a star of nth magnitude will be photographed in t seconds if

$$n = 2{\cdot}5 \log t + 5 \log D - 5 \log d - 9{\cdot}9$$

if plates are used that are sensitive to $\frac{1}{30}$th of a candle-power second. It will be seen that, putting $d = (10)^{-2}$, $D = 150$ cm. and $t = 4\ (10)^4$ seconds, $n = 22$, in agreement with results at Mount Wilson.

In order to photograph stars in the daytime, the light from the stars must be comparable to the light from an equivalent solid angle of sky. The blackening of a plate may be expressed as a

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function of the light × time by a curve of the type shown in fig. 1. The most effective exposure is obviously that at which the chemical action of the sky light is about at the point marked A, *i.e.* at which the plate is just about to become fogged. At this point the small extra amount of light from the stars makes most difference.

The scattered light from the sky may be represented with considerable accuracy by Rayleigh's formula as long as the particles

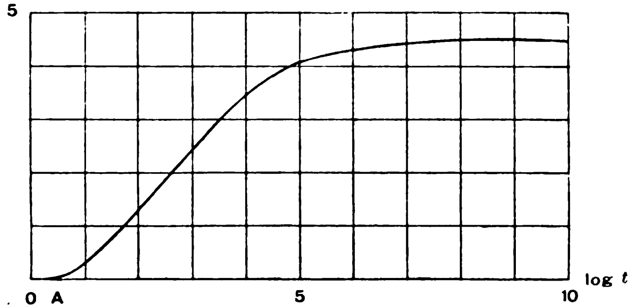

Fig. 1.

are small in comparison to the wave-length of light, *i.e.* as long as the sky is free from dust or clouds. In a blue sky the light from unit solid angle of sky at an angle β from the sun and γ above the horizon is given as *

$$i = \pi^2 \mathrm{I} \frac{m}{\mathrm{N}} \cdot \frac{p}{g} \cdot \frac{1 + \cos^2\beta}{\cos\gamma} \left(\frac{\mu^2 - 1}{\rho}\right)^2 \frac{1}{\lambda^4} = \frac{1 \cdot 01 \,.\, 10^{-19}}{\lambda^4} \cdot \frac{1 + \cos^2\beta}{\cos\gamma} \mathrm{I},$$

I being the illumination due to the sun, m the mean molecular weight of the air, N the number of atoms in a gramme atom, p the pressure, g the gravitational acceleration, μ the refractive index, and ρ the density of air and λ the wave-length. It will be seen from this that in a perfectly clear sky the light from a star of magnitude n is equal to the light from an equivalent area of sky when

$$\frac{\pi^2}{16} \cdot \frac{d^2\mathrm{D}^2}{\mathrm{L}^2} \cdot \frac{1 \cdot 01 \,.\, 10^{-19}}{\lambda^4} \cdot \frac{1 + \cos^2\beta}{\cos\gamma} \mathrm{I}$$

is of the same order as

$$\frac{\mathrm{D}^2}{d^2} \cdot 2 \cdot 512^{-(26 \cdot 6 + n)} \mathrm{I},$$

assuming that the sun is of stellar magnitude −26·6, and further, that the energy distribution in the star's spectrum is something

* Strictly speaking, this quantity should be increased by the fraction of the total incident light reflected by the surface of the earth. This correction, which may of course be quite appreciable, has been neglected, as the formula is used only to give a rough idea of what is to be expected.

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like that in the solar spectrum. In other words, the part of the photographic plate on which the image of the star falls will receive twice as much light as the surrounding parts if

$$n = 5 \log L + 10 \log \lambda - 10 \log d - 2{\cdot}5 \log \frac{1 + \cos^2 \beta}{\cos \gamma} + 21{\cdot}4.$$

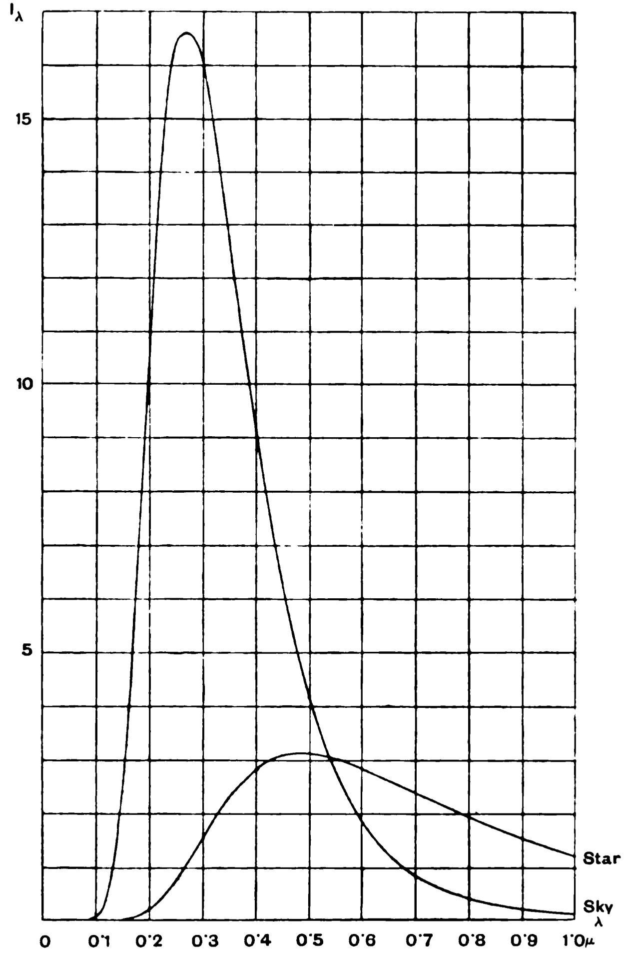

FIG. 2.

If the size of the image d is fixed, it is obviously of advantage to use as large a focal length as possible. If the size of d is determined by the scintillation of the star, $\frac{d}{L} = \delta$ will be independent of

the instrument and will represent the apparent angular displacement of the star. In that case

$$n = 10 \log \lambda - 5 \log L - 10 \log \delta - 2\cdot 5 \log \frac{1 + \cos^2 \beta}{\cos \gamma} + 21\cdot 4,$$

so that a short focus objective is to be recommended. The one change which always improves the contrast is to increase the wave-length λ. Thus a clear gain of 1·5 magnitude is effected by increasing the wave-length some 40 per cent. This becomes evident when the diagram (fig. 2) is considered. The curves are arranged to show the distribution of radiant energy in the spectrum of the sky and of a star of solar type, the scale of the ordinates being fixed so that the total light between 4000 Å. and 8000 Å., *i.e.* the total visible light from either source, is equal. It is clear how much is to be gained by using the red end of the spectrum.

The experiments were carried out with the above-mentioned equatorial telescope in the spring and summer of 1916. The apparatus consisted of a dark slide arranged so that different parts of the plate could be exposed in succession. A slide containing a number of different filters was mounted in front of the plate, so that various filters could be interposed and their relative usefulness compared. The plates used were Wratten Panchromatic B, and the filters were supplied by the same firm.* Those found most advantageous were those numbered 70 and 91. Filter 88 demanded rather long exposures. The percentages of light of different wave-lengths transmitted, as measured by Messrs. Wratten & Wainwright, are given in Table I.

TABLE I.

Filter No. . .	70.	91.	88.
	per cent.	per cent.	per cent.
6000			
6100			
6200			
6300			
6400			
6500	0·4		
6600	7·6		
6700	19·6	0·4	
6800	34·0	2·8	
6900	39·0	12·0	0·9
7000	45·5	15·5	6·5
7500	about 70	about 70	about 70

* We are indebted to the Wratten Division of Messrs. Kodak for the trouble they took in obtaining information about the constants of these plates and filters.

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Fig. 3 indicates, on a scale five times the scale of fig. 2, the relative energy of the various wave-lengths in the light from a star and from the sky after passing through filter 70. Fig. 4 represents their effect in arbitrary units upon a Wratten Panchromatic B plate, assuming the sensitiveness to different wave-lengths

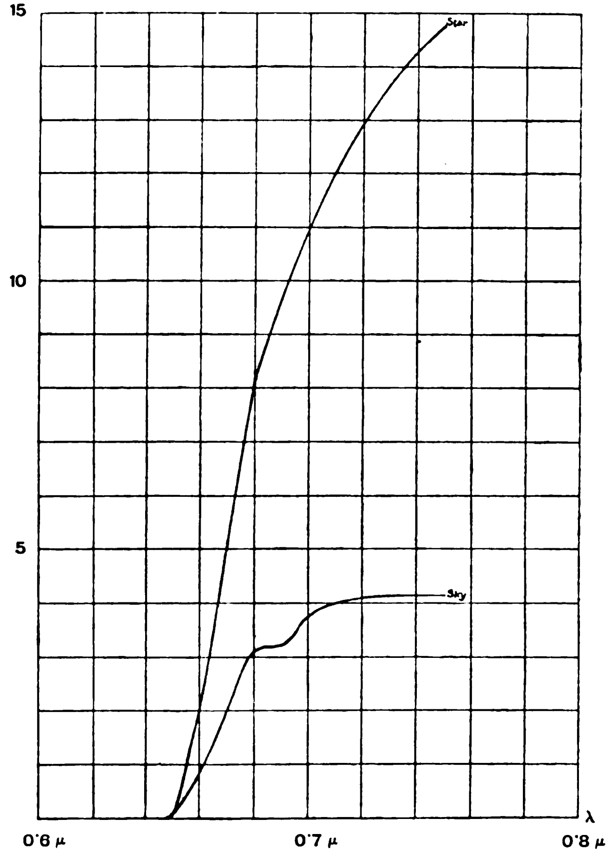

FIG. 3.

claimed by the makers. Fig. 5 shows the effect on an ordinary plate without a filter, using the figures given by Eder. It is apparent that whereas in fig. 4 the effect on the plate due to the star is about 2·75 times as great as that due to the light from the sky, it is about 2·75 times weaker in fig. 5, *i.e.* over two magnitudes have been gained by using a red sensitive plate and filter.

In practice it was found that the theoretical predictions were borne out with fair accuracy. Thus it was possible to photograph

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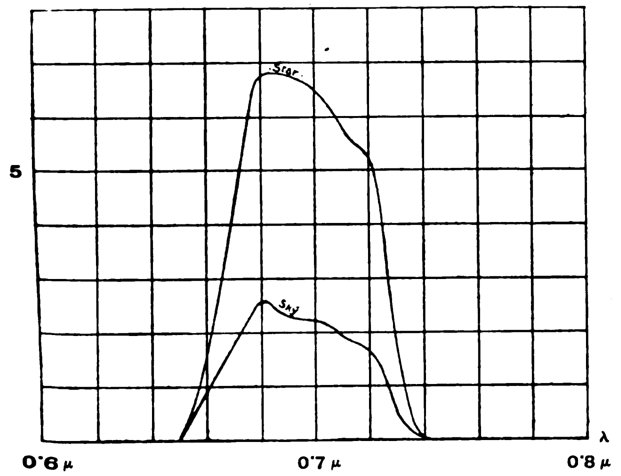

FIG. 4.

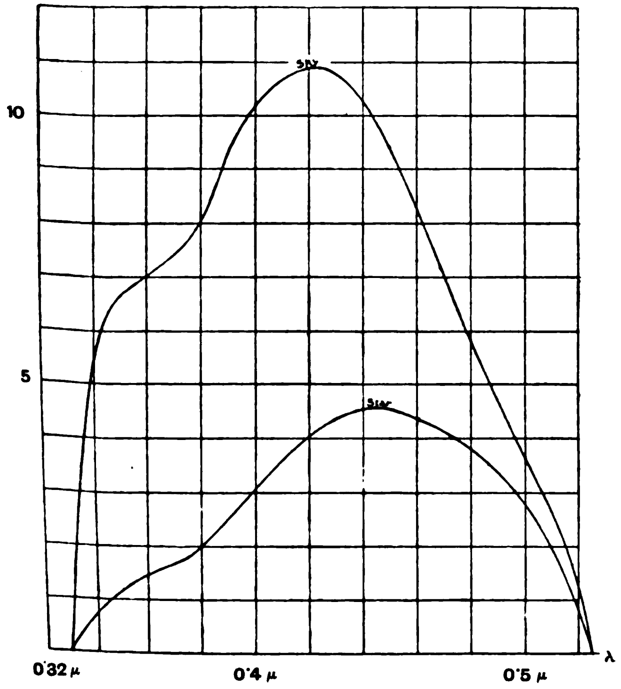

FIG. 5.

oth magnitude stars at any time of the day, even in hazy weather, whereas second and third magnitude stars required a very clear sky and calm air. The accompanying Table II. contains lists of a number of stars photographed, with details as to time of day, exposure, etc., which may be of use to anybody who may carry on this research.

TABLE II.

Date.	G.M.T.	Star.	Magnitude.	Filter.	Exposure (in secs.).	Sky.	Image.	No. of Plate.
7/6/16	5.55 p.m.	Arcturus	0·2	70	12, 16	Medium	Clear	15
18/6/16	11.19 ,,	,,	0·2	88	720, 480	,,	,,	17
19/6/16	6.49 ,,	,,	0·2	88	540	,,	Faint	20
30/7/16	12.15 ,,	Capella	0·2	70	5, 10, 15	,,	Clear	25
31/7/16	11.0 a.m.	,,	0·2	70	1,2,3,4,5	,,	4 & 5 clear	26
1/8/16	9.15 p.m.	Arcturus	0·2	70	4, 5, 6	Good	All ,,	29
1/8/16	9.20 ,,	,,	0·2	91	{10, 15, 20, 25, 30, 35}	,,	,, ,,	29
2/8/16	7.15 ,,	Polaris	2·2	70	{10, 15, 20, 25, 30, 35, 40, 45, 50}	Medium	All visible	30
19/8/16	5.15 ,,	,,	2·2	70	5,7,9,11	,,	,,	33
19/8/16	5.20 ,,	,,	2·2	91	{120, 100, 80, 40, 30}	,,	40 & 30 not visible	33
30/8/16	2.15 ,,	α Ursæ Maj.	2·0	70	{5, 7, 9, 11, 13}	Good	All visible	35
21/9/16	12.5 ,,	Moon	...	70	5, 7, 10	Clouds	Clear	38
21/9/16	12.10 ,,	Venus	...	70	3	,,	,,	38
21/9/16	12.15 ,,	,,	...	91	30	,,	,,	38
26/10/16	10.40 a.m.	ψ Ursæ Maj.	3·2	70	20, 25, 30	,,	Faint	43
26/10/16	12.50 p.m.	γ ,,	2·5	70	15, 20	,,	,,	43

The images are clear and well defined, as indeed they should be, being taken in almost monochromatic light. It is, of course, essential to set the plate in focus with the filter in position.

Unfavourable weather unfortunately prevented any attempts to photograph Regulus when near the sun, though this star would be very well adapted for testing Einstein's theory.* In a really clear sky, however, there should be no particular difficulty, as the scattered light is proportional to $1 + \cos^2 \beta$, *i.e.* it can only be twice as great near the sun as at 90 degrees from it. There seems to be every reason to hope that stars even fainter than the first and second magnitude could be photographed quite near the sun

* Venus and Mercury might be almost equally advantageous at certain periods.

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in a fine climate, *e.g.* in Kashmir or perhaps at Mount Wilson. It would be most desirable that these experiments be carried through by some observatory which possesses a suitable instrument,* and which is favoured by really clear air.

In spite of the increased exposure attending the use of filters, their value in stellar photography may be considerable. Thus, for instance, it may be possible to photograph the comparatively cold companions of some double stars by using a red filter, whereas the brighter star prevents this if no filter is employed. It may even be possible to discover dark stars if sufficiently red and infra-red sensitive plates, such as those described by Sir William Abney, are employed.

The use of these light filters is often of great advantage for observing. In really good weather the image one observes is apparently the true diffraction disc, as is proved by the fact that one can separate double stars almost up to the theoretical limiting power of the telescope. It is for this reason that it is advisable to choose a magnifying power such that the image of the star just covers one cone in the eye; this is about half the diameter of the objective measured in millimetres. The image of the star subtends an angle equal to $2{\cdot}44\frac{\lambda}{D}$, so that the solid angle is $4{\cdot}68\frac{\lambda^2}{D^2}$. The light from this solid angle of blue sky is

$$\frac{4{\cdot}72 \cdot 10^{-19}}{\lambda^2 D^2} \cdot \frac{1+\cos^2\beta}{\cos\gamma} I,$$

whereas the light from an nth magnitude star is $I \cdot 2{\cdot}51^{-(26\cdot 6+n)}$. The two are equal when

$$n = 5 \log \lambda + 5 \log D - 2{\cdot}5 \log \frac{1+\cos^2\beta}{\cos\gamma} + 19{\cdot}2.$$

Putting $\lambda = 5000$ Å., D = 15 cm., and $\frac{1+\cos^2\beta}{\cos\gamma} = 2$, the light from the star is equal to the light from the sky when $n = 2{\cdot}8$. As one can probably see a fifth magnitude star in daytime through a six-inch telescope, it would seem as if one could distinguish a bright point on an evenly illuminated surface when the amount of light falling upon one cone is about 10 per cent. greater than the amount falling upon all the others. It is obvious that the contrast may be improved by using greater wave-lengths, though of course the eye imposes a limit at the red end of the spectrum.† The curves using normal sensitiveness for the eye are drawn in fig. 6, but they can of course only be applied to give a rough idea, as the constant of Fechner's law varies with the wave-length by a different amount for each individual eye.

A fact which these experiments brought out very clearly was

* Probably the object-glass need not be achromatised.

† In certain parts of the sky considerable advantage may be gained by using a Nicol prism, either with or without a filter.

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the enormous difference in the amount of scattered light from apparently blue sky on different days. It is clear that particles of dust or moisture, whose size is not small in comparison to a wave-length, will not conform to Rayleigh's λ^{-4} law. The amount of red light scattered may thus be used for constructing an instrument to give a quantitative measure of the transparency of the air. All that is necessary is to construct an instrument in which the sky can be compared photographically or visually with some known artificial light. A convenient form which might be developed into a standard instrument consists in a vertical camera for photographing the zenith, which enables a strip of sky forming an image some 10 cm. long and 2 cm. broad to be focussed on the plate. (In a standard instrument a pin-hole of well-defined dimensions

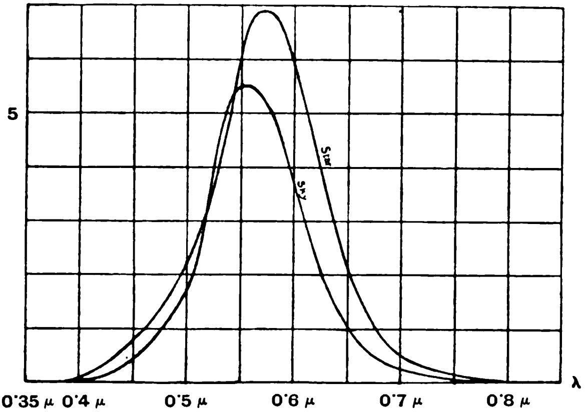

Fig. 6.

might be used.) Next to the strip formed by the light from the sky a strip is exposed to the radiation of a tungsten-filament lamp, burning at a definite voltage and producing a known amount of light. A shutter is pulled across both strips at a constant rate such that the one end of each is slightly overexposed, whilst the other ends are not exposed at all. The current through the lamp is interrupted when the shutter reaches its full extension, so that the blackening of both strips diminishes gradually to zero. All light, of course, passes through standard filters. The point on the two strips at which the blackening is equal defines the amount of light of the wave-length determined by the filter, which is scattered by the air. The zenith must, of course, be photographed at such a time that the sun is some definite distance from it, say 45 degrees. An instrument of this type would enable the transparency of the air at widely different times and localities to be compared quantitatively, at any rate in the daytime. The steadiness of the air must, of course, be measured as heretofore by observing or photographing the apparent movement of a star. Quite apart from the

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astronomical value of being able to compare different atmospheric conditions quantitatively, the transparency of the air to radiation may be of considerable interest in various meteorological investigations. As radiation is supposed to be the most important factor in producing equilibrium in the upper air, any measurements which have a bearing on this should be of value. It is probable, however, that the haze and dust are usually at lower levels, in which convection quite overpowers radiation. Only in those cases in which minute crystals of ice are the cause of scattering would the quantities interest meteorologists, but it is not improbable that the polarisation of different parts of the sky might be used to distinguish such days.

Summary.

Methods are indicated for testing by photography Einstein's hypothesis of the refraction of light by a gravitational field.

The most favourable conditions for photographing stars in the daytime are discussed. Experiments are described showing that stars may be photographed in daytime by using red filters and red sensitive plates, and their value for observing stars during the day is emphasised.

An instrument is described by means of which the transparency of the air at different times and places may be compared quantitatively.

It is suggested that experiments on daylight photography of stars be undertaken by some observatory possessing a suitable instrument, and enjoying a fine climate, with a view to testing Einstein's theory.

Sidmouth :
1916 *Dec.* 4.

The Choice of an Origin for Galactic Longitudes.

By C. D. Perrine, Assoc. R.A.S.

The increasing use of galactic co-ordinates renders desirable the consideration of the best origin for the longitudes.

Innes has adopted for his tables of conversion * the position of the solar apex. The choice of a fixed point as origin is, as he points out, preferable to one which is variable. It is doubtful, however, if the effect of precession would be large enough for a long period to make any great difference in many of the problems for which galactic co-ordinates are used. As the basis, however, for a system of co-ordinates for accurate star positions, the case is different. The choice of any natural origin, such as the solar apex, is fraught with the uncertainties inherent to the determination of

* *Union Obs. Circular*, No. 29.

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its exact position, which may be quite as great as the effects of precession. In the choice of any such origin it will be necessary to be more or less arbitrary, as Innes has been.

The object of this note is to raise the question whether there is not a more convenient and logical origin than the solar apex. Recent investigations have shown that there is a range of something like 60° or more between the positions of the apex as derived from different groups of stars, and that these discordances appear to be systematic. It appears, therefore, not to be as fixed as has been assumed. There is the further objection that the accepted position of the solar apex is at some distance from the galactic plane. This is not serious, but it is preferable that the origin lie in or close to the fundamental plane.

It is, I think, constantly becoming more clear that the so-called ellipsoidal axis has great significance in our stellar system. While the positions found for it vary somewhat, and it will probably prove to be variable to some extent, there seems to be little doubt that it does, in fact, lie in the galactic plane. Altogether it seems to me to offer peculiar advantages, and I would, therefore, propose the adoption of the southern end of that axis, the extremity nearest to the present assumed apex of solar motion, as the origin from which to reckon galactic longitudes.

As the vertex

$$\alpha \quad 274^\circ$$
$$\delta \quad -12^\circ$$

found by Eddington * from the Boss stars lies almost exactly in the galactic plane (as determined by Newcomb), this would seem to be a most natural position to adopt, assuming the epoch 1900·0 where necessary.

Observatorio Nacional Argentino,
Cordoba:
1916 *October* 16.

Photographic Determination of the Parallax of Three Southern Binary Systems. By J. Voûte.

(*Communicated by H.M. Astronomer at the Cape of Good Hope.*)

The plates for these determinations were taken with the Cape astrophotographic refractor during the years 1914, '15, '16. The observations were made with the view of finding whether a longer list of stars might be taken in hand with that instrument after the completion of the parallax observations made with the Transit Circle. The results obtained from instruments with short focus

* *Stellar Movements*, p. 102.

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are not without value, as was shown by Kapteyn in his reductions of the Donner plates and also by Kostinsky's parallax work. The results obtained for these three stars having proved satisfactory, a list of twenty-three stars has been taken in hand.

On every plate exposures are recorded at two consecutive epochs of maximum parallactic factor. In general three exposures of five minutes were made at each epoch, the star-images being shifted in declination. A longer exposure than that found necessary to obtain measurable images of faint stars was chosen in order to reduce the effect of a guiding error. All plates were taken within an hour before the meridian transit of the parallax-star, and as far as possible the second exposure was made at the same hour-angle.

A rotating sector for reducing the images of the bright stars was not available, and consequently the proposal suggested by Grossmann was adopted, viz. of fixing in front of the object-glass a wire (in this case a 2·4 mm. watch-spring) in the direction of the parallel, thus producing a diffraction figure in the direction of the declination. Without this wire it would be impossible to point accurately on the images of a bright double star such as α Crucis, while pointings on the first order diffraction figures may be made with sufficient accuracy.

The plates have been measured with the measuring arrangement of the Zeiss stereo-comparator by a method which will be described in a future paper, where a description of the results derived from the complete list of stars will be published.

κ Toucani System.—The two stars κ Toucani and Lac. 353 form a physical system, as evidenced by the agreement of their proper motions, and, as is now confirmed, also of their parallaxes. Both stars are binaries; the component of κ Toucani has been observed through an arc of about 15°, and that of Lac. 353 about 85°; their periods are therefore still unknown.

The mean magnitude of the eleven comparison stars used for determining the plate constants is 10·4.

From this region four plates were obtained, every set of two consecutive plates giving a value of the parallax after eliminating the proper motions.

The two independent determinations are:

	Plates: *a* and *b*.	Plates: *c* and *d*.	Relative π.
	″	″	″
κ Toucani	+0·16	+0·17	+0·165
Lac. 353	+0·19	+0·11	+0·150

Hence the parallax of the system relative to that of the comparison stars is +0″·157.

Hertzsprung has suspected a hypothetical parallax of 0″·09 (*A.N.*, **190**, 113).

α Crucis System.—The three stars forming this system are

bright stars α_1 and α_2 Crucis accompanied by Lac. 5147. According to Boss the three stars have nearly the same P.M.

Four plates were obtained of this region.

The mean magnitude of the ten comparison stars is 9·6.

The two independent determinations are:

	Plates		Relative π.
	a and *c*.	*b* and *d*.	
	″	″	″
α_1 and α_2 Crucis	+0·01	−0·01	0·000
Lac. 5147	+0·01	−0·01	0·000

Hence the relative parallax of the system is 0″·000.

Gill's value for the parallax of α_1 and α_2 Crucis is +0″·055 (absolute).

Boss 4386 = *Melbourne* 4 = *β* 416.—This is one of the most interesting systems. The tenth magnitude component, which is at a distance of 34″ from the bright star, was found by Herschel; since discovery it has described only a few degrees in its orbit; the other component found in Melbourne, rediscovered by *β* in 1876, has a period of forty-three years. The exposures, taken under splendid condition of definition, show an elongation in the *β*-pair, the Herschel star being quite isolated, so that the parallax can be obtained for both images.

Six plates were obtained.

The mean magnitude of the ten comparison stars is 10·0.

The three independent determinations are:

	Plates			Relative π.
	a and *b*.	*c* and *e*.	*d* and *f*.	
	″	″	″	″
β 416	+0·18	+0·20	+0·14	+0·173
h's star	+0·12	+0·18	+0·18	+0·160

Hence the relative parallax of the system is +0″·166.

The hypothetical parallax computed by Luplau-Janssen is +0″·182 (*A.N.*, **202**, 5).

Royal Observatory,
Cape of Good Hope:
1916 *Oct.* 27.

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On Einstein's Theory of Gravitation, and its Astronomical Consequences. By W. de Sitter, Assoc. R.A.S. Second Paper.*

Contents of Second Paper.

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20. We will now derive the gravitational field of n bodies, which have arbitrarily given motions relatively to the system of co-ordinate axes. We will suppose the bodies to be rigid spheres. By this we mean that in a system of co-ordinates relatively to which any one body is at rest, its outer surface would be described as a sphere. In a system relatively to which it moves it then undergoes the well-known Lorentz-contraction. By the attribute "rigid" I mean that the form of the bodies will not be influenced by their mutual gravitation. We further suppose that their dimensions are small compared with their mutual distances, and also their velocities are supposed to be small compared with the velocity of light. The different bodies will be distinguished by indices μ or ν; the indices i and j refer to the co-ordinate axes as before.

We will take rectangular Cartesian co-ordinates, $x_1 = x$, $x_2 = y$, $x_3 = z$, and $x_4 = ct$, t being the time. The g_{ij} will be

$$\begin{aligned} g_{ii} &= -1 + \gamma_{ii}, & g_{ij} &= \gamma_{ij}, \\ g_{i4} &= \chi_i, & g_{44} &= 1 + \gamma, \end{aligned} \qquad (i, j = 1, 2, 3).$$

The γ_{ij} and γ will be of the first order, χ_i will be of the order $\frac{3}{2}$. Only in γ we require the terms of the second order.

We must now form the equations (first paper, p. 706)

$$\text{(II.)} \qquad G_{ij} = -\kappa T_{ij} + \tfrac{1}{2}\kappa g_{ij} T.$$

* See first paper, *M.N.*, vol. lxxvi. p. 699. For this second paper I have, beyond the help from my colleagues and friends already acknowledged in the first paper, had the great privilege of several conversations with Professor Einstein himself, during a visit of his to Leiden.

To the list of papers given in the footnote to page 700 must be added:

VII. Einstein, "Hamiltonsches Prinzip und allgemeine Relativitätstheorie," *Sitzungsber. Berlin*, 1916 Oct., p. 1111.

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To the required order of accuracy we have

$$(63)\qquad \begin{cases} T_{ij} = \rho \dot{x}_i \dot{x}_j, & T_{i4} = -\rho \dot{x}_i, \\ T_{44} = \rho(1 + \gamma + \phi^2), & T = \rho, \end{cases}$$

where we have again put

$$\dot{x}_i = \frac{dx_i}{cdt}, \qquad \phi^2 = \sum_\alpha{}' \dot{x}_\alpha^2,$$

The velocities $\dot{x}_i$ are of the order $\frac{1}{2}$. We have neglected all terms derived from the inner constitution of the gravitating bodies. By the same reasoning as in art. 10 (first paper) we can show that they can be taken into account by slightly altering the meaning of the symbol ρ.

We will suppose that γ_{ij}, χ_i, and γ do not depend explicitly on $x_4 = ct$, but only through the co-ordinates x_1, x_2, x_3 of the n bodies, which are functions of t. Consequently, a differentiation with respect to x_4 increases the order of smallness by $\frac{1}{2}$, and such a term as, *e.g.*, $\frac{\partial^2 \gamma}{c^2 \partial t^2}$, is of the second order. To the required order of accuracy the equations are found to be

$$(64)\quad \begin{cases} G_{ij} = \frac{1}{2}\frac{\partial^2 \gamma}{\partial x_i \partial x_j} - \frac{1}{2}\sum_\alpha{}' \frac{\partial^2 \gamma_{\alpha\alpha}}{\partial x_i \partial x_j} + \frac{1}{2}\sum_\alpha{}' \left[\frac{\partial^2 \gamma_{i\alpha}}{\partial x_\alpha \partial x_j} + \frac{\partial^2 \gamma_{j\alpha}}{\partial x_\alpha \partial x_i} - \frac{\partial^2 \gamma_{ij}}{\partial x_\alpha^2}\right] = -\frac{1}{2}\delta_{ij}\kappa\rho, \\ G_{i4} = -\frac{1}{2}\sum_\alpha{}' \frac{\partial^2 \gamma_{\alpha\alpha}}{\partial x_i c\partial t} + \frac{1}{2}\sum_\alpha{}' \left[\frac{\partial^2 \gamma_{i\alpha}}{\partial x_\alpha c\partial t} + \frac{\partial^2 \chi_\alpha}{\partial x_i \partial x_\alpha} - \frac{\partial^2 \chi_i}{\partial x_\alpha^2}\right] = \kappa\rho\dot{x}_i, \\ G_{44} = -\frac{1}{2}\sum_\alpha{}' \frac{\partial^2 \gamma}{\partial x_\alpha^2} - \frac{1}{2}\sum_\alpha{}' \frac{\partial^2 \gamma_{\alpha\alpha}}{c^2 \partial t^2} + \sum_\alpha{}' \frac{\partial^2 \chi_\alpha}{\partial x_\alpha c\partial t} + \frac{1}{4}\sum_\alpha{}' \left(\frac{\partial \gamma}{\partial x_\alpha}\right)^2 \\ \qquad + \frac{1}{4}\sum_\alpha{}' \sum_\beta{}' \frac{\partial \gamma}{\partial x_\alpha}\frac{\partial \gamma_{\beta\beta}}{\partial x_\beta} - \frac{1}{2}\sum_\alpha{}' \sum_\beta{}' \frac{\partial}{\partial x_\alpha}\left[\gamma_{\alpha\beta}\frac{\partial \gamma}{\partial x_\beta}\right] = -\frac{1}{2}\kappa\rho(1 + \gamma + 2\phi^2). \end{cases}$$

It will be remembered that $\delta_{ii} = 1$ and $\delta_{ij} = 0$ for $i \neq j$, and that Σ' denotes a sum over the values 1, 2, 3 only.

If we neglected all but the terms of the lowest order, the equations would be linear. Then the field of the n bodies would be got by superposing the fields of the separate bodies, each computed as if the others did not exist. The field of a single body has been found in art. 11. Thus, if we take the system B of art. 11 we find that the γ_{ij} must be of the form

$$\gamma_{ii} = \gamma, \quad \gamma_{ij} = 0 \text{ for } i \neq j.$$

21. If this is introduced into the equations for G_{ij} and the terms of the first order of G_{44}, these equations all reduce to

$$(65)\qquad \sum_\alpha{}' \frac{\partial^2 \gamma}{\partial x_\alpha^2} = \kappa\rho.$$

The integral is

$$\gamma = -\frac{\kappa}{4\pi}\int \frac{\rho dV}{r}.$$

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Here r is the three-dimensional distance from the point (x_1, x_2, x_3) at which the value of γ is required to the point (x_1', x_2', x_3') at which the density is ρ. Thus $r^2 = \sum_a'(x_a - x_a')^2$. Further, dV is the three-dimensional element of volume. The integral must be taken over the whole of three-dimensional space. Outside the n bodies, however, $\rho = 0$, and as the bodies are supposed never to coincide, the integral consists of n separate integrals, each over one of the bodies. Since the bodies are small, we can take for r the distance to any point within the body, say the centre. In performing the integration $\int \rho dV$ we must remember that the body moves with the velocity ϕ_ν. In a system of co-ordinates in which the body would be at rest $\int \rho dV$ would be the mass. Now, however, we have in consequence of the Lorentz-contraction *

$$\int_\nu \rho dV = m_\nu \sqrt{1 - \phi_\nu^2}.$$

Therefore, for points outside the n bodies,

$$(66) \qquad \gamma = -2\lambda^2 \sum_\nu \frac{m_\nu}{r_\nu}(1 - \tfrac{1}{2}\phi_\nu^2).$$

Here we have put, as before, $\kappa = 8\pi\lambda^2$ and $r_\nu^2 = \sum_a'(x_a - x_{a\nu})^2$, $x_{1\nu}, x_{2\nu}, x_{3\nu}$ being the co-ordinates of the body with the index ν, m_ν its mass, and ϕ_ν its velocity. For points inside one of the bodies we shall have the same value in which the sum is to be taken over the $n - 1$ other bodies, the term relating to the body itself being replaced by a term of the form (29) [first paper, art. 11].

The last term in (67) is of the second order. It will be combined below with other terms of the second order, and we will put

$$(67') \qquad \begin{cases} \gamma = \gamma_0 + \gamma', \\ \gamma_0 = -2\lambda^2 \sum_\nu \dfrac{m_\nu}{r_\nu}. \end{cases}$$

The second order term γ' is a sum of several terms, one of which is thus

$$\gamma_1' = \lambda^2 \sum_\nu \frac{m_\nu \phi_\nu^2}{r_\nu}.$$

* For any transformation $\sqrt{g}\, d\tau$ is invariant, if $d\tau$ is the four-dimensional element of volume. Thus for the three-dimensional element of volume we have

$$\left(\frac{dV'}{dV}\right)^2 = \left(\frac{dt}{dt'}\right)^2 \frac{g}{g'}.$$

If we use a Lorentz-transformation, then $g = g'$, and to the required order $\frac{dt}{dt'} = \sqrt{(1 - \phi_\nu^2)}$, therefore $dV' = dV\sqrt{(1 - \phi_\nu^2)}$, where the letters without accents refer to the system in which the body is at rest.

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We now use this value of γ in the equation for G_{i4}, which then becomes

$$\sum_a{}' \frac{\partial}{\partial x_a}\left[\frac{\partial \chi_a}{\partial x_i} - \frac{\partial \chi_i}{\partial x_a}\right] = 2\kappa\rho\dot{x}_i + 2\frac{\partial^2 \gamma_0}{\partial x_i c\partial t}.$$

In the partial differentiation of γ_0 with respect to t, we must consider $x_{i\nu}$ as a function of t, thus $\frac{\partial x_{i\nu}}{c\partial t} = \dot{x}_{i\nu}$, but x_i does not depend explicitly on t. The integral is found to be

(68) $$\chi_i = \frac{\kappa}{2\pi}\int\frac{\rho\dot{x}_i}{\mathrm{r}}dV = 4\lambda^2\sum_\nu\frac{m_\nu\dot{x}_{i\nu}}{\mathrm{r}_\nu}$$

The correction for the Lorentz-contraction can here be neglected, as it would give a term of the order $\frac{5}{2}$. From (68) we find

$$\sum_a{}' \frac{\partial^2 \chi_a}{\partial x_a c\partial t} = 2\frac{\partial^2 \gamma_0}{c^2\partial t^2}.$$

This we introduce into the equation G_{44}. We then divide by $(1+\gamma)$, and subtract (65). Then we find

(69) $$\sum_a{}' \frac{\partial^2(\gamma' - \gamma_1')}{\partial x_a^2} = 2\kappa\rho\phi^2 + \frac{\partial^2\gamma_0}{c^2\partial t^2} + \sum_a{}'\left(\frac{\partial\gamma_0}{\partial x_a}\right)^2.$$

The first term on the right gives

$$\gamma_2' = -4\lambda^2\sum_\nu\frac{m_\nu\phi_\nu^2}{\mathrm{r}_\nu},$$

which can be combined with γ_1'. The second term gives

$$\gamma_3' = -\lambda^2\sum_\nu m_\nu\frac{\partial^2\mathrm{r}_\nu}{c^2\partial t^2},$$

The last term can, by means of (65), be transformed to

$$\sum_a{}' \frac{\partial^2}{\partial x_a^2}(\tfrac{1}{2}\gamma_0^2) - \kappa\rho\gamma_0,$$

We have thus

$$\gamma_4' = \tfrac{1}{2}\gamma_0^2,$$

and

$$\sum_a{}' \frac{\partial^2\gamma_5'}{\partial x_a^2} = -\kappa\rho\gamma_0.$$

Now whenever ρ differs from zero, say for a point within the body ν, γ_0 is of the form

$$\gamma_0 = \gamma_{0\nu} - 2\lambda^2\sum_{\mu\neq\nu}\frac{m_\mu}{\mathrm{r}_\mu},$$

The first part does not depend on the other bodies; for any one point it is a constant, and it can thus be taken into account at

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once in (65) by writing $\rho' = \rho(1 - \gamma_{0\nu})$ for ρ. We will suppose that this has been done, and then we will call ρ' again ρ. There remains

$$\sum_a{}' \frac{\partial^2 \gamma_5'}{\partial x_a^2} = 2\lambda^2 \kappa \rho \sum_{\mu \neq \nu} \frac{m_\mu}{\Delta_{\mu\nu}},$$

where $\Delta_{\mu\nu}$ has been written for r_μ, since the latter is required only for points within the body ν. We then find

$$\gamma_5' = -4\lambda^2 \sum_\nu \frac{m_\nu}{\mathrm{r}_\nu} \sum_{\mu \neq \nu} \frac{m_\mu}{\Delta_{\mu\nu}}.$$

Collecting results we have

$$(70) \quad \begin{cases} \gamma_{ii} = \gamma_0, \quad \gamma_{ij} = 0, \\ \gamma_0 = -2\lambda^2 \sum_\nu \frac{m_\nu}{\mathrm{r}_\nu}, \\ \chi_i = 4\lambda^2 \sum_\nu \frac{m_\nu \dot{x}_{i\nu}}{\mathrm{r}_\nu}, \\ \gamma = \gamma_0 + \gamma', \\ \gamma' = \frac{1}{2}\gamma_0^2 - 3\lambda^2 \sum_\nu \frac{m_\nu \phi_\nu^2}{\mathrm{r}_\nu} - 4\lambda^2 \sum_\nu \frac{m_\nu}{\mathrm{r}_\nu} \sum_{\mu \neq \nu} \frac{m_\mu}{\Delta_{\mu\nu}} - \lambda^2 \sum_\nu m_\nu \frac{\partial^2 \mathrm{r}_\nu}{c^2 \partial t^2}. \end{cases}$$

If we put

$$(71) \qquad \sum_a{}'(x_{i\nu} - x_i)\dot{x}_{i\nu} = \mathrm{r}_\nu \epsilon_\nu, \qquad \sum_a{}'(x_{i\nu} - x_i)\ddot{x}_{i\nu} = \mathrm{r}_\nu \eta_\nu,$$

and if by P_ν we denote the angle at the body ν between the velocity of that body and the radius r_ν, then we have

$$\epsilon_\nu = -\phi_\nu \cos \mathrm{P}_\nu$$

and

$$(72) \qquad \frac{\partial^2 \mathrm{r}_\nu}{c^2 \partial t^2} = \frac{\phi_\nu^2 - \epsilon_\nu^2}{\mathrm{r}_\nu} + \eta_\nu.$$

22. The analysis of the preceding article is due to Dr. J. Droste.* Practically simultaneously Einstein † has given another solution of the same equations. Einstein does not introduce the condition that $\dot{x}_i$ shall be small; on the other hand, he gives his solution only to the first order in κ. It is, however, easy to complete it by adding the terms of the second order in g_{44}. Einstein takes as co-ordinates $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = ict$ $(i = \sqrt{-1})$. Then he puts

$$g_{pq} = -\delta_{pq} + \gamma_{pq}.$$

* J. Droste, *Proceedings Academy of Sciences at Amsterdam*, June 1916, (vol. xix. p. 447).

† Einstein VI., *Sitzungsber. Berlin*, June 1916.

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Thus for i and j different from 4 these γ_{ij} are the same as those in the preceding article. Further,

$$\gamma_{p4} = i\chi_p, \quad \gamma_{44} = -\gamma.$$

Einstein then puts

$$(73) \qquad \gamma_{pq} = \gamma_{pq}' + \delta_{pq}\psi,$$

and he introduces the conditions

$$\sum_a \frac{\partial \gamma_{pa}'}{\partial x_a} = 0 \quad \text{or} \quad \sum_a \frac{\partial \gamma_{pa}}{\partial x_a} = \frac{\partial \psi}{\partial x_p}.$$

The function ψ which has been arbitrarily introduced requires one more condition, which is

$$(74) \qquad \psi = -\tfrac{1}{2}\sum_a \gamma_{aa}'.$$

The equations then become, neglecting terms of the second order,

$$(75) \qquad \sum_a \frac{\partial^2}{\partial x_a^2}\Big(\gamma_{pq}' - \tfrac{1}{2}\delta_{pq}\sum_\beta \gamma_{\beta\beta}'\Big) = 2\kappa\Big(\mathrm{T}_{pq} - \tfrac{1}{2}\delta_{pq}\sum_\beta \mathrm{T}_{\beta\beta}\Big),$$

or

$$\sum_a \frac{\partial^2}{\partial x_a^2}\gamma_{pq}' = 2\kappa \mathrm{T}_{pq};$$

or, if we return to the variable $ct = -ix_4$,

$$(76) \qquad \sum_a \frac{\partial^2 \gamma_{pq}'}{\partial x_a^2} - \frac{\partial^2 \gamma_{pq}'}{c^2 dt^2} = 2\kappa \mathrm{T}_{pq}.$$

The integral is

$$(77) \qquad \gamma_{pq}' = -\frac{\kappa}{2\pi}\int \frac{\mathrm{T}_{pq}'}{\mathrm{r}'}d\mathrm{V}.$$

Here $\mathrm{r}'^2 = \sum_a (x_a - x_a')^2$, T_{pq}' is the value at the point (x_1', x_2', x_3') *and at the time* $t - \frac{\mathrm{r}'}{c}$. For p and q different from 4, T_{pq}' are the same as T_{pq} in the preceding article. Further,

$$\mathrm{T}_{p4}' = -i\mathrm{T}_{p4}, \quad \mathrm{T}_{44}' = -\mathrm{T}_{44}.$$

The integration in (77) must be performed with care, the time $t - \frac{\mathrm{r}'}{c}$ being different for each point in any of the bodies. The space over which the integral must be extended—*i.e.* in which ρ differs from zero—therefore does not coincide with the space occupied by the body at any particular time. For r' we can take

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its value for the centre of the body. Further, we find,* neglecting quantities of the order of the square of the velocity,

$$\int_\nu \rho dV = \frac{m_\nu}{1+\epsilon_\nu}.$$

Therefore we have, correct to the order $\frac{3}{2}$,

$$\gamma_{pq}' = 0, \quad \gamma_{p4}' = -4i\lambda^2 \sum_\nu \frac{m_\nu \dot{x}_{i\nu}}{r_\nu'},$$

$$\gamma_{44}' = 4\lambda^2 \sum_\nu \frac{m_\nu}{r_\nu'}(1-\epsilon_\nu).$$

Consequently,

$$(78) \quad \begin{cases} \psi = -\frac{1}{2}\gamma_{44}' = -2\lambda^2 \sum_\nu \frac{m_\nu}{r_\nu'}(1-\epsilon_\nu), \\ \gamma_{ii} = \psi, \; \gamma_{ij} = 0, \; \gamma = -\gamma_{44} = \psi, \\ \chi_i = 4\lambda^2 \sum_\nu \frac{m_\nu \dot{x}_{i\nu}}{r_\nu'}(1-\epsilon_\nu). \end{cases}$$

It will be seen that this solution differs from (70)—if there also we neglect the terms of the second order—by the substitution of r_ν' for r_ν and by the factor $(1-\epsilon_\nu)$. Now, to the first order in the velocities we have

$$r' = r - \frac{1}{2}r'\frac{\partial r}{c\partial t} = r(1-\epsilon).$$

Consequently, the factor $(1-\epsilon)$ is exactly cancelled, and (78) agrees with (70), neglecting only the terms of the second order (and in χ_i of the order $\frac{5}{2}$). We could now add the terms of the second order in the same way as was done in the preceding article, and we should find the same result, with the exception of the term γ_3', which is here included in the first approximation. The two systems may differ in the terms of the second order owing to the difference in the additional conditions introduced to fix the system of reference.

In both systems the velocity of light is

$$v = c\left[1 + \gamma + 4\lambda^2 \sum_\nu \frac{m_\nu \phi_\nu}{r_\nu} \cos V_\nu\right],$$

where V_ν is the angle between the element $d\sigma$ of the ray of light (taken positive in the direction of the propagation of light) and the velocity of the body ν.

In the system (78) the g_{ij} are functions of the distance r' between the point at which gravitation acts at the time t and the body from which it emanates at the time $t - \frac{r'}{c}$. This shows that

* See Lorentz, *Theory of Electrons*, note 19 (p. 254), where $\frac{v_r}{c} = -\epsilon$.

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gravitation is propagated with the velocity of light. Nevertheless, in the system (70) the g_{ij} depend only on the distance r between the *simultaneous* positions of the point acted upon and the source of the action, and we should apparently be forced to make the conclusion that the velocity of propagation is instantaneous. The paradox is, however, only apparent. If we speak of propagation of gravitation at all, we must conceive the g_{ij} as variable with the time, and in the given problem this variability can only arise from the motions of the bodies producing the gravitational field. Consequently, by the method of art. 21, where these velocities are assumed to be small, and third differentials with respect to the time are neglected entirely, it is *a priori* impossible to derive anything at all about the velocity of propagation.

23. In the planetary theories we wish to determine the motion of one of the n bodies in the field produced by the others and itself. If the body is of finite dimensions, the accelerations impressed on its different points will not be the same. The body being rigid, its points must, however, all have the same motion. There are thus produced strains in it which affect the T_{ij}, and through these again the gravitational field. These effects are, of course, of the second order and, if the dimensions of the bodies are small compared with their mutual distances, they will be negligible. Then the equations of motion of a body are the same as those of any point in it, say the centre. The equations can be written in any of the forms given in the first paper. We will use the formulas (47), and for χ_i and γ we use the values (70).

If the body whose motion is considered has the index μ, then in γ_0 we must leave out the term with that index. The same remark applies to the single sums in χ_i and in the second and fourth terms of γ'. In the double sum, which forms the third term of γ', we must in the first sum omit the body μ, but in the second (inner) sum it must be included. We will bring the term so arising into evidence by taking it out of the sum. Then the index μ must be excluded from *all* sums. The complete equation then becomes:

$$
\begin{aligned}
(82)\quad \frac{d^2 x_{i\mu}}{c^2 dt^2} + \lambda^2 \sum_\nu m_\nu \frac{x_{i\mu} - x_{i\nu}}{\mathrm{r}_\nu^2} &= 4\lambda^2 \sum_\nu \sum_\kappa m_\nu m_\kappa \frac{x_{i\mu} - x_{i\nu}}{\mathrm{r}_\nu^3 \mathrm{r}_\kappa} \\
&+ 4\lambda^2 \sum_\nu m_\nu \sum_p{}' \frac{x_{p\mu} - x_{p\nu}}{\mathrm{r}_\nu^3} \dot{x}_{i\mu} \dot{x}_{p\mu} \\
&- \lambda^2 \sum_\nu m_\nu \frac{x_{i\mu} - x_{i\nu}}{\mathrm{r}_\nu^3} \left\{ \phi_\mu^2 + 2\phi_\nu^2 - \tfrac{3}{2}\epsilon_\nu^2 + \tfrac{1}{2}\mathrm{r}_\nu \eta_\nu \right\} \\
&+ \lambda^2 \sum_\nu m_\nu \left\{ \frac{7}{2} \frac{\ddot{x}_{i\nu}}{\mathrm{r}_\nu} + 3 \frac{\dot{x}_{i\mu} - \dot{x}_{i\nu}}{\mathrm{r}_\nu^2} \epsilon_\nu + 4 \sum_p{}' \frac{(x_{i\mu} - x_{i\nu})\dot{x}_{p\nu} - (x_{p\mu} - x_{p\nu})\dot{x}_{i\nu}}{\mathrm{r}_\nu^3} \dot{x}_{p\mu} \right\} \\
&- 2\lambda^4 \sum_\nu m_\nu \frac{x_{i\mu} - x_{i\nu}}{\mathrm{r}_\nu^3} \sum_{\kappa \neq \nu} \frac{m_\kappa}{\Delta_{\kappa\nu}} - 4\lambda^4 m_\mu \sum_\nu m_\nu \frac{x_{i\mu} - x_{i\nu}}{\mathrm{r}_\nu^4}.
\end{aligned}
$$

It should be noted that r_ν is identical with $\Delta_{\mu\nu}$.

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The left-hand members put equal to zero give the motion according to Newton's law of gravitation. The first three terms on the right are the generalisation of the terms already found in the first paper for the motion of an infinitesimal planet.

In applying this equation to the solar system we must take account of the smallness of most of the masses. The sun will be given the index $\mu = 0$.

We will introduce co-ordinates relatively to the sun by putting

$$x_i' = x_i - x_{i0},$$

from which

$$\frac{d^2x_i'}{c^2dt^2} = \frac{d^2x_i}{c^2dt^2} - \frac{d^2x_{i0}}{c^2dt^2}.$$

In the right-hand member we must then put

$$\dot{x}_i = \dot{x}_i' + \dot{x}_{i0}, \quad \ddot{x}_i = \ddot{x}_i' + \ddot{x}_{i0}.$$

In these right-hand members we can use the values given by the Newtonian theory. The system of co-ordinates to which (82) is referred was fixed in art. 20 by the additional conditions introduced during the integration. One of these was that at a large distance from all masses the geodetic lines are straight lines. In Newtonian mechanics this condition is satisfied relatively to a system of co-ordinates of which the origin is in the centre of gravity of all masses. Thus for the Newtonian values we will have to take

$$\dot{x}_{i0} + m_\mu \dot{x}_{i\mu} + \sum_\nu m_\nu \dot{x}_{i\nu} = 0,$$

or, if we put

$$M = 1 + m_\mu + \sum_\nu m_\nu,$$

$$\dot{x}_{i\mu} = \left(1 - \frac{m_\mu}{M}\right)\dot{x}_{i\mu}' - \sum_\nu \frac{m_\nu}{M}\dot{x}_{i\nu}',$$

and the same for second differentials. The sums exclude the values 0 and μ. After the substitution we drop the accents.

We must now write the equation (82) first for the index μ and then for the index 0, and subtract the latter from the former. I will slightly alter the notation of the mutual distances, to bring it into harmony with the usual notation of planetary theories. In equation (82) we first replace r_ν by $\Delta_{\mu\nu}$. Then we replace

$$\Delta_{0\mu}, \ \Delta_{0\nu}, \ \Delta_{\mu\nu}, \ \Delta_{\kappa\nu}\ ; \ m_0, \ m_\mu, \ m_\nu$$

by

$$r, \quad r_\nu, \ \Delta_\nu, \ \Delta_{\kappa\nu}\ ; \ 1, \quad m, \ m_\nu$$

respectively, and we also drop the index μ of the co-ordinates, and we replace λ^2 by λ_0^2. The masses m and m_ν are very small. We will suppose them to be of the order $\frac{1}{2}$. Then in the right-hand

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members all terms beyond the first three will be of the order $2\frac{1}{2}$, and the resulting equation becomes, correct to the second order,

$$(83)\qquad \frac{d^2x_i}{c^2dt^2}+\lambda_0^2(1+m)\frac{x_i}{r^3}+\lambda_0^2\sum_\nu m_\nu\frac{x_i-x_{i\nu}}{\Delta_\nu^3}-\lambda_0^2\sum_\nu m_\nu\frac{x_{i\nu}}{r_\nu^3}$$
$$=4\lambda_0^4\frac{x_i}{r^4}+4\lambda_0^2\sum_p{}'\frac{x_p}{r^3}\dot{x}_p\dot{x}_i-\lambda_0^2\frac{x_i}{r^3}\phi^2.$$

The left-hand member gives the unperturbed motion and the perturbing forces according to Newton's law. The right-hand member contains nothing beyond the terms which have already been found in the first paper (art. 15) for an infinitesimal planet.

If we wish to take account of the terms of the order $\lambda^4 m$, then we must: 1st, in the ordinary perturbing function on the left hand use the co-ordinates increased by the "perturbations" produced by the right-hand member, *i.e.* we must use the formula (52, B) instead of Kepler's third law, and we must take account of the motion of the perihelion found in art. 16; 2nd, we must in the right-hand member of (83) use the co-ordinates increased by their Newtonian perturbations of the first order; and 3rd, we must add to this right-hand member the following terms:—

$$(83')\quad\left\{\begin{aligned}&\lambda_0^2 m\left\{2\frac{\dot{x}_i\dot{r}}{r^2}-\frac{x_i}{r^3}\left[4\phi^2-\tfrac{3}{2}\dot{r}^2\right]\right\}+8\lambda^4 m\frac{x_i}{r^4}\\&+\lambda_0^2\sum_\nu m_\nu\left\{\frac{x_i-x_{i\nu}}{\Delta_\nu^3}\left[4\chi_\nu+\tfrac{3}{2}\epsilon_\nu^2-\phi^2-2\phi_\nu^2\right]+\epsilon_\nu\left[\frac{3\dot{x}_i+\dot{x}_{i\nu}}{\Delta_\nu^2}+\frac{7\Delta_\nu\dot{x}_i}{r^3}\right]\right.\\&\qquad\left.+\dot{x}_i\left[\frac{4\epsilon_\nu'}{\Delta_\nu^3}-\frac{7r_\nu\dot{r}_\nu}{r^3}\right]+6\frac{x_i}{r^3}\chi_\nu+\frac{x_{i\nu}}{r_\nu^3}\left[\tfrac{3}{2}\dot{r}_\nu^2-2\phi_\nu^2\right]+3\frac{\dot{r}_\nu\dot{x}_{i\nu}}{r_\nu^2}\right\}\\&+\lambda_0^4\sum_\nu m_\nu\left\{\frac{x_i-x_{i\nu}}{4\Delta_\nu^3}\left[\frac{3}{r_\nu}-\frac{2r\cos\psi_\nu}{r_\nu^2}\right]-\frac{2x_i}{r_\nu^3\Delta_\nu}-\frac{x_i\cos\psi_\nu}{2r^2r_\nu^2}\right.\\&\qquad\left.+\frac{7}{2}\frac{x_{i\nu}}{r_\nu^3}\left[\frac{2}{r_\nu}+\frac{1}{r}-\frac{1}{\Delta_\nu}\right]\right\},\end{aligned}\right.$$

where for abbreviation we have put

$$\Delta_\nu\epsilon_\nu=\sum_p{}'(x_{p\nu}-x_p)\dot{x}_{p\nu},\qquad \Delta_\nu\epsilon_\nu'=\sum_p{}'(x_p-x_{p\nu})\dot{x}_p,$$
$$rr_\nu\cos\psi_\nu=\sum_p{}'x_px_{p\nu},\qquad \chi_\nu=\sum_p{}'\dot{x}_p\dot{x}_{p\nu}.$$

All these terms are, of course, extremely small, and none of them will give rise to an effect which is at all within the reach of observations.

24. For the lunar theory * I will take a system of reference which has its origin in the sun. The masses of the sun, the earth, and the moon will be m, 1, and μ respectively. We will further

* See also De Sitter, "Planetary Motion and the Motion of the Moon according to Einstein's Theory," *Proceedings Acad. Amsterdam*, June 1916 (vol. xix. p. 367).

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put $\lambda_0{}^2 = \lambda^2 m$, $\lambda_1{}^2 = \lambda^2$, if λ^2 is expressed with the mass of the earth as unit. Consequently, $\lambda_0{}^2$ will have the same numerical value as before, and $\lambda_1{}^2 = \lambda_0{}^2/m$. We will put

x_i, Δ = heliocentric co-ordinates and distance of the moon,
ξ_i, ρ = " " " " earth,
x_i, r = geocentric " " " moon.

Further, we put

$$\sum_a{}' \dot{x}_a{}^2 = \phi^2, \quad \sum_a{}' \dot{\mathrm{x}}_a{}^2 = \phi_1{}^2, \quad \sum_a{}' \dot{\xi}_a{}^2 = \phi_0{}^2,$$

whence

$$\phi_1{}^2 = \phi_0{}^2 + \phi^2 + 2\sum_a{}' \dot{x}_a \dot{\xi}_a .$$

We now must compute the difference

$$\frac{d^2\mathrm{x}_i}{c^2dt^2} - \frac{d^2\xi_i}{c^2dt^2},$$

both differential quotients being computed by (82). The terms of (82) will be divided into three groups, viz.:—

a. Those which only contain the masses 1 and μ.

b. Those in $\frac{d^2\mathrm{x}_i}{c^2dt^2}$ containing m or $m \, . \, \mu$, and in $\frac{d^2\xi_i}{c^2dt^2}$ containing m and $m \, . \, 1$,

c. The others, *i.e.* in $\frac{d^2\mathrm{x}_i}{c^2dt^2}$, the terms with $m \, . \, 1$, and in $\frac{d^2\xi_i}{c^2dt^2}$ those with $m \, . \, \mu$.

The first group corresponds to the problem of the *two* bodies, earth and moon. The second group would give the perturbations by the sun if the complete field of the earth and sun were equal to the sum of the superposed fields of each of these bodies computed as if the other did not exist. The third group contains the terms which arise from the "interference" of the two fields.

a. In the first group I take, in the terms of the second order,

$$\ddot{x}_i = -\lambda_1{}^2(1 + \mu)\frac{x_i}{r^3} .$$

Further, I put

$$\epsilon = \sum_p{}' x_p \dot{\xi}_p .$$

We then find, after some reductions,

$$(84) \quad \frac{d^2x_i}{c^2dt^2} + \lambda_1{}^2(1+\mu)\frac{x_i}{r^3} = 4\lambda_1{}^4(1 - \mu + 2\mu^2)\frac{x_i}{r^4} + 4\lambda_1{}^2(1 + \tfrac{3}{4}\mu)\frac{\dot{x}_i\dot{r}}{r^2} - \lambda_1{}^2(1 + 2\mu)\frac{x_i}{r^3}\phi^2$$
$$+ \lambda_1{}^2\frac{x_i}{r^3}\left[(1+\mu)\phi_0{}^2 + 2\sum_p{}'\dot{x}_p\dot{\xi}_p + 3\lambda_1{}^2\mu\frac{\dot{r}\epsilon}{r} + \tfrac{3}{2}(1+\mu)\frac{\epsilon^2}{r^2} + \tfrac{1}{2}(1-\mu)\sum_p{}' x_p\ddot{\xi}_p\right]$$
$$+ \lambda_1{}^2\left[(1+2\mu)\frac{\dot{x}_i\epsilon}{r^3} + \tfrac{7}{2}(1-\mu)\frac{\ddot{\xi}_i}{r}\right].$$

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If we took $\mu = 0$, $\dot{\xi}_i = \ddot{\xi}_i = 0$, this would become the formula for the motion of an infinitesimal planet. We can take

$$\phi_0^2 = \dot{\rho}^2 + \rho^2\dot{\theta}_0^2, \quad \ddot{\xi} = -\lambda_0^2\frac{\xi_i}{\rho^3}.$$

If we neglect the excentricity of the earth's orbit, we have

$$\dot{\rho} = 0, \quad \dot{\theta}_0 = \nu_0,$$

therefore

$$\phi_0^2 = \frac{\lambda_0^2}{\rho}.$$

The formula (84) then becomes, if we neglect in the second member the terms having μ as a factor,

$$(85) \quad \frac{d^2x_i}{c^2dt^2} + \lambda_1^2(1+\mu)\frac{x_i}{r^3} = 4\lambda_1^4\frac{x_i}{r^4} + 4\lambda_1^2\frac{\dot{x}_i\dot{r}}{r^2} - \lambda_1^2\frac{x_i}{r^3}\phi^2$$

$$+ \lambda_1^2\frac{x_i}{r^3}\sum_p{}'\left(2\dot{x}_p\dot{\xi}_p - \frac{\lambda_0^2x_p\xi_p}{2\rho^3}\right) + \lambda_1^2\frac{x_i\epsilon}{r^3} + \tfrac{7}{2}\lambda_1^2\lambda_0^2\frac{\xi_i}{r\rho^3}$$

$$+ \lambda_1^2\frac{x_i}{r^3}\left[\frac{\phi_0^2}{\rho} + \frac{3}{2}\frac{\epsilon^2}{r^2}\right].$$

We can, in the terms of the second order, safely neglect the excentricity and the inclination of the moon's orbit. Then, if

$$D = l_☾ - l_☉$$

be the difference of the mean longitudes of the moon and sun, we have

$$\sum_p{}' x_p\xi_p = r\rho\cos D, \quad \epsilon = r\rho\nu_0\sin D, \quad \sum_p{}'\dot{x}_p\dot{\xi}_p = r\rho\nu\nu_0\cos D,$$

where we can take

$$\nu = \lambda_1 a^{\frac{3}{2}}, \quad \nu_0 = \lambda_0\rho^{\frac{3}{2}}.$$

The terms in the second line of (85) are periodic in D, and can therefore certainly be neglected. The terms in the last line contain secular terms. The secular part of ϵ^2 is

$$\epsilon^2 = \tfrac{1}{2}r^2\rho^2\nu_0^2 = \tfrac{1}{2}\lambda_0^2\frac{r^2}{\rho}.$$

The two terms can therefore be combined, and can be transported to the left-hand member. We then have

$$(86) \quad \frac{d^2x_i}{c^2dt^2} + \lambda_1^2\left(1 + \mu - \tfrac{7}{4}\frac{\lambda_0^2}{\rho}\right)\frac{x_i}{r^3} = 4\lambda_1^4\frac{x_i}{r^4} + 4\lambda_1^2\frac{\dot{x}_i\dot{r}}{r^2} - \lambda_1^2\frac{x_i}{r^3}\phi^2.$$

The right-hand member is the same as for an infinitesimal planet. The only secular effect is thus a motion of the perigee amounting to

$$\delta\varpi = 3\frac{\lambda_1^2}{p}v.$$

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The motion in one century is

$$0''\cdot 06.$$

b. Many of the terms of the second group are zero because the origin of the system of reference has been taken in the sun. We find

$$(87)\quad \frac{d^2x_i}{c^2dt^2}+\lambda_1{}^2m\left(\frac{\mathrm{x}_i}{\Delta^3}-\frac{\xi_i}{\rho^3}\right)=4\lambda_1{}^4m\frac{\xi_i}{\rho^4}+4\lambda_1{}^4m^2\left(\frac{\mathrm{x}_i}{\Delta^4}-\frac{\xi_i}{\rho^4}\right)$$
$$+4\lambda_1{}^2m\left(\frac{\dot\Delta\dot{\mathrm{x}}_i}{\Delta^2}-\frac{\dot\rho\dot\xi_i}{\rho^2}\right)-\lambda_1{}^2m\left(\frac{\mathrm{x}_i}{\Delta^3}\phi_1{}^2-\frac{\xi_i}{\rho^3}\phi_0{}^2\right)-4\lambda_1{}^4m\mu\frac{\mathrm{x}_i}{\Delta^4}.$$

The left-hand member contains the ordinary perturbing function. In the usual developments of this function the factor $\lambda_0{}^2/\rho^3$ is replaced by $\nu_0{}^2$. This, however, is not correct in the new theory, but we must take, according to (52, B),

$$\lambda_1{}^2m=\lambda_0{}^2=\rho^3\nu_0{}^2\left(1+\frac{3\lambda_0{}^2}{\rho}\right).$$

The first term on the right contains no co-ordinates of the moon, and can thus only give rise to periodic terms, which we neglect. The last term could be added to the second. It is, however, about 27 million times smaller than this, and can be neglected.

The other terms in the right-hand member produce perturbations in the motion of the moon, which will be investigated in the next article.

c. The terms of the third group are

$$(88)\quad \frac{d^2x_i}{c^2dt^2}=4\lambda_1{}^4m\left(\frac{\mathrm{x}_i}{r\Delta^3}-\mu\frac{\xi_i}{r\rho^3}\right)-2\lambda_1{}^4m\left(\frac{\mathrm{x}_i}{\rho\Delta^3}-\mu\frac{\xi_i}{\Delta\rho^3}\right)$$
$$+4\lambda_1{}^4m\frac{x_i}{r^3}\left(\frac{1}{\Delta}+\frac{\mu}{\rho}\right)-2\lambda_1{}^4m\frac{x_i}{r^3}\left(\frac{1}{\rho}+\frac{\mu}{\Delta}\right).$$

The last four terms have x_i/r^3 as a factor. If we develop the coefficient in powers of the parallax a/ρ, we can neglect all but the lowest power, and we can thus in these small terms take $\Delta=\rho$. The four terms together then give

$$2\lambda_1{}^2\frac{\lambda_0{}^2}{\rho}\;\frac{x_i}{r^3}(1+\mu).$$

This can be transported to the left-hand member. The coefficient of x_i/r^3 then becomes

$$\lambda_1{}^2\left(1+\mu-\frac{15}{4}\frac{\lambda_0{}^2}{\rho}-2\mu\frac{\lambda_0{}^2}{\rho}\right),$$

for which we can write

$$\lambda_1{}^2(1+\mu'),$$

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and the difference between μ and μ' is very much smaller than the uncertainty of the numerical value of μ. Of the other terms of (88) the third is of the same nature as the second of (87), but 328000 times as small, and therefore entirely negligible. The fourth term is still smaller, and if we replace Δ by ρ, it is of the same nature as the first of (87), and can be neglected for the same reason. The second term, which contains the factor μ and is therefore very small, can, moreover, produce only periodic perturbations. Of all the terms in (88) the first is the largest. It is still about 800 times smaller than the second of (88), and can be neglected. It can, however, also very easily be taken into account. Developing in powers of $1/\rho$, we find for the lowest power

$$-2\lambda_1^2\lambda_0^2\frac{x_i}{r\rho^3} = -2\lambda_1^2\nu_0^2\frac{x_i}{r}.$$

The resulting radial and transversal perturbing forces are (if we neglect the inclination of the moon's orbit),

$$S = -2\nu_0^2, \qquad T = 0.$$

The effect on the motion of the perigee is

$$ed\varpi = 2\nu_0^2 r^2 \cos v dv,$$

which gives only extremely small periodic terms, that can be neglected.

There thus remains only the equation (87), *i.e.* the effect of the pure superposition of the fields of the earth and the sun.

25. We can write this equation in the following form :—

$$(89) \qquad \frac{d^2x_i}{c^2dt^2} + \lambda_1^2 m\left(\frac{\mathrm{x}_i}{\Delta^3} - \frac{\xi_i}{\rho^3}\right) = \lambda_1^2[4A_i + 4B_i - C_i],$$

where

$$A_i = \lambda_1^2 m^2\left(\frac{\mathrm{x}_i}{\Delta^4} - \frac{\xi_i}{\rho^4}\right),$$

$$B_i = \lambda_1^2 m\left(\frac{\Delta\dot{\Delta}\dot{\mathrm{x}}_i}{\Delta^3} - \frac{\rho\dot{\rho}\dot{\xi}_i}{\rho^3}\right),$$

$$C_i = \lambda_1^2 m\left(\frac{\mathrm{x}_i}{\Delta^3}\phi_1^2 - \frac{\xi_i}{\rho^3}\phi_0^2\right).$$

We must develop A, B, and C in powers of r/ρ. We will only retain such terms as can give secular perturbations, and of these only those of the lowest degree in r/ρ. We will write x, y, z for x_1, x_2, x_3, and similarly x, y, z, etc.; and for A_1, A_2, A_3 we will write A_x, A_y, A_z, and similarly B_x, . . ., etc. As plane of xy we take the orbital plane of the moon, thus $z = 0$. Then the radial, transversal, and orthogonal perturbing forces are

$$S_A = \frac{x}{r}A_x + \frac{y}{r}A_y, \quad T_A = \frac{x}{r}A_y - \frac{y}{r}A_x, \quad W_A = A_z,$$

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and similarly for B and C. Let i and Ω be the inclination and ascending node of the moon's orbit on the ecliptic. We will neglect i^2. Also we neglect the excentricity of the earth. Then we have:

$$\xi = \rho\cos L, \qquad \dot\xi = -\rho\nu_0 \sin L.$$
$$\eta = \rho \sin L, \qquad \dot\eta = \rho\nu_0\cos L.$$
$$\zeta = -\rho\sin i \sin(L-\Omega), \qquad \dot\zeta = -\rho\nu_0 \sin i\cos(L-\Omega).$$

$$\rho^3\nu_0^{\,2} = \lambda_0^{\,2} = \lambda_1^{\,2} m.$$

Further, we have

$$\Delta^2 = \rho^2\left[1 + 2\frac{x\cos L + y\sin L}{\rho} + \frac{r^2}{\rho^2}\right],$$

and

$$\mathrm{x} = \xi + x, \quad \mathrm{y} = \eta + y, \quad \mathrm{z} = \zeta.$$

Taking first A_x, we find for the terms of the lowest degree in r/ρ,

$$\frac{\mathrm{x}}{\Delta^4} - \frac{\xi}{\rho^4} = \frac{x}{\rho^4} - 4\frac{\xi(x\cos L + y\sin L)}{\rho^5}.$$

The secular part (*i.e.* the part independent of L) of the numerator of the second term is $\frac{1}{2}\rho x$, therefore

$$A_x = -\lambda_0^{\,2} m\frac{x}{\rho^4} = -\frac{m\nu_0^{\,2}}{\rho}x.$$

Similarly we find

$$A_y = -\frac{m\nu_0^{\,2}}{\rho}y.$$

Further,

$$\frac{\mathrm{z}}{\Delta^4} - \frac{\zeta}{\rho^4} = \zeta\left(\frac{1}{\Delta^4} - \frac{1}{\rho^4}\right) = 4\rho\sin i\sin(L-\Omega)\frac{x\cos L + y\sin L}{\rho^5}.$$

The numerator of the last fraction is

$$r\cos(w - L),$$

where w is the moon's longitude. The secular part thus becomes

$$\frac{2r\sin i\sin(w-\Omega)}{\rho^4}.$$

Now we put

$$z' = r\sin i\sin(w-\Omega),$$

so that z' is the co-ordinate of the moon perpendicular to the ecliptic. Then we have

$$A_z = 2\frac{m\nu_0^{\,2}}{\rho}z'.$$

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Consequently

$$(90)\quad \begin{cases} S_A = -\dfrac{mv_0^2}{\rho}r, \\ T_A = 0, \\ W_A = 2\dfrac{mv_0^2}{\rho}z'. \end{cases}$$

It is not necessary to relate the computations of the terms B and C at the same length. In B we have $\dot\rho = 0$, so there remains only one term to be developed. We have $\Delta\dot\Delta = \mathrm{x}\dot{\mathrm{x}} + \mathrm{y}\dot{\mathrm{y}} + \mathrm{z}\dot{\mathrm{z}}$. After development we find:

$$B_x = \tfrac{1}{2}\frac{mv_0^2}{\rho}x - \tfrac{1}{2}\frac{mv_0}{\rho}\dot y,$$

$$B_y = \tfrac{1}{2}\frac{mv_0^2}{\rho}y + \tfrac{1}{2}\frac{mv_0}{\rho}\dot x,$$

$$B_z = -\tfrac{1}{2}\frac{mv_0^2}{\rho}z' + \tfrac{1}{2}\frac{mv_0}{\rho}z'\nu.$$

The factor z' in the last term of B_z is found by neglecting the moon's excentricity in the expressions for $\dot x$ and $\dot y$, which is allowed on account of the factor sin i. With the same accuracy we can take $\nu = \dot\theta$. Then, since

$$x\dot y - y\dot x = r^2\dot\theta, \quad x\dot x + y\dot y = r\dot r,$$

we find

$$(91)\quad \begin{cases} S_B = \tfrac{1}{2}\dfrac{mv_0^2}{\rho}r - \tfrac{1}{2}\dfrac{mv_0}{\rho}r\dot\theta, \\ T_B = \qquad\qquad \tfrac{1}{2}\dfrac{mv_0}{\rho}\dot r, \\ W_B = -\tfrac{1}{2}\dfrac{mv_0^2}{\rho}z' + \tfrac{1}{2}\dfrac{mv_0}{\rho}z'\dot\theta. \end{cases}$$

Finally, for C we find similarly

$$C_x = -\tfrac{1}{2}\frac{mv_0^2}{\rho}x + \frac{mv_0}{\rho}\dot y,$$

$$C_y = -\tfrac{1}{2}\frac{mv_0^2}{\rho}y - \frac{mv_0}{\rho}\dot x,$$

$$C_z = \tfrac{3}{2}\frac{mv_0^2}{\rho}z' - \frac{mv_0}{\rho}z'\nu,$$

and

$$(92)\quad \begin{cases} S_C = -\tfrac{1}{2}\dfrac{mv_0^2}{\rho}r + \dfrac{mv_0}{\rho}r\dot\theta, \\ T_C = \qquad\qquad -\dfrac{mv_0}{\rho}\dot r, \\ W_C = \tfrac{3}{2}\dfrac{mv_0^2}{\rho}z' - \dfrac{mv_0}{\rho}z'\dot\theta. \end{cases}$$

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Collecting results, we find for the total perturbing forces (4A + 4B − C),

$$(93)\quad \begin{cases} S = -\frac{3}{2}\frac{mv_0^2}{\rho}r - 3\frac{mv_0}{\rho}r\dot{\theta}, \\ T = \qquad\qquad + 3\frac{mv_0}{\rho}\dot{r}, \\ W = +\frac{9}{2}\frac{mv_0^2}{\rho}z' + 3\frac{mv_0}{\rho}z'\dot{\theta}. \end{cases}$$

It has already been remarked that the ordinary Newtonian perturbing function, which stands in the left-hand member of (89), also requires a correction. We call the last two terms in this left-hand member

$$-\lambda_1^2 R_i,$$

and we develop R_x, R_y, R_z in powers of r/ρ as we have done for A, B, C. Then transporting to the right-hand member, we find for the secular terms of the lowest degree,

$$R_x = \frac{1}{2}\frac{mx}{\rho^3}, \qquad R_y = \frac{1}{2}\frac{my}{\rho^3}, \qquad R_z = -\frac{3}{2}\frac{mz'}{\rho^3}.$$

Therefore

$$S_R = \frac{1}{2}\frac{m}{\rho^3}r, \qquad T_R = 0, \qquad W_R = -\frac{3}{2}\frac{m}{\rho^3}z'.$$

These are the perturbing functions as used in the ordinary lunar theories. Usually m/ρ^3 is replaced by n_0^2, by Kepler's third law. Now, however, we must take by (52) of the first paper

$$\frac{\lambda_1^2 m}{\rho^3} = v_0^2\left(1 + \frac{3\lambda_0^2}{\rho}\right).$$

The correction therefore is

$$\delta\frac{m}{\rho^3} = 3\frac{\lambda_0^2}{\lambda_1^2}\frac{v_0^2}{\rho} = 3\frac{mv_0^2}{\rho}.$$

Therefore

$$(94)\qquad \delta S_R = \frac{3}{2}\frac{mv_0^2}{\rho}r, \qquad \delta T_R = 0, \qquad \delta W_R = -\frac{9}{2}\frac{mv_0^2}{\rho^3}z'.$$

It will be seen that these exactly cancel the first terms of (93). There then remain only the second terms, viz.:

$$(95)\qquad S = -3\frac{mv_0}{\rho}r\dot{\theta}, \quad T = 3\frac{mv_0}{\rho}\dot{r}, \quad W = 3\frac{mv_0}{\rho}z'\dot{\theta}.$$

26. We found in art. 24 that the new terms in the motion of the moon are completely given by the superposition of the fields of the sun and the earth, both computed as if the other did not exist. The expression for these fields which has been used above is that given by the formulas B of art. 11 (first paper). If we had

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used the system A of the same article, we would have found, instead of (89),

$$(96) \qquad \frac{d^2x_i}{c^2dt^2} + \lambda_1^2 m\left(\frac{x_i}{\Delta^3} - \frac{\xi_i}{\rho^3}\right) = \lambda_1^2[2A_i + 2B_i - 2C_i + 3D_i],$$

where

$$D_i = m\left[\frac{x_i\dot{\Delta}}{\Delta^3} - \frac{\xi_i\dot{\rho}}{\rho^3}\right].$$

It is easily found that the term D is of a higher order in r/ρ than A, B, and C, and can consequently be neglected. The other terms give for S, T, and W exactly the values (95). On the other hand, in this system Kepler's third law is exact (if we neglect the moon's excentricity), as is shown by (52, A), and therefore the terms (94) do not exist in this case. The two systems thus give the same result, as they ought to do.

We must now find the effect of the perturbing forces (95) on the elements of the moon's orbit. We can in the perturbing forces use the Keplerian values for the co-ordinates and velocities. We thus have

$$r\dot{\theta} = \frac{\lambda_1\sqrt{p}}{r}, \qquad \dot{r} = \frac{\lambda_1 e \sin v}{\sqrt{p}}.$$

Substituting these values, we find that secular terms arise in the elements ϖ and Ω. We find

from S: $\quad e\,d\varpi = 3\dfrac{mv_0}{\rho}\lambda_1\sqrt{p}\,.\,r\cos v\,dv,$

from T: $\quad e\,d\varpi = 3\dfrac{mv_0}{\rho}\dfrac{\lambda_1}{\sqrt{p}}r^2\left(1+\dfrac{r}{p}\right)e\sin^2 v\,dv,$

from W: $\quad \sin i\,d\Omega = 3\dfrac{mv_0}{\rho}\dfrac{\lambda_1}{\sqrt{p}}r^2\sin i\sin^2(w-\Omega)dv,$

where $w = v + \varpi$. It would not be difficult to integrate these equations rigorously. We get a sufficient approximation, however, by neglecting the excentricity and the inclination. We then find the following secular terms—

$$(97) \qquad \delta\varpi = \delta\Omega = \tfrac{3}{2}\frac{mv_0\nu}{\rho}\,.\,nt,$$

where we have put $a^{\frac{3}{2}}\lambda_1 = \nu$. The numerical value is

$$+1''\cdot91 \text{ per century},$$

to which for the perigee we must add the 0″·06 found above.

The observed values are *

{ Brown, Cowell	. .	$\delta\varpi =$	$+14643536'' \pm 2''$
{ Newcomb, de Vos	. .		$+14643530'' \pm 2''$
Newcomb, Brown	. .	$\delta\Omega =$	$-6967944'' \pm 2''$

* See De Sitter, "The Motions of the Lunar Perigee and Node, and the Figure of the Moon," *Proc. Acad. Amsterdam*, vol. xvii. p. 1309 (April 1915).

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The theoretical values, including the new terms found here, are

$$\delta\varpi = +14643534'' \pm 2''$$
$$\delta\Omega = -6967939'' \pm 2''$$

It is to be regretted that it is not possible from Brown's lunar theory to find the theoretical motion with one more decimal place. The residuals

$$\Delta\varpi = \begin{cases} +2'' \pm 3'' \\ -4'' \pm 3'' \end{cases}$$
$$\Delta\Omega = \quad -5'' \pm 3''$$

are undoubtedly partly due to the inaccuracy of the theoretical value, owing to accumulation of errors by the neglect of the decimal of the second in the many terms of which the complete value is the sum. The residuals as they stand are therefore not decisive either for or against Einstein's theory.

27. In the equations of art. 21 we will assume that there is only one body, the sun, to which we will assign the index 0, Further, we put

$$\mathrm{x}_i = x_i - x_{i0}, \quad x_{i0} = a_i t, \quad \mathrm{r}^2 = \sum_p{}' \mathrm{x}_p{}^2,$$
$$\sum_p{}' a_p{}^2 = q^2, \quad \sum_p{}' a_p \mathrm{x}_p = -\mathrm{r}\epsilon.$$

Then

$$\frac{\partial^2 \mathrm{r}}{c^2 \partial t^2} = \frac{q^2 - \epsilon^2}{\mathrm{r}}$$

and the g_{ij} become

$$(98) \quad (D) \quad \begin{cases} -1+\gamma & 0 & 0 & -2a_1\gamma \\ 0 & -1+\gamma & 0 & -2a_2\gamma \\ 0 & 0 & -1+\gamma & -2a_3\gamma \\ -2a_1\gamma & -2a_2\gamma & -2a_3\gamma & 1+\gamma+\tfrac{1}{2}\gamma^2+\gamma(2q^2-\tfrac{1}{2}\epsilon^2) \end{cases}$$

where

$$\gamma = -\frac{2\lambda_0{}^2}{\mathrm{r}}.$$

This will be called the system D.

In the first paper we found the field of the sun at rest at the origin of the system of co-ordinates, *i.e.* for $a_i = 0$. The line-element in this gravitational field was

$$(99) \qquad ds^2 = -(1-\gamma_0)d\sigma^2 + (1+\gamma_0+\tfrac{1}{2}\gamma_0{}^2)c^2dt^2,$$

where

$$\gamma_0 = -\frac{2\lambda_0{}^2}{r}, \quad r^2 = \sum_p{}' x_p{}^2, \quad x_{i0} = 0.$$

If to (99) we apply a Newton-transformation

$$x_i' = x_i - a_i t,$$

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and then after the transformation we drop the accents, we find $\gamma_0 = \gamma$, and the g_{ij} become

$$(100)\quad (N)\left\{\begin{array}{cccc} -1+\gamma & 0 & 0 & a_1(1-\gamma) \\ 0 & -1+\gamma & 0 & a_2(1-\gamma) \\ 0 & 0 & -1+\gamma & a_3(1-\gamma) \\ a_1(1-\gamma) & a_2(1-\gamma) & a_3(1-\gamma) & 1+\gamma+\frac{1}{2}\gamma^2-q^2(1-\gamma) \end{array}\right.$$

This will be called the system N. It will be seen that it does not agree with the system D (98). In (N) the g_{i4} are of the order $\frac{1}{2}$, and the difference of g_{44} with its value in (99) is of the first order; in (D) these terms are of the orders $\frac{3}{2}$ and 2 respectively. The velocity of light in the two systems is

$$(101)\quad \left\{\begin{array}{ll} (D) & v = c(1+\gamma), \\ (N) & v = c(1+\gamma+q\cos V+\frac{1}{2}q^2\cos^2 V), \end{array}\right.$$

where V is the angle between the ray of light and the velocity q.

If, on the other hand, we transform (99) by a Lorentz-transformation,* we find exactly the system D. This is due to the fact that the Lorentz-transformation is an *orthogonal* transformation of the four co-ordinates x_1, x_2, x_3, x_4.

The equations of motion in the systems (D) and (N) are different, also the course of rays of light is different. The *observable* phenomena must be exactly the same in the two systems. It will be interesting to demonstrate this by a simple example.

Let a planet move in a circle, and let it be observed by an observer on the sun. If the radius of the circle be a, then the differential equation for θ is in the two systems

$$(102)\quad \left\{\begin{array}{ll} (D) & \dfrac{d^2\theta}{c^2dt^2} = -\dfrac{\lambda^2\epsilon}{a^2}\dot{\theta}, \\ (N) & \dfrac{d^2\theta}{c^2dt^2} = 0. \end{array}\right.$$

* The formulas for a Lorentz-transformation are given in my paper of 1911 on the (old) principle of relativity, *M.N.*, vol. lxxi. p. 391. It must be remarked that what is here called a_1, a_2, a_3 is the same as αq, βq, γq in that paper. By the application of these formulas, and the transformation formula (3) of the first paper (vol. lxxvi. p. 701) the g_{ij} become

$$A\left\{\begin{array}{cccc} -1+\gamma_0 & 0 & 0 & -2a_1\gamma_0 \\ 0 & -1+\gamma_0 & 0 & -2a_2\gamma_0 \\ 0 & 0 & -1+\gamma_0 & -2a_3\gamma_0 \\ -2a_1\gamma_0 & -2a_2\gamma_0 & -2a_3\gamma_0 & 1+\gamma_0+\frac{1}{2}\gamma_0^2+2q^2\gamma_0 . \end{array}\right.$$

Now, however, γ_0 is not the same as γ, since in γ_0 the co-ordinates x_i and x_{i0} are simultaneous relatively to the time t, and in γ relatively to the time t'. We must then apply the formula (6) of vol. lxxi. p. 393, where the index 1 must be taken to refer to the planet and 2 to the sun. Then $-aq_1\Delta_q = \frac{1}{2}a_1 r\epsilon$ in our present notation, and, since $\sum\limits_{p}{}' a^p x^p = -r\epsilon$, we have, accurate to the second degree in q, $r^2 = r^2(1-\epsilon^2)$, and consequently $\gamma_0 = \gamma(1-\frac{1}{2}\epsilon^2)$. This reduces (A) to (98).

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The light which leaves the planet at the time t reaches the observer at a time t_0, which is different in the two systems, since the velocity of light is different. In both systems, however, we can apply the rule of planetary aberration, which asserts that the true direction at the time t is the apparent direction at the time t_0. In both systems, therefore, the observed value θ_0 is the same as the true value θ belonging to the time t, as derived from the equation (102). We can thus omit the index 0 of θ_0.

The observer does not know the time t, but only the time t_0. We must therefore write the differential equation with respect to this time. We thus find

$$\frac{d^2\theta}{c^2dt_0^{\,2}} = \frac{d^2\theta}{c^2dt^2}\left(\frac{dt}{dt_0}\right)^2 + \frac{d\theta}{c^2dt}\frac{d^2t}{dt_0^{\,2}}.$$

In the system (D) the velocity of light is the same in all directions, consequently $t_0 - t$ is a constant, and

(103) $$\text{(D)}\quad \frac{d^2\theta}{c^2dt_0^{\,2}} = \frac{d^2\theta}{c^2dt^2} = -\frac{\lambda^2\epsilon}{a^2}\dot{\theta}.$$

In the system (N) we have, to the first order in q,

$$v = c(1 + q\cos V) = c(1 + \epsilon),$$

and consequently

$$t_0 = t + \int_0^a \frac{dr}{v} = t + \frac{a}{c}(1 - \epsilon).$$

Therefore

$$\frac{d^2t}{dt_0^{\,2}} = \frac{a}{c}\frac{d^2\epsilon}{dt_0^{\,2}} = ac\frac{d^2\epsilon}{c^2dt^2} = -\frac{c\lambda^2\epsilon}{a^2}.$$

Consequently in the system (N) we find

(103′) $$\text{(N)}\quad \frac{d^2\theta}{c^2dt_0^{\,2}} = -\frac{\lambda^2\epsilon}{a^2}\dot{\theta},$$

or exactly the same as in the system (D).

28. In this article we will consider the gravitational field produced by the fixed stars. We will suppose that the distribution of the stars in space is symmetrical round the sun. Of course this is very far from being true in the actual galactic system, but, if we took a distribution in ellipsoidal, instead of spherical, shells we would find practically the same result, and also for more irregular distributions the results would undoubtedly be of the same order of magnitude. Since exact numerical results are, of course, entirely out of the question, and the order of magnitude is all we can hope to derive, it is not necessary at present to consider any but the simplest possible case.

We will thus compute the field produced by a hollow spherical shell, whose total mass may be M, and whose inner and outer radii are R_1 and R_2 respectively. The analysis of art. 11 of the

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first paper can be used unaltered, and—as it will appear that we can neglect terms having λ^4 as a factor—we will find

$$ds^2 = -d\sigma^2 + c^2dt^2 + \gamma(d\sigma^2 + c^2dt^2),$$

where

$$\gamma = -\frac{2\lambda^2}{r}m(r) + 2\lambda^2[n(r) - \text{N}].$$

Integrating by parts we find

$$m(r) = rn(r) - s(r),$$

where

$$s(r) = \int_0^r n(r)dr.$$

Therefore

$$\gamma = -2\lambda^2\text{N} + \frac{2\lambda^2}{r}s(r). \tag{104}$$

Inside the shell we have $s(r) = 0$, and outside $s(r) = r\text{N} - \text{M}$. If the density ρ were constant we would have $s(r) = \frac{1}{2}m(r)$. If ρ decreases with increasing r, the ratio $s(r)/m(r)$ will increase. We thus have always

$$m(r) \leqq 2s(r), \tag{105}$$

and $m(r)$ and $s(r)$ will remain of the same order of magnitude so long as ρ differs appreciably from zero. It will be remembered that $m(r)$ is the total mass within the sphere of radius r.

Inside the shell we have

$$\gamma_0 = -2\lambda^2\text{N}.$$

Therefore γ_0 is a constant, and there will be *no* gravitational effects. For, if we perform the transformation

$$x_i' = x_i\sqrt{(1-\gamma_0)}, \quad t' = t\sqrt{(1+\gamma_0)},$$

the g_{ij} will be reduced to the values given by

$$ds^2 = -d\sigma^2 + c^2dt^2.$$

The units of space and time will be changed, but this cannot be verified by observations of the motions of bodies inside the shell relatively to each other.

The difference in the unit of time can, however, be verified by observing light which emanates from a point within the mass, or outside the shell. The frequency of light-vibrations is proportional to $\sqrt{(1+\gamma)}$ [see first paper, art. 13]. Now $s(r)$ is always positive, therefore all light coming from distances exceeding R_1 must appear displaced *towards the violet* as compared with light from a source inside the shell. If γ_1 be the value of γ at the source of the light, then the ratio of the frequencies is

$$\frac{\text{N}_1}{\text{N}_0} = \sqrt{\frac{1+\gamma_1}{1+\gamma_0}}. \tag{106}$$

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Now there is certainly no systematic displacement of the stellar spectral lines towards the violet. Of the very faint stars, of which no measures of spectral lines can be made, we know, of course, nothing; but of stars within a certain distance, say r_1, we can say with certainty that a systematic displacement towards the violet is smaller than, say, $\frac{1}{3}$ km./sec. On the displacement towards the violet produced by the general field is, of course, superposed the displacement towards the red, produced by each star separately (*cf.* art. 13). This may easily cancel the displacement towards the violet. It is well known that stars of all spectra actually show a small systematic displacement towards the red. Still, to be quite safe, we will increase the limit, and only assert that the general field of the stars certainly does not cause a displacement exceeding 1 km./sec. towards the violet. We can then say

$$\sqrt{\frac{1+\gamma_1}{1+\gamma_0}} < 1 + 3 \,.\, 10^{-6}$$

or

$$\gamma_1 - \gamma_0 < 6 \,.\, 10^{-6}(1+\gamma_0). \tag{107}$$

In (106) no assumption is made as to the smallness of γ_1 or γ_0; (107) shows, however, that they must be small, and consequently the neglect of the terms of the second order is justified. Now we have

$$\gamma_1 - \gamma_0 = \frac{2\lambda^2}{r} s(r).$$

Therefore, dividing by $\lambda^2 = 10^{-8}$, and using (105), we have

$$m(r_1) < 600\, r_1.$$

Here r_1 is expressed in astronomical units. If we express it in parsecs we have, roughly,

$$m(r_1) < 10^8 \,.\, r_1. \tag{108}$$

If we take, *e.g.*, for r_1 the distance corresponding to a parallax of 0″·01 we would find

$$m(0''\cdot 01) < 10^{10}.$$

It appears probable that the actual mass within this distance is much smaller. It should be pointed out that, although the limit has been derived from the observations of stellar spectra, $m(r)$ includes *all* mass within a sphere of radius r. It thus also includes all faint or dark stars, and all nebulæ and meteoric or other matter.

29. Having now discussed the principal astronomical consequences of the new theory, we will return briefly to the fundamental concepts. The principle of general relativity asserts that the space co-ordinates and the time are entirely irrelevant, and have no physical meaning whatever. In fact, we never observe anything but a combination of time and space. Nobody has ever measured a

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pure distance, nor a pure interval of time. Always the distance, or the interval of time, is measured from a material point (event) m_1 at time t_1 and at the place (x_1, y_1, z_1) to another point m_2 at another time t_2 and another place (x_2, y_2, z_2).

Let us, by way of illustration, consider the concept "velocity." The differential quotient $\frac{d\sigma}{dt}$ does not correspond to any physical reality. It is a consequence of the theory, or of the old theory of relativity, that no material velocity can exceed the velocity of light. This does not mean that $\frac{d\sigma}{dt}$ can never exceed the value c, but that its value for a material body at any point of the four-dimensional time-space and in any direction is always smaller than the value of the same differential quotient for a ray of light at the same point and in the same direction. In other words, no material body can ever overtake a ray of light, and every moving body can be overtaken by light.

Take, *e.g.*, two systems of co-ordinates A and B. The system A we will suppose to have been so chosen relatively to existing material bodies that at large distances from these bodies the geodetic lines are practically straight lines. Such a system is by Einstein called a *Galilean* system. Let the system B be non-Galilean. Then B must be derived from A, and A from B, by a non-linear transformation. In both systems we take cylindrical co-ordinates r, θ, z, and let the transformation which reduces the one system to the other be a rotation. Thus, if θ' refers to B and θ to A, we have

$$\theta' = \theta - \omega c t.$$

The values of the line-element in the two systems, if we neglect the gravitational effect of material bodies, are

$$(109) \qquad \left\{ \begin{array}{ll} \text{A.} & ds^2 = -d\sigma^2 + c^2 dt^2, \\ \text{B.} & ds^2 = -d\sigma'^2 - 2r^2\omega d\theta' c dt + (1 - r^2\omega^2)c^2 dt^2, \end{array} \right.$$

where $d\sigma^2 = dr^2 + r^2 d\theta^2 + dz^2$, and similarly $d\sigma'$.

Now consider a "velocity" perpendicular to the radius-vector in the plane $z = 0$, thus $d\sigma = \pm r d\theta$. The velocity of light in the two systems is

$$\text{A.} \quad v = c,$$
$$\text{B.} \quad v' = c(1 \mp r\omega),$$

where the upper sign must be taken for light travelling in the direction of increasing θ'. It follows that for values of r exceeding $1/\omega$ light is propagated tangentially in only one direction. Similarly we can show that for $r\omega > 1$ it is a physical impossibility for a material body to be at rest in the system B. It must necessarily move towards the negative θ' with a velocity exceeding $(r\omega - 1)c$, and smaller than $(r\omega + 1)c$. This shows the irreality of the co-ordinates and of the "velocity." Still, it may very well be that,

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for smaller values of r, the system B is more appropriate for the description of the actual phenomena than A.

The difference between the two systems is characterised by the value of g_{24}.* In A we have $g_{24}=0$ and in B $g_{24} = -r^2\omega$. Both values are solutions of the equation

$$G_{24}=0,$$

which, of course, is the same in all systems of co-ordinates. The right-hand member is zero, since we neglect the presence of matter. It can be shown that the equation is satisfied by

$$g_{24}=kr^2,$$

k being an arbitrary constant. The theory only gives the differential equation, and does not prescribe the constant of integration. This must, of course, be so determined that the solution represents the observed relative motions of material bodies and light-rays as described in the adopted system of co-ordinates. The particular solution which does so in one system must, if this system is transformed into another, by the same transformation be reduced to the particular solution which fits the observed phenomena in the new system. This implies that the constants of integration are also subjected to the transformation, and are therefore as a rule different in different systems of co-ordinates.†

In classical mechanics the constants of integration are prescribed, since only one system of co-ordinates is admitted. This prescription is usually given in the form of the values of the variable quantities at infinity. Thus the solution of Poisson's equation is subjected to the condition that the potential shall be zero at infinity.‡ In the theory of general relativity the values at infinity are, of course, different in different systems of co-ordinates.

If we consider physical phenomena at the surface of the earth, and describe them in a system of co-ordinates in which this surface

* We need only consider g_{24}. We could, *e.g.*, introduce the condition (12) $g=-1$, or any other, then g_{44} would follow immediately from g_{24}.

† We must be careful to distinguish between the indeterminateness of the constants of integration (or rather the arbitrary functions, or boundary conditions) corresponding to the fact that the g_{ij} are given by *differential* equations, and the fundamental indeterminateness involved by the principle of general relativity, which has been explained in art. 8 (first paper), and which allows a free choice of the system of co-ordinates, and of the measure of space and time. Thus, *e.g.*, in art. 10 the assumption $g_{i4}=0$ is of mixed character. The primary assumption was that we should have a system possessing three-dimensional spherical symmetry, and in which the field should be stationary. This imposes certain conditions on the g_{i4}, as well as on the other g_{ij}, which might, if desired, be expressed in the form of differential equations. That among all possible solutions of these equations we chose $g_{i4}=0$ is of the nature of a boundary condition.

‡ Of course we know nothing about infinity, and never will. The true condition determining the arbitrary constants or functions is that the actually observed phenomena shall be correctly represented, and only for mathematical convenience this condition is brought in the form of a boundary condition for infinity.

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is fixed, we find that the formula (109, B) is the one we need to represent the observations [apart from gravitational effects], and not (109, A), if only we take for ω the appropriate value. This value we call the rotational velocity of the earth.

Suppose we use a system of co-ordinates relatively to which the earth has a velocity of rotation ω_1. This means that the co-ordinate θ of any point on the earth would in that system be represented by the formula $\theta = \theta_0 + \omega_1 t$. This ω_1 is, of course, entirely arbitrary. It is also not ascertainable by observations, since the co-ordinate axes cannot be observed. It must thus in a true theory of relativity disappear from the final equations describing the observed or observable phenomena. The differential equation gives $g_{24} = kr^2$, and the constant k is also undetermined, and must also disappear from the final equations. Now, if we come to compare the theory with the observations, we will find that, in order to make it fit, we have to take

$$k = \omega_1 - \omega,$$

and if this is introduced, both k and ω_1 will disappear and only the observable quantity ω will remain.*

Rotation in its purely kinematical aspect is thus relative in the new theory, quite as relative as linear translation. We can transform it away as we can transform away linear translation. But there is a difference. If a linear translation is transformed away (by a Lorentz-transformation), it is utterly gone; no trace of it remains. Not so in the case of rotation. The transformation which does away with a rotation at the same time alters the equation of relative motion in a definite manner. This shows that rotation is not a purely kinematical fact, but an essential physical reality. Its amount ω is a physical constant, proper to the earth, like its mass.

I have purposely in the above only referred to such methods of ascertaining and measuring ω which depend exclusively on observations made on the surface of the earth (Foucault's pendulum, gyroscopes, the ellipsoidal figure of the earth), without any reference to celestial bodies. That by all these methods, and also by the apparent daily motion of the stars, we find the same value of ω, is a confirmation of the correctness of the theories by means of which the various phenomena, which are used in the different methods, are explained, especially of the theory of inertia. But all this is entirely independent of the system of co-ordinates.

The case is somewhat analogous with the gravitational effect on planetary motion of a rotation of the sun. Whether the solar system has a translation or not can only be ascertained by observations of bodies outside that system. Therefore such a translation can have no effect on the relative motions within the system.

* See also De Sitter, "The Relativity of Rotation in Einstein's Theory," *Acad. of Sci., Amsterdam*, Sept. 1916 (vol. xix. p. 527).

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But whether the sun rotates or not can be ascertained by observations within the solar system, and accordingly the principle of relativity does admit an effect of the rotation of the sun on the relative motions of the planets.

30. Newton in his *Principia* gave only formal laws for gravitation and inertia. He made no attempt at explanation: "Hypotheses non fingo." When Newton's integral law of gravitation was replaced by Poisson's equation, the concept of a *field* of gravitation, of which the material bodies were the sources, was introduced. At the same time the absolute space was substantialised and called "ether." At very large distances from any material body the potential is zero: the whole of the numerical value of the potential at any point can thus be derived from the material sources.

Expressed in the language of Einstein's theory, we would say, at infinity the g_{ij} are:

$$(110) \qquad \left\{ \begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{array} \right.$$

These are the natural values, or the g_{ij} of absolute space, or of the ether. In classical mechanics the g_{ij} in the neighbourhood of matter are the same as (110), with the exception of g_{44}, which becomes

$$g_{44} = 1 + \gamma,$$

and γ is produced by the material bodies through Poisson's equation. In Einstein's theory all g_{ij} differ from the values (110), and they are all determined by differential equations, of which the right-hand members (κT_{ij}) depend on matter. Thus matter here also appears as the source of the g_{ij}, *i.e.* of inertia. But can we say that the *whole* of the g_{ij} is derived from these sources? The differential equations determine the g_{ij} apart from constants of integration, or rather arbitrary functions, or boundary conditions, which can be mathematically defined by stating the values of g_{ij} at infinity. Evidently we could only say that the whole of the g_{ij} is of material origin if these values at infinity were *the same for all systems of co-ordinates.* It is not necessary to insist on the values (110). We can prescribe any other set of degenerated values which the g_{ij} must have at infinity, only they must be the same for all systems of reference, *i.e.* they must be invariant for all transformations, or at least for a so comprehensive group of transformations that restriction to this group does not mean giving up the principle of relativity.

The values (110) are certainly not invariant. Einstein has, however,* pointed out a set of degenerated g_{ij} which are actually

* In a conversation with the writer.

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invariant for all transformations in which, at infinity, x_4 is a pure function of x_4'. They are:

$$(111) \qquad \left\{ \begin{matrix} 0 & 0 & 0 & \infty \\ 0 & 0 & 0 & \infty \\ 0 & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty^2 \end{matrix} \right.$$

Einstein makes the hypothesis that at infinity the g_{ij} actually have the values (111). These are then the "natural" values, and any deviation from them must be due to material sources. The meaning of this degenerated set of g_{ij} is that at infinity the four-dimensional time-space is dissolved into a three-dimensional space and a one-dimensional time. In the three-dimensional space all transformations are still allowed, also those involving arbitrary functions of t: rotations, accelerated motions, etc. At infinity we would thus have an absolute time,* but no absolute space. Of course, so far as our observations reach (in space and time), there would still be complete four-dimensional relativity, but at very large distances from all matter the g_{ij} would gradually converge towards the degenerated values (111).

The hypothesis can thus be said to be consistent with the formal principle of relativity. It is, however, not a consequence of it, and also denying it is not contrary to the principle. If we adopt the hypothesis, then in any system of co-ordinates the g_{ij} would have exactly determined values, there being no constants of integration left by which they can be made to fit the observed phenomena. Now it is certain that, in many systems of reference (*e.g.* in all Galilean systems), the g_{ij} at large distances from all material bodies known to us actually have the values (110). On Einstein's hypothesis these are special values which, since they differ from (111), must be produced by some material bodies. Consequently there must exist, at still larger distances, certain unknown masses which are the source of the values (110), *i.e.* of all inertia. These hypothetical masses would form the boundary of the universe, which would thus be necessarily finite and limited. Outside them there would be *nothing* but the field of g_{ij} gradually degenerating to the values (111).

We must insist on the impossibility that any of the known fixed stars or nebulæ can form part of these hypothetical masses. The light even from the farthest stars and nebulæ has approximately the same wave-length as light produced by terrestrial sources.

* This must be so understood that we are free to introduce any other time t', which is an arbitrary function of t, but not of x_1, x_2, x_3. For finite values of x_1, x_2, x_3, x_4 *all* transformations are still allowed, but they must all at infinity degenerate in such a way that t' becomes independent of the space-co-ordinates. Thus, *e.g.*, an ordinary Lorentz-transformation would be no longer allowed, since it does not degenerate at infinity; but a transformation which for all finite values of x_1, x_2, x_3, x_4 coincides with a Lorentz-transformation, while degenerating at infinity in the required manner, would be allowable.

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Consequently the ratio of the units of time and space is in these nebulæ approximately the same as here; the deviation of the g_{ij} from the Galilean values (110) is of the same order as here, and they must therefore be still inside the limiting envelope which separates our universe from the outer parts of space, where the g_{ij} have the values (111).

If the hypothesis is accepted, then rotation is relative not only in its kinematical aspect, but also physically. The constant ω in the value of g_{24} is then induced by, or derived from, these distant masses, and it is at the same time the velocity of rotation relatively to these masses. These thus take up the rôle of the absolute space in the old theory of inertia. And, to my mind at least, they are quite as objectionable as this absolute space, if not more so. If we believe in the existence of these supernatural masses, which control the whole physical universe without having ever been observed, then the temptation must be very great indeed to give preference to a system of co-ordinates relatively to which they are at rest, and to distinguish it by a special name, such as "inertial system" or "ether." * Formally the principle of relativity would remain true, but as a matter of fact we would have returned to the absolute space under another name.

Theoretically it is, of course, important that the *possibility* † has been shown to ascribe the whole of the deviation of the actual g_{ij} from certain standard values to material sources, but evidently this "explanation" of the origin of inertia by hypothetical masses which must be imagined for that purpose and have not been otherwise observed, is practically equivalent to no explanation at all, or to a confession of our ignorance. The hypothesis is not an essential element of Einstein's theory. We can accept that theory without the hypothesis. We need not, therefore, give up the hope that some day a satisfactory explanation of inertia will be found which, of course, must, as all physical theories, be consistent with the principle of relativity. In the meantime, we can be content to put ourselves again on the point of view of Newton, accepting the formal law and making no hypotheses.

Even if Einstein has not explained the origin of inertia and gravitation, his theory represents an enormous progress over the physics of yesterday. Perceiving the irrelevance of the representation by co-ordinates in which our science was clothed, he has penetrated to the deeper realities which lay hidden behind it, and not only has he entirely explained the exceptional and universal nature of gravitation by the principle of the identity of gravitation and inertia, but he has laid bare intimate connections between

* Of course, once the source of inertia had been pointed out, the need would be felt to explain *how* the inertia is transferred from these distant masses to the known material bodies, and a hypothetical mechanism would probably be built up quite analogous to the ether in the old theories.

† At least the *logical* possibility. Whether it will be possible to imagine an actual distribution of ordinary physical matter having the desired effect, remains to be investigated. If at all finite, they must certainly enormously exceed the whole known physical universe.

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branches of science which up to now were considered as entirely independent from each other, and has thus made an important step towards the unity of nature. Finally, his theory not only explains all that the old theory of relativity could explain (experiment of Michelson, etc.), but, *without introducing any new hypothesis or empirical constant*, it explains the anomalous motion of the perihelion of Mercury, and it predicts a number of phenomena which have not yet been observed. It has thus at once proved to be a very powerful instrument of discovery.

Leiden:
1916 *Sept.-Oct.*

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

The Hon. Sir Charles Algernon Parsons, K.C.B., M.A., D.Sc., LL.D., F.R.S., 6 Windsor Terrace, Newcastle-on-Tyne; and

Manning Robertson, A.R.I.B.A., 50 Norfolk Square, London, W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

John Beebe, "La Rocca," East Bendigo, Victoria, Australia (proposed by C. J. Merfield);

George Rusby Kaye, Curator of the Bureau of Education of the Government of India, Simla, India (proposed by H. G. Tomkins); and

John Stewart, Raleigh Street 45, Essendon, Melbourne, Australia (proposed by C. J. Merfield).

The President announced that the Council had awarded the Gold Medal of the Society to Mr. Walter Sydney Adams, of the Mount Wilson Observatory, for his investigations in Stellar Spectroscopy, including the Determination of absolute magnitudes.

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Fifty presents were announced as having been received since the last meeting, including, amongst others:—

Vol. I. of the Annals of the Dearborn Observatory, Evanston, Illinois (Measures of Double Stars), presented by the Observatory.

The Part played by Rotation in Cosmic Evolution.

By J. H. Jeans, M.A., F.R.S.

1. The present paper contains a condensed account of some investigations on the rotational theory of cosmic evolution. Two ways are known in which a mass can be broken up under increasing rotation. In the first way (Poincaré-Darwin) the mass first flattens, then loses its symmetry, becoming first ellipsoidal and then pear-shaped, and then throws off a satellite. In the second way (Laplace-Roche) the flattened stage is directly followed by the shedding of a ring of matter at the equator, and this ring may break up into satellites. The results obtained in the present paper indicate that these are representative (although very special) cases of two distinct general methods of break-up. These two general methods are found to be the only possible methods of break-up, and neither of them seems to be able to explain the genesis of our solar system. The first method of break-up appears to lead to the normal binary star formation, while the second seems likely to lead to the spiral nebula formation. Under this second method of break-up it is found that the ejected matter will only condense into detached masses when the parent mass is much greater than our Sun.

Mathematical Preliminaries.

2. The equations of equilibrium for any mass in rotation with a uniform angular velocity ω about the axis of z are

$$\frac{\partial p}{\partial x} = \rho \frac{\partial V}{\partial x} + \omega^2 \rho x = \rho \frac{\partial \Omega}{\partial x} \quad . \quad . \quad . \quad (1)$$

and two similar equations, where

$$\Omega = V + \tfrac{1}{2}\omega^2(x^2 + y^2) \quad . \quad . \quad . \quad . \quad (2)$$

In these equations V denotes the total gravitational potential including that of tidal forces, if any are present. We may put

$$V = V_M + V_T,$$

when V_M is the gravitational potential of the rotating mass, and V_T is the potential of any tidal forces. The condition that a figure shall be one of equilibrium for a mass rotating freely in space is that V_T shall vanish.

Let P be any point inside the mass; let R be the distance

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from P to a point x, y, z, also inside the mass; let dS' be an element of surface of a small sphere surrounding P, and let dS be an element of surface of the boundary. Then since $\nabla^2 V = -4\pi\rho$, we have

$$V_x = -\frac{1}{4\pi}\iiint \frac{\nabla^2 V}{R}\,dx\,dy\,dz$$

$$= \frac{1}{4\pi}\iint\left\{\frac{\partial V}{\partial n}\left(\frac{1}{R}\right) - V\frac{\partial}{\partial n}\left(\frac{1}{R}\right)\right\}(dS + dS')$$

$$= V + \frac{1}{4\pi}\iint\left\{\frac{\partial V}{\partial n}\left(\frac{1}{R}\right) - V\frac{\partial}{\partial n}\left(\frac{1}{R}\right)\right\}dS.$$

If the conditions of equilibrium (1) are satisfied, Ω is constant over the boundary, so that on the boundary

$$V = C - \tfrac{1}{2}\omega^2(x^2 + y^2),$$

where C is a constant, and hence we obtain, as the value of V_T at the point P,

$$V_T = V - V_x = -\frac{1}{4\pi}\iint\left\{\frac{\partial V}{\partial n}\left(\frac{1}{R}\right) - V\frac{\partial}{\partial n}\left(\frac{1}{R}\right)\right\}dS$$

$$= -\frac{1}{4\pi}\iint\frac{\partial V}{\partial n}\left(\frac{1}{R}\right)dS + \tfrac{1}{2}\omega^2\iint(x^2 + y^2)\frac{\partial}{\partial n}\left(\frac{1}{R}\right)dS,$$

omitting a constant which has no physical importance. It appears that if V_T, ω, and the shape of the boundary are given, then

$$\iint\frac{\partial V}{\partial n}\left(\frac{1}{R}\right)dS$$

is determined at every point P inside the mass. Hence $\partial V/\partial n$ is determined at every point of the boundary, so that of course $\partial\Omega/\partial n$ is determined at every point of the boundary.

3. From equations (1) we readily obtain

$$\frac{\partial\rho}{\partial y}\frac{\partial\Omega}{\partial x} = \frac{\partial\rho}{\partial x}\frac{\partial\Omega}{\partial y}, \text{etc.},$$

whence it readily follows that the surfaces $\Omega = $ cons. coincide with the surfaces of constant density, and therefore also, by equation (1), with the surfaces of constant pressure.

We also obtain from equations (1)

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) = \nabla^2\Omega = 2\omega^2 - 4\pi\rho \qquad (3)$$

Multiply both sides of the equation by $dx\,dy\,dz$ and integrate throughout any closed surface inside the matter and we obtain

$$\left.\begin{aligned} -\iint\frac{1}{\rho}\frac{\partial p}{\partial n}dS &= \iiint(2\omega^2 - 4\pi\rho)dx\,dy\,dz \\ &= 2\omega^2 W - 4\pi M \end{aligned}\right\} \quad . \qquad (4)$$

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where W is the volume and M the mass enclosed within the surface.*

Compressible Masses.

4. The material now obtained puts us in a position to discuss the configurations of equilibrium of compressible and non-homogeneous masses. To separate the difficulties, let us first consider masses of compressible matter which are homogeneous in structure, although not of course in density, so that the pressure is a function of the density. For such matter we may put

$$\int \frac{dp}{\rho} = \phi(\rho),$$

and equation (3) assumes the form

$$\nabla^2 \phi(\rho) = 2\omega^2 - 4\pi\rho \quad . \quad . \quad . \quad . \quad (5)$$

the differential equation which must be satisfied by ρ for equilibrium to be possible. The boundary is determined by the condition $p = 0$, and therefore by a condition of the form $\rho = \sigma$, where σ is a constant which may be zero. For a special problem in which V_T and ω are given, and in which the boundary is also given, it follows from § 2 that ρ and $\partial\rho/\partial n$ are given at all points of the boundary, whence from equation (5) it follows that ρ can be uniquely determined at every point inside the mass. It follows that configurations of equilibrium may be classified by their boundaries.

When V_T and ω^2 are given for any problem, any given boundary configuration will permit of an infinite number of vibrations in which the internal configuration changes while the boundary does not. Our result shows that none of these vibrations can be of zero frequency, whence, from the general theory of points of bifurcation, it follows that all internal vibrations are necessarily stable.

For a freely rotating mass there must always be a series of configurations, analogous to the Maclaurin spheroids, in which the boundary is a figure of revolution. This series begins with the same configuration as the Maclaurin series, namely, a sphere; it also ends with the same configuration as the Maclaurin series, namely, a flat spheroidal disc of uniform density σ. Obviously the principal vibrations and stabilities of these end configurations must be the same on both series, so that the same points of bifurcation must occur on both. The general physical principle that vibrations of longest wave-length are the first to become unstable shows that the two series must both first lose their stability through displacements in which the circular cross-section gives place to an elliptic one. In this way we discover the existence of a series of what we may call pseudo-ellipsoidal configurations analogous to the Jacobian ellipsoids. This series begins in a figure of revolution on

* *Cf.* Poincaré, *Les Hypothèses Cosmogoniques*, p. 22.

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the principal series; it ends in a long drawn-out filament which is identical with a Jacobian ellipsoid of uniform density σ. The stability of the end configuration on the pseudo-ellipsoidal series is the same as that of the end configuration on the Jacobian ellipsoid, whence, by the argument previously used, it follows that the first point of bifurcation on the pseudo-ellipsoidal series must lead to a pear-shaped deformation.

It does not appear to be possible to predict whether the pseudo-ellipsoidal series will, at its start, be stable or unstable; nor, if it is stable, is it possible to predict whether the pear-shaped series will be stable or unstable. But these questions are of minor importance. If the pseudo-ellipsoidal series is stable, the motion along it will be statical; if it is unstable, there will be dynamical motion either along it, or along configurations adjacent to it. In this latter case the pear-shaped point of bifurcation will be of the kind which may be described as a "dynamical point of bifurcation." * In any case the general sequence of changes in a compressible mass will be very similar to that in an incompressible mass, and appears likely to lead to a breaking up of the mass into two (or possibly more) detached bodies rotating round one another, as in the normal binary star formation.

Non-homogeneous Masses.

5. The discussion of the configurations of a non-homogeneous mass is naturally more difficult. Let the matter be of different types a, b, c . . ., these letters referring either to chemically distinct types of matter or to mixtures of such types in varying proportions.

Consider a special problem in which V_T, ω, and the shape of the boundary are given, and in which the shells of matter come in order a, b, c . . . from the boundary inwards. From § 3, it follows that surfaces of transition from one type of matter to another must be identical with surfaces $\Omega = \text{cons.}$ An equation such as (5) will be satisfied throughout each layer of matter, but will not be true at the interface between two layers.

As in § 4, ρ and $d\rho/dn$ are determined at the boundary, so that, from equation (5), the arrangement of density is determined throughout the layer of matter of type a. Matter of type b comes next, and Ω and $\partial\Omega/\partial n$ must be continuous at the interface, so that ρ and $\partial\rho/\partial n$ are determined for matter of type b at this interface, and therefore through the whole layer. In this way we can build up the configuration layer by layer, and the configuration is unique when the order of the layers is determined. It follows that all internal vibrations are stable so long as the layers do not change places.

* *Cf.* a paper "On the Motion of Tidally distorted Masses," now in the press (*R.A.S. Memoirs*). If the pear-shaped point of bifurcation is dynamical, it appears to be possible for the mass to break into more than two detached pieces.

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For given values of V_T and ω there will, however, be an infinite number of configurations corresponding to the different ways of arranging the layers. It must in general be supposed that convective movements in which the layers change places are physically possible, and when this is the case only one of all the possible configurations can be stable, namely, that in which the potential energy is least. But if convective movements are impossible, then every configuration is stable. In either case the argument can proceed as in § 4.

6. Two conditions must be satisfied by a configuration of equilibrium. One, which has already been discussed, is that the equations of equilibrium shall be satisfied; the other is, that p shall be positive everywhere. If p changes its sign at any point on a series of equilibrium configurations, the series must be supposed to be cut off abruptly at that point; configurations beyond, although satisfying the analytical conditions of equilibrium, are not physically possible.

It is easily seen that p cannot change sign at a point in the interior of the mass, for the preliminary condition for p to change sign is that p shall vanish. When p is just about to change sign we must have $\partial p/\partial n = 0$ at a point on the boundary. When this occurs, centrifugal and tidal forces outweigh the gravitational attraction at the point in question, and a stream of matter comes off.

The point at which $\partial p/\partial n$ vanishes must be a point of equilibrium on the equipotentials $\Omega = \text{cons.}$ Except when σ, the density at the boundary, vanishes, it is readily shown that the surface at such a point must be shaped like a sharp edge or a needle point. For instance, on the principal series of figures of revolution (analogous to the Maclaurin spheroids) $\partial p/\partial n$ cannot possibly vanish except on the edge of the limiting flat disc, while similarly on the pseudo-ellipsoidal series, $\partial p/\partial n$ cannot vanish except at the pointed ends of the limiting infinitely long ellipsoid. It is readily verified that $\partial p/\partial n$ does in actual fact vanish in both these instances.

When σ, the density at the boundary, vanishes, there is no such limitation on the points at which $\partial p/\partial n$ vanishes.

7. From the results just obtained it follows that for a solid or fluid or other medium in which the density does not vanish at the boundary, the birth of satellites can only take place through the first mechanism; the series of flattened figures of revolution gives place first to a series of pseudo-ellipsoidal figures, after which the satellite is thrown off.

For a gas either mechanism may come into play. On the principal series there are two points of importance,—P, the point of bifurcation at which the series loses its stability, and B, the breaking-up point. If B happens to be nearer to the spherical configuration than P, the second mechanism will come into operation, while if the converse is the case the figure of revolution will lose its symmetry after passing P, after which either mechanism

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may come into operation.* But before a satellite can be born by rotation, the configuration must have passed through one or other of the points B and P.

At the point P we have

$$\frac{\omega^2}{2\pi\gamma\rho} = \cdot 18712 \quad . \quad . \quad . \quad . \quad (6)$$

when the mass is homogeneous and incompressible. Both compressibility and non-homogeneity appear to increase this value ;† they permit of the more dense matter crowding near the centre where the centrifugal force is least, so that more rotation is required to produce a given effect. At the point B we have

$$\frac{\omega^2}{2\pi\gamma\bar{\rho}} = \cdot 36075 \quad . \quad . \quad . \quad (7)$$

when the mass is highly concentrated near its centre, $\bar{\rho}$ being the mean density M/W. When the mass is less concentrated the angular momentum appears to be greater than that determined by equation (7), and so *à fortiori* greater than that determined by equation (6).

Hence it appears almost certain that before a rotating mass can have thrown off a satellite it must have passed through a configuration of angular momentum greater than that defined by equation (6). This is not quite definitely proved, but the argument which follows will not be invalidated even if the number ·18712 is considerably reduced, while all available evidence goes to show that it ought to be increased.

The Solar System.

8. Let us now turn our attention specially to the solar system. For a body of mass M, moment of momentum *M*, and radius of gyration *k*, we have

$$\frac{\omega^2}{2\pi\gamma\bar{\rho}} = \frac{2M^2}{3\gamma \mathrm{M}^3}\frac{r_0^{\,3}}{k^4} \quad . \quad . \quad . \quad . \quad (8)$$

It appears that as a body shrinks, keeping its constitution the same, so that $k^2/r_0^{\,2}$ remains the same, $\omega^2/2\pi\gamma\bar{\rho}$ will *increase*, being proportional to $1/r_0$. The present value of this quantity for our Sun is

$$\frac{\omega^2}{2\pi\gamma\bar{\rho}} = \cdot 0000148 \quad . \quad . \quad . \quad . \quad (9)$$

the question before us is whether with shrinkage $\omega^2/2\pi\gamma\bar{\rho}$ can have *decreased* from the value (6) to the value (9).

* This at any rate represents the theoretical possibilities which are open. I have not succeeded in finding a case in which the second mechanism can operate when once the point of bifurcation P has been passed.

† *Phil. Trans.*, 213 A, p. 457.

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Equation (8) shows that a decrease in $\omega^2/2\pi\gamma\bar{\rho}$ can be produced by an increase in M or by a decrease in M. Now M will doubtless have increased for our system, but a rough calculation shows that on no reasonable hypotheses can the increase in M have been enough to account for more than a very small fraction of the difference now before us. We shall accordingly assume M constant and equal to 2×10^{33} gms.

An interstellar atmosphere would decrease M, for the Sun would rotate in a viscous medium. The density of this medium is doubtless small, but then the viscosity of a gas is independent of its density. Calculations show that this agency would produce only an infinitesimal change in M in an interval of 10^{10} years. Tidal friction and other interactions between members of the solar system will decrease M, but an upper limit to this decrease is obtained, as Babinet pointed out,* by supposing the moment of momentum of the whole solar system all concentrated in the Sun. The value of this total moment of momentum is

$$M = 3{\cdot}3 \times 10^{50} \text{ C.G.S. units} \qquad (10)$$

If this were all concentrated in the present Sun, regarded as homogeneous, the value of $\omega^2/2\pi\gamma\bar{\rho}$ would be increased to

$$\frac{\omega^2}{2\pi\gamma\bar{\rho}} = {\cdot}0117 \qquad (11)$$

which is still far below the value given by equation (6).

9. Assigning to M its maximum possible value, as given by equation (10), the general equation (8) becomes

$$\frac{\omega^2}{2\pi\gamma\bar{\rho}} = 34 \times 10^8 \frac{r_0^3}{k^4}.$$

Assuming the primæval nebula to have had a mean radius r_0 equal to n times the present radius of the Sun, it appears that equation (6) could only have been satisfied if k^2 were as small as ${\cdot}101 n^{-\frac{1}{4}} r_0^2$. For instance, with r_0 equal to the radius of Neptune's orbit, the value of k^2 must have been ${\cdot}0013 r_0^2$.

In this calculation we have certainly taken an excessive value for M. If we take M equal to the present angular momentum of the Sun, M must be reduced to about one-sixtieth † of its former value, and k^2/r_0^2 is reduced to the same fraction of the values just calculated.

10. For an incompressible mass rotating so that $\omega^2/2\pi\gamma\bar{\rho}_i$ is equal to ${\cdot}18712$, the value of k^2 is ${\cdot}5731 r_0^2$, so that it is clear that if our Sun ever rotated with sufficient angular momentum to throw off a satellite, there must have been very extreme concentration of mass near the centre.‡ Practically all the mass must have been

* *C.R.*, lii. (1861), p. 481.

† This is approximately the ratio 1 : 61 . 163 . . . given by See (*Ast. Nach.*, 4053).

‡ See Poincaré, *Les Hypothèses Cosmogoniques*, p. 20.

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concentrated within a sphere of radius comparable with k, and so only a small fraction of r_0; and for such a mass the potential at regions near the edge may, with almost perfect accuracy, be taken to be

$$V_M = \frac{M}{R}.$$

The configurations of equilibrium of such a mass rotating freely under no tidal forces have been studied by Roche.* The principal series of configurations consists of figures which are at first spheroids, but give place finally when $\omega^2/2\pi\gamma\bar{\rho} = \cdot 36$ to a lens-shaped figure with a sharp edge. At this point a break-up occurs through $\partial p/\partial n$ vanishing over the edge. Up to this point the configurations are all stable, and there is no point of bifurcation.

It follows that if our system has ever broken up through rotation, the mechanism must have been of what we have called the second kind, matter being thrown off round the equator of the nebula, and the value of ω^2 must have been given by

$$\frac{\omega^2}{2\pi\gamma\bar{\rho}} = \cdot 36 \quad . \quad . \quad . \quad . \qquad (12)$$

With this definite knowledge of the value of ω^2 we can correct the calculations of § 9, and the results stand as in the following table :—

Values of k^2/r_0^2 *when* $\omega^2/2\pi\gamma\bar{\rho} = \cdot 36$.

$M =$ / Assumed Value for r_0.	Total Moment of Momentum of Solar System.	Present Angular Momentum of Sun.
Radius of present Sun . . .	I ·072	·0012
,, orbit of Earth . .	·005	·00008
,, ,, Neptune . .	·00090	·000015

11. If a break-up of the kind we are now considering occurred, we may suppose that when it happened the density of the Sun continually increased as we passed inwards. No other arrangement can be permanently stable, so that exterior matter which has cooled until it has become denser than interior matter must after a short interval change place with the latter.

Let the ejected matter be supposed to have come from near the surface of the Sun, and to have been of density $\beta\bar{\rho}$ before ejection; then the vast majority of matter in the Sun may be supposed to have been of density greater than $\beta\bar{\rho}$. The value of Mk^2 is

$$Mk^2 = \iiint (x^2 + y^2)\, \rho\, dx\, dy\, dz,$$

and this will be greater than

$$\beta \iiint (x^2 + y^2)\, \bar{\rho}\, dx\, dy\, dz,$$

* Poincaré, *loc. cit.*, chap. iii.

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or $\beta M k_0^2$, where k_0 is the radius of gyration of a homogeneous mass bounded by the lens-shaped figure. The value of k_0^2 is found to be about $\frac{1}{2}r_0^2$,* so that we necessarily have

$$\tfrac{1}{2}\beta < \frac{k^2}{r_0^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

It now appears that the values calculated in the table above provide upper limits to the values of $\frac{1}{2}\beta$. Thus if the Sun ejected matter by rotation, the density of this matter before ejection must have been less than a certain calculable density.

For instance, imagine the Sun ejecting matter when of a mean radius equal to that of Neptune's orbit, its angular momentum being equal to that of the whole present solar system. The value of $\bar{\rho}$ is $5{\cdot}5 \times 10^{-12}$ and $\frac{1}{2}\beta < {\cdot}00090$, so that $\beta\bar{\rho}$ must have been less than 10^{-14}. If the angular momentum was only that of the present Sun, $\beta\bar{\rho}$ must have been less than $1{\cdot}8 \times 10^{-16}$.

Conditions for Condensations to Occur.

12. Matter of very low density, ejected slowly from a rotating mass, is not likely to condense at all; it will scatter into space or fall back into the Sun. For a rotational theory of planetary evolution to be tenable it must show that the ejected matter would be of sufficient density and amount to condense into planets.

Poincaré,† following Roche, assumes that the ejected matter, if it condenses at all, will condense into a ring rotating with the same velocity ω as the primary. He recognises that a condition for condensation to occur at all is that the density shall be above a certain limit, and proves that the ring can only exist without breaking up if its mean density ρ' is greater than $\omega^2/2\pi\gamma$: "Nous avons ainsi une limite inférieure de la densité de l'anneau, et *à fortiori* de la nébuleuse." But the value of $\omega^2/2\pi\gamma$ is known to be $\cdot 36\bar{\rho}$, and so this lower limit is enormously greater than the upper limit allowed by equation (13). Our Sun can only have ejected matter by rotation if its density were less than, say, $\cdot 002\bar{\rho}$, while ejected matter can only have formed into planets if its density were greater than $\cdot 36\bar{\rho}$. The two limits occur in the wrong order, and we conclude that, subject to the assumptions on which we have been working, a rotational theory of planetary formation is untenable as regards our solar system.

The argument can perhaps be seen more forcibly by being reversed. Assume that our Sun broke up under rotation and that condensation occurred. Then $\omega^2/2\pi\gamma$ must have been equal to $\cdot 36\bar{\rho}$, and must have been less than ρ'. Thus β must have been greater than $\rho'/\bar{\rho}$, and so *à fortiori* greater than $\cdot 36$. Hence k^2/r_0^2 must have been greater than $\frac{1}{2}\beta$, and so *à fortiori* greater than $\cdot 18$. Hence from equation (8), r_0 must have been less than $\cdot 16$ times

* The exact value is $\cdot 52313 r_0^2$.

† *Les Hypothèses Cosmogoniques*, p. 23.

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the present radius of the Sun, and the mean density of the primæval nebula must have been greater than 244 times that of the present Sun, or greater than 30 times the density of lead. This assumes the whole present angular momentum of the system to have been concentrated in the central nebula when the last satellite was born; if we suppose the angular momentum to have been double that of the present Sun, the requisite density is found to be many million times that of lead.

13. This conclusion rests on two assumptions. First, there is the assumption, apparently regarded as obviously true by Poincaré, that the ejected matter will expand on ejection; second, there is the assumption that if the matter condenses it will form a ring rotating in equilibrium.

As regards the first assumption, it is natural to suppose that to a first approximation the gas changes adiabatically on ejection, and so necessarily expands. There may be a certain loss of energy by radiation, but seeing that the ejected matter is only supposed to move from just inside the Sun to just outside the Sun, it seems unthinkable that there should be a contraction sufficient to bridge over the great gap between the two limits of density. In any case we shall find (§ 14) that further arguments which do not involve this assumption also show the rotational hypothesis to be untenable.

The second assumption is of a severely restrictive nature, for it implies perfect symmetry about the axis of rotation, both before and after ejection. Every star or mass of matter in the neighbourhood of the rotating body exerts tidal forces which disturb this symmetry. In general the distance of even the nearest stars will be great compared with the dimensions of the rotating mass. The tidal forces may be expanded in a series of harmonics; the terms of first degree will as usual provide the acceleration of the rotating mass in space, and of the remaining terms, those of second degree will adequately represent the tidal potential. Equation (2) now assumes the form

$$\Omega = \frac{M}{R} + \tfrac{1}{2}\omega^2(x^2 + y^2) + p_{11}x^2 + p_{12}xy + \ \cdot \ \cdot \ \cdot$$

and on drawing the surfaces $\Omega =$ cons., which are possible boundaries for the mass, it becomes clear that $\partial\Omega/\partial n$ (or $\partial p/\partial n$) will not vanish simultaneously round a ring, but will first vanish at two points at opposite ends of a diameter. Thus small tidal forces such as occur in nature will localise the area over which matter is ejected, and the ejected matter will form two jets streaming out from antipodal points.

As being applicable to the ejection of matter in this way, it is easy to obtain the equation (a generalisation of equation (4))

$$\frac{d^2 W}{dt^2} = \iint \frac{1}{\rho} \frac{\partial p}{\partial n} dS + 2\omega^2 W - 4\pi M \qquad . \qquad . \quad (14)$$

where M is a detached mass of ejected matter enclosed in a volume W,

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and the surface integral extends over the boundary of this volume. In obtaining this equation, the ejected matter is supposed to move in any way whatever, subject to the condition that velocities of ejection are small compared to the velocity of rotation (ωr), and also to the condition that the vorticity remains everywhere equal to (0, 0, 2ω), the vorticity of the movement of uniform rotation before ejection.*

In this equation the first term on the right represents the effect of gas-pressure on changes in W; since p must necessarily increase on passing inwards, it appears that gas-pressure tends to increase dW/dt. Similarly the next two terms represent the expanding effect of rotation and the condensing effect of gravitation.

Viscosity is not taken into account in arriving at equation (14). To include the most general system of stresses possible, we must replace the first term on the right by

$$\iint \frac{1}{}\left\{ l\left(\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{xy}}{\partial y}+\frac{\partial p_{xz}}{\partial z}\right)+m(\ \ldots\ )+n(\ \ldots\ )\right\} dS,$$

where l, m, n are the direction-cosines of the normal; and this must again be positive, each element representing the outward thrust on a certain surface layer of gas.

If the matter expands at first on ejection, dW/dt will begin by being positive. Equation (14) shows that unless $M > \omega^2 W/2\pi$, the value of dW/dt will continually increase, so that W will increase beyond limit, and the matter scatters freely into space. Thus we are brought back to the same limit $\rho' > \omega^2/2\pi$ which we used in § 12, and the conclusions of that section are re-established.

14. The foregoing considerations have been concerned only with the possibility of rotation preventing condensation from occurring. But we have seen that gas-pressure is also an agency opposing condensation. Let us now consider the effect of gas-pressure upon the possibility of a stream of ejected matter condensing into (i) a long filament, and (ii) a series of detached masses.

It is readily shown that a long cylinder or filament of gas in equilibrium under its own gravitation must necessarily have a mass $2\kappa/\gamma$ per unit length, when γ is the gravitation constant and κ is the ratio of pressure to density. This formula assumes the

* Poincaré (*Hypothèses Cosmogoniques*, p. 31) has studied the possibility of a mass persisting permanently without expansion when the vorticity is not uniform. He finds that ω must be a function of $\bar{\omega}$ (where $\bar{\omega}^2 = x^2 + y^2$), and a necessary condition for permanence is that

$$2\pi\rho' - \left(\omega^2 + \bar{\omega}\omega\frac{\partial\omega}{\partial\bar{\omega}}\right) > 0 \qquad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

so that ρ' can only be very small if $\bar{\omega}\omega$ decreases as $\bar{\omega}$ increases. Such a condition as this, even if it could ever be established, could only be permanently satisfied in a perfectly non-viscous medium. Viscosity must always tend to equalise values of ω, so that if the inequality (i) is satisfied at one instant, it cannot be so permanently unless $\rho' > \omega^2/2\pi$. So that if ρ' is very small, the mass cannot remain in a condensed state. *Cf.* Poincaré, *l.c.*, p. 49.

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matter to be at uniform temperature and away from the influence of tidal forces, but temperature variations and tidal forces will not alter the order of magnitude of the result. In terms of the mean-square molecular velocity C, the mass per unit length becomes

$$\frac{2}{3}\frac{C^2}{\gamma} \quad . \quad . \quad . \quad . \quad (15)$$

As we should expect, the mass per unit length necessary for condensation is greater for high molecular velocities than for low. Gravitation, represented by γ in formula (15), tends to produce condensation, while energy of molecular motion, represented by C^2, tends to resist it. The ratio of these quantities determines whether condensation can occur or not.

A lower limit to the velocity of projection will be fixed by the velocity of effusion into a vacuum, and this is known to be* $C/\sqrt{(6\pi)}$. Thus for a column of gas to condense as it comes out, its rate of emission must be greater than

$$\cdot 153\, C^3/\gamma \text{ gms. per second} \quad . \quad . \quad . \quad (16)$$

Calculations relative to the formation of globular or other detached masses lead by a different path to precisely similar results. It is known that a body of gas can only condense into a permanent globular mass when both its mass and radius are infinite. There are, however, possibilities of large bodies of gas forming into almost permanent masses, the determination of the rate at which these dissolve being similar to that of the dissipation of planetary atmospheres in the kinetic theory of gases.

A molecule moving with velocity c will escape from the surface of a globular mass of mass M and radius R if

$$\tfrac{1}{2}c^2 > \gamma M/R,$$

so that very rapid dissolution can only be avoided if

$$\pi\bar{\rho}R^2 > \frac{3C^2}{8\gamma}$$

The term on the left is the mass of the globe per unit length at its equator; that on the left only differs by a factor $\frac{9}{16}$ from expression (15). Hence the discussion can proceed as before, the factor $\frac{9}{16}$ naturally not materially influencing the result.

From expression (16) it is clear that the rate of emission necessary for condensation will be least when C is least. It is again a question of balancing molecular velocity against gravitation. Taking $C = 3 \times 10^4$, the velocity appropriate for hydrogen at 7° abs., or for oxygen at −156° C., or iron at 120° C., we find that formula (16) demands an emission of matter at a rate greater than 6×10^{19} gms. a second, so that a mass equal to that of the Earth must be ejected in less than three years. The more likely value, $C = 6 \times 10^4$,

* Jeans, *Dynamical Theory of Gases*, p. 133 (2nd edition).

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would require the Earth's mass to be emitted in five months. These rates clearly indicate something very different from the slow emission of matter contemplated by the rotational theory. It appears that matter emitted in a series of violent explosions could condense, but the main agency in such a process would be explosion and not rotation. To get condensation from a rotational theory the molecular velocity would have to be far below 3×10^4, and so far below the velocity at the boiling point of most substances.

The only way in which we can get a molecular velocity low enough for condensation to take place is by replacing our molecules by meteorites or particles of dust. But when this is done, and the matter is supposed to have a density comparable with that calculated in § 11, the free path becomes so large that the medium no longer possesses the physical properties of a gas. The particles or meteorites describe orbits freely without mutual interference. The mathematical analysis appropriate for the discussion of this case is identical with that for the motion of a cluster of stars, and there is no possibility of such a cluster breaking into detached masses by rotation.*

15. A nebula rotating at the critical speed might emit matter at a sufficient rate for condensation to occur if the nebula were on a much greater scale than our Sun. If the nebula were acted on by slight tidal forces, we might get matter emitted at two antipodal points and condensing into two long filaments as suggested in § 13. These filaments, when of sufficient length, would be longitudinally unstable, and would condense into nuclei at an average distance apart comparable with †

$$\sqrt{\frac{1}{\pi\gamma\rho}\frac{dp}{d\rho}} = \frac{C}{\sqrt{3\pi\gamma\rho}}.$$

Multiplying this by expression (15), we find as the mass condensing about each nucleus ‡

$$\frac{2C^3}{3\gamma\sqrt{3\pi\gamma\rho}} = \cdot 21 C^3 \gamma^{-\frac{3}{2}} \rho^{-\frac{1}{2}} \quad . \quad . \quad . \quad (17)$$

Taking $\rho = 10^{-17}$ and $C = 6 \times 10^4$ we obtain a mass of about 10^{33} grammes, and therefore comparable with our Sun. To obtain a mass as small as Uranus or Neptune, ρ would have to be about 10^{-9}, about 200 times greater than the mean density of the mass of the Sun spread through a sphere of radius that of Neptune's orbit, or again about 200,000 times the maximum density allowed by our calculations of § 11, on supposing the nebula to extend to Neptune's orbit.

* *M.N.*, lxxvi. (1915), p. 78.

† See the paper already referred to "On the Motion of Tidally distorted Masses."

‡ This formula, except for the numerical coefficient, can be obtained from a consideration of physical dimensions only, so that it may be taken to express the mass formed by any process whatever of gravitational condensation.

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Conclusion.

16. To sum up, it has been found that the rotational theory fails to explain the genesis of the solar system in three distinct ways:—

(i) The present momentum of the system fixes a limit to the density near the edge of the primæval nebula, and this density is so low that matter thrown off from the edge could not condense into planets.

(ii) The only form of rotational theory which is applicable to our system postulates matter being thrown off continuously and slowly, and the matter would be ejected so slowly that it could not condense.

(iii) Even if these objections were not valid, and matter had condensed, the masses formed would have been much greater than the planets of our system.

The discussion has necessarily been dependent upon numerical estimates, but we have been able to argue in terms of upper and lower limits. Some of these limits have, however, been very wide, and each time we have used the limit most favourable to the rotational hypotheses, we have in effect weighted the beam of the balance so as to favour this hypothesis unduly. Weighed in this balance, the hypothesis has been found wanting. It is impossible to conjecture whether or not in the course of time new factors or new physical agencies may appear of which our argument has taken no account. If such agencies appear, the balance may conceivably again swing in the other direction. In particular, it has been assumed throughout that the angular momentum of the primitive nebula cannot have been greater than the present total angular momentum of the whole solar system. Thus if any sufficiently effective means of leakage of angular momentum comes to light in the future, the whole argument will require reconsideration. But, on the present evidence, we can say with some confidence that the rotational theory cannot explain the origin of the solar system.

In general, it has become clear that detached masses can be formed by rotation in two cases only:—

(i) When the angular momentum is much greater than that in our system, a condition which appears to lead to the binary-star formation.

(ii) When the mass is much greater than that in our system. This condition appears likely to lead to the spiral nebula formation.

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Note on the Action of Viscosity in Gaseous and Nebular Masses.
By J. H. Jeans, M.A., F.R.S.

1. The general equations of motion of a viscous mass are

$$\rho\frac{Du}{Dt}=\rho X-\frac{\partial p}{\partial x}+\frac{1}{3}\kappa\frac{\partial\theta}{\partial x}+\kappa\nabla^2 u, \text{ etc.} \quad . \quad . \quad (1)$$

where κ is the coefficient of viscosity, p is the mean pressure, and θ the expansion, these being defined by

$$p=\tfrac{1}{3}(p_{xx}+p_{yy}+p_{zz}),$$
$$\theta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z},$$

and therefore being of course invariant for changes of axes. For a gas p is roughly proportional to ρ, so that the terms in equation (1) divide themselves into terms multiplied by κ and terms multiplied by ρ. The well-known result follows,* that the effect of viscosity in modifying a given motion is proportional to κ/ρ, the quantity which Maxwell calls the "kinematic" coefficient of viscosity. For a gas at low density the value of κ is independent of ρ, so that κ/ρ will vary inversely as ρ, and may become enormously large at low densities. It follows that viscosity may quite conceivably preponderate over gravitation in determining the motion of a nebulous mass of gas, just as, for instance, it does in determining the motion of particles of dust in the atmosphere.

We shall be concerned mainly with the internal motions in a rotating mass of gas, so that it will be convenient to transform equation (1) to rotating axes, which we shall suppose to rotate with a uniform velocity ω about the axis of z. The values of the velocities and accelerations are

$$u=\dot{x}-y\omega, \quad v=\dot{y}+x\omega, \quad w=\dot{z},$$
$$\frac{Du}{Dt}=\ddot{x}-2\dot{y}\omega-\omega^2 x, \quad \frac{Dv}{Dt}=\ddot{y}+2\omega\dot{x}-\omega^2 y, \quad \frac{Dw}{Dt}=\ddot{z}.$$

Let us assume the velocities $\dot{x}$, $\dot{y}$, $\dot{z}$ relative to the moving axes to be small, and let us now denote these by u, v, w, then it is found that equations (1) transform into

$$\rho\frac{du}{dt}=\rho(X+\omega^2 x)-\frac{\partial p}{\partial x}+\frac{1}{3}\kappa\frac{\partial\theta}{\partial x}+\kappa\nabla^2 u, \text{ etc.} \quad . \quad (2)$$

Let the vorticities of the motion relative to the moving axes be denoted by ξ, η, ζ, so that

$$\xi=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}, \text{ etc.}$$

* *Cf.* Lamb, *Hydrodynamics* (4th edition), p. 470.

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Assuming that κ is constant throughout the mass, we readily obtain from equations (2), three equations of the type

$$\frac{\partial p}{\partial y}\left(\frac{du}{dt} - X - \omega^2 x\right) - \frac{\partial p}{\partial x}\left(\frac{dv}{dt} - Y - \omega^2 y\right) = \rho\frac{d\zeta}{dt} - \kappa\nabla^2\zeta \quad . \quad (3)$$

Multiplying these three equations by $\partial p/\partial x$, $\partial p/\partial y$, $\partial p/\partial z$, and adding, we obtain

$$\sum\frac{\partial p}{\partial z}\left(\rho\frac{d\zeta}{dt} - \kappa\nabla^2\zeta\right) = 0 \quad . \quad . \quad . \quad . \quad (4)$$

2. The simplest case arises when the whole vorticity is everywhere about the axis of z. We then have $\xi = \eta = 0$, and equation (4) becomes

$$\frac{d\zeta}{dt} = \frac{\kappa}{\rho}\nabla^2\zeta \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which is the well-known Fourier equation.

Thus the vorticity ζ is propagated according to the same law as governs the conduction of heat or diffusion. The similarity extends beyond the general nature of the law, for the coefficient κ/ρ is very nearly equal to the corresponding coefficient $\mathfrak{D}$ in the equation of diffusion of gases,* and also to the coefficient θ/c in the equation of conduction of heat in gases.†

For instance, if the outer layers of a nebula shrink through cooling, ζ will be increased at the surface, and this excess value of ζ will be propagated inwards until the value of ζ is uniform, just in the same way as an excess of temperature at the boundary would be propagated inwards until the temperature was uniform. A continual shrinkage through cooling at the surface will be analogous to a continual increase of temperature at the surface, and the vorticity ζ will acquire a steady gradient exactly similar to a temperature gradient.

Unfortunately this special case of $\xi = \eta = 0$ is of only limited interest, as we shall now see. For if $\xi = \eta = 0$, then equation (5) must be satisfied, and hence the right-hand members of all the three equations (3) disappear. It follows that we must have

$$\frac{du}{dt} - X - \omega^2 x = q\frac{\partial p}{\partial x} \quad . \quad . \quad . \quad . \quad (6)$$

and two similar equations, from which we readily deduce

$$\frac{d\xi}{dt} = \frac{\partial p}{\partial z}\frac{\partial q}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial q}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and two symmetrical equations. The vanishing of $d\xi/dt$ and of $d\eta/dt$ is now seen to require that $d\zeta/dt$ shall also vanish, except in very special cases, so that in general the motion reduces to a

* Jeans, *Dynamical Theory of Gases* (2nd edition), p. 326.
† *Ibid.*, p. 307.

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steady motion in which propagation of vorticity does not occur at all.

We are accordingly compelled to fall back on the general equation (4), of which no exact interpretation presents itself. It will, however, be clear that in a general way vorticity is propagated at a rapidity comparable with that expressed by equation (5), and therefore comparable with the propagation by diffusion or conduction of heat.

In the special case of ρ being constant everywhere, it is of course obvious from equation (3) that each component of vorticity satisfies equation (5) exactly.

3. It is obvious without discussion that the general motion determined by equations (2) and (3) must represent a compromise between the motion when there is no viscosity ($\kappa = 0$) and the motion when viscosity is predominant ($\kappa = \infty$). Let us examine the abstract limiting case of $\kappa = \infty$.

Equations (3) assume the form $\nabla^2 \xi = 0$, etc., and it is clear that the case of main practical interest occurs when ξ, η, ζ are all constants. By a new and suitable choice of the axes and amount of their rotation ω, these values of ξ, η, ζ can all be made to vanish, and we are left with an irrotational motion, relative to these moving axes, of components u, v, w given by

$$u = -\frac{\partial \phi}{\partial x}, v = -\frac{\partial \phi}{\partial y}, w = -\frac{\partial \phi}{\partial z} \qquad . \quad . \qquad (8)$$

When $\kappa = \infty$ equations (2) assume the form

$$\frac{1}{3}\frac{\partial \theta}{\partial x} + \nabla^2 u = 0, \text{ etc.,}$$

and in virtue of relations (8), these are equivalent to

$$\frac{\partial}{\partial x}\nabla^2 \phi = 0, \text{ etc.}$$

Thus $\nabla^2 \phi$ must be a constant, say $-A$, and ϕ must be of the form

$$\phi = -\tfrac{1}{6}A(x^2 + y^2 + z^2) + \text{a spherical harmonic.}$$

We have

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -A,$$

so that each element of gas expands at a uniform rate A, undergoing at the same time a change of shape which depends on the spherical harmonic part of ϕ.

4. According to the suggestion of the preceding paper, a spiral nebula may be regarded as made up of a lens-shaped nucleus, surrounded by an atmosphere of gas emitted from the periphery of the lens, the emission being almost entirely localised at two antipodal points, and the atmosphere being accordingly almost confined to the

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equatorial plane of the mass. At most only the motion of this outer atmosphere will approximate to that just considered.

The value of the spherical harmonic in ϕ appropriate to such a case is readily seen to be

$$-\tfrac{1}{12}A(x^2+y^2-2z^2)$$

so that

$$\phi = -\tfrac{1}{4}A(x^2+y^2),$$

and the velocities are given by

$$\dot{x} = \tfrac{1}{2}Ax, \quad \dot{y} = \tfrac{1}{2}Ay, \quad \dot{z} = 0.$$

If the rate of emission of matter is uniform, A will be independent of the time, and the equations of the orbit described by any element of the emitted gas will be

$$x = x_0 e^{\frac{1}{2}At}, \quad y = y_0 e^{\frac{1}{2}At}.$$

Thus, relative to the moving axis, each element describes a straight line with a velocity which increases exponentially with the time. The orbit in space will be

$$r = r_0 e^{\frac{1}{2}A\theta/\omega},$$

an equiangular spiral. If the matter is emitted from two antipodal points, the main mass of the ejected matter will of course describe two equiangular spirals starting from these two points, the matter expanding as it proceeds so that the boundaries of the jet are themselves equiangular spirals.

5. The motion just considered is only the limiting case in which viscosity preponderates entirely over gravitation, etc. Actual motion must be intermediate between this and the motion obtained by neglecting viscosity altogether and supposing the orbits determined by gravitation and gas-pressure. But inasmuch as the arms of many spiral nebulæ approximate much more closely to equiangular spirals * than to gravitational orbits, the question arises as to what numerical conditions must be satisfied in order that the purely viscous motion may give a good approximation to the truth.

We have seen that the vorticities are propagated approximately in accordance with equation (5), so that the time necessary for the vorticity approximately to equalise itself over a distance r must be comparable with

$$\frac{\rho r^2}{\kappa} \qquad . \quad . \quad . \quad . \quad . \qquad (9)$$

Remembering that κ is independent of the density in a gas, it might be thought that propagation could be made as rapid as we please by taking ρ small enough. But this does not prove to be the case. The value of κ is approximately $\frac{1}{3}\rho cl$, where c, l are molecular velocity and free-path, so that formula (9) reduces to $3r^2/cl$. This formula is, however, only valid when l is small

* v. Pahlen, *Ast. Nach.*, 4503.

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compared with r, and for values of l greater than r the effect of viscosity falls off with extreme rapidity. Thus the time of equalisation must always be greater than r/c, and there can be no approach to equalisation unless the radial velocity of the arms is much smaller than the molecular velocity, a condition which is not likely to be satisfied in a real nebula.

The result just obtained is only an illustration of the obvious principle that the angular velocity in the arms can only tend to approximate to that of the central mass through some agency which moves with a speed greater than the radial velocity of the arms. Molecular motion, then, is almost certainly inadequate; the motion of free electrons might possibly provide the needed agency, as also might electric forces (acting, for instance, through electric charges in the medium).

Perhaps, however, it is wiser to leave discussion of the general question until something definite is known, or something reasonable has been conjectured, as to the physical constitution of nebular masses. For the present it may be recorded that it has been found that viscosity (possibly of a somewhat generalised kind) can be highly effective in influencing the paths of matter in a nebula, and that the paths, if determined by viscosity alone, would be equiangular spirals.

Greenwich 1910 *Catalogue of Stars, Zone* + 24° 0′ *to* + 32° 0′: *List of Stars the P.M. of which is* 20″ *or over per century; and Analysis of Numbers and Percentages of Proper Motions.* By W. G. Thackeray.

(*Communicated by the Astronomer Royal.*)

The Greenwich 1910 Catalogue is a complete survey of the zone of the sky covering the region + 24° 0′ to + 32° 0′ of all stars down to B.D. magnitude 9·0.

The proper motions have been determined of over 12,000 stars. About 7500 of these were observed by Bessel about 1825, and 5500 by Lalande about 1800, and all of them were observed in the Astronomische Gesellschaft catalogues of Leiden, Cambridge, and Berlin B. These catalogues have been all reduced to the basis of Boss's system. Full particulars of the systematic corrections applied will be given in the introduction to the Catalogue when the exigencies of war allow of the printing. The following table gives the stars of largest proper motion. It requires no explanation beyond the * in column 1, which denotes that the star is in Boss's Preliminary General Catalogue, and the proper motion is taken from that source. In the column A.G.C. No. the numbers without a letter refer to the Cambridge Catalogue, the letters B and L to Berlin and Leiden Catalogues respectively.

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Table I.

List of Stars the P.M. of which is 20″ *or more per Century.*

B.D. No.		A.G.C. No.		Mag.	Approximate. R.A. 1910.			Dec. 1910.		P.M. R.A. Time (ˢ·0001).	P.M. R.A. Arc. (″·001).	P.M. Dec. ″·001.
					h	m	s	°	′			
28°	4704*		3	7·5	0	1	56	28	32	+288	+380	− 176
28	4*		19	2·0		3	44	28	36	+106	+139	− 161
24	4	B	15	7·8		4	36	24	47	+143	+195	− 142
29	14		37	8·5		6	42	29	57	+163	+212	+ 35
26	8		47	8·5		7	23	26	36	+222	+298	− 60
29	31		95	8·7		11	29	30	2	+142	+172	+ 49
30	34		120	8·6		13	38	30	28	+102	+133	− 169
25	77		328	9·0		30	0	25	34	+285	+388	− 57
29	105		344	8·8		31	14	29	30	+125	+164	− 419
28	103*		370	4·1		33	48	28	49	− 173	− 227	− 248
30	89		372	9·0		34	1	30	32	− 170	− 218	− 70
29	141		486	7·5		45	0	29	58	+169	+220	− 31
30	125	L	288	8·0		47	46	30	58	+175	+225	− 46
29	158		574	8·2		53	7	29	50	+162	+212	− 120
30	206	L	494	8·8	1	16	23	30	52	+380	+487	− 95
27	225		800	7·7		20	11	28	6	+205	+271	− 165
31	247	L	521	8·7		20	27	31	31	− 261	− 334	− 102
30	233	L	546	8·1		24	47	30	33	+163	+209	0
28	249		849	7·3		26	16	28	57	+149	+195	− 78
28	253		867	8·8		28	17	28	48	+189	+248	− 45
24	239	B	490	7·4		32	27	24	43	− 155	− 211	− 238
30	252		904	8·0¹		32	58	30	20	+194	+252	+ 5
28	271		906	8·8		33	17	29	7	+316	+414	− 238
27	262		918	7·8		34	30	27	39	+371	+490	+ 168
28	312*		1022	4·3		47	57	29	8	+ 13	+ 17	− 232
30	308	L	733	8·9		52	32	31	9	+171	+219	− 76
27	335		1146	9·0	2	3	12	27	53	+129	+170	− 304
29	366		1158	8·7		5	10	29	23	+217	+284	− 261
26	373		1200	8·0		10	15	26	56	+234	+313	− 136
30	421	L	990	7·3		33	10	30	26	− 368	− 478	− 404
25	449		1436	8·0		41	0	25	16	+177	+240	− 183
26	465		1454	8·1		42	50	26	42	+218	+291	− 119
30	448	L	1049	7·2		43	18	30	45	+163	+209	− 168
26	484		1512	7·6		50	19	26	31	+192	+257	− 162
29	503		1523	9·0		52	22	29	18	+157	+205	− 54

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TABLE I.—*List of Stars the P.M. of which is 20″ or more per Century*—contd.

B.D. No.		A.G.C. No.		Mag.	Approximate. R.A. 1910.			Dec. 1910.		P.M. R.A. Time (s·0001).	P.M. R.A. Arc (″·001).	P.M. Dec. ″·001.
°					h	m	s	°	′			
26	503*		1553	7·1	2	57	6	26	15	+179	+241	− 168
25	495		1581	8·1	3	3	8	26	0	−131	−176	− 836
25	580		1758	8·0		31	46	25	42	+185	+250	− 259
29	597		1805	8·6		37	28	29	54	+142	+185	− 157
28	593		1858	8·5		43	54	28	22	+ 48	+ 63	− 225
31	660	L	1451	8·8		46	30	31	45	+ 87	+111	− 354
30	609	L	1516	8·2		56	45	31	5	− 76	− 98	− 196
28	614		1967	8·1		58	14	28	57	+213	+279	− 122
29	678		2025	7·0	4	9	31	29	41	+ 85	+111	− 170
30	653	L	1646	8·7		14	24	30	56	+ 88	+113	− 181
24	659	B	1436	8·9		22	24	24	14	+263	+360	+ 102
26	730		2118	8·6		31	23	26	57	+158	+211	− 142
27	688		2150	8·0		37	59	27	31	+ 55	+ 73	− 244
28	726		2254	8·8		54	51	29	4	+209	+274	− 134
24	726	B	1602	8·3		56	7	24	31	+ 76	+103	− 263
27	733		2321	8·9	5	4	22	27	32	+145	+192	− 49
27	734		2322	7·3		4	25	27	27	+153	+205	− 104
24	772		2330	8·3		5	13	25	2	− 26	− 35	− 210
27	881		2718	8·9		44	7	27	6	+ 68	+ 90	− 256
31	1159	L	2392	9·0		54	30	31	26	+129	+166	− 286
26	1067		2935	8·9	6	0	20	26	34	−119	−159	− 357
29	1154*		3064	4·4		9	30	29	31	− 49	− 64	− 266
27	1124		3265	8·7		23	30	27	5	−151	−202	− 429
24	1357	B	2526	8·5		35	45	24	3	+139	+190	− 280
25	1460		3536	7·2		45	27	25	52	− 12	− 15	− 210
30	1359	L	2875	8·1		49	42	30	17	+191	+248	− 245
29	1441*		3695	6·3		57	47	29	29	+121	+158	− 823
30	1423		3753	8·6	7	2	22	29	59	−111	−144	− 319
25	1594		3774	7·3		4	4	25	53	− 81	−109	− 203
25	1613		3831	8·6		8	26	25	10	−294	−399	− 104
27	1356		3880	8·1		12	52	27	25	+119	+159	− 139
32	1562*	L	3150	4·8		23	19	31	57	+117	+141	− 183
29	1539		3991	8·8		24	43	29	34	− 1	− 1	− 206
25	1709		4051	8·0		29	21	25	10	+ 96	+130	− 352
29	1590*		4132	5·0		37	41	29	6	+ 53	+ 69	− 237
28	1463*		4150	1·3		39	49	28	14	−471	−622	− 58

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TABLE I.—*List of Stars the P.M. of which is 20″ or more per Century*—contd.

B.D. No.		A.G.C. No.		Mag.	Approximate. R.A. 1910.			Approximate. Dec. 1910.		P.M. R.A. Time (ˢ·0001).	P.M. R.A. Arc (″·001).	P.M. Dec. ″·001.
					h	m	s	°	′			
31°	1684	L	3313	8·2	7	47	48	30	53	+563	+721	−1848
29	1664		4285	7·5		54	58	29	29	−102	−134	−1179
25	1865*		4397	6·0	8	5	2	25	46	− 51	− 58	− 354
31	1781	L	3472	8·3		12	37	30	54	−227	−291	− 814
27	1589*		4482	5·7		14	36	27	30	− 9	− 12	− 388
24	1956		4603	9·0		31	30	24	47	−159	−216	+ 44
26	1816		4618	7·5		32	47	26	22	− 85	−115	− 186
30	1739		4619	9·0		32	52	29	57	−183	−238	− 105
25	1981		4674	8·5		39	6	25	8	− 96	−130	− 337
28	1660*		4722	6·0		47	14	28	40	−365	−479	− 245
26	1865		4732	7·0		49	35	26	33	+ 54	+ 72	− 437
27	1715*		4838	6·0	9	3	30	27	0	− 87	−116	− 387
25	2059		4860	8·9		6	39	25	26	−169	−229	− 104
29	1883		4896	7·0		12	33	28	57	+ 58	+ 75	− 509
31	1978	L	3860	7·5		20	7	30	53	+ 74	+ 95	− 190
27	1775		5001	7·0		27	30	27	24	−108	−144	− 236
26	2019*		5130	4·0		47	39	26	25	−163	−220	− 63
28	1826		5173	7·9		54	9	27	57	−263	−347	− 57
25	2191		5179	7·5		54	58	24	59	−190	−258	− 53
27	1844		5202	8·7	10	0	22	26	56	−164	−219	− 90
29	2000		5216	8·0		2	5	29	41	+ 22	+ 29	− 244
24	2193	B	3953	8·3		6	49	24	12	−291	−398	+ 62
26	2065		5280	7·5		11	53	26	19	+114	+153	− 299
25	2247		5341	8·1		20	2	25	4	+ 15	+ 21	− 203
25	2255		5365	8·9		23	20	25	20	+147	+199	− 130
27	1914		5430	8·5		33	11	27	5	+161	+215	− 88
28	1923		5478	8·5		38	57	27	53	−175	−232	− 80
26	2148		5557	8·9		51	12	26	10	−156	−210	− 94
28	1952		5559	8·4		51	25	28	13	−341	−450	− 145
29	2108		5563	8·8		52	13	29	25	+247	+323	− 169
25	2335*		5620	7·9	11	0	27	25	41	−297	−400	− 82
31	2238*			8·7		6	6	30	56	+452	+579	− 212
31	2240	L	4377	8·5		6	8	30	56	+452	+579	− 212
29	2156		5740	8·7		20	14	29	30	−234	−306	− 155
24	2360		5764	8·5		22	48	24	32	−156	−212	− 111
30	2163	L	4449	6·8		25	18	30	28	− 70	− 91	− 192

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TABLE I.—*List of Stars the P.M. of which is 20" or more per Century*—contd.

B.D. No.	A.G.C. No.	Mag.	Approximate 1910. R.A.			Approximate 1910. Dec.		P.M. R.A. Time (s·0001).	P.M. R.A. Arc (″·001).	P.M. Dec. ″·001.
			h	m	s	°	′			
30° 2164	L 4450	9·0	11	25	21	30	29	− 81	−105	− 223
28 2032	5845	8·8		38	2	27	45	−153	−202	− 57
31 2290	L 4509	8·5		39	59	31	28	− 44	− 56	− 391
29 2253	5983	8·0	12	1	1	28	48	+124	+162	− 121
30 2238	L 4619	8·7		8	8	30	30	−136	−177	− 129
28 2106	6067	6·5		14	59	28	40	−151	−197	− 142
25 2493	6079	7·3		16	33	25	32	−169	−227	+ 158
32 2241	L 4666	8·6		20	2	31	47	−156	−199	+ 16
27 2135	6121	8·2		22	43	27	31	+ 30	+ 40	− 242
30 2310	6194	8·7		34	48	29	44	+112	+146	− 200
31 2401	L 4752	8·9		38	58	31	18	+ 50	+ 64	− 202
24 2493	6241	6·7		42	7	24	39	− 91	−123	− 241
27 2173	6252	8·1		44	11	27	5	−199	−267	− 75
25 2568	6253	6·0		44	24	25	20	−244	−331	− 118
29 2330	6267	8·4		46	59	29	21	−116	−153	− 209
27 2187	6280	7·4		48	40	27	17	−172	−229	− 95
24 2539	B 4698	8·0	13	3	20	24	29	−222	−304	+ 121
24 2540	B 4699	8·2		3	21	24	50	−208	−282	+ 119
30 2371	6368	8·1		4	20	29	52	+147	+191	− 80
28 2193*	6390	4·1		7	40	28	19	−604	−795	+ 875
28 2205	6416	8·3		12	3	28	13	+ 39	+ 52	− 357
25 2619	6430	8·0		14	35	25	5	−223	−303	+ 44
29 2405	6460	8·6		19	23	29	42	−347	−451	+ 242
28 2232	6500	8·7		26	11	28	7	− 59	− 78	− 188
25 2643	6514	6·0		28	32	24	49	+ 48	+ 64	− 219
31 2540	L 5000	6·3		42	11	31	21	−196	−252	− 90
27 2296	6607	7·0		44	58	27	26	−365	−483	− 81
28 2272	6633	8·5		48	57	28	16	+164	+216	− 412
29 2499	6729	8·9	14	4	30	29	25	−165	−217	− 41
30 2494	L 5127	7·3		10	2	30	38	−307	−393	+ 158
30 2512	L 5169	8·3		18	4	30	3	−515	−670	− 322
24 2733	B 5072	9·0		21	33	24	3	+566	+775	−1099
25 2782	6848	8·0		24	14	24	54	−174	−236	− 141
30 2536*	L 5230	5·0		30	46	30	8	+149	+194	+ 120
24 2757	B 5130	8·7		37	17	24	16	−159	−218	+ 6
27 2411	6948	9·0		39	24	27	8	−216	−289	+ 5
26 2606	6981	8·8		45	4	26	5	− 25	− 34	− 326

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TABLE I.—*List of Stars the P.M. of which is 20″ or more per Century*—contd.

B.D. No.	A.G.C. No.	Mag.	Approximate. R.A. 1910.	Approximate. Dec. 1910.	P.M. R.A. Time (s·0001).	P.M. R.A. Arc (″·001).	P.M. Dec. ″·001.
°			h m s	° ′			
25 2873°	7084	4·9	15 3 21	25 13	+138	+187	− 183
26 2677	7155	8·0	15 10	26 1	−401	−541	− 116
31 2724	L 5461	7·3	17 11	31 48	−162	−206	+ 143
30 2653*	L 5478	5·2	19 29	30 36	+101	+129	− 198
28 2503	7420	8·0	54 55	27 59	−575	−760	+ 304
25 3020	7468	7·5	16 0 20	25 29	−390	−528	+ 698
26 2854	7688	8·5	27 52	26 38	− 73	− 97	− 253
25 3097	7703	7·8	29 37	25 53	+168	+226	− 65
31 2873	L 5861	7·2	32 57	31 8	− 5	− 7	− 502
29 2860	7742	8·0	34 44	29 51	− 33	− 43	− 198
31 2884*	L 5889	2·6	37 54	31 45	−365	−464	+ 385
24 3137	B 5874	8·2	17 6 56	24 38	−164	−222	+ 223
29 2979	8118	8·0	14 10	29 19	− 53	− 69	+ 208
31 3027	L 6200	8·0	23 21	31 9	−233	−298	+ 89
26 3023	8207	7·9	23 59	26 52	− 55	− 74	+ 280
29 3029	8236	9·0	25 52	29 28	−164	−215	− 288
27 2888°	8419	3·3	42 56	27 46	−244	−322	− 750
29 3165	8592	7·5	57 11	29 34	− 92	−120	+ 163
26 3151	8618	7·3	58 51	26 20	+277	+373	− 581
30 3113	L 6474	6·7	18 0 28	30 23	− 15	− 20	− 288
29 3213	8758	7·0	11 14	29 11	+ 22	+ 29	− 240
26 3215	8797	9·0	14 11	26 38	+243	+325	+ 99
28 3039	9090	8·2	34 45	28 51	− 49	− 64	− 461
31 3330	L 6797	8·7	37 30	31 28	+ 61	+ 78	− 835
25 3780	9797	8·0	19 14 10	25 12	+142	+193	+ 226
24 3737°	B 7012	6·3	21 42	24 45	−137	−186	− 631
31 3618	L 7391	7·0	25 53	31 26	− 13	− 17	− 425
31 3727	L 7589	8·3	38 59	31 32	− 8	− 10	− 216
24 3849	B 7229	7·2	39 19	24 23	+ 59	+ 81	− 293
27 3516	10423	7·1	44 21	27 38	− 4	− 5	+ 225
27 3523	10444	7·0	45 27	27 31	+ 59	+ 78	− 223
29 3815	10636	8·8	53 12	29 18	− 86	−113	− 184
29 3818	10644	8·9	53 34	29 41	+ 85	+111	− 194
29 3820	10646	8·2	53 36	29 35	+ 88	+114	+ 231
29 3872*	10780	6·2	59 55	29 39	+515	+682	− 528
25 4090	10814	7·9	20 1 8	25 48	+ 22	+ 30	− 378
31 3926	L 7926	9·0	1 13	31 47	+157	+208	− 96

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Table I.—*List of Stars the P.M. of which is 20″ or more per Century*—contd.

B.D. No.	A.G.C. No.	Mag.	Approximate. R.A. 1910. h	m	s	Dec. °	′	P.M. R.A. Time (ˢ·0001).	Arc (″·001).	Dec. ″·001.
29° 4135	11623	8·7	20	37	33	29	52	+170	+221	− 34
24 4251	B 7952	8·8		45	13	24	37	−133	−181	− 220
31 4230	L 8505	8·3		45	37	31	57	−140	−178	− 279
25 4422	11941	7·0		53	44	26	3	+187	+252	+ 86
26 4070	12099	8·5	21	1	30	26	55	−142	−190	− 141
26 4156	12483	8·4		21	32	26	50	−111	−150	− 158
24 4411	12550	8·7		25	14	25	3	− 93	−126	− 164
26 4237	12786	7·2		37	2	26	21	+263	+355	− 87
25 4607	12834	8·2		40	5	26	6	+151	+203	+ 60
28 4169*	12835	4·5		40	7	28	20	+214	+282	− 240
28 4210	12951	8·9		47	0	28	45	+165	+217	+ 191
31 4574	L 9192	7·6		50	47	31	54	+159	+203	− 250
25 4648	13020	8·8		52	14	26	1	+155	+209	+ 23
29 4550*	13062	7·4		54	42	29	24	−295	−387	− 378
27 4252	13179	8·0	22	1	48	27	32	+ 39	+ 52	− 209
24 4533*	B 8525	4·0		2	49	24	54	+220	+299	+ 18
28 4312	13269	8·7		7	59	28	37	+176	+231	+ 119
24 4563	B 8578	8·9		11	44	24	29	−149	−204	− 85
30 4774	L 9624	8·7		37	56	31	14	+100	+128	− 197
29 4742	13664	8·1		39	25	29	37	+187	+243	+ 23
29 4753	13685	7·2		41	22	29	59	−192	−250	− 348
25 4808	13695	9·0		41	55	25	29	−157	−213	− 12
27 4480*	13858	2·2–2·7		59	25	27	35	+144	+192	+ 133
26 4580*	13952	6·5	23	7	27	26	21	−144	−194	− 122
26 4586	13967	8·9		8	46	26	24	+132	+178	− 118
29 4890	13996	8·0		11	53	30	11	+270	+351	+ 92
28 4562	14030	8·5		15	28	28	23	+497	+655	− 45
30 4961	L 9973	8·4		25	46	31	13	+140	+179	− 164
30 4982	L 10006	7·5		30	57	30	31	+430	+554	+ 239
30 4983	L 10007	7·9		30	59	30	40	−164	−210	− 269
29 4971	14180	7·7		32	36	30	10	+152	+198	+ 63
27 4588	14195	6·8		34	59	27	45	+231	+306	+ 231
28 4634	14250	8·6		40	32	29	4	+701	+918	+ 22
29 5007	14313	9·0		45	53	29	51	+200	+260	− 3
27 4642	14346	7·0		50	26	28	8	+438	+578	+ 2
27 4663	14402	9·0		56	23	27	23	+173	+231	+ 5
26 4734*	14413	6·9		57	28	26	36	+622	+837	− 986

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The following tables of numbers and percentages of proper motion are arranged in order of magnitude and size of proper motion. They practically explain themselves, but perhaps attention may be called again to the fact that the stars with proper motion ranging between 5″ and 10″ per century are the most symmetrically distributed among all magnitudes, and that out of 2805 stars with proper motions over 5″ per century, only 77 of these stars are brighter than $6^{m}\cdot0$, and consequently that there are a very large number of faint stars whose mean parallactic motion is equivalent to those of the brightest stars and whose parallax must be sensible. These stars can only be determined by a rigid examination of all faint stars with large P.M.'s. Roughly, between 20 and 25 per cent. of stars down to the $9^{m}\cdot0$ have proper motions over, and 75 to 80 per cent. under, 5″ per century.

TABLE II.

Proper Motions of Stars arranged according to Magnitude and Size of Proper Motion.

Limits of Magnitude.	0″–5″.	5″–10″.	10″–15″.	15″–20″.	20″–30″.	30″–50″.	50″–.	Total No. of Stars.
				Numbers.				
m m								
0–5·9	88	32	19	9	9	4	4	165
6·0–9	290	87	52	15	8	4	5	461
7·0–9	1141	268	92	18	23	17	10	1569
8·0–5	2473	493	120	41	34	23	8	3192
8·6–9	2956	569	123	39	31	14	4	3736
9·0–	2290	495	85	31	14	5	1	2921
Sums	9238	1944	491	153	119	67	32	12044
				Percentages.				
m m								
0–5·9	54	19	11	5·4	5·4	3·0	1·8	
6·0–9	63	19	11	3·2	1·7	0·9	1·1	
7·0–9	73	17	6	1·2	1·5	1·0	0·7	
8·0–5	77·5	15·5	4	1·3	1·1	0·7	0·2	
8·6–9	79	15	3	1·4	0·8	0·4	0·1	
9·0–	78·5	17	3	1·1	0·5	0·1	0·1	

The following table shows the number of stars of mags. $6^{m}\cdot0$ to $9^{m}\cdot0$ with proper motions between 5″ and 10″ per century compared with the total number of stars for each hour of right ascension. The zone cuts the Milky Way about 6^{h} and 19^{h}.

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TABLE III.

Number of Stars of Magnitudes $6^m \cdot 0$ *to* $9^m \cdot 0$ *the Proper Motions of which are within the limits* 5″ *to* 10″ *for each hour of R.A.*

Limits of R.A.	$6^m \cdot 0$–9.	$7^m \cdot 0$–9.	$8^m \cdot 0$–5.	$8^m \cdot 6$–9.	$9^m \cdot 0$–	Sum.	Total No. of Stars $6^m \cdot 0$–$9^m \cdot 0$.	Per-cent-age.
h h								
0–1	4	15	21	30	22	92	486	19
1–2	3	14	23	17	21	78	440	18
2–3	4	10	19	29	26	88	393	22
3–4	6	14	23	21	20	84	407	21
4–5	3	10	14	21	14	62	332	19
5–6	1	12	16	29	17	75	645	12
6–7	4	13	31	25	19	92	731	13
7–8	2	8	13	25	19	67	512	13
8–9	4	11	16	16	20	67	419	16
9–10	1	10	14	23	8	56	381	15
10–11	4	11	19	32	15	81	359	23
11–12	6	16	14	15	22	73	301	24
12–13	3	14	19	26	13	75	338	22
13–14	8	7	26	17	21	79	313	25
14–15	3	11	23	30	26	93	339	27
15–16	2	7	29	13	23	74	345	21
16–17	5	16	16	21	11	69	371	19
17–18	3	13	23	16	23	78	530	15
18–19	1	8	28	27	32	96	686	14
19–20	5	14	21	32	30	102	972	10
20–21	4	7	29	26	22	88	849	10
21–22	1	9	16	22	24	72	653	11
22–23	2	9	22	32	30	95	571	17
23–0	8	9	18	24	17	76	507	15
Sums	87	268	493	569	495	1912	11880	
Means	4	11	21	24	21	80		

A Statistical Discussion of the Proper Motions of the Stars in the Greenwich Catalogue for 1910. By Sir F. W. Dyson, F.R.S.

In this number of the *Monthly Notices* Mr. Thackeray gives statistics of the total proper motions of stars up to and including $9^m \cdot 0$ on the scale of the *Bonn Durchmusterung* which lie between the limits of declination $+24°$ and $+32°$. The area of this zone

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is about one-sixteenth of the whole sky, and the zone both cuts the Milky Way and passes near its pole. It thus gives a very good sample of the whole sky, and affords ample material for the investigation of the distribution of proper motions for stars of different magnitudes.

The numbers given by Mr. Thackeray have been rearranged so as to show the number of stars with proper motions above certain limits out of all the stars down to various limiting magnitudes. This rearrangement is shown in Table I.

TABLE I.

Number of Stars with Proper Motions above certain Limits.

p.m. \ Mag.	0m–5m·9.	0m–6m·9.	0m–7m·9.	0m–8m·5.	0m–8m·9.	0m–9m·0.
>50″	4	8	19	27	31	33
>30	8	17	44	75	92	98
>20	17	34	84	148	196	215
>15	26	58	126	231	318	368
>10	45	129	289	514	724	868
> 5	77	348	776	1494	2273	2902
> 0	165	626	2195	5387	9123	12046

It will be seen from inspection that the figures in any column obey approximately the law $\frac{n}{N} = \frac{c^2}{c^2 + s^2}$, where n is the number of stars with proper motions $>s$; N is the total number of stars, and c is a constant for each column.

TABLE II.

Values of c *calculated from formula* $n/N = c^2/(c^2 + s^2)$.

p.m. \ Mag.	0m–5m·9.	0m–6m·9.	0m–7m·9.	0m–8m·5.	0m–8m·9.	0m–9m·0.
	″	″	″	″	″	″
>50″	7·74	5·66	4·65	3·53	2·91	2·61
>30	6·78	5·01	4·24	3·55	3·02	2·71
>20	6·78	4·79	3·91	3·35	2·96	2·70
>15	6·47	4·79	3·70	3·21	2·85	2·66
>10	6·12	5·09	3·91	3·24	2·94	2·78
> 5	4·68	5·59	3·70	3·10	2·88	2·82
Mean	6·46	5·16	4·02	3·32	2·93	2·71
>20 to >10	6·46	4·89	3·84	3·27	2·92	2·71

It will be seen that for the bright stars there are more large proper motions than are indicated by the formula, but as the range is widened the formula fits the observations very satisfactorily.

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It will be shown that a formula of this character follows from the assumption that the stars whose apparent magnitude $> m$ are distributed in space according to the law $re^{-a^2h^2r^2}dr$. The number of stars in any region of the sky with velocities between u_1 and u_1+du_1 in a direction perpendicular to the solar motion and to the line of sight is taken as proportional to $e^{-h_1^2u_1^2}du_1$. Similarly, $e^{-h_2^2(u_2-\mathrm{U})^2}du_2$ is proportional to the number with velocities between u_2 and u_2+du_2 perpendicular to the line of sight and in the plane of the solar motion, where U is the resolved part of the solar motion in that part of the sky. The values of h_1 and h_2 differ in accordance with Schwarzschild's ellipsoidal hypothesis. The use of different values makes the analysis complicated, and as the formula aimed at is to apply to total proper motions over a large region of the sky, h_1 has been taken equal to h_2 and the subscripts dropped.

Putting $u_1=v\sin\theta$ and $u_2=v\cos\theta$ where v is the total velocity perpendicular to the line of sight, the distribution

$$re^{-a^2h^2r^2}dr\ e^{-h^2u_1^2}du_1\ e^{-h^2(u_2-\mathrm{U})^2}du_2$$

is the same as

$$re^{-a^2h^2r^2}dr\ e^{-h^2(v^2-2v\mathrm{U}\cos\theta+\mathrm{U}^2)}v\,dv\,d\theta.$$

Putting $v=sr$ where s is the total p.m., this becomes

$$r^3s\,dr\,ds\,d\theta\ \exp.\ \{-(a^2+s^2)h^2r^2+2h^2rs\mathrm{U}\cos\theta-h^2\mathrm{U}^2\}.$$

Expanding exp. $(2h^2rs\mathrm{U}\cos\theta)$ and integrating with respect to θ, this becomes

$$e^{-h^2\mathrm{U}^2}s\,ds\sum_0^\infty\frac{h^{2n}\mathrm{U}^{2n}}{n!^2}s^{2n}\ .\ e^{-h^2(a^2+s^2)r^2}\ .\ h^{2n}\ .\ r^{2n+3}\ .\ dr.$$

Integrating next with reference to r, the number of proper motions between s and $s+ds$

$$=\frac{s\,ds}{(a^2+s^2)^2}e^{-h^2\mathrm{U}^2}\left\{1+\frac{2}{1!}y+\frac{3}{2!}y^2+\ .\ .\ .\right\}$$

where

$$y=\frac{h^2\mathrm{U}^2s^2}{a^2+s^2}.$$

This

$$=\frac{s\,ds}{(a^2+s^2)^2}\left(1+\frac{h^2\mathrm{U}^2s^2}{a^2+s^2}\right)\exp.\left(-\frac{a^2h^2\mathrm{U}^2}{a^2+s^2}\right).$$

Integrating with reference to s, the proportion of total proper motions less than s

$$=\frac{s^2}{a^2+s^2}\ .\ \exp.\left(-\frac{a^2}{a^2+s^2}\ .\ h^2\mathrm{U}^2\right).$$

This formula may be compared with the formula $\dfrac{\tau}{\sqrt{(\tau^2+a^2)}}$ for

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the proportion of the resolved proper motion $<\tau$ in a direction perpendicular to a stream velocity U, and the proportion $\frac{v}{\sqrt{v^2+a^2}}e^{-\frac{a^2}{a^2+v^2}h^2U^2}$ between $+v$ and $-v$ in the direction of a stream velocity U. (*M.N.*, vol. lxxiii. pp. 337 and 403.)

If $\bar{v}$ be the mean peculiar velocity in one co-ordinate $h\bar{v}=\frac{1}{\sqrt{\pi}}$, thus $hU=\frac{1}{\sqrt{\pi}}\frac{U}{\bar{v}}$; $U=V\sin p$ where V is the velocity of the solar motion and p the angular distance from the apex. It will be found that $\bar{v}$ is not very different from the solar motion V.

Thus h^2U^2 lies between 0 and ·3. The exponential may accordingly be expanded, and the formula for the proportion of total proper motions less than s is

$$\frac{s^2}{a^2+s^2}\left\{1-\frac{a^2}{a^2+s^2}\cdot h^2U^2+\frac{1}{2!}\left(\frac{a^2}{a^2+s^2}\right)^2\cdot h^4U^4-\text{etc.}\right\}.$$

This is approximately equal to

$$\frac{s^2}{s^2+a^2(1+h^2U^2)}.$$

The difference between this and the more exact form is

$$\frac{s^2}{a^2+s^2}\cdot\frac{1}{2}\cdot\left(\frac{a^2}{a^2+s^2}\right)^2 h^4U^4.$$

The maximum value of this for $s=a$ is $\frac{1}{16}\times\frac{1}{9}$ or $\frac{1}{144}$ of the total number of stars, or $\frac{1}{72}$ of the number with proper motion $>s$.

For the zone $+24°$ to $+32°$ the mean value of U^2 is $\cdot42V^2$. Thus the value of c^2 found from the observations may be equated to $a^2+\frac{\cdot42}{\pi}\frac{V^2}{\bar{v}^2}$ or $a^2+\frac{\cdot42}{\pi}M^2$ where M is the parallactic motion.

Provided that M and a are proportional, the quantities c of Table II. are proportional to the mean parallaxes of the different groups of stars. Using the means in the last line (*i.e.* the quantities obtained from stars >20″, 15″, and 10″ per century, omitting those >50 because of the small numbers on which they depend, and those >5″ because accidental errors in the p.ms. will affect the figures appreciably), and the total numbers of stars between certain limits of magnitude given in Table I., quantities are found which are proportional to the parallaxes of stars $6^m{\cdot}0$–$6^m{\cdot}9$, $7^m{\cdot}0$–$7^m{\cdot}9$, $8^m{\cdot}0$–$8^m{\cdot}5$, $8^m{\cdot}6$–$8^m{\cdot}9$, and $9^m{\cdot}0$.

For example, for 165 stars $0-5^m{\cdot}9$ CM = 6″·46.
,, 626 ,, $0-6^m{\cdot}9$ CM = 4″·89.
Therefore ,, 461 ,, $6^m{\cdot}0-6^m{\cdot}9$ CM = 3″·85.

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Thus we find

TABLE III.

Limits of Mag. B.D.	Mean Mag. H.C.	Value of C×M.	Parallactic Motion. M.	Ratio. C.
m m	m	″	″	
6·0 – 6·9	6·7	4·34	3·85	1·13
7·0 – 7·9	7·6	3·42	3·00	1·14
8·0 – 8·5	8·4	2·87	2·40	1·20
8·6 – 8·9	9·0	2·41	2·04	1·18
9·0	9·4	2·05	1·82	1·13

where the values of M are derived from the formulæ $M = 410\varpi$ and $\log \varpi = -1·224 - ·120m$, given in the *Monthly Notices* for November. The mean value of C is 1·156.

Thus

$$a^2 + \frac{·42}{\pi} M^2 = 1·156^2 M^2$$

or

$$a = 1·10 M,$$

and

$$c = \sqrt{\left(a^2 + \frac{·42}{\pi} M^2\right)} = 1·05a.$$

The constancy of the values of C given in Table III. is sufficiently good. The value $a = 1·10$ M gives a larger value for the peculiar motions as compared with parallactic motion than is given by line-of-sight observations. It agrees with the results obtained from the Carrington stars when allowance is made for the direction of the axes of Schwarzschild's ellipsoid with reference to the latter stars. This may be due to a defect in the assumed formula, which is derived from stars of moderately large proper motion, and may not suit those less than 5″ per century so well; but, on the other hand, it is possible that the peculiar velocities of the bright stars which have been determined from spectroscopic observations may be smaller than those of stars of larger apparent magnitude.

For the distribution of stars in space, our assumed formula is $n = Ne^{-a^2h^2r^2}$, where N is the total number of stars brighter than a certain limiting magnitude, and n is the number of these at a distance greater than r. The value of

$$ah = \frac{1}{\sqrt{\pi}} \cdot \frac{a}{v} = \frac{1}{\sqrt{\pi}} \times \frac{1}{1·10} \times \frac{a}{V},$$

and measuring distance in parsecs, and proper motions in seconds per century, V = 410.

Thus

$$ah = \frac{a}{1·10 \times 410 \sqrt{\pi}} = \frac{c}{1·156 \times 410 \sqrt{\pi}}$$

where c is the quantity given in Table II. and

$$\log n = \log N - 10^{-6} \times 1·417 c^2 r^2,$$

from which the following table is derived:—

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TABLE IV.

Limits of Magnitude. m m	Log n.	$N-n$ (r small).	r_0.	Limit.
0 – 5·9	$2·2174 - 25·7r^2 \times 10^{-6}$	$·010r^2$	92	260
0 – 6·9	$2·7966 - 14·7r^2 \times 10^{-6}$	$·021r^2$	121	436
0 – 7·9	$3·3414 - 9·07r^2 \times 10^{-6}$	$·046r^2$	155	607
0 – 8·5	$3·7313 - 6·52r^2 \times 10^{-6}$	$·081r^2$	182	757
0 – 8·9	$3·9601 - 5·25r^2 \times 10^{-6}$	$·110r^2$	204	869
0 – 9·0	$4·0808 - 4·52r^2 \times 10^{-6}$	$·125r^2$	219	953

The distance r_0 at which the number of stars in the shell between r and $r+dr$ is a maximum is given by $1 - 2a^2h^2r^2 = 0$. The corresponding values of r are given in the fourth column. The last column gives the extreme limit to which the stars of the several groups reach, measured in parsecs.

Putting $r = 20$ in the third column, it is seen that there are, according to this formula, 4, 8, 18, 32, 44, 50 stars with parallax $>''·05$ within these limits of magnitude in this part of the sky. For the whole sky the numbers should be multiplied by 16. For the brighter stars the numbers appear to be much too small. The number 50 down to $9^m·0$ B.D. in this zone or 800 from the whole sky does not seem at all improbable. Out of 60 stars with proper motions $>20''$ per century measured for parallax at Greenwich, 12 or 1 in 5 were found to have parallaxes $>''·050$. This proportion gives about 700 in the whole sky, leaving 100 for stars of proper motion less than $20''$ per century. The extreme limits of the stars as given by the formula are too close. For the brighter stars a better agreement with the observed numbers of proper motions could be obtained, following Professor Eddington, by use of two terms $n = N_1 \frac{s^2}{c_1^2 + s^2} + N_2 \frac{s^2}{c_2^2 + s^2}$, the constants being chosen so as to suit the large and small proper motions.

The distribution of the stars according to the formulæ already obtained are given in Table V.

TABLE V.

Distribution in Distance of Stars of different Magnitudes.

Parallax.	Distance.	0–$5^m·9$.	$6^m·0$–$6^m·9$.	$7^m·0$–$7^m·9$.	$8^m·0$–$8^m·5$.	$8^m·6$–$8^m·9$.	$9^m·0$.
$>''·10$	<10	1	1	$2\frac{1}{2}$	$3\frac{1}{2}$	3	$1\frac{1}{2}$
''·100 – ''·050	10 – 20	3	3	$7\frac{1}{2}$	$10\frac{1}{2}$	9	$4\frac{1}{2}$
·050 – ·025	20 – 40	11	13	30	42	34	20
·025 – ·010	40 – 100	59	89	176	300	241	125
·010 – ·005	100 – 200	76	242	511	852	778	448
005 – ·003	200 – 333	15	102	500	942	992	670
·003 – ·002	333 – 500	0	11	312	945	1361	1200
·002 – ·001	500 – 1000	0	0	12	115	317	450

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The luminosity law was determined by dividing the stars into half magnitudes and determining the numbers between limits of distance whose logs progressively increased by ·1. The result obtained was

$$\log N \cdot dM = \{5{\cdot}430 - {\cdot}0091(M - 17{\cdot}8)^2\}dM,$$

where M is the absolute magnitude. The result cannot be considered satisfactory, because the coefficient of M^2 obtained from values M = 0 to M = 5·0 was nearly − ·02, and nearly zero as found from 5·0 to 10·0.

List of Stars in Zone $+0° 0'$ *to* $+5° 0'$, *the Proper Motions of which appear to be large, and are not in Boss's or Porter's Catalogues.*

(*Communicated by the Astronomer Royal.*)

A comparison between the Albany Catalogue for 1875 and the Abbadia Catalogue for 1900 gave 7000 stars common to both. The following list gives those stars which appear to have large proper motions. The proper motions are only approximate and not corrected for systematic error, and are given to three places of decimals in Right Ascension and two in Declination.

List of Stars with apparently large Proper Motions (not in Boss's or Porter's Catalogues) from a Comparison of Albany Catalogue (A.G., 1875) and Abbadia, 1900.

(These P.M.'s are only approximate, and not corrected for systematic errors.)

B.D. No.	Albany No.	Approximate R.A. 1900.	Approximate Dec. 1900.	Magnitude.	P.M. R.A. (s·001).	P.M. Dec. (″·01).
°		h m s	° ′			
3·68	115	0 29 48	3 50	8·8	+ 4	−21
2·72	117	30 4	2 40	8·8	+ 6	−17
0·181	289	1 0 56	1 11	8·8	+10	−15
2·315	569	56 40	2 24	8·8	−10	−27
2·321	577	57 49	2 52	6·9	+11	+16
3·339	680	2 22 32	3 59	8·5	+ 8	+22
2·393	704	26 20	3 0	7·9	− 5	−20
1·543	886	3 1 13	1 36	8·9	+23	−85
4·519	941	11 30	4 40	8·8	0	−29
1·577	956	13 45	1 57	8·8	+ 5	−16
3·515	1076	35 57	3 13	9·1	− 5	−23
1·661	1110	41 30	2 1	9·1	+ 4	+17
0·659	1120	44 24	1 4	8·4	+13	−65
5·678	1332	4 28 32	5 11	8·2	− 8	−25

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B.D. No.	Albany No.	Approximate R.A. 1900.	Approximate Dec. 1900.	Magnitude.	P.M. R.A. (s·001).	P.M. Dec. (″·01).
°		h m s	° ′			
2·816	1484	4 50 56	2 47	8·8	+ 8	−30
2·1041	1860	5 37 30	2 39	8·8	+13	−55
1·1677	2613	6 59 54	1 33	8·5	− 4	−27
4·1690	2812	7 19 44	3 57	8·9	− 7	−17
4·1714	2846	23 10	4 21	8·3	−13	− 4
4·1743	2893	28 18	3 59	8·8	−11	0
4·2025	3496	8 37 42	4 47	8·7	−11	− 6
4·2050	3548	44 0	3 51	7·6	−14	+10
4·2051	3549	44 9	4 3	8·2	−23	− 6
5·2143	3713	9 9 40	4 51	8·4	−11	+ 3
3·2300	3922	53 49	2 53	8·9	−12	−11
3·2331	3983	10 7 38	3 36	9·1	− 6	−19
3·2344	3997	11 46	2 55	8·6	−17	− 3
4·2510	4328	11 35 21	4 13	8·6	+ 9	−10
2·2516	4430	12 2 35	2 10	9·1	−16	−14
5·2592	4441	5 5	4 53	8·9	− 5	−31
2·2625	4647	13 2 27	2 42	9·0	+ 6	−20
3·2757	4692	14 4	3 26	8·0	−16	− 7
0·3065	4728	24 7	0 38	7·3	−15	− 3
3·2802	4765	33 24	2 50	9·0	−11	− 9
2·2783	4895	14 6 28	1 50	6·5	−12	+ 5
3·2896	4965	23 30	3 14	7·1	−16	+ 6
1·2959	5013	33 10	1 19	8·9	−13	0
3·2926	5030	37 54	3 7	8·8	−14	−11
4·3023	5231	15 27 1	4 24	8·7	−11	− 2
2·3157	5529	16 37 19	2 16	9·0	+ 1	−18
1·3347	5587	51 6	1 16	9·0	−15	+ 2
4·3288	5591	52 4	4 5	8·9	− 1	−14
4·3509	5924	17 42 51	4 59	8·7	−46	−21
4·3574	6049	59 37	4 26	8·8	−13	−26
4·4157	6750	19 29 38	4 21	8·9	− 4	+29
4·4165	6758	31 18	4 33	8·9	− 3	−23
3·4191	6905	50 4	3 48	9·2	−21	−22
2·4127	7067	20 13 59	2 54	8·6	+18	0
2·4203	7194	30 42	2 58	7·8	−12	− 5
5·4874	7615	21 44 8	5 15	8·8	+35	− 4
4·4762	7636	49 8	4 30	8·5	+16	+ 1
2·4453	7658	54 3	2 43	8·5	+13	+ 8
3·4691	7739	22 11 17	3 33	8·6	−11	−11
4·4879	7817	30 32	4 52	8·6	+23	−18

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The Magnitudes of the Cordoba "Catalogo de las Zonas de Exploracion," −52° to −62°. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The volume here dealt with is dated 1914, but was only received at Oxford in 1916. It contains about 90,000 stars, the places being given to $0^s{\cdot}1$ in R.A. and $0'{\cdot}1$ in Dec. and the magnitudes to 0·1 as far as 9·9; after that to quarter magnitudes as far as 12. Very few particulars are given in the all-too-brief introduction. It appears that the greater part of the work was done under Dr. Thome's direction with a 5-inch telescope, the object-glass of which was found broken by Dr. Perrine when he took over the directorship in 1909. He had the O.G. repaired and completed the work, "carrying out Dr. Thome's plans of work as far as they could be known," and has now earned our gratitude by publishing it. It will be seen from the following analysis that a few more particulars would be very welcome: especially valuable would be a list of the observers and the zones which they observed, with dates, for some of the anomalies noted below are probably due to personality.

2. The observations of magnitude are visual, and are therefore not comparable directly with the photographic magnitudes of the *Astrographic Catalogue*. Nevertheless if there is an "obscured patch" in the sky, or any equivalent systematic change in the relative numbers of bright and faint stars, it ought to be in evidence with both visual and photographic magnitudes. The main object of the present paper is to obtain some information about the course of the "obscured spiral" indicated in former papers, in a region of the sky where we have as yet no astrographic measures and seem likely to be without them for some time, according to such information from the Sydney Observatory as is forthcoming.

3. The method of counts under each heading of the magnitude scale is of course just as applicable to visual as to photographic records. Zones −52°, −53°, −54°, −60°, −61° were all counted in full detail, in order to test the magnitude scale as fully as possible. But it will suffice to give here part of one of these zones only. For magnitudes up to 9·9 the general run of the different hours seems to show that anomalies of estimate are small, but from 10 onwards there are serious discordances, as shown in Table I., which gives the counts of fainter magnitudes in Zone −52°.

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TABLE I.

Counts of fainter magnitudes in Zone -52°.

Mag. / R.A.	9·8.	9·9.	10.	10¼.	10½.	10¾.	11.	11¼.	11½.	11¾.	12.
0^h	3	2	22	25	37	35	29	2	2	0	0
1	5	5	16	11	25	29	23	9	30	9	0
2	6	7	24	21	14	16	11	12	14	5	0
3	21	13	107	5	5	2	8	0	0	0	0
4	8	11	30	29	41	24	19	0	0	0	0
5	13	11	69	33	46	47	41	0	0	0	0
6	15	17	170	20	22	6	21	0	0	0	0
7	23	16	249	10	16	4	8	0	0	0	0
8	25	13	48	42	77	63	98	0	1	0	0
9	30	12	61	71	116	114	144	15	20	4	4
10	50	17	43	63	91	48	53	12	12	1	1
11	38	46	85	88	124	59	54	5	2	0	0
12	18	16	50	64	109	84	96	9	6	0	0
13	14	11	38	36	66	68	93	26	24	1	4
14	12	11	27	35	66	77	108	18	32	5	0
15	26	10	69	61	102	106	147	34	24	1	0
16	28	22	59	67	89	75	112	57	35	1	0
17	14	10	36	38	57	69	110	43	19	0	0
18	25	7	117	16	42	31	73	19	17	0	0
19	24	26	247	1	5	3	0	1	0	0	0
20	9	23	180	7	11	18	13	11	19	2	0
21	11	10	37	18	24	44	47	22	28	3	0
22	14	7	15	17	25	26	32	26	23	3	0
23	6	4	19	20	49	27	41	3	5	0	0
Totals.	438	327	1818	798	1259	1075	1381	324	313	35	9

4. It is clear that the count in 19^h, for instance, is conducted on a different plan from that in 21^h. Probably a different observer was at work, but without knowing details we can only regard the comparative counts as of no value. The only question is whether, as found in some other cases, there are certain points in the scale which can be trusted, although in between them the scale becomes untrustworthy. To examine this point, the cases in zones -52°, -53°, and -54° were collected where the number of stars registered as 10 is specially in excess. These are shown in Table II. First the actual counts are given, those under head 9·9 denoting all the stars with mags. 9·9, 9·8, 9·7, and 9·6, greater than the previous heading 9·5. For 10·0 the numbers represent the single heading 10; under 11·0 are given all with 10¼, 10½, 10¾, and 11.

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5. Adding up the various columns we form the cumulative total N at the foot; and then log N is compared with a similar quantity derived from *all* the hours in these three zones. It will be seen that the difference is nearly constant, except for the heading 10·0, where there is a sudden jump. It is inferred, with some probability, that the undoubted anomaly in the scale for certain hours will not affect the comparison, through log N, of the points "All" and 9·9, or brighter. But it would be more satisfactory if these anomalies could be traced to their source.

TABLE II.

Cases of excessive 10 *counts in Zones* − 52°, − 53°, *and* − 54°.

Mag. / R.A.	6·0.	7·0.	8·0.	8·5.	9·0.	9·5.	9·9.	10·0.	11·0.	All.
					Zone − 52°.					
3^h	0	0	7	6	13	31	48	107	20	0
6	2	5	11	8	20	38	52	170	69	0
7	2	5	12	14	16	39	68	249	38	0
18	1	2	15	10	24	45	60	117	162	37
19	0	0	10	10	15	35	73	247	9	1
20	2	2	10	11	13	24	52	180	49	32
					Zone − 53°.					
2	1	1	6	13	8	24	28	71	52	17
7	0	2	10	7	21	49	74	236	22	0
18	1	1	10	14	14	34	60	139	185	2
19	1	3	8	16	13	34	54	118	107	27
20	0	0	8	11	13	32	47	115	81	21
21	2	1	8	13	5	29	57	112	48	25
					Zone − 54°.					
6	1	2	8	9	21	30	44	157	73	0
7	0	4	13	13	16	33	84	174	65	0
18	0	1	8	12	15	54	76	263	58	4
Total.	13	29	144	167	227	531	877	2455	1038	166
N.	13	42	186	353	580	1111	1988	4443	5481	5647
Log N.	1·11	1·62	2·27	2·55	2·76	3·05	3·30	3·65	3·74	3·75
All.	1·28	1·81	2·48	2·77	3·02	3·30	3·54	3·71	3·97	4·00
Diff.	·17	·19	·21	·22	·26	·25	·24	·06	·23	·25

6. The importance of the point represented by 9·9 is that, for the remaining zones, we need not count in detail in order to watch the behaviour of the difference

$$\log(\text{All}) - \log 9{\cdot}9.$$

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The total number of stars, "All," is given for each hour by the running number; and it is easy to pick out those brighter than 10 by their having a decimal of magnitude, which catches the eye. The results for all the ten zones are given in Table III. The constant shown at the foot of each column has been subtracted from the differences, so as to reduce each column to mean value zero.

7. The phase of the first harmonic has in the last few papers (on the astrographic magnitude scales) been adopted as the reference point for tracing the backbone of the spiral, the effect being regarded as an "obscuration."

In Table IV. are given, first, the coefficients A and B (of sin θ and cos θ) as calculated directly. In the first column the declination is increased by 0°·5, since each zone is only one degree wide and not two degrees, as for the *Astrographic Catalogue.*

Table III.

Values of log All – log 9·9 for different Zones.

(The unit is 0·01).

	-52°.	-53°.	-54°.	-55°.	-56°.	-57°.	-58°.	-59°.	-60°.	-61°.
0h	+ 5	- 11	- 1	+ 1	- 5	- 1	+ 4	+ 6	+ 1	+ 14
1	+ 9	+ 2	+ 2	+ 5	- 1	+ 2	+ 11	+ 12	+ 15	+ 8
2	- 6	- 1	- 5	- 3	+ 8	+ 4	+ 9	+ 6	+ 12	+ 13
3	- 13	- 6	- 3	+ 2	+ 4	+ 11	+ 2	+ 4	+ 22	+ 22
4	- 8	- 4	- 10	- 8	0	+ 8	+ 8	+ 4	+ 9	+ 18
5	+ 5	- 5	- 8	+ 5	+ 9	+ 3	0	+ 3	+ 15	+ 21
6	- 4	- 3	+ 2	+ 3	- 3	+ 2	- 3	+ 3	+ 2	+ 17
7	- 2	- 3	- 7	- 2	- 3	- 1	0	- 1	- 1	+ 11
8	- 3	+ 1	+ 8	- 6	+ 6	+ 14	+ 16	+ 1	+ 12	+ 11
9	+ 2	+ 5	- 8	- 10	- 6	- 4	0	+ 9	+ 9	+ 4
10	- 13	- 3	+ 1	+ 1	- 12	- 10	- 10	- 14	- 19	- 7
11	- 7	- 6	- 9	- 6	- 7	- 16	- 12	- 18	- 15	- 20
12	+ 3	+ 5	+ 8	- 2	- 9	+ 2	+ 2	0	- 4	- 22
13	- 1	+ 3	- 2	+ 12	- 6	- 4	+ 4	- 3	- 3	- 14
14	+ 12	+ 3	- 2	+ 1	- 3	- 4	0	- 5	- 6	- 8
15	+ 6	- 4	+ 7	+ 8	+ 4	- 7	- 24	- 9	+ 3	0
16	- 1	+ 6	+ 4	- 1	- 3	- 7	- 18	- 6	- 3	+ 8
17	+ 4	+ 4	+ 7	- 1	+ 3	- 17	- 27	- 14	- 21	- 21
18	- 1	+ 9	+ 1	+ 3	+ 2	+ 4	+ 8	- 10	- 21	- 16
19	- 4	+ 3	+ 5	- 2	- 3	+ 5	+ 11	+ 6	- 5	- 13
20	+ 3	+ 3	+ 1	+ 4	+ 9	+ 14	+ 18	+ 11	+ 19	+ 24
21	+ 3	- 2	+ 4	0	+ 12	+ 3	+ 6	+ 12	+ 10	- 21
22	- 1	+ 4	- 3	0	+ 2	+ 4	+ 6	+ 10	- 5	- 20
23	+ 2	- 5	+ 6	- 3	+ 11	- 2	- 14	- 7	- 21	- 7
Const.	48	44	46	44	40	41	38	39	34	28

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TABLE IV.

Particulars of "Obscuration" for the Zones.

(The unit for A, B, and R is 0·01).

Dec.	A.	B.	R.	θ_0.	ψ_0.
° −52·5	−3·3	+0·3	3·3	96°	+17°
−53·5	−3·4	−1·8	3·7	63	−20
−54·5	−3·6	+0·3	3·6	95	+9
−55·5	−2·0	+0·2	2·0	95	+5
−56·5	−0·6	+5·6	5·7	174	+81
−57·5	+3·2	+5·0	5·9	212	+115
−58·5	+3·9	+5·5	6·7	215	+114
−59·5	+2·9	+6·9	7·5	203	+100
−60·5	+7·8	+5·0	9·2	238	+131
−61·5	+12·7	+5·3	13·8	248	+137
−65	−3·5	−4·9	6·0	36	[+271]

Next the coefficient R, where $R^2 = A^2 + B^2$. There is a distinct increase in R with declination in the Cordoba results. The results for −65° are quoted from the note on the Melbourne Counts given in *M.N.*, lxxvii. p. 40, being determination (*a*), which is obtained by a comparison of columns for magnitude 13·20 and 12·25 on the C. and M. scale. This scale is only doubtfully applicable to the Cordoba Visual Counts (see below), but assigns 12·3 and 11·3 as the magnitudes compared, the interval being thus close to 1·0 magnitude as at Melbourne, though the stars used are about one magnitude brighter.

Next is given θ_0, the R.A. of greatest obscuration. If we write

$$A \sin\theta + B \cos\theta = -R \cos(\theta - \theta_0)$$

we see that $\tan(\theta_0 - 180°) = A/B$.

8. In previous papers it has been suggested that the values of θ_0 indicate a spiral

$$\psi_0 \equiv \theta_0 + 3{\cdot}66\delta - 247° = 0,$$

where δ is the declination of the zone analysed. The values of ψ_0 are near zero at first, but then suddenly increase. The result for Melbourne suggests that the increase is maintained, though it is unprofitable to attempt an exact formula at present. The equation to the spiral is, of course, only intended as a rough approximation, and since the spiral is probably not related to our Earth's axis, the equation is clearly liable to modification as we approach high values of δ, of either sign. But without further confirmation we cannot exclude also the possibility of systematic error of some kind.

9. One piece of evidence, of a reassuring kind, appears at the other end of the zones where they join on to the Cape astrographic zones. In Table V. are given all the consecutive determinations

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of A and B. Accidental error being clearly rather large, means of every consecutive three zones have been taken, and from these means smoothed values of $-R$ and θ_0 have been determined. It will be seen that the junction with the Cape is quite satisfactory.

Table V.

Juxtaposition of Cape and Cordoba series.

δ.	A.	B.	A′.	B′.	R_0'.	θ_0'.	ψ_0
-41°	- 5·4	-4·6					
-42	- 9·7	-3·7	-4·4	-5·1	6·7	41°	0
-43	+ 1·8	-6·9	-4·1	-4·5	6·1	43	- 2
-44	- 4·3	-2·8	-2·2	-4·4	4·9	27	- 21
-45	- 4·2	-3·3	-3·9	-4·0	5·6	45	- 7
-46	- 3·3	-6·0	-5·7	-5·5	7·9	46	- 10
-47	- 9·7	-7·3	-6·9	-6·6	9·5	46	- 13
-48	- 7·6	-6·6	-7·9	-4·1	8·9	63	0
-49	- 6·3	+1·7	-5·4	-2·2	5·8	68	+ 1
-50	- 2·4	-1·7	-2·6	-0·9	2·7	71	+ 1
-51	+ 0·9	-2·6	-1·6	-1·3	2·1	53	- 21
-52·5	- 3·3	+0·3	-1·9	-1·4	2·4	55	- 24
-53·5	- 3·4	-1·8	-3·4	-0·4	3·4	83	0
-54·5	- 3·6	+0·3	-3·0	-0·4	3·0	82	- 4
-55·5	- 2·0	+0·2	-2·1	+2·0	3·0	132	+ 42
-56·5	- 0·6	+5·6	+0·2	+3·6	3·6	183	+ 90
-57·5	+ 3·2	+5·0	+2·2	+5·4	5·8	200	+103
-58·5	+ 3·9	+5·5	+3·3	+5·8	6·7	210	+109
-59·5	+ 2·9	+6·9	+4·9	+5·8	7·6	220	+117
-60·5	+ 7·8	+5·0	+7·8	+5·7	9·7	234	+127
-61·5	+12·7	+5·3					

There is one element of uncertainty at the junction. The differences of log N are taken for two magnitudes separated by about a unit magnitude in each case, but the separation is not very accurately determined. It should be consistently the same throughout each of the two series, but the Cape value may differ a little from the Cordoba value. It will be seen that this would make a misfit in R, but would not affect θ and ψ. At present attention has been chiefly concentrated on θ and ψ, which trace out the backbone of the suggested spiral ; and only approximate estimates, on which no great stress is laid, have been made of the intensity R, so that there may be a misfit in R of the kind indicated. But the run of the numbers shows that it cannot be serious. There is a drop in R to the junction, and then a rise, but the drop and rise are clearly indicated within the two separate series of results, which are on scales consistent within themselves.

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10. The values of θ and ψ are sensibly unaffected by any such change of scale, which could only assign slightly different weights to the three components, of which the mean is taken as a smoothing process: and we see that the values of ψ, which should be zero if the suggested equation for the spiral, viz.

$$\psi = \theta + 3{\cdot}66\delta - 247^{\circ} = 0,$$

is absolutely correct, are satisfactorily small as far as -54° declination. Thus not only do the Cordoba results fit satisfactorily on to the Cape results, but they confirm the spiral as far as -54°. After that there is a comparatively sudden increase in ψ. To keep it near zero we must largely increase the coefficient (3·66) of δ. This coefficient indicates the angle M at which the spiral cuts the meridians, the value of M corresponding to tan M = 3·66 being 74°·7. To fit the portion from $\delta = -54^{\circ}$ to $\delta = -60^{\circ}$ we must have tan M = 20 or 30, *i.e.* M = 87° or 88°, so that the spiral is only very slightly inclined to a parallel of declination. This would afford a natural explanation of the increase in the coefficient K, for the backbone of the spiral, *i.e.* the region of greatest obscuration, would remain longer in close contact with the zone under analysis as we reach higher declinations.

11. We may now examine the distribution of the numbers in Table III. according to ψ; as was done for the Cape results on p. 153 of *M.N.*, lxxvi. For this purpose the different zones were started as below.

Dec.	−52°	−53°	−54°	−55°	−56°	−57°	−58°	−59°	−60°	−61°
ψ_0	4h	5h	6h	9h	12h	13h	14h	15h	16h	17h

The means of the zones thus arranged are given in Table VI., those for the Cape being repeated (from the above reference) for comparison.

TABLE VI.

ψ.	Cord.	Cape.	ψ.	Cord.	Cape.	ψ.	Cord.	Cape.	ψ.	Cord.	Cape.
h			h			h			h		
(0)	−6·7	−3·0	(6)	+2·2	−1·3	(12)	+4·4	+7·2	(18)	−2·3	−2·6
(1)	−8·1	−1·2	(7)	+1·5	+1·5	(13)	+5·9	+3·8	(19)	−5·5	−6·2
(2)	−8·1	−2·3	(8)	+2·3	+2·3	(14)	+4·3	+7·8	(20)	−3·7	−6·3
(3)	−3·2	−0·7	(9)	+4·8	+1·9	(15)	+2·9	+8·8	(21)	−3·1	−4·0
(4)	+0·7	−0·3	(10)	+6·0	+2·7	(16)	+1·8	+2·4	(22)	−5·9	−6·5
(5)	+1·0	−3·1	(11)	+9·0	+5·4	(17)	+2·4	−3·8	(23)	−2·6	−3·4

12. There are systematic differences between the two sets, a large part of which can be represented by the second harmonic

$$3{\cdot}0 \cos(\psi - 16^{h}),$$

but there is no reason why the two sets should be the same. It is a little disappointing not to find better marked in the Cordoba results the "sharp edge" on the following side of ψ, noticed for

the Cape, and to some extent for Melbourne (*M.N.*, lxxvii. p. 41). Between $\psi = 15^h$ and $\psi = 19^h$ the Cape results drop from $+8{\cdot}8$ to $-6{\cdot}2$, the Cordoba results only from $+2{\cdot}9$ to $-5{\cdot}5$. Perhaps the edge is softened by the spiral clinging closer to the zones of declination.

13. We may remark how little influence the galaxy seems to have on these results. Any influence it has should appear as a first harmonic with either maximum or minimum steadily near 12^h, since the galaxy reaches its greatest declination south at $12^h - 60°$, and for smaller declinations the two points of intersection with the zone are symmetrically placed with respect to 12^h, and are not far from it. But we have seen that the maximum obscuration, so far from remaining sensibly constant for these zones, changes very rapidly with the declination. It is difficult to see how such a phenomenon could be related to the galaxy: it can certainly have nothing to do with galactic latitude, though possibly it might be connected with galactic longitude.

14. Another point of interest is the march of the obscuration according to magnitude. Taking the three zones $-52°$, $-53°$, and $-54°$, which were counted in full detail, and adding together the portions 12^h–22^h, and then the portions 23^h–11^h, we get the following results for log N for different magnitudes.

TABLE VII.

Log N for "obscured" and "unobscured."

Mag. (Cordoba)	6·0	7·0	8·0	8·5	9·0	9·5	9·9	All
Mag. (C. & M.)	6·2	7·2	8·5	9·2	9·7	10·4	11·0	12·2
12^h–22^h	1·342	1·909	2·610	2·932	3·170	3·461	3·696	4·185
23^h–11^h	1·556	2·065	2·695	2·967	3·201	3·493	3·732	4·161
"obs."–"un."	+·214	+·156	+·085	+·035	+·031	+·032	+·036	–·024

The change with magnitude is thus far from uniform throughout. It is nearly uniform at 0·070 per magnitude from magnitude (Cordoba) 6·0 to 8·5. Then from 8·5 to 9·9 it is sensibly constant; and then in the next 1·2 mag. (on scale C. & M.) it falls ·055. This intermittence in the phenomenon seems scarcely compatible with the idea of *obscuration*, which has been provisionally adopted as an explanation: but nothing essential turns on this particular hypothesis.

15. The C. and M. magnitudes in Table VII. are taken from the table given on p. 145 of *M.N.*, lxxv., reducing the hourly counts in zones of 1° to four square degrees by the factor $4/15 \cos\delta$.

16. To throw further light on the march of the obscuration the first harmonics were calculated for various magnitudes, using the sum of zones $-52°$, $-53°$, and $-54°$. The results are as shown in Table VIII.

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TABLE VIII.

Zones $-52°$, $-53°$, *and* $-54°$. *First harmonics of log N.*

Mag.				Means.		Differ. Mag.	
Cord.	C. and M.	A.	B.	A.	B.	A.	B.
6·0	6·2	+7·0	−11·5				
7·0	7·2	+4·7	−17·3	+3·2	−14·2		
8·0	8·5	+1·0	−13·7				
8·5	9·2	−3·3	−15·8			−4·0	−3·4
9·0	9·7	−3·5	−19·2				
9·5	10·4	−4·3	−20·9	−4·1	−20·9		
9·9	11·0	−4·5	−22·7			−3·4	−1·9
All	12·2	−8·2	−23·2	−8·2	−23·2		

17. Here, again, we see the curious halt, in both A and B, from mags. 9·0 or earlier to 9·9. For the bright stars the determinations of A and B are of course subject to greater uncertainties, owing to the smallness of the available numbers; indeed, for the first group (mag. 6·0 Cordoba) the determination is rendered theoretically impossible, since one of the twelve terms submitted to analysis contains no stars at all (3^h and 4^h). For purposes of computation one star was assumed in this group, making the logarithm zero; but this is of course not a strictly legitimate process.

18. Taking the means of A and B for the first three groups, we may consider that they correspond to magnitude 7·3 (C. and M.). The slopes in A and B are fairly steady down to somewhere between 9·2 and 9·7 (C. and M.), say 9·3, so that the differences correspond to a difference of two magnitudes. This is only rough, but gives the numbers −4·0 and −3·4 as the change per mag.

19. The slope then disappears from mag. 9·3 to 11·0 (C. and M.). If we assume that it then reappears from 11·0 to 12·2, the differences divided by 1·2 give us −3·4 and −1·9, nearly the same numbers as before. This representation by two similar slopes separated by a horizontal line is probably only an approximation, but exhibits the main features of the change.

SUMMARY.

1. Detailed hourly counts of the magnitudes of the Cordoba zones −52°, −53°, and −54° show that after magnitude 9·9 the magnitude scale is quite untrustworthy; but the limit indicated by the faintest stars included is satisfactorily uniform.

2. Using the points "9·9" and "All" for the whole of the ten zones (−52° to −61°) the differences of log N are found to vary round the circuit of each zone in a manner similar to that found elsewhere. The junction with the Cape Astrographic Counts (−42 to −51°) is very satisfactory, carrying on the suggested "spiral" to −54°. After this, however, the spiral becomes suddenly much flatter.

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The Irregular Variable RS Camelopardalis.
By H. C. Plummer, M.A.

Professor Turner's criticism of the paper on this subject by C. Martin and H. C. Plummer deals entirely with the discussion of the observations, and not with the observations themselves. As the responsibility of one author alone is thus involved, it will be convenient for him to make a short reply. Professor Turner's practical experience in applying periodic analysis has been so extensive that his views certainly deserve every respect.

He takes an early opportunity of remarking that "nothing here said challenges the analysis" of our paper. This is eminently reassuring, for it covers no less than thirteen out of fourteen sections of the paper. As a matter of fact, the final paragraph was added as an after-thought, and it may be that its brevity is partly responsible for a misunderstanding of its meaning. The criticism turns very largely on this unfortunate paragraph, and even on a single sentence, which is quoted no less than three separate times. Let it be said generally that, so far as Professor Turner's remarks will act as a warning against *any* too dogmatic deduction from the results of analysis of this kind, they have the writer's fullest sympathy and approval. On the other hand, it seems clear that he is disappointed that the star does not conform to a certain ideal type of variation. In that the blame must be placed on the star, not on us. The particular pitfall which Professor Turner describes in such pathetic terms could never have the slightest allurement for anybody who was seriously investigating the variation of the star. The one thing certain from the beginning was that a single ruling period, however long, was most improbable, and that such an assumption must be absolutely avoided in the representation.

It is a little hard to see how the spirit of this final paragraph could have been more completely misrepresented. In fact, its simple argument has been exactly inverted. With every allowance for any possible obscurity, it is obviously directed against the representation by means of an Argelander-Chandler formula. It suggests a more satisfactory alternative, the analysis of the recorded variation as a whole. But unfortunately, as it now appears, it showed that a colourable imitation of the Argelander-Chandler formula could be forced out of the more complete representation. The object was to exhibit the relations between the different methods, obviously not to support the one which is held to be inferior and which, according to Professor Turner, has never proved satisfactory. With this explanation the details of the adjusted formula, which in any case apply strictly only to this particular star, are seen to be of trivial importance. They do not deserve the critical attention which they have received.

This explanation also covers the phrase "a simple oscillation in the period 88·7 days," which obviously cannot be taken out of its

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context. Is it reasonable to suppose that after taking the trouble to disentangle three periods (whatever their meaning may be), the work has been virtually thrown away, and any single period has been nominated in substitution? It is disappointing, after pointing out a means of escape from an old rut, to be brought back to it and to be told that it is a bad one.

To deny any intention of returning to that particular path is to render a considerable part of Professor Turner's criticism irrelevant. But that is not to say that the path actually followed is necessarily the right one. Here, naturally, there must be much latitude of opinion. A simple aggregate of four harmonic terms has been suggested for the variations of short period. Presumably, three questions are now involved: (1) How far is the representation of the observed past satisfactory? (2) With what probability can it be expected to extend to the future? (3) What physical reality lies behind its precise form? With regard to (1), Professor Turner is good enough to speak of "the beautiful accordance" between the observations and the formula. This is over-generous, but it may be taken to dispose of the first question. But this covers the modest aim of the main inquiry. To represent the past behaviour of the star at all did not prove the easiest of tasks, though that of course may argue lack of skill. It is, however, to be remarked that investigations of similar cases seem to have failed to reach even this preliminary stage.

When we come to the other questions a critic is in rather difficult case, for the simple reason that no attempt was made to decide them. The second is a question of probability which the future alone can settle. The representation may fail—perhaps one should say that it will very likely fail; and Professor Turner would probably be even more emphatic. But something may be added as to the reason. The critic may contend that the method is in defect: that another method would lead to more certain results. That is not believed. The failure will arise because the past record does not contain the whole of the required evidence, just as it is certainly defective in regard to the longer periodicities. In that case every method is doomed to definite failure, and the burden of the criticism is that an impossible task has been undertaken. Such criticism, in the present state of knowledge in this department, would surely carry its own condemnation.

The third question, important as it is, is even more remote. It was not even considered, and it seems idle to discuss it until the future has given its verdict on the preliminary issue. It will become profitable only when the permanent existence of the terms in question is assured, and the study of other stars has given a broader basis for discussion. In the meantime, it is not proposed to deal further with the argument contained in the middle of Professor Turner's note (§§ 6–11) beyond endorsing any warning against hasty conclusions, whether favourable or unfavourable to either side of the case. Above all, uncertain analogies are not in the least helpful and are to be deprecated.

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Some reason for Professor Turner's misconception, and the difficulty of guarding against it by anticipation, may be seen in his description of the type of irregular variation shown by RS Camelopardalis. Apparently he regards it as a mild form of departure from regularity, with an approximate tendency to a regular period. Now from the beginning of the discussion such a view was never even contemplated. The irregularity of this star is so pronounced that so far from regarding it as a mere variant of the long-period regular variable, any affinity it may have with this class must be considered purely conjectural.

The possibility of making this reply at an early date is due to Professor Turner's courtesy in forwarding a copy of his note in proof.

Observations made at Adelaide during the Annular Eclipse of the Sun on 1916 *July* 30. Communicated by G. F. Dodwell.

The central line of the eclipse passed from the N.W. to the S.E. of South Australia, the eclipse being annular for a distance of about 75 miles on each side of the central line. The annular phase was well observed at Adelaide, approximately 40 miles S.W. of that line. The atmospheric conditions were all that could be desired, the sky in the vicinity of the sun remaining clear throughout the eclipse. A series of photographs was obtained at the Observatory with the 8-inch Cooke equatorial refractor by Mr. H. B. Willsmore, and visual observations of contacts, etc., made with the finder of that instrument. The first contact was at approximately July 30 $10^h\ 12^m\ 42^s$ (Adelaide Standard Civil Time). This is considered fairly exact, although the Moon's edge was not noticed until it was just inside the Sun's disc. Allowance was therefore made for its rate of progress. There were two or three spots visible on the Sun's surface, and the edge of the Moon reached the centre of the largest of these at $10^h\ 28^m\ 55^s$ a.m. A.S.T. Diminution of light became noticeable before 11 o'clock, and was accompanied by a perceptible fall in the air temperature. At $11^h\ 57^m\ 22^s$ a.m. the eclipse became annular. Mid-eclipse was at $0^h\ 0^m\ 6^s$. From measurements on the photographs the area of the annulus was 12·63 per cent. The appearance of the Sun during this stage of the eclipse was most interesting and impressive. The light was much enfeebled, and it was possible to look at the Sun with the naked eye, but the irradiation was still so great that without a dark glass the body of the Moon could not be distinguished, the Sun's appearance resembling that of a small lake of molten silver. The sky was a very deep blue, and a slight tinge of purple colour was noticed in the clouds near the horizon. The light was estimated as about equal to that immediately after sunset on a clear winter day. The sunlight streaming through the foliage of trees cast beautiful rings of light and shade on the ground. Birds

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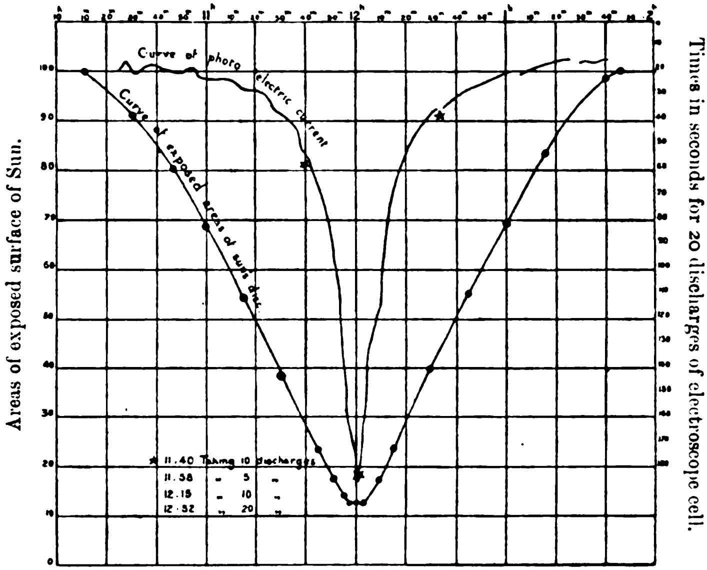
and animals were not uniformly affected by the eclipse. Some apparently took no notice, others appeared uneasy or behaved as if they expected the approach of night. The annular phase ended at $0^h\ 2^m\ 50^s$, thus lasting $5^m\ 28^s$. Immediately before the commencement and end of this phase the phenomenon of Bailey's Beads was well seen, and a photograph taken at $11^h\ 57^m\ 12^s$ recorded an example of it. The receding edge of the Moon reached the middle of the largest sun-spot at approximately $0^h\ 12^m\ 30^s$. The last contact was well observed at $1^h\ 44^m\ 39^s$. Observations of the eclipse made by many observers in the city and country were received at the Adelaide Observatory. Mr. A. E. Messent, of the

Areas of exposed surface of Sun (total surface = 100), and measurements of photo-electric current during eclipse.



Observatory staff, obtained photographs at Angaston on the central line.

Professor Kerr Grant has supplied the following account of actinic measurements made by himself and Mr. R. S. Burdon at the Adelaide University :—

"Observations on the photo-electric power of the Sun's light were made throughout the duration of the eclipse by means of a photo-electric cell similar in construction to that described by Hughes, *Phil. Mag.*, xxv. pp. 679–682. The chief difference was that the tube was made of glass and not quartz, which means that the effect was limited to light of wave length greater than 3300 A.U. The current through the tube was measured by means of a flapping gold-leaf electroscope as described by Zeleny. This method had been found to give very consistent results when tested by using

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the light of an electric lamp; and, except for some irregularities, almost certainly observational, at the commencement of the eclipse, gave a continuous variation throughout the eclipse. When compared with the variation of exposed area of the Sun's disc, the graph of photo-electric effect shows in a very marked way the decrease of actinic value of sunlight as we pass from the centre to the edge of the disc."

The following are the temperatures recorded by Mr. E. Bromley, of the Meteorological Bureau, during the eclipse:—

Time a.m.	Temperature. Shade.	Sun.	Time p.m.	Temperature. Shade.	Sun.
10.0	53·0	101·2	12.0	50·3	54·0
10.15	53·0	104·0	12.5	50·0	52·4
10.30	53·1	103·5	12.10	49·7	52·28
10.45	53·8	101·0	12.15	49·5	54·2
11.0	53·8	95·0	12.20	49·6	57·3
11.10	53·4	89·0	12.25	50·0	61·0
11.20	53·0	83·3	12.30	50·3	64·4
11.30	52·6	76·4	12.40	51·2	72·5
11.40	52·1	72·9	12.50	52·0	80·5
11.45	51·7	65·0	1.0	52·7	87·6
11.50	51·1	61·6	1.15	53·9	97·5
11.55	50·9	58·1	1.30	55·2	105·0
			1.45	55·7	108·8
			2.0	56·0	109·2

Light northerly wind; sky clear except for a few clouds near horizon.

Probable Motions in the Spiral Nebula Messier 51 (Canes Venatici) found with the Stereo-comparator. Preliminary Communication. By Dr. S. Kostinsky, Astronomer at the Central Observatory, Pulkovo.

1. The great nebula Messier 51 is the most typical object of this kind and the best illustration to the cosmogonical hypothesis of Chamberlin-Moulton. For a long time observers have been interested in the question of possible changes in its form.

Lord Rosse, who detected the spiral structure of the nebula early in 1845, doubted whether such an object could exist in statical equilibrium without internal movements (*Phil. Trans.*, 1850, p. 504). Therefore his assistant, J. Stoney, in 1849–50 measured the positions of some stars and knots of the nebula with regard to the centre. Later, R. Copeland repeated these measures with the great reflector of the same Rosse observatory

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at Birr Castle; unfortunately these visual observations (very difficult on account of the faintness of the object) were not exact enough for the purpose.

From 1889 to 1904 Isaac Roberts obtained a series of excellent photographs of the nebula with his 20-inch reflector; some of them were measured by him, especially for the purpose of studying the possible changes of the single parts of the nebula, and also more completely by his widow, Mrs. Dorothea Roberts. Certain indications of such changes seemed to result (*Rivista di Astronomia*, 1910, pp. 31–41 and 62–79), but the authoress herself believes her materials and her means for the study of it insufficient to solve the question of minute displacements.

2. Beginning with the year 1896, I succeeded in obtaining with the great astrograph at Pulkovo 12 plates of the above-mentioned nebula with exposures from 47 minutes to 3 hours; on some of them are to be seen many faint and well-defined details, which are effaced on account of over-exposure on the photographs of Roberts and the later ones of Keeler. This gives rise to the hope that at a sufficient difference of epochs small movements of the single knots will be distinguishable, if they exist at all.

As a matter of fact, on comparing stereoscopically the first of my photographs with the last ones, taken in 1896 March and 1916 April, I remarked almost indisputable displacements of some characteristic knots lying on the spirals; also movements were noticed of many stars—in the nebula itself and in its neighbourhood—some of which are probably physically connected with it.

Preliminary stereoscopic measures on the above-mentioned couple of plates of 36 single knots showed me that their observed proper motions with regard to the centre of the nebula seem to have a systematic character in the different parts. They led me to the following preliminary conclusions:—

(*a*) On the outer spiral the motion proceeds as though in general the single parts were moving away from the centre, so that the spiral has a tendency to draw itself together in the direction opposite to the hands of a clock;

(*b*) On the contrary, on the inner spiral in its eastern part the motion towards the centre prevails, and if there is any tendency to draw together it is rather in the opposite direction, *i.e.* with the hands of a clock;

(*c*) In the mean the annual proper motions of the single knots and stars are of the order $0''\cdot04$–$0''\cdot05$, but vary within wide limits.

I hope that a detailed working up of my material will soon allow me to prove these preliminary results.

Pulkovo:
1916 *May.*

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Occultations of Stars by the Moon observed at the Royal Observatory, Greenwich, in the year 1916.

(*Communicated by the Astronomer Royal.*)

Date and G.M.T. 1916.	d	h	m	s	Star and Phase of Occultation.	Magnitude.	Observer.	Aperture.
								inches.
Jan.	15	12	5	1·43	χ Tauri, D	5·3	A.C.	2¾
	,,			1·14	,, ,,	,,	R.C.	6¾
	,,			1·13	,, ,,	,,	F.J.	10
Feb.	7	9	27	22·57[a]	W.B. O., 579, D	6·5	A.C.	2¾
May	8	11	31	25·58	*d'* Cancri, D	5·9	A.C.	2¾
	,,			25·12	,, ,,	,,	W.	10
Sept.	15	15	22	18·06[b]	ε Arietis, R	4·6	A.C.	2¾
	16	11	41	50·90	26 Tauri, D*	6·6	H.	10
	16	12	3	48·30[c]	27 Tauri, D*	3·7	H.	10
	16	12	23	35·76	28 Tauri, D*	5·2	H.	10
	16	12	55	5·21	27 Tauri, R	3·7	H.	10
Nov.	2	8	35	56·85	94 B Capricorni, D	5·7	A.C.	2¾
	7	6	47	23·74	212 B Piscium, D	7·1	H.	10

Phenomena marked with an asterisk occurred at the Moon's Bright Limb; all other phenomena at the Dark Limb. The telescopes used are as follows:—

Aperture 10 in., Astrographic Equatorial (Guiding Telescope).
,, 6¾ in., Sheepshanks Equatorial.
,, 2¾ in., Portable telescope in Altazimuth Dome.

The initials H., A.C., W., R.C., F.J., are those of Mr. Hollis, Mr. Crommelin, Mr. Witchell, Mr. Cullen, and Mr. Jeffreys respectively.

Notes.

a. The star was too faint to hold steadily.

b. Clouds passing; the power employed was too low to show the duplicity of the star.

c. The observer noted that the count of seconds was doubtful.

Royal Observatory, Greenwich:
1917 *Jan.* 11.

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Micrometrical Measures of Double Stars (11*th Series*) *and New Double Stars.* By Rev. T. E. Espin, M.A.

The measures in the following list have been made with the 24-in. Calver reflector, with the exception of a few stars measured in the early part of 1914. Of the following stars of *h* there are, as far as I am aware, no other measures :—

h 1014	*h* 1715	*h* 2557
1017	1765	2559
1021	1831	2564
1031	1932	2568
1385	1978	2803
1571	2516	2869

Two of *h*'s stars I have been unable to find :—

(1) *h* 1679 was searched for on 1916 December 15. It is certainly identical with *h* 1673, the R.A. being one minute too great, and the position angle being reversed.

(2) *h* 2593, searched for on 1916 April 22, 28, May 3, and not found.

Name.	R.A. 1900.		Decl.		P.	D.	Mags.		Nts.	Date.	Year.	
	h	m	°	′	°	″						
+44°·4550	0	0·0	+44	40	235·1	20·37	6·5	9·2	2	·991	14	
A 110		0·6	42	6	122·1	1·77	9·2	9·3	2	·902	16	
Espin 153		11·2	40	44	242·2	2·37	9·5	10·6	3	·004	17	
h 1014		13·4	41	56	50·5	11·51	9·4	9·5	2	·939	16	
h 1017		14·8	41	59	263·1	16·43	9·8	11·2	2	·939	16	
h 1021		16·6	41	40	246·7	6·08	9·1	9·5	4	·950	16	
h 1978		24·9	43	36	223·2	5·61	9·5	9·7	4	·925	16	
h 1031		27·1	41	10	142·8	6·16	9·6	11·5	3	·973	16	
ρ 62		29·7	44	17	310·5	6·79	9·1	10·0	2	·961	14	
Espin 444		31·4	44	57	193·0	2·63	9·2	9·5	2	·843	14	
Espin 445		32·6	44	54	348·7	3·82	9·3	10·3	1	·865	14	
Σ 44		33·0	40	26	266·8	10·30	8·6	9·2	2	·961	16	
Espin 157	1	7·8	40	37	92·1	15·85	7·0	12·8	3	·978	16	AB
					48·0	19·68		13·2	2	·981		AC
Arg. vi. 3		8·9	42	23	288·9	11·60	8·8	9·0	2	·861	16	BC
					101·9	34·36	A=	8·3	2	·861	16	AB
Espin 4		25·2	43	6	97·1	3·18	7·9	8·8	2	·100	16	
+43°·322		28·8	42	45	94·8	44·38	8·6	8·8	2	·018	16	
A 1527		55·2	43	11	238·8	5·21	8·2	10·1	3	·079	16	
Espin 48	2	3·4	42	23	200·1	9·27	7·4	11·0	3	·102	16	
ρ 66		13·0	42	32	187·4	6·10	9·3	10·2	2	·891	16	

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Name.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.	Year.	
	h m	° ′	°	″					
Ho 319	3 14·6	+45 1	46·4	11·74	7·0 11·5	1	·873	14	AB
			308·1	18·13	15	1	·873	14	AC
ν Persei	38·4	42 16	46·6	31·40	4·0 12·0	2	·152	16	
A 1543	48·3	42 36	182·0	4·29	9·0 9·3	2	·105	16	
Espin 13	4 38·2	43 37	216·1	15·35	7·2 13·9	2	·139	15	AB
			346·1	16·79	14·2	2	·139	15	AC
Σ 599	43·8	44 47	337·3	10·23	7·9 8·6	2	·940	14	AB
			65·7	14·74	C=13·7	2	·940	14	BC
Σ 613	51·6	43 59	103·2	15·99	8·5 9·0	3	·136	15	AB
			17·6	15·60	C=11·0	3	·136	15	BC
OΣ 189	8 7·9	43 20	292·3	4·21	6·6 9·7	2	·168	15	
Espin 18	20·2	42 26	236·1	12·62	8·3 9·2	2	·258	16	
Σ 1369	9 29·1	40 25	149·1	24·71	6·8 7·8	3	·276	16	
h 2516	57·4	40 4	98·5	6·93	11·4 11·6	2	·272	16	
Σ 1405 rej.	59·9	40 4	252·2	22·11	8·0 10·5	2	·293	16	
Σ 1430 rej.	10 19·8	41 25	{268·0	21·99	8·0 12·0	2	·326	16	AB
			235·9	22·91	C= 9·5	2	·326	16	AC
h 2531	22·5	40 43	3·0	8·76	8·5 8·8	3	·261	16	
h 2534	27·4	40 58	326·2	22·24	5·0 11·8	2	·265	16	
OΣ 223 rej.	32·3	40 57	133·7	22·68	7·5 10·7	2	·312	16	
Σ 1475 rej.	43·7	41 55	199·6	23·80	8·0 11·1	3	·316	16	
h 2557	11 2·0	44 2	25·9	23·32	9·4 9·6	2	·321	16	
h 2559	3·0	43 3	266·8	14·72	11·7 12·1	2	·293	16	
h 494	7·5	40 44	149·7	29·45	9·4 9·6	2	·361	16	
h 2564	11·2	42 49	129·0	32·45	8·5 10·5	2	·301	16	
h 2568	16·4	44 10	244·0	24·03	11·0 12·5	2	·321	16	
h 2596	12 2·3	43 39	238·5	33·94	8·2 10·7	2	·373	16	
Σ 1606	5·7	40 27	325·8	0·91	6·8 7·0	4	·377	16	
Σ 1624	11·7	40 9	149·4	6·13	7·2 9·4	2	·345	16	
Σ 1638 rej.	19·3	43 36	280·5	8·28	9·0 10·5	2	·372	16	
+43°·2276	50·4	43 7	72·1	38·67	9·3 9·5	2	·345	16	
Espin 125	13 0·9	42 13	119·9	2·39	8·6 9·6	2	·395	16	
h 2639	1·6	41 28	160·4	31·61	7·8 11·0	3	·377	16	AB
			136·6	56·18	C= 9·2	3	·377	16	AC
			291·5	31·34		2	·406	16	CB
h 1244	49·2	42 41	129·2	7·04	7·5 11·0	1	·427	16	
Espin 625	15 17·7	44 40	251·6	1·64	9·5 9·6	3	·355	16	
Σ 1956	29·7	42 8	36·7	1·61	7·7 8·9	3	·505	16	
Espin 631	16 32·1	43 38	161·4	10·66	8·0 10·0	2	·512	16	
h 2803	57·3	40 34	263·9	18·11	9·6 11·6	3	·592	16	

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Name.	R.A. 1900. h	m	Decl. °	′	P. °	D. ″	Mags.		Nts.	Date.	Year.	
Espin 634	17	0·8	+42	20	97·8	2·25	9·5	10·5	1	·556	16	
Espin 636		33·4	41	46	127·1	2·31	9·3	10·1	2	·577	16	
+43°·2774		34·2	43	12	154·1	9·12	8·8	8·9	2	·681	15	
ρ 18	18	3·4	42	12	174·3	15·58	9·3	10·1	2	·571	16	
ρ 5		8·6	42	43	108·8	14·75	9·4	10·0	3	·668	15	
ρ 6		8·8	42	46	286·4	10·70	9·2	9·8	3	·668	15	
Espin 473		10·0	42	49	161·0	11·63	9·0	10·5	2	·768	15	AB
					280·1	31·01	C=	9·0	2	·768	15	AC
Σ 2393		41·7	38	12	23·4	13·92	7·3	9·5	2	·748	14	
h 1385	19	16·4	43	51	205·7	6·59	9·7	12·7	2	·799	14	
h 2869		18·3	42	2	357·9	8·54	10·8	11·7	2	·577	16	
A 594		24·1	42	53	332·6	2·36	9·0	9·2	2	·753	15	
Espin 23		46·8	44	54	134·8	7·94	7·0	11·5	3	·714	15	AB
					103·3	10·71	C=	14·8	3	·714	15	AC
h 1470	20	0·0	38	2	337·5	29·25	7·0	9·0	4	·741	14	
Espin 85		0·6	43	54	34·4	2·59	9·7	9·9	3	·655	15	
ρ 41		6·4	42	54	337·8	5·92	9·3	9·6	3	·730	15	
ρ 42		11·8	42	37	270·1	5·82	9·5	9·6	2	·723	15	
+43°·3627		26·8	43	23	231·2	7·01	9·1	9·1	2	·784	15	
+44°·3552		39·8	44	34	297·4	41·19	8·5	9·0	2	·863	14	
h 1571		39·8	41	9	55·1	7·04	9·6	9·9	3	·839	16	
h 1615	21	3·9	44	51	271·3	10·80	10·3	10·9	3	·746	14	
h 1618		6·4	43	35	132·7	2·41	12·0	12·5	2	·842	15	BC
					172·6	19·79	A=	8·0	3	·867	15	AB
Σ 2773		6·5	43	35	115·4	3·24	8·7	9·5	2	·796	15	AB
					64·8	22·23	C=	13·0	3	·814	15	AC
					261·3	29·65	D=	13·9	3	·814	15	AD
h 1634		14·8	42	19	137·2	26·88	8·8	9·0	3	·783	16	
h 1639		18·3	43	42	102·3	14·41	11·2	12·1	2	·744	14	
h 1640		18·8	43	43	43·0	9·32	9·5	10·7	3	·746	14	
Espin 99		25·5	44	33	199·9	5·11	8·5	11·5	2	·710	14	
Espin 100		25·7	44	47	157·9	4·73	8·6	9·0	3	·774	14	
Espin 683		54·6	44	13	249·0	8·75	8·5	9·2	2	·719	14	
h 1715		57·7	44	46	254·1	14·02	11·0	11·3	2	·744	14	
A 2497	22	13·4	42	2	196·4	5·14	9·0	12·0	2	·945	15	
Σ 2902		19·3	44	51	87·6	6·29	7·0	7·8	2	·702	14	
h 1765		21·4	42	46	187·6	14·71	9·4	9·7	2	·924	16	
A 415		46·6	43	33	331·4	4·07	8·7	10·2	3	·870	15	
h 1822		47·2	40	48	136·4	5·39	8·5	9·4	2	·995	16	
h 1831		51·2	42	31	91·3	14·79	9·7	10·1	2	·938	16	
A 788	23	19·7	45	13	270·0	4·20	8·8	13·7	1	·014	14	
h 1883		21·8	45	51	146·6	18·95	9·4	9·6	1	·014	14	
h 1932		59·2	42	2	302·2	7·01	8·4	8·6	2	·902	16	

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Notes.

h 1978. *h* gives P 217°.
Espin 4. If the assumed P.M. is correct, this must be a physical pair.
Espin 13. Star C, added with the 24-inch.
Σ 599. Star C, added with the 24-inch.
h 2534. The measures give P.M. 0″·131 at 260°·8.
h 2559. *h*'s place is some 30 sec. too small in R.A.
h 2639. β's measure in 1906 should probably be 57″·23 for AC.
h 2869. A 14 mag. about the same distance N.
Espin 23. C was first seen in 1914.
h 1615. *h* reverses angle.
B.D. +43°·3627. A.G. (Bonn) double? Not in β's G.C.
h 1640. *h*'s angle 54°·0.
Σ 2902. Two distant *comites*, 13 mag., and a little pair in field at P 49° mags. 12·0, 12·8; P 79°·9; D 4″·97. 2 nts.
h 1831. *h*'s place 2′ too far N.

New Double Stars.

No.	B.D.	R.A. 1920.	Decl.	P.	D.	Mags.	Nts.	1916.	
		h m	° ′	°	″				
1480	+41° 4935	0 0·4	+42 3	90·9	6·33	9·1 12·5	4	·912	
1481	+43° 40	11·6	43 53	80·8	7·70	8·6 9·1	2	·017	
1482	+43° 41	12·2	43 45	187·9	3·76	9·5 14·0	2	·017	
1483	+42° 50	13·7	42 28	346·5	2·56	9·5 9·6	3	·923	AB
				14·7	2·27	12·0 12·1	2	·944	CD
				165·2	28·90		1	·925	AC
1484	+42° 52	13·9	42 56	341·8	2·35	9·5 12·7	2	·814	BC
				42·6	32·38	A = 8·5	2	·814	AB
1485	+42° 61	15·7	43 10	94·4	9·60	8·2 12·3	3	·844	
1486	+40° 77	19·9	40 48	350·7	6·06	8·8 12·0	3	·994	
1487	+42° 81	22·9	42 42	50·8	3·14	9·5 13·0	2	·861	
1488	+41° 133	41·9	41 41	280·4	6·78	8·6 8·9	2	·955	
1489	+40° 181	46·8	40 59	24·6	6·00	9·5 10·8	2	·999	
1490	+43° 180	51·0	44 3	144·1	2·75	9·0 12·0	3	·056	
1491	+43° 188	53·0	43 20	127·9	2·16	9·5 9·6	3	·035	
1492	+43° 198	56·0	43 23	75·3	3·04	9·1 11·5	2	·023	
1493	+43° 323	1 28·2	43 35	55·7	7·97	7·7 13·7	3	·014	
1494	+40° 323	29·4	40 52	5·0	5·99	9·4 9·7	2	·986	
1495	+42° 327	30·5	43 5	66·3	1·24	9·2 9·4	2	·814	
1496	+42° 333	31·7	43 10	191·0	7·09	8·5 12·0	2	·072	
1497	+43° 340	33·0	43 19	158·9	5·61	9·3 11·9	3	·079	
1498	+43° 369	41·9	43 23	359·1	3·50	9·5 9·6	2	·100	
1499	+42° 403	50·9	42 55	298·7	5·99	9·4 10·9	2	·825	
	+43° 440	2 5·4	43 39	181·6	10·20	8·8 10·0	2	·011	
1500	+42° 488	11·7	42 31	25·4	6·69	9·5 11·4	2	·891	
1501	+41° 450	16·8	41 46	95·7	6·56	8·6 13·2	2	·941	

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No.	B.D.	R.A. 1920.	Decl.	P.	D.	Mags.		Nts.	1916.	
		h m	° ′	°	″					
1502	+40° 503	2 17·8	+41 10	162·7	6·70	9·4	10·8	2	·999	AB
				24·1	15·35	C=	9·5	2	·999	AC
1503	+41° 463	21·3	41 40	61·1	5·83	9·6	11·2	4	·974	
1504	+41° 491	29·9	41 29	100·0	5·95	9·2	13·5	2	·974	
1505	+42° 558	31·2	42 32	9·4	3·29	9·4	12·7	4	·682	
1506	+42° 601	35·9	42 20	353·4	5·71	8·4	14·0	2	·105	
1507	+41° 536	40·1	42 1	319·3	2·66	9·5	10·7	2	·941	
1508	+41° 566	49·2	41 51	57·7	3·43	9·5	9·7	2	·910	
1509	+41° 582	52·2	41 23	175·2	6·81	9·5	11·2	2	·958	
1510	+40° 637	53·0	40 46	324·2	4·97	9·3	9·3	2	·987	
1511	+42° 675	53·5	42 59	280·8	6·44	9·5	10·7	2	·072	
1512		56·0	41 13	116·9	3·75	9·5	9·7	3	·977	
1513	+41° 630	3 5·2	41 23	91·4	7·22	9·1	10·2	2	·999	
1514	+42° 739	12·7	42 44	48·7	1·19	9·5	9·6	2	·509	
1515	+41° 705	26·8	41 27	312·6	4·91	9·5	9·6	2	·974	
1516	+41° 713	29·2	41 54	87·1	4·20	9·3	11·0	2	·941	
1517	+42° 818	39·6	42 59	181·6	4·72	9·3	11·5	3	·397	
1518	+42° 825	41·7	42 19	240·0	5·95	9·4	11·2	2	·911	
1519	+42° 849	48·4	43 3	117·6	1·56	9·5	9·7	5	·060	
1520	+42° 888	59·7	43 0	4·4	6·10	9·5	9·6	2	·102	
1521	+42° 960	4 19·1	42 25	292·9	6·31	8·0	13·0	2	·958	
1522	+43° 967	19·6	43 15	311·0	8·89	7·5	13·7	2	·138	
1523	+43° 984	23·1	43 55	39·0	2·61	9·5	10·6	5	·072	
1524	+42° 1068	43·8	42 49	83·1	6·18	8·5	13·7	2	·903	
1525	+42° 1073	45·1	42 34	269·8	5·63	9·2	12·2	2	·987	
1526		45·4	42 51	103·3	1·93	9·8	10·0	3	·920	
1527	+43° 1137	50·1	43 12	138·3	2·87	9·5	10·7	3	·114	AB
				320·4	10·51	C=	13·0	3	·114	AC
1528	+44° 1211	5 21·5	44 12	45·3	5·61	9·4	9·5	2	·104	
1529	+44° 1229	25·7	44 27	236·3	2·59	9·3	13·7	3	·114	
1530	+43° 1346	39·5	43 20	14·5	5·19	9·1	9·1	2	·987	
1531	+43° 1382	49·0	43 52	301·5	4·03	9·3	9·4	2	·104	
1532	+43° 1482	6 5·5	43 40	231·8	7·61	9·5	12·0	3	·114	
1533	+43° 1499	9·1	43 42	254·3	6·47	8·5	12·5	2	·104	
1534	+43° 1531	18·4	43 26	92·1	1·78	9·4	9·7	2	·120	
1535	+42° 1595	37·6	42 37	74·2	6·95	8·6	13·2	2	·247	
1536	+41° 1615	7 6·0	41 46	11·8	1·42	9·1	9·4	2	·114	
1537	+41° 1718	38·4	40 55	354·9	6·73	9·3	13·2	2	·258	
1538	+42° 1794	53·7	42 16	216·6	2·53	9·2	10·0	2	·175	
1539	+41° 1766	54·0	41 14	100·8	1·32	9·2	9·4	3	·262	

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No.	B.D.	R.A. 1920.		Decl.		P.	D.	Mags.		Nts.	1916.	
		h	m	°	′	°	″					
1540	+40° 2014	8	9·3	+40	52	1·3	7·04	8·8	11·5	3	·269	
1541	+40° 2196	9	15·8	40	24	21·2	7·84	9·0	9·8	3	·303	
1542		10	18·6	40	35	196·7	3·52	9·5	9·6	2	·258	
1543	+40° 2370		42·2	40	30	12·1	3·20	9·5	12·5	2	·265	
1544	+40° 2642	13	14·0	40	25	267·6	6·97	8·1	10·0	2	·328	
1545	+40° 2643		14·5	40	25	88·9	6·34	9·5	13·5	2	·328	
1546	+40° 2648		16·8	40	16	221·9	2·87	9·2	13·5	2	·345	
1547			18·8	43	49	86·6	1·93	11·8	12·7	4	·420	BC
						327·2	27·53	A=	10·2	3	·408	AB
1548	+44° 2272		20·7	44	44	361·1	1·06	9·5	9·6	2	·387	
1549	+44° 2284		29·7	43	48	N.P.	5±	9·5	11	1	·427	
1550	+40° 2839	14	58·2	40	26	161·2	2·19	9·4	10·9	2	·387	
1551	+41° 2584	15	13·1	41	22	139·7	1·67	9·3	12·0	2	·417	
1552	+44° 2450		24·0	44	34	275·1	4·96	9·5	13·5	2	·372	
1553	+42° 2620		31·6	42	10	150·6	2·89	9·4	9·6	2	·454	
1554	+44° 2510		46·5	44	7	357·8	4·92	9·4	9·6	2	·372	
1555	+43° 2548		56·5	43	23	73·6	2·29	9·1	10·3	2	·407	
1556	+44° 2532		57·7	43	55	217·2	9·69	9·0	10·7	3	·374	
	+40° 3194	17	36·8	40	49	50·6	10·79	9·2	9·4	2	·588	
	+40° 3205		41·1	40	29	240·0	13·10	7·4	12·2	2	·618	
1557	+41° 2928		52·3	41	9	4·4	9·35	8·7	12·0	2	·602	
1558	+41° 2954		57·6	41	45	283·1	5·26	9·2	12·7	3	·587	
1559		18	26·4	41	54	181·2	1·85	9·8	10·5	3	·573	
1560	+41° 3144		44·6	41	53	340·9	7·95	9·5	11·6	2	·720	
1561	+41° 3147		44·8	41	15	122·2	6·97	9·2	10·5	2	·750	
1562	+42° 3309	19	16·1	42	6	323·7	5·75	8·5	8·9	2	·577	
1563	+42° 3446		42·4	42	15	206·9	7·57	9·2	10·8	2	·715	
1564	+42° 3476		45·4	42	26	200·9	7·91	9·2	10·6	2	·779	
1565	+42° 3481		45·8	42	22	230·2	3·41	11·5	12·3	4	·793	BC
						36·2	38·23	A=	9·2	3	·790	AB
	+42° 3514		50·9	42	18	340·3	9·51	8·7	9·3	2	·834	
1566	+42° 3630	20	8·0	42	14	350·2	5·60	9·5	9·6	2	·715	
1567	+42° 3656		12·4	42	15	177·4	2·02	9·3	9·5	2	636	
1568	+41° 3685		13·3	41	39	186·7	2·62	12·2	12·3	4	·894	BC
						299 0	11·45	A=	9·5	2	·899	A
1569			29·8	42	15	268·6	2·68	9·3	10·6	3	·697	
1570	+41° 3852		38·1	41	41	119·1	4·68	9·5	9·6	2	·795	
1571	+41° 3862		39·2	41	10	36·0	6·41	9·5	9·6	3	·810	
1572	+41° 3870		40·6	41	43	166·5	5·99	9·2	13·5	2	·779	
1573	+41° 3890		44·2	41	42	74·1	4·52	9·0	12·7	3	·839	

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No.	B.D.	R.A. 1920. h	m	Decl. °	′	P. °	D. ″	Mags.		Nts.	1916.	
1574	+41° 3922	20	49·7	+41	17	155·6	1·64	9·6	9·7	3	·936	
1575	+42° 3924		54·8	42	38	357·6	5·39	9·5	13·7	2	·632	
1576	+42° 3930		55·6	42	32	6·7	4·49	9·5	14·0	3	·647	
1577	+41° 3967		58·5	42	10	38·9	6·12	9·3	9·8	2	·670	
1578	+42° 3948		58·7	42	12	228·9	2·29	9·3	9·8	2	·670	
1579	+42° 3961	21	1·8	42	32	269·8	3·52	9·4	12·0	2	·728	
1580	+42° 3974		3·2	42	32	88·1	3·31	9·5	9·6	2	·728	
1581	+41° 4047		10·6	41	54	49·2	4·66	9·5	9·7	2	·961	
1582	+41° 4063		12·3	41	59	133·4	3·89	9·2	10·5	2	·899	
	+42° 4055		15·6	42	28	306·7	13·31	9·3	10·0	3	·697	
1583	+41° 4093		15·9	42	3	76·1	5·89	9·6	10·2	2	·851	BC
						239·6	14·28	A=	9·4	2	·851	AB
1584	+41° 4096		16·4	41	42	113·3	3·43	9·5	10·8	2	·913	
1585	+42° 4065		17·6	42	35	265·8	4·97	9·3	12·5	3	·762	
1586	+40° 4721	22	1·3	41	2	39·2	4·37	9·5	9·6	3	·866	
1587			14·7	40	37	299·7	3·91	9·5	9·6	2	·961	
1588	+40° 4781		15·1	40	40	96·0	5·89	9·4	9·5	2	·961	
1589	+41° 4464		17·0	41	57	178·3	9·30	8·6	9·7	3	·796	
1590	+41° 4472		18·1	41	23	167·0	7·95	9·5	10·7	3	·895	
1591	+42° 4382		19·9	42	38	160·1	2·31	9·4	10·0	2	·672	
1592	+41° 4536		28·5	41	32	312·0	7·25	9·1	11·2	2	·787	
1593	+40° 4868		34·0	41	13	145·4	6·73	9·3	12·7	2	·899	
1594	+41° 4650		53·5	41	24	94·8	8·97	8·7	9·7	2	·891	
1595	+40° 4972		58·0	41	11	337·9	2·24	9·4	9·5	2	·961	
1596	+41° 4680	23	2·0	42	7	76·4	5·70	11·7	12·7	3	·789	BC
						21·9	35·77	A=	7·6	2	·787	AB
1597	+41° 4802		26·5	41	31	90·0	7·70	8·4	12·5	2	·814	
1598			30·8	40	25	273·5	2·76	9·4	9·5	2	·981	
1599	+40° 5125		34·5	40	20	347·0	3·81	9·3	9·4	2	·987	
1600	+41° 4835		34·8	42	14	301·8	6·03	9·5	9·7	3	·021	

Notes.

1522. Also a 10 mag., P 102°·9, D 31″·5.
1526. Forms a distant *comes* to B.D. +42°·1079 at P 237°.
1531. There is a 9·3 mag. at P 292°·6, D 41″, and four other stars in the group.
1542. P of two stars, S of B.D. +40°·2322.
1557. An 11 mag. at P 37°·8, D 28″, a more distant 13 mag. at P 309°.
1559. Forms a distant *comes* to B.D. +41°, 3063 at 87°·2.
1562. A 12 mag. at P 56°·3, D 30″.
1566. On the second night, the N star seemed to be slightly the brighter.
1568. Middle of three stars on N border of a nebula. R.A. about 20 sec. less than in Argelander.
69. This agrees in R.A. with B.D. +42°, 3780, but is 4′ S in declination. There is no star in Argelander's place.
1575. SF of two.
1576. N star of a pair in Argelander.
1589. Also a 12 mag. at P 240°, D 22″.

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On the Question of Extinction of Light in Space and the Relations between Stellar Magnitudes, Distances, and Proper Motions. By Dr J. Halm, Ph.D. (Plates 1, 2.)

The standardisation of the photographic magnitudes of stars near the South Pole, and subsequent differential comparisons between the south-polar sequence and stars in selected areas of the zone 45° southern declination, have established a sufficiently reliable framework of standard photographic magnitudes for an examination of the magnitudes of the C.P.D. catalogues with the view of ascertaining their relation to a fundamental photographic system. A detailed account of this work will be published in the *Annals of the Cape Observatory.* The present communication is limited to the announcement and discussion of some results whose bearing on questions of the structure of the universe may be of general interest.

§ 1. *The Statistical Relation between Magnitudes and Star Counts.*

The work of standardising the magnitudes of the C.P.D. was primarily undertaken with the view of testing the accuracy of its photometric basis through a comparison with results derived from stars of the northern hemisphere. As the medium of comparison the relation between magnitudes and star counts was utilised. With regard to the southern hemisphere, the records of the star numbers in the C.P.D. catalogues discussed by Stratonoff in his "Etudes sur la structure de l'univers," *Publications de l'Observatoire astronomique et physique de Tachkent*, No. 3, form a suitable basis, while the extensive work of Chapman and Melotte (*Memoirs R.A.S.*, vol. lx., part iv.), based chiefly on the Greenwich Astrographic Catalogues and the northern portion of the Franklin-Adams charts, establishes the relation between photographic magnitudes and star numbers for the northern portion of the heavens.

The first question was to examine how far the relationship between star numbers and magnitudes agrees between the two hemispheres.

In Appendix II. of his publication Stratonoff gives the numbers of C.P.D. stars per square degree in groups comprised between the C.P.D. magnitudes 1 to 6·0; 6·1 to 6·5; 6·6 to 7·0; 7·1 to 7·5; 7·6 to 8·0; 8·1 to 8·5; 8·6 to 9·0; 9·1 to 9·5 and 9·6 >.

These "star densities" are quoted for contiguous areas whose positions are recorded in Appendix I. of Publication No. 2. The material utilised in the present investigation is comprised within the zone between − 20° and − 50° declination, containing 432 areas numbered from 1189 to 1620. After reducing the C.P.D. magnitudes to the Cape standard system, the numbers of stars,

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N,* brighter than certain specified magnitudes were found as shown in Table I.

TABLE I.

Limiting Magnitudes. C.P.D.	Stand. Ptg.	log N (Whole Sky).
6·5	6·15	3·565
7·0	6·83	3·910
7·5	7·57	4·265
8·0	8·36	4·658
8·5	9·20	4·984
9·0	10·04	5·359
9·5	10·85	5·653
10·15	11·87	6·105

In comparing these results with those derived by Chapman and Melotte, it should be noted that their magnitudes are mainly based on the Harvard system of the north-polar sequence published in H.C. 170. Professor Seares has pointed out that the scale of this system differs sensibly from that derived by fundamental observations made at Mount Wilson. From the figures published in table iii. of his paper on "A Comparison of the Harvard and Mount Wilson Scales of Photographic Magnitude" (*Astrophysical Journal*, vol. xli., 1915), the differences (Mount Wilson—H.C. 170) in order of magnitude are shown in Table II.

TABLE II.

m.	(Mt. W. - H.).	*m.*	(Mt. W. - H.).
6·0	−0·02	12·0	+0·31
7·0	+0·02	13·0	+0·32
8·0	+0·09	14·0	+0·33
9·0	+0·18	15·0	+0·31
10·0	+0·25	16·0	+0·25
11·0	+0·29	17·0	+0·17

According to Seares, the results of the Göttingen and Yerkes actinometry support the Mount Wilson results.

In Table III. are shown for specified values of *m* (standard photographic)—

(*a*) the values of log N derived from the C.P.D. star counts (Stratonoff) and interpolated graphically from the observed values of Table I.;

(*b*) the values of log N derived by Chapman and Melotte and based on the photographic system of H.C. 170;

(*c*) the values of log N derived from the same material as under (*b*), but based on the Mount Wilson photographic system.

* The numbers N are related to the numbers B counted in the area ω (1 square degree) by

$$N = \frac{4\pi}{\omega} . B,$$

$$\log \frac{\pi}{\omega} = 4·616.$$

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TABLE III.

m.	log N (C.P.D.).	log N (C. & M., H.C. 170).	log N (C. & M., Mt. Wilson).
4·0	...	(2·48)	...
5·0	...	(2·99)	(3·02)
6·0	3·50	3·50	3·51
7·0	4·00	3·99	3·98
8·0	4 47	4·51	4·47
9·0	4·92	4·99	4·92
10·0	5·33	5·43	5·33
11·0	5·73	5·84	5·73
12·0	6·12	6·22	6·12
13·0	...	6·57	6·47
14·0	...	6·88	6·79
15·0	...	7·19	7·09
16·0	...	7·47	7·39
17·0	...	7·74	7·68

The figures of this table indicate that the Cape photographic system is in agreement with the Mount Wilson system both as regards scale and zero-point. This fact lends support to the correctness of the Mount Wilson scale, in addition to the evidence in its favour shown by the Göttingen and Yerkes observations. The subsequent discussion will therefore be based on the values of log N given in the second and fourth columns of Table III.

Next it seemed important to examine the relation between m and log N with reference to the galaxy. Two regions were selected, the one comprising stars in close proximity to the Milky Way (galactic latitude, $\beta = \pm 2^\circ \cdot 5$), and the other containing the stars near the southern galactic pole ($\beta = -74^\circ$). The resulting figures are shown in Table IV., first and second column. The third column of the same table shows the values of log N_0 taken from Table III. for the corresponding magnitudes, the fourth column giving the logarithm of the ratio N/N_0.

TABLE IV.

Galaxy: $\beta = \pm 2^\circ \cdot 5$.				Galactic Pole: $\beta = -74^\circ$.			
m (Ptg.).	log N.	log N_0.	log N/N_0.	*m* (Ptg.).	log N.	log N_0.	log N/N_0.
6·14	4·02	3·57	+0·45	7·58	3·85	4·28	−0·43
7·00	4·40	4·00	+0·40	8·28	4·18	4·60	−0·42
7·85	4·86	4·40	+0·46	9·01	4·53	4·93	−0·40
8·71	5·24	4·80	+0·44	9·75	4·80	5·24	−0·44
9·70	5·68	5·22	+0·46	10·42	5·08	5·51	−0·43
10·64	6·04	5·60	+0·44	11·06	5·27	5·76	−0·49
11·85	6·54	6·07	+0·47	11·86	5·57	6·07	−0·50

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The ratios N/N_0 show a remarkable constancy throughout the available range of magnitudes, in agreement with the result derived by Pickering from the visual magnitudes, and also with the general conclusions arrived at by Chapman and Melotte in their discussion of the photographic magnitudes of the northern hemisphere. The ratio (Galaxy : Pole) is, however, much in excess of that found by Chapman and Melotte, viz. 7·8 as against 3·1. The difference is doubtless to a large extent real, and due to an apparently greater contrast in star wealth. It appears also to be borne out by the Johannesburg Franklin-Adams plates (table xvi. of Chapman and Melotte's paper). If, within the zones centred on $-30°$, $-45°$, and $-60°$ declination, the numbers of stars down to the 17th magnitude are compared between regions $\beta \leqq 5°$ and $\beta > 70°$, the ratio 6·3 is found, *i.e.* double the amount found from the northern stars. It is quite likely, however, that the ratio derived from Table IV. may be somewhat exaggerated by seasonal changes in the zero-point of the magnitude scale, the probability of such a defect being suggested by the fact that the regions near the galactic pole were always observed at the same (hottest) time of the year.

Attention was next directed to a comparison of the standardised photographic magnitudes with a visual system. Professor Kapteyn has kindly placed at our disposal a detailed comparison between the C.P.D. magnitudes and the visual magnitudes of the Harvard system for the zone here examined. Through the medium of the C.P.D. we may thus compare the visual and photographic systems. Six regions, according to galactic position, were examined. In each region the stars were collected in groups arranged in order of C.P.D. magnitude. The third column of Table V. shows the differences (Photographic-Visual) derived from the general mean of all the regions, while columns 4–9 exhibit the departures from these mean differences for each region and magnitude.

Table V.

m. (C.P.D.)	*m.* (Photogr.)	(Ptg.-Vis.)	$\beta=67°.9$	$55°.4$	$38°.9$	$20°.3$	$13°.1$	$5°.1$
7·5	7·60	+0·56	+0·32	+0·23	+0·17	−0·11	−0·26	−0·35
8·0	8·38	+0·55	+0·37	+0·21	+0·17	−0·13	−0·26	−0·37
8·5	9·15	+0·55	+0·41	+0·27	+0·11	−0·10	−0·29	−0·38
9·0	10·00	+0·67	+0·43	+0·33	+0·10	−0·16	−0·29	−0·40
9·5	10·85	+0·86	+0·40	+0·34	+0·03	−0·11	−0·29	−0·38
10·0	11·59	+0·89	+0·37	+0·29	+0·04	−0·11	−0·27	−0·30
		Mean :	+0·38	+0·28	+0·10	−0·12	−0·28	−0·36

The figures of this table point to several interesting facts. First, it appears from the third column that the difference (Ptg.-Vis.) increases with increasing magnitude, *i.e.* fainter stars are somewhat redder than bright stars. Secondly, the constancy of the figures shown in the vertical columns 4–9 indicates that the rate at which

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fainter stars become redder is the same for all regions. Thirdly, the means at the bottom of these columns show that the stars of al magnitudes become increasingly *bluer* as the galaxy is approached.

§ 2. *The Physical Relation between Magnitudes and Star Numbers.*

Having established in the figures of Table III. the statistical relation between m and N on what may be considered a consistent photometric basis, an attempt was made to examine this relation from the physical point of view.

It is well known that in a universe in which the stars are uniformly distributed, and in which the star's light is not impeded by space absorption, the relation between apparent magnitudes and star numbers is expressed by the equation:

$$m = \tfrac{5}{3} \log N + \text{constant} \quad . \quad . \quad . \quad (1)$$

This relation holds whatever may be the law according to which the absolute luminosities are distributed, provided that the luminosity law is independent of the distance.

It is quite evident from the figures of Table III. that equation (1) does not represent the observed relation between m and N. Opinions as to the possible cause of the observed differences are divided, the majority of astronomers being inclined to attribute them to a gradual thinning out of star matter with increasing distances from us. The alternative assumption of extinction of the star's light during its passage through space, though its possibility is universally admitted, has hitherto received little support. Perhaps the only important attempt to reconcile observation with this point of view is that of Comstock in his paper in No. 558 of the *Astronomical Journal.* The criticism of Comstock's views by Kapteyn in No. 566 of the same publication centres on the fact that extinction of the amount suggested by Comstock would lead to an enormous and obviously quite inadmissible increase of the star density with distance. While the force of his arguments against the amount of extinction must be admitted, the trend of his reasoning is, however, not directed against the assumption itself. In fact, some of his calculations appear to bear out a coefficient of extinction about one-third of that derived by Comstock.

A significant light appeared to be thrown upon the question by the following attempt at representing the observed relation between m and N.

Instead of equation (1) the modified form

$$m = \tfrac{5}{3} \log N + a/\bar{\pi} + \text{constant} \quad . \quad . \quad . \quad (2)$$

was adopted, where $\bar{\pi}$ represents the mean parallax of stars of magnitude m.

The relation between magnitude and mean parallax being

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known through Kapteyn's and Comstock's investigations of parallactic motions for stars brighter than 12th magnitude, we are enabled to test the applicability of equation (2), at least within this range.

Kapteyn's general results are given on page 12 of the Groningen Publications, No. 8, the derived parallaxes being based on the value 16·7 km./sec. for the solar motion. They extend over a range of magnitudes from 2·7 to 8·9. A further value of $\bar{\pi}$ for fainter stars of average magnitude 10·5 may be added to Kapteyn's series through an investigation of Comstock (*Astron. Journ.*, No. 558). Reducing Kapteyn's parallaxes to Campbell's value of solar motion, 19·5 km./sec., the values of $\bar{\pi}$ in the second column of Table VI. were obtained corresponding with the magnitudes of the first column. The third column of the same table shows the values of log N related to the same magnitudes. They were interpolated from the column headed "Harvard" in table xiii. of Chapman and Melotte's paper. Since the magnitudes to which the parallaxes refer, except the last, which is somewhat uncertain, are based on the Potsdam system, a small constant correction for reduction to Harvard should have been added to the values of log N. This correction, however, is irrelevant for our purpose, and has therefore been omitted.

Table VI.

m.	$\bar{\pi}$.	log N.
2·73	0″·0362	2·00
4·14	0·0194	2·73
5·07	0·0139	3·21
6·01	0·0122	3·68
6·93	0·0085	4·13
8·89	0·0060	5·08
10·5	0·0046	5·77

In Table VIa. the observed magnitudes are compared with values computed from the equation:

$$m = \frac{5}{3} \log N + 0{\cdot}00764/\bar{\pi} - 0{\cdot}82.$$

Table VIa.

m (Obs.).	*m* (Comp.).	O – C.
2·73	2·72	+ ·01
4·14	4·12	+ ·02
5·07	5·08	– ·01
6·01	5·94	+ ·07
6·93	6·96	– ·03
8·89	8·92	– ·03
10·5	10·47	+ ·03

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The agreement is surprisingly close, and suggests a physical interpretation of equation (2), viz. that the observed facts may perhaps be explained on the assumption of a universe in which the stars are uniformly distributed, their light suffering a minute amount of extinction during its passage through space.

The question then arises whether such an assumption would be sufficient to explain the observed relationship between m and N down to the faintest stars included in Chapman and Melotte's investigation.

The evidence afforded by Table VII. must, however, not be taken to indicate uniformity of star density throughout the whole of the universe. In fact, the pronounced crowding of stars near the galaxy emphasises the futility of an attempt to explain the structure of the universe on the basis of such a simple assumption. It must be considered, however, that in summing up the stars over the whole sky, those contributed by the regions near the galaxy must necessarily preponderate. Consequently, the resulting relation between star numbers and magnitudes may be considered typical of *low* galactic latitudes, but cannot be taken without further proof to represent the conditions prevailing near the galactic poles.

The course adopted in the subsequent investigation is, first, to examine how far the combined assumptions of uniform density and of extinction of light in space account for the statistical facts relating to the stars as a whole, and afterwards to compare the theoretical conclusions with the conditions prevailing in special areas selected with reference to the galaxy.

§ 3. *Relations between Magnitudes, Star Numbers, and Distances on the Assumptions of Uniform Density and Extinction of Light.*

The assumption of *uniform* density leads to a simple analytical aspect of the problem. Consider the stars enclosed in a spherical shell of thickness dr of which the sun is the centre. The volume of the shell is

$$4\pi r^2 dr.$$

Let $\nu(r)$ represent the number of stars contained in unit volume at distance r, then the total number of stars contained between the radii $(r \pm \frac{1}{2}dr)$ is

$$4\pi\nu(r)r^2 dr.$$

Let further the proportion of stars whose absolute magnitudes lie between $(\mathrm{M} \pm \frac{1}{2}d\mathrm{M})$ be represented by

$$\phi(\mathrm{M})d\mathrm{M}.$$

Consequently the number of stars whose distances lie between $(r \pm \frac{1}{2}dr)$ and whose absolute magnitudes are between $(\mathrm{M} \pm \frac{1}{2}d\mathrm{M})$ is represented by

$$4\pi\nu(r)r^2 dr\phi(\mathrm{M})d\mathrm{M} \quad . \quad . \quad . \quad . \quad (3)$$

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We may further introduce the apparent magnitudes, m, by the relation:

$$M = m - 5 \log r - ar \quad . \quad . \quad . \quad (4)$$

where ar represents the effect of extinction in space assumed proportional to the distance.

We may replace dM in (3) by its equivalent form in terms of m through differentiating (4), considering r constant. Hence

$$dM = dm,$$

so that (3) expressed in terms of m and r takes the form:

$$4\pi\nu(r)r^2 dr\phi(m - 5 \log r - ar)dm \quad . \quad . \quad (5)$$

and represents therefore the number of stars whose distances lie between $(r \pm \frac{1}{2}dr)$ and whose apparent magnitudes are between $(m \pm \frac{1}{2}dm)$.

So far the density $\nu(r)$ has been treated as an unknown function of the distance. We shall now assume that

$$\nu(r) = \nu_0 = \text{constant},$$

and hence write instead of (5):

$$4\pi\nu_0 r^2 dr\phi(m - 5 \log r - ar)dm \quad . \quad . \quad . \quad (6)$$

Further, writing:

$$r_1 = r - \tfrac{1}{2}dr\ ;\ r_2 = r + \tfrac{1}{2}dr,$$

(6) may be represented in the form:

$$\tfrac{4}{3}\pi\nu_0(r_2{}^3 - r_1{}^3)\phi(m - 5 \log r - ar)dm \quad . \quad . \quad (7)$$

where obviously $\frac{4}{3}\pi\nu_0$ is the total number of stars contained in a sphere of radius 1 (1 parsec).

With regard to the function $\phi(M)$, its analytical form seems to be well established by the investigations of Seeliger, Kapteyn, and Hertzsprung. These researches agree in assigning to $\phi(M)$ the form:

$$\frac{h}{\sqrt{\pi}} e^{-h^2(M - M_0)^2},$$

an expression indicating that the absolute magnitudes are distributed about a mean value M_0 in accordance with the law of accidental errors. This analytical expression for $\phi(M)$ will therefore be adopted, and hence we shall finally write (7) in the form:

$$\left.\begin{aligned} &N_0(r_2{}^3 - r_1{}^3)\frac{h}{\sqrt{\pi}} e^{-h^2[m - 5 \log r - ar - M_0]^2} dm \\ &N_0 = \tfrac{4}{3}\pi\nu_0. \end{aligned}\right\} \quad . \quad (8)$$

This expression enables us to compute the number of stars whose apparent magnitudes lie between any specified limits $(m \pm \frac{1}{2}dm)$, and whose distances are between any specified and sufficiently

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narrow limits r_1, r_2. The four unknowns N_0, h, a, and M_0 may be ascertained so as best to satisfy the data of Table III.

After a number of somewhat laborious trials the following constants were selected:

$$N_0 = 0{\cdot}3076\,;\ h = 0{\cdot}249\,;\ a = 0{\cdot}00305\,;\ M_0 = 3{\cdot}95 \quad . \quad (9)$$

The scheme according to which the star numbers were computed was adopted in accordance with that used by Kapteyn. The successive steps in distance were so chosen that the interval between two successive boundaries was determined by the condition:

$$\log r_2 - \log r_1 = 0{\cdot}200.$$

As the mean radius, r, within each of these shells the value

$$r = \frac{3}{4}\frac{r_2^4 - r_1^4}{r_2^3 - r_1^3}$$

was adopted. This radius obviously represents the mean radius for *all* the stars contained within the shell. Table VII. shows the distances and corresponding values of ar used in the computations, as well as the quantities:

$$A = N_0(r_2^3 - r_1^3)\frac{h}{\sqrt{\pi}}.$$

TABLE VII.

$\log r_2$.	$\log r_1$.	$\log r$.	ar.	$\log$ A.
3·64	3·44	3·566	11·22	9·430
3·44	3·24	3·366	7·08	8·830
3·24	3·04	3·166	4·47	8·230
3·04	2·84	2·966	2·82	7·630
2·84	2·64	2·766	1·78	7·030
2·64	2·44	2·566	1·12	6·430
2·44	2·24	2·366	0·71	5·830
2·24	2·04	2·166	0·45	5·230
2·04	1·84	1·966	0·28	4·630
1·84	1·64	1·766	0·18	4·030
1·64	1·44	1·566	0·11	3·430
1·44	1·24	1·366	0·07	2·830
1·24	1·04	1·166	0·05	2·230
1·04	0·84	0·966	0·03	1·630
0·84	0·64	0·766	0·02	1·030
0·64	0·44	0·566	0·01	0·430

With the data of Table VII. the number of stars of apparent magnitudes lying between $(m \pm \frac{1}{2})$ have been computed by means of (8) and (9).

The resulting figures, computed with three-place logarithms, are shown in Table VIII. for the magnitudes from 1·5 to 17·5.

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TABLE VIII.

Apparent magnitude = *m* (phot.).

log r_2.	log r_1.	1·5 2·5.	2·5–3·5.	3·5–4·5.	4·5–5·5.	5·5–6·5.	6·5–7·5.	7·5–8·5.	8·5–9·5.	9·5–10·5.	10·5–11·5.	11·5–12·5.	12·5–13·5.	13·5–14·5.	14·5–15·5.	15·5–16·5.	16·5–17·5.
3·64	3·44													1	5	45	347
3·44	3·24									2	15	114	766	4560	23880	110100	451900
3·24	3·04						2	13	94	582	3200	15490	66680	252900	847200	2500000	6532000
3·04	2·84				2	12	78	449	2270	10160	40360	140900	436500	1188000	2871000	6110000	11480000
2·84	2·64	0·1	0·4	3	21	121	608	2710	10660	37150	114300	310400	744700	1577000	2952000	4887000	7145000
2·64	2·44	0·4	2·8	17	89	416	1710	6208	19860	56360	140900	311100	608200	1047000	1600000	2153000	2559000
2·44	2·24	1·5	8·6	43	189	740	2547	7762	20900	49660	104500	194100	317600	460200	588800	665300	665300
2·24	2·04	3·3	16·0	69	259	865	2553	6654	15310	31190	56100	88920	124800	154500	169400	163700	140000
2·04	1·84	5·2	21·8	81	264	762	1941	4375	8730	15350	23870	32810	39810	42660	40360	33810	24950
1·84	1·64	6·2	22·9	74	211	532	1185	2333	4055	6223	8452	10120	10720	10030	8280	6487	3899
1·64	1·44	6·3	20·1	57	142	313	611	1054	1603	2158	2565	2691	2495	2040	1479	949	533
1·44	1·24	5·3	14·8	37	81	157	270	408	547	647	675	622	508	366	233	131	65
1·24	1·04	3·8	9·4	21	40	69	104	138	163	170	156	127	91	58	32	16	7
1·04	0·84	2·4	5·3	10	17	26	35	41	43	39	32	23	14	8	4	2	1
0·84	0·64	1·3	2·6	4	7	9	10	11	10	8	6	4	2	1			
0·64	0·44	0·8	1·1	2	2	3	3	3	2	1	1	1					
0·44	0·24	0·3	0·4	1	1	1	1	1									
0·24	0·04	0·1	0·1														

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From these figures the following quantities may be calculated :—

(*a*) The number of stars, N, whose magnitude is brighter than *m*.

(*b*) The mean parallax of stars of magnitude *m*.

(*c*) The mean distance of stars of magnitude *m*.

(*d*) The mean absolute magnitude, $\bar{M}$, of stars of magnitude *m*.

These four quantities, with additional data which will be discussed further on, are collected in Table IX.

TABLE IX.

m.	log N.	log ϖ.	log $\bar{r}$.	$\bar{M}$.	log ($\varpi N^{0.234}$).	log ($\bar{r}N^{-0.183}$).	$\frac{\Delta \log N}{\Delta \log \varpi}$	$\frac{\Delta \log N}{\Delta \log \bar{r}}$
1.50	1.16	−1.237	1.737	−6.16	−0.97	1.52		
2.50	1.708	−1.395	1.856	−5.99	−0.995	1.543	3.52	4.78
3.50	2.248	−1.546	1.965	−5.69	−1.020	1.554	3.66	5.06
4.50	2.775	−1.686	2.067	−5.35	−1.037	1.559	3.77	5.23
5.50	3.283	−1.817	2.163	−4.99	−1.049	1.562	3.95	5.38
6.50	3.774	−1.939	2.253	−4.61	−1.056	1.562	4.07	5.53
7.50	4.245	−2.053	2.337	−4.22	−1.060	1.560	4.17	5.63
8.50	4.697	−2.160	2.417	−3.82	−1.061	1.558	4.26	5.69
9.50	5.127	−2.260	2.492	−3.41	−1.060	1.554	4.32	5.79
10.50	5.536	−2.354	2.562	−2.98	−1.059	1.549	4.36	5.78
11.50	5.924	−2.443	2.630	−2.55	−1.057	1.547	4.38	5.67
12.50	6.289	−2.526	2.696	−2.13	−1.055	1.545	4.33	5.54
13.50	6.633	−2.607	2.759	−1.71	−1.055	1.546	4.28	5.42
14.50	6.956	−2.682	2.819	−1.31	−1.055	1.546	4.29	5.26
15.50	7.259	−2.753	2.878	−0.92	−1.054	1.550	4.22	5.08
16.50	7.541	−2.822	2.934	−0.54	−1.057	1.554	4.08	4.95
17.50	7.804	−2.888	2.988	−0.19	−1.062	1.560	3.96	4.82
18.50	8.052	−2.951	3.040	+0.15	−1.067	1.567	3.86	4.61
19.50	8.279	−3.011	3.091	+0.45	−1.074	1.576	3.75	4.38
20.50	8.491	−3.068	3.141	+0.73	−1.081	1.587		

We proceed to discuss the results in succession by comparison with observation.

With regard to the relation between *m* and N, Table X. shows the comparison between the theoretical values and the results of observation given in Table III.

TABLE X.

m.	log N (Obs.).	log N (Theory).	Diff.	*m*.	log N (Obs.).	log N (Theory).	Diff.
5.0	3.02	3.029	−.009	12.0	6.12	6.107	+.013
6.0	3.51	3.528	−.018	13.0	6.47	6.461	+.009
7.0	3.99	4.009	−.019	14.0	6.79	6.794	−.004
8.0	4.47	4.471	−.001	15.0	7.09	7.107	−.017
9.0	4.92	4.912	+.008	16.0	7.39	7.400	−.010
10.0	5.33	5.332	−.002	17.0	7.68	7.673	+.007
11.0	5.73	5.730	.000				

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The agreement between observation and theory may be considered highly satisfactory, even to the faintest stars so far included in the statistical investigation of Chapman and Melotte.

The constants of (9) have been adapted to represent the relation between N and the *photographic* magnitudes. If this relation refers to *visual* magnitudes and if the difference (Photg. – Visual) is known, then by converting the visual into photographic magnitudes the observed relation should be in agreement with that of Table IX. Or *vice versa*, if the magnitude corresponding with the observed log N is taken from this table, the difference between the tabular and observed magnitude should be identical with the colour index (Photg. – Visual).

Chapman and Melotte in their table xiii. show the relation between the visual magnitudes and N according to Harvard. The figures quoted there are given in the first two columns of Table XI. On the other hand, from Table IX. we find the photographic magnitudes of the third column, and also the logarithms of the mean parallaxes.

Table XI.

m (Visual).	log N.	*m* (Phot.).	log π.	(Phot. - Visual.)
2·0	1·61	2·32	− 1·37	+ 0·32
3·0	2·14	3·30	− 1·52	+ 0·30
4·0	2·66	4·28	− 1·66	+ 0·28
5·0	3·17	5·28	− 1·79	+ 0·28
6·0	3·68	6·32	− 1·92	+ 0·32
7·0	4·17	7·34	− 2·04	+ 0·34
8·0	4·66	8·42	− 2·15	+ 0·42
9·0	5·13	9·50	− 2·26	+ 0·50
10·0	5·57	10·58	− 2·36	+ 0·58
11·0	5·98	11·65	− 2·45	+ 0·65
12·0	6·35	12·67	− 2·54	+ 0·67
13·0	6·68	13·64	− 2·62	+ 0·64

The last column shows a slight increase of the colour index with increasing magnitudes, of the same amount as that indicated by the photometric results in the third column of Table V.

It is readily shown that the relation between magnitudes and mean parallaxes is likewise correctly represented. Fig. 1 presents this relation in graphical form, the abscissæ being represented by the apparent visual magnitudes and the ordinates by the values of log $\bar{\pi}$. The observed values (Kapteyn) derived from the parallactic motions (Table VI.) are indicated by small circles. It will be noticed that the curve does not deviate much from a straight line, thus confirming the approximate correctness of the empirical law of Kapteyn within the range of these observations. Reduced to

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Campbell's solar motion, the numerical relation between magnitude and mean parallax, according to Kapteyn, is

$$\log \bar{\pi} = -1{\cdot}869 - 0{\cdot}108(m - 5{\cdot}5) \quad . \quad . \quad (10)$$

This equation is represented by the straight line of fig. 1. The

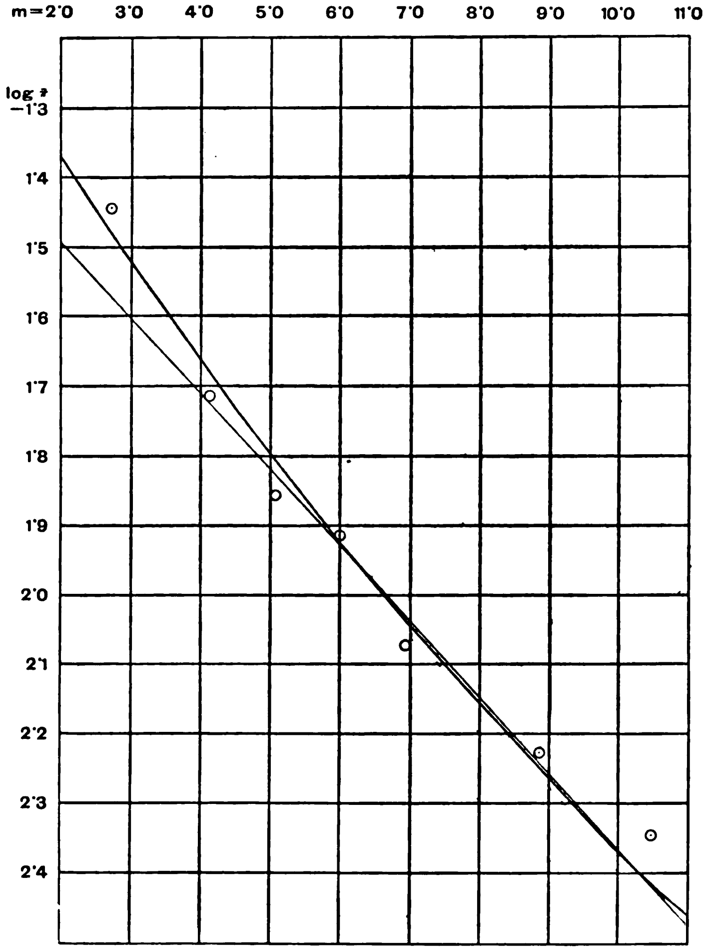

Fig. 1.

theoretical curve adapts itself well to the slope of this line in the region of the fainter stars, but deviates appreciably from it for the brighter stars, following fairly closely the curve derived from observation.

The agreement between the theoretical curve and Kapteyn's equation is specially interesting on account of recent criticism directed against the magnitude coefficient of the latter, which, it

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is maintained, is too large. This point will be dealt with in a subsequent section (§ 5). It will be pointed out that determinations of this coefficient based on the *motus peculiares* must differ from those based on parallactic motions on account of the fact that the peculiar motions depend in some degree on the absolute magnitudes.

We may further compare the conditions of a universe in which light undergoes extinction during its passage with those of a universe in which $a = 0$, but the assumed laws of density and luminosity are the same. Table XII. contains the values of log N, log $\bar{\pi}$, and $\bar{M}$ for the two assumptions $a = 0$ and $a = 0.00305$.

Table XII.

m (Ptg.).	$a=0.$ log N.	log $\bar{\pi}$.	$\bar{M}$.	$a=0.00305.$ log N.	log $\bar{\pi}$.	$\bar{M}$.
1·5	1·28	−1·37	−7·15	1·16	−1·24	−6·1
2·5	1·88	−1·57	−7·15	1·71	−1·40	−6·0
3·5	2·48	−1·77	−7·15	2·25	−1·55	−5·7
4·5	3·08	−1·97	−7·15	2·78	−1·69	−5·3
5·5	3·68	−2·17	−7·15	3·28	−1·82	−5·0
6·5	4·28	−2·37	−7·15	3·77	−1·94	−4·6
7·5	4·88	−2·57	−7·15	4·24	−2·05	−4·2
8·5	5·48	−2·77	−7·15	4·70	−2·16	−3·8
9·5	6·08	−2·97	−7·15	5·13	−2·26	−3·4
10·5	6·68	−3·17	−7·15	5·54	−2·35	−3·0
11·5	7·28	−3·37	−7·15	5·92	−2·44	−2·5
12·5	7·88	−3·57	−7·15	6·29	−2·53	−2·1
13·5	8·48	−3·77	−7·15	6·63	−2·61	−1·7
14·5	9·08	−3·97	−7·15	6·96	−2·68	−1·3
15·5	9·68	−4·17	−7·15	7·26	−2·75	−0·9
16·5	10·28	−4·37	−7·15	7·54	−2·82	−0·5

According to these figures, the presence of extinction causes a considerable diminution in the number of stars brighter than any specified magnitude. The comparison of the mean parallaxes $\bar{\pi}$ for the same magnitudes also reveals an enormous reduction in the extent of our vision.

Finally, the table shows that, while in a universe without extinction the mean absolute magnitudes are the same for all apparent magnitudes, the presence of extinction causes the intrinsically fainter stars to become on the average also apparently fainter.

The numbers shown in Table VIII. enable us to find the distribution with regard to distance for stars brighter than a given magnitude. We are thus in a position to compare the "frequency law" resulting from the present theory with that derived by Dyson and Eddington from the proper motions.

The law found by these investigators is given by

$$2k^2re^{-k^2r^2}dr \quad . \quad . \quad . \quad . \quad . \quad (11)$$

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which expresses the fraction of the total number of stars whose distances lie between $(r \pm \frac{1}{2}dr)$. The analytical form is empirical, and chosen purely on account of its adaptability to the observed data on the one hand, and mathematical treatment on the other.

For this reason it is not to be expected that the frequency curves derived from the two different points of view should show absolute agreement. Also, before comparing the frequency curves attention may be drawn to the fact that the constant term in Kapteyn's equation (10) differs appreciably from that determined by more recent values of the parallactic motions. Comparing, for instance, the parallactic motion of the sun derived from the discussion of the Boss stars as published in *Astron. Journ.*, No. 614, p. 112, with that derived by Dyson from the Carrington stars, we find the following data:—

Boss : mean magnitude 5·7 ; $V_0 = 3''·85$ (per century),
Carrington : ,, ,, 9·3 ; $V_0 = 1''·71$,

V_0 representing the angular velocity of the sun at distance $\bar{\pi}^{-1}$.

The corresponding logarithms of the mean parallaxes are

Boss : $\log \bar{\pi} = -2·02$; Carrington : $\log \bar{\pi} = -2·38$,

and hence:

$$\log \bar{\pi} = -2·00 - 0·100(m - 5·5).$$

Comparing with (10), we notice a small diminution of the magnitude coefficient which, however, is scarcely outside the p.e. of its determination. But there is an appreciable difference in the constant term, indicating that the unit of distance according to Kapteyn is 1·35 times that derived from more recent determinations.

This difference, however, is not relevant for our purposes. It would be a simple matter to adapt the preceding results to the more recent parallax unit by increasing the quantities $\log r$, on which the calculations are based, by the constant 0·13, and by applying corresponding corrections to the constants ν_0 and a, which would leave the relation between m and $\log N$ unaffected.

For comparing the frequency curves, the investigations of the Boss stars by Eddington and of the Carrington stars by Dyson were consulted. The limiting magnitudes (visual) are 5·8 and 10·3 respectively, which according to Table V. correspond with the photographic magnitudes 6·3 and 11·2.

In the case of the Boss stars, the percentages given by Eddington in *M.N.*, vol. lxxii. p. 384, were adopted. A few per cent. of the stars remain unaccounted for; the percentages here given are based on the actual numbers shown in Eddington's table. For the Carrington stars the percentages were computed from (11), adopting Dyson's value for k.

In the subjoined table the argument $\log r$ represents the logarithm of the radius of the sphere with centre at the sun which encloses the stated proportion of stars.

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TABLE XIII.

Boss Stars. log *r*.	Boss Stars. Per cent.	Carrington Stars. log *r*.	Carrington Stars. Per cent.
1·221	2·1	1·20	0·1
1·395	5·1	1·40	0·3
1·569	9·5	1·60	0·9
1·742	17·8	1·80	2·1
1·916	32·7	2·00	5·4
2·090	53·4	2·20	12·9
2·264	74·6	2·40	29·3
2·438	89·6	2·60	58·2
2·611	96·0	2·80	88·9
2·785	98·7	3·00	99·6
2·959	100·0		

Thus, in the case of the Carrington stars, for example, 5·4 per cent. of all the stars of the catalogue are contained within a sphere of radius 100 parsecs, and 99·6 per cent. within the sphere of radius 1000 parsecs.

The difference between the limiting photographic magnitudes is very nearly 5·0. In comparing with the present theory we may therefore calculate from Table VIII. the distribution in distance for stars brighter than 6·5 and 11·5. The resulting percentages are shown in Table XIV. for the adopted value $a = 0·00305$ on the one hand, and for the assumption $a = 0·00$ on the other.

TABLE XIV.

Stars brighter than 6·5 (Ptg.). log *r*.	Percentages. $a = 0·00305$.	Percentages. $a = 0·0$.	Stars brighter than 11·5 (Ptg.). log *r*.	Percentages. $a = 0·00305$.	Percentages. $a = 0·0$.
0·64	0·1	0·1			
0·84	0·6	0·2			
1·04	1·6	0·5			
1·24	4·0	1·3	1·24	0·1	
1·44	9·0	2·9	1·44	0·5	
1·64	18·0	5·9	1·64	1·5	0·1
1·84	32·3	11·2	1·84	4·2	0·2
2·04	51·4	19·1	2·04	10·9	0·5
2·24	72·0	29·8	2·24	24·7	1·3
2·44	88·5	42·7	2·44	46·8	2·9
2·64	97·3	56·3	2·64	73·4	5·9
2·84	99·8	69·4	2·84	93·1	11·2
3·04	100·0	80·4	3·04	99·5	19·1
3·24		88·6	3·24	100·0	29·8
3·44		94·0	3·44		42·7
3·64		97·3	3·64		56·3
3·84		98·9	3·84		69·4
4·04		99·7	4·04		80·4
4·24		100·0	4·24		88·6

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The curves representing these figures are shown in fig. 2, the argument log r representing the abscissæ, and the percentages the ordinates. A comparison with the curves representing the data of Table XIII. showed that a constant correction 0·06 must be deducted from the argument log r of the latter table to bring the two tables into agreement. The limiting magnitudes of Table XIV. being 0·3 larger than those of Table XIII., we should expect the values of log r to be approximately larger by 0·03. Instead they are found to be smaller by 0·06. Hence the difference Δ log r between Table XIII. and theory amounts to + 0·09 instead of 0·13 as found before from the parallactic motions.

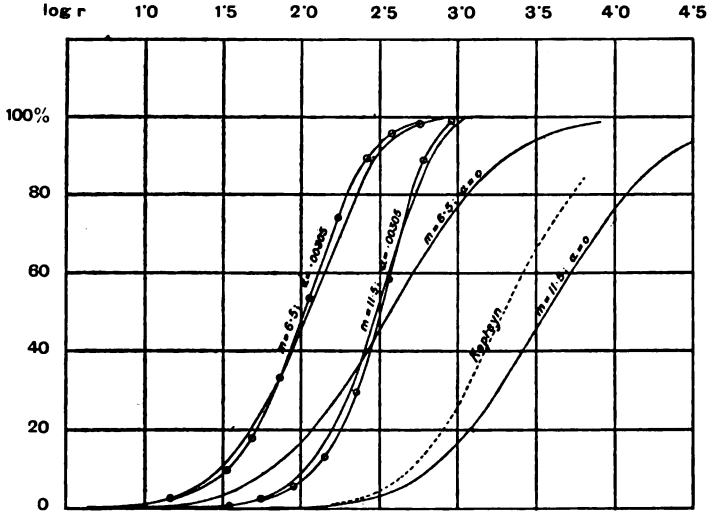

Fig. 2.

The curves representing the data of Table XIII., after applying the correction $\Delta \log r = -0{\cdot}06$, are shown in fig. 2 as the lines drawn through the small circles. These curves adapt themselves to those derived from the present theory as well as can be expected from material deduced from entirely different aspects of the problem and based on assumptions which must be considered as first approximations rather than strict analytical expressions of the facts.

The comparison with the curves derived on the assumption of the absence of space extinction reveals emphatically the enormous "shrinkage" of the visible universe due to the minute rate of extinction, on which the present theory is based.

It is further interesting to compare the frequency law here obtained with that derived from Kapteyn's data, given in table vi. on page 119 of *Astron. Journ.*, No. 566. The percentages of stars brighter than 9·5 (visual), according to Kapteyn, are shown in the dotted line curve of fig. 2, which, however, to avoid

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confusion, has been shifted 0·8 unit towards the right. The slope of this curve is considerably less than that of the Carrington stars, with which it may be compared as regards the apparent magnitudes of the stars (see § 8).

Summarising the results of this section, it has been found on the assumption of a universe which fulfils the conditions

(*a*) of a star density independent of the distance;
(*b*) of a distribution of the absolute magnitudes about a mean magnitude according to the law of errors; and
(*c*) of the presence of some agency by which the absolute magnitudes are increased in proportion to the distance,

that the following facts of observation are accounted for, viz.: *first*, the statistical relation between apparent magnitudes and star numbers, based on material extending over the whole sky; *secondly*, the relation between apparent magnitudes and mean parallaxes; *thirdly*, the relation between star frequency and distance for stars brighter than a given magnitude.

§ 4. *Distribution of Stars in different Galactic Regions. Structure of the Galaxy.*

We shall now examine how far the conditions prevailing in different regions of the sky deviate from the general conditions derived on the assumption that the star density is uniform throughout the universe.

Table IV., and also the results of Chapman and Melotte and Pickering, show that for corresponding magnitudes the ratio of the star numbers, N, is approximately the same for galactic and non-galactic regions. This fact can be accounted for by assuming that in the luminosity law (7*a*)

$$M - M_0 = m - 5 \log r - ar - f(\lambda, \beta),$$

where $f(\lambda, \beta)$ is a function of the angular co-ordinates λ (galactic longitude) and β (galactic latitude), but is independent of r.

Such an assumption, as Mr. Hough pointed out to me, could be explained on the hypothesis that the law of extinction is of the form:

$$ar + f(\lambda, \beta).$$

The last term can be explained on the assumption that, in addition to the agency which causes the general extinction *ar*, the space immediately surrounding the sun is filled with absorbing matter whose density depends on the *direction*. On this assumption, the galaxy would be a purely optical phenomenon caused by a *stronger local* absorption of light in the direction of the galactic poles. The suggestion recommends itself on account of its simplicity, and deserves to be tested in the light of observations.

When the distribution of stars along the galactic circle, *i.e.*

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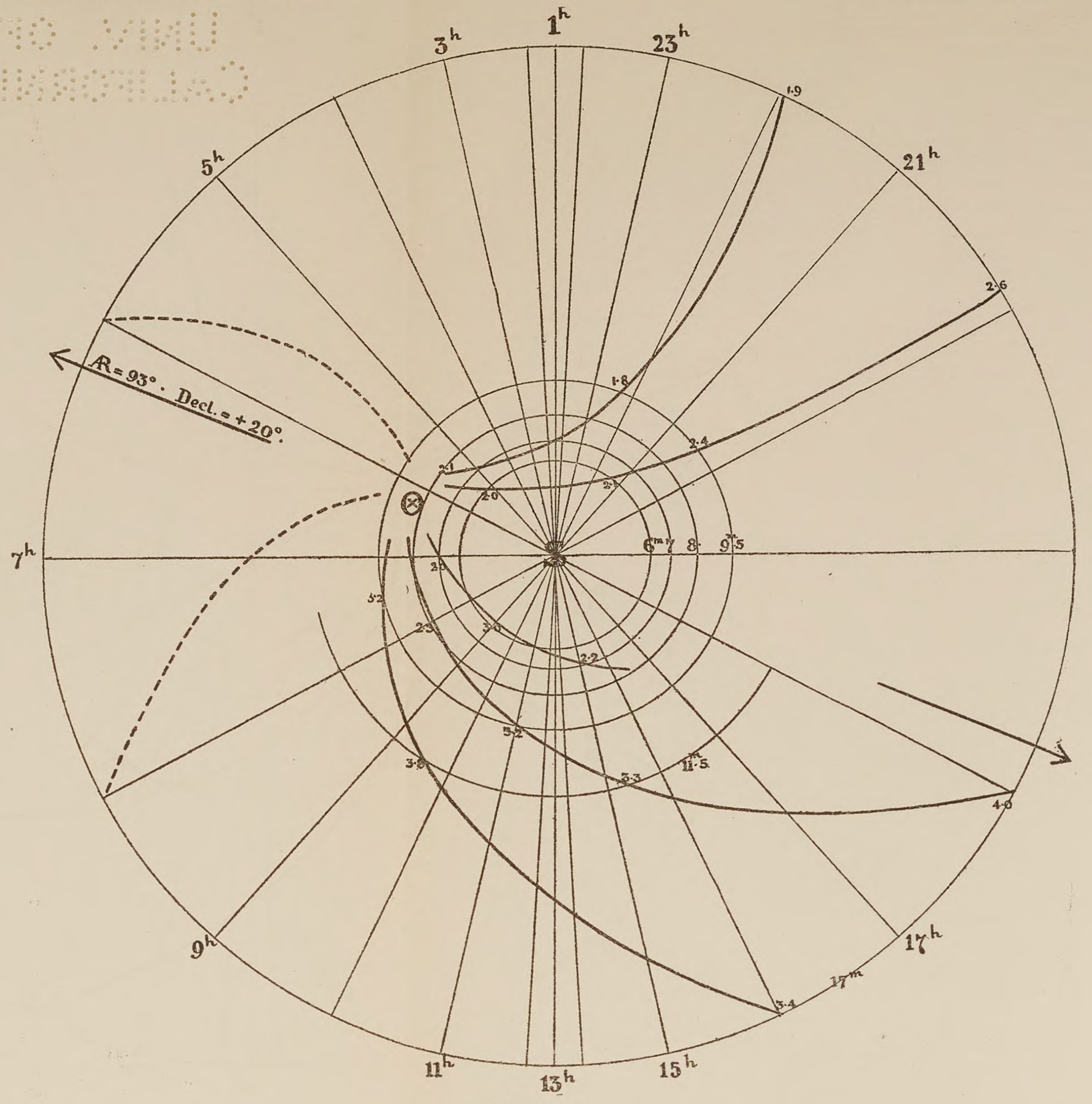

Galactic Plane.

Fig. 3*a*.

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according to galactic longitudes, is examined, the existence of strongly marked local condensations becomes at once apparent. The galaxy does not represent the appearance of a uniform ring, but rather that of an assemblage of "clusters." Stratonoff's charts (Atlas to Publications, Nos. 2 and 3) bear out this feature in both hemispheres. But if the phenomenon of the Milky Way is due to local absorption, the radial directions towards the clusters represent vistas along which this absorption is least effective. Hence the galactic longitudes of the clusters should be the same for stars of all magnitudes.

To take an example, the southern chart of Stratonoff for stars whose C.P.D. magnitude is $> 9{\cdot}6$ (mean photographic magnitude about $11{\cdot}5$) shows two well-pronounced condensations on the galactic circle at R.A. $9^h{\cdot}5$ and $15^h{\cdot}5$. The assumption of local absorption requires that similar condensations should be found on the galactic circle at the same R.A. for brighter as well as fainter stars.

For the fainter stars, table xvi. of Chapman and Melotte's paper gives the data required for ascertaining the positions of the galactic condensations. The table contains the numbers of all stars found within 225 square degrees of the Franklin-Adams plates for regions covering the whole sky. The *mean* number is 500×10^3, which corresponds with a mean magnitude $17 \pm$. In plotting the star numbers on a chart similar to those of Stratonoff, six well-defined galactic condensations in R.A. $6^h{\cdot}1$, $8^h{\cdot}0$, $16^h{\cdot}0$, $18^h{\cdot}0$, $20^h{\cdot}0$, $22^h{\cdot}8$, with relative densities $1{\cdot}6$, $2{\cdot}6$, $3{\cdot}4$, $4{\cdot}0$, $2{\cdot}6$, $1{\cdot}8$ (density of whole sky = 1) were located. Apparently the two condensations of the C.P.D. stars cannot be reconciled with corresponding condensations of the fainter stars on the assumption of local absorption. Similar failures are met with when the brighter stars are examined. The divergences are generally so great that the hypothesis must be abandoned.

A different view, however, seems to lead to significant results. Supposing that the luminosity law and the law of extinction are the same throughout the universe, we may assign to stars of a given apparent magnitude the mean distances shown in Table IX. (This assumption is approximately warranted, as the spread in distance for stars of given magnitude is comparatively small.) Describing round the sun in the galactic plane circles with these distances as radii and locating the condensations of stars of given magnitude on these circles, it was found possible to connect these points from one circle to another by curves which are of the nature of the wings of a spiral whose centre is situated at O (see fig. 3*a*, Plate 1). The densities within these wings, relative to the mean density, have been marked in the figure. They appear to be fairly uniform throughout the wing. [The drawn-out curves are fairly well established; the dotted curves, however, must be considered hypothetical, since only the intersections with the outer circle are known.] The material is still scanty, and definite conclusions must be deferred until more is known about

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the distribution of stars between the 11th and 17th magnitudes. But the figure certainly *suggests* a structure of the galaxy in the form of a *spiral*, the sun being situated at some distance from its centre. The most noteworthy result, however, is the position of this centre as viewed from the sun. The co-ordinates of the centre of the spiral are found to be:

$$\text{Æ} = 93^\circ, \ \text{decl.} = +20^\circ.$$

But this is almost exactly the position of the true vertex derived from the proper motions (see Eddington's *Stellar Movements*, p. 124). Such a coincidence doubtless points to a *physical* significance of the point O as a centre, not only as regards the arrangement of star matter, but also with regard to the general movements of the celestial bodies.

The same method may be applied to the arrangement of stars in a plane at right angles to the Milky Way. In table xi. of Chapman and Melotte the ratios of the star numbers found in zones of different galactic latitudes to the mean numbers (taken over the whole sky) are shown for stars brighter than specified magnitudes ranging from 5 to 17. From these numbers may be found the ratios of the numbers in each zone to the mean numbers (whole sky) between any two consecutive magnitudes m and $(m+1)$.

Drawing, therefore, concentric circles round the sun in a plane passing through the galactic poles with radii equal to the mean distances corresponding with the magnitudes $(m+\frac{1}{2})$, and mapping the ratios at the points of intersection between these circles and the radii drawn in the directions of the galactic regions I. to VIII. (Chapman and Melotte), lines of equal ratios may be constructed. These lines are shown in fig. 3*b*, Plate 2. They convey an impression of the structure of the galactic condensation in cross-section, which, however, cannot be claimed to be more than an approximate representation of the actual conditions on account of the excentric position of the sun.

Broadly speaking, the quasi-radial direction of these lines of equal density may be accepted, especially in middle latitudes and for stars brighter than 12^m. It is evident that if the arrangement of these lines were *strictly* radial, the theory of the preceding section could be rigorously applied along each radius, and the only change from one galactic region to another would be a change in the constant N_0 in (8). Hence the ratio of the numbers of bright to the numbers of faint stars would be the same in all parts of the sky, in accordance with the evidence of Table IV. and with similar results obtained by Pickering and Chapman and Melotte.

§ 5. *Relation between Proper Motion and Absolute Magnitude.*

Recent researches by Kapteyn and Adams have demonstrated the probability of a relation between proper motion and luminosity. It appears that intrinsically faint stars possess on the average larger

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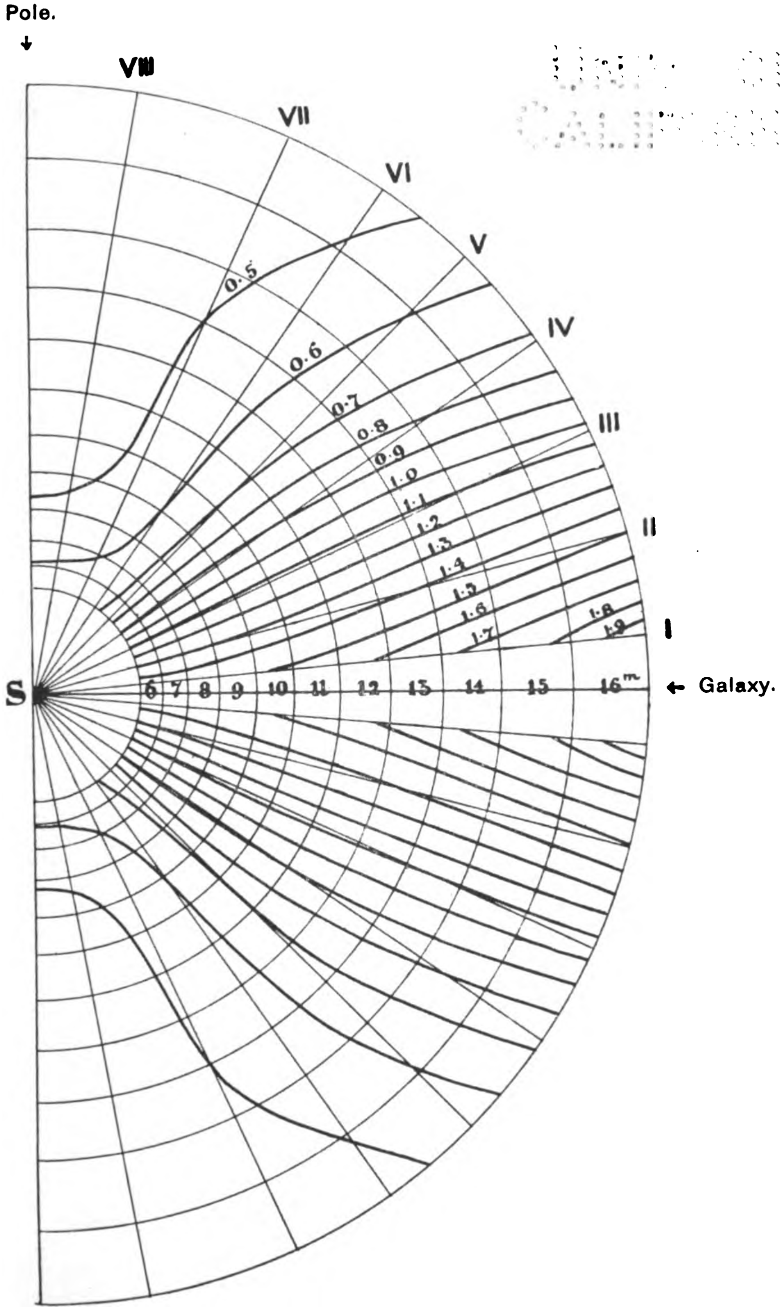

Fig. 3*b*.

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motions than intrinsically bright stars. The question can be examined in the light of the preceding results. It was shown that the average absolute magnitudes increase appreciably with the apparent magnitudes. Hence a corresponding increase in the average motions of the stars with apparent magnitude should be anticipated. Observations confirm this conclusion, the relation between the proper motion, μ, and the average absolute magnitude being expressed by the equation:

$$\log \bar{\mu} - \log \bar{\pi} = 0{\cdot}06\bar{M} + 1{\cdot}10 \quad . \quad . \quad . \quad (12)$$

In Table XV. columns 2 and 3 are shown for specified values of m (visual), the theoretical values of the average parallax, $\bar{\pi}$, and absolute magnitude, $\bar{M}$: the fourth column contains the values of $\bar{\mu}$ expressed in seconds of arc and computed from equation (12).

TABLE XV.

m (Vis.).	log $\bar{\pi}$.	$\bar{M}$.	$\bar{\mu}$.	m (Vis.).	log $\bar{\pi}$.	$\bar{M}$.	$\bar{\mu}$.
2·0	−1·37	−6·0	0″·225	8·0	−2·15	−3·8	0″·052
3·0	−1·52	−5·7	0·174	9·0	−2·26	−3·4	0·044
4·0	−1·66	−5·4	0·132	10·0	−2·36	−2·9	0·037
5·0	−1·79	−5·0	0·102	11·0	−2·45	−2·5	0·032
6·0	−1·92	−4·6	0·079	12·0	−2·54	−2·0	0·028
7·0	−2·04	−4·2	0·064	13·0	−2·62	−1·6	0·025

For comparing the theoretical values of $\bar{\mu}$ with observation, the results of Kapteyn's investigation (*Ast. Nach.* 3487, p. 110, table iii.) and for fainter stars Comstock's results (*Astron. Journ.*, No. 655, p. 55, table ix.) may be used. Table XVI. shows these results of observation in the first two columns, while the third column gives the theoretical values of $\bar{\mu}$ interpolated from Table XV.

TABLE XVI.

m.	$\bar{\mu}$ (Obs.).	$\bar{\mu}$ (Theor.).	Diff.	Authority.
2·73	0″·196	0″·190	+0″·006	Kapteyn
4·14	0·147	0·129	+0·018	,,
5·07	0·101	0·103	−0·002	,,
6·01	0·079	0·079	0·000	,,
6·93	0·066	0·065	+0·001	,,
8·5	0·045	0·048	−0·003	Comstock
11·5	0·030	0·030	0·000	,,

Equation (12) therefore represents the observed proper motions with a high degree of accuracy throughout the whole range of apparent magnitudes.

The p.m. on which Table XVI. is based have been collected in order of magnitude only, taking no account of the distances. It is

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important to show that equation (12) also applies to stars selected according to observed parallaxes. The number of stars for which reliable parallaxes are available is still very scanty. For the present purpose two main sources have been utilised, viz. the results quoted by Kapteyn in Groningen Publications, No. 8, and the results of Russell's investigations (*Astron. Journ.*, No. 618–19). From these the following correlated values of $\bar{M}$ and $\log \mu/\pi$ were derived and compared with equation (12*a*).

$\bar{M}$.	$\log \mu/\pi$ (Obs.).	No. of Stars.	$\log \mu/\pi$ (Comp.).
−3·0	0·78	22	0·82
−0·4	1·01	27	0·99
+1·7	1·19	38	1·12
+4·1	1·21	29	1·26

$$\log \mu = 0{\cdot}063\bar{M} + \log \pi + 1{\cdot}01 \quad . \quad . \quad . \quad (12a)$$

The essential agreement between these two different sets of observations renders the conclusion highly probable that equation (12) expresses a general law applicable to stars under all conditions, the departures of individual stars being in accordance with the law of errors.

Considering that for any star

$$M = m + 5 \log \pi - a/\pi,$$

we may then write equation (12) in the form:

$$\log \mu = 1{\cdot}05 + 0{\cdot}06m + 1{\cdot}30 \log \pi - 0{\cdot}06a/\pi \quad . \quad (12b)$$

the constant term being the mean of the two equations (12) and (12*a*). The last term of this equation, however, is negligible within the range of observed parallaxes and proper motions. [Even at the distance of the B-stars the term a/π amounts to only 0·4 of a magnitude, and hence the last term of equation (12*b*) to only 0·024.] Neglecting, therefore, this term, we may write:

$$\log \pi_{m,\mu} = -0{\cdot}81 - 0{\cdot}046m + 0{\cdot}77 \log \mu \quad . \quad . \quad (13)$$

This equation is in close agreement with Kapteyn's empirical formula:

$$\log \pi_{m,\mu} = -0{\cdot}77 - 0{\cdot}0434m + 0{\cdot}71 \log \mu \quad . \quad . \quad (14)$$

which consequently may be considered as an alternative form of equation (12).

An important point with regard to the latter equation must, however, be considered. The proper motion, μ, is the resultant of the transverse *motus peculiaris* of the star and the parallactic displacement due to the motion of the sun. Obviously an influence of the star's luminosity on the latter component is impossible, and hence, if a connection between motion and luminosity exists, it has been introduced into equation (12) through the medium of

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the peculiar motions. Now, in the case of such broad averages as are dealt with in Table XV., we may assume:

$$\bar{\mu}^2 = (V_0 \overline{\sin \lambda \cdot \pi})^2 + \bar{\epsilon}^2,$$

where V_0 expresses the sun's motion at unit distance, λ the arc of the great circle between apex and star, and ϵ the transverse component of the *motus peculiaris*. [The value of $\overline{\sin \lambda}$ derived from Kapteyn's general discussion is 0·800 and $V_0 = 4''\cdot12$ per annum.]

Computing $\bar{\epsilon}$ from the values of $\bar{\mu}$ in Table XV., it is found that these quantities are closely represented by the formula:

$$\log \bar{\epsilon} = 0\cdot08\bar{M} + \log \bar{\pi} + 1.12 \quad . \quad . \quad . \quad (15)$$

the only essential change in the transition from μ to ϵ being a slight increase in the factor of M from 0·06 to 0·08.

Further, assuming that the directions of the motions are distributed at random, the radial velocity, V_r, in km./sec. is expressed by

$$\bar{V}_r = \frac{9\cdot476}{\pi} \cdot \bar{\epsilon},$$

and hence

$$\log \bar{V}_r = 1\cdot53 + 0\cdot08m + 0\cdot40 \log \bar{\pi} \quad . \quad . \quad (16)$$

and also, substituting from (13):

$$\log \bar{V}_r = 1\cdot20 + 0\cdot062m + 0\cdot308 \log \bar{\mu} \quad . \quad . \quad (17)$$

The equation in this form leads to an interesting comparison with observations recently made by Kapteyn and Adams (*Proc. National Academy of Sciences*, vol. i., 1915 January and July). If the radial velocities of stars whose apparent magnitudes vary within narrow limits are collected in groups in order of their proper motions, μ, then the mean apparent magnitudes of these groups may be assumed practically constant, and equation (17) reduces to

$$\log \bar{V}_r = \text{const.} + 0\cdot308 \log \bar{\mu} \quad . \quad . \quad . \quad (18$$

which shows that the radial velocity increases with the proper motion.

If the mean magnitude is assumed $m = 4\cdot5$, which is approximately that of the stars in Campbell's lists, we have from (17):

$$\log \bar{V}_r = 1\cdot48 + 0\cdot308 \log \bar{\mu},$$

from which the following values of V_r for specified values of μ are computed:

TABLE XVII.

μ.	V_r.	μ.	V_r.
"	km.	"	km.
0·008	6·8	0·30	20·8
0·02	9·0	0·40	22·8
0·05	12·0	0·50	24·4
0·10	14·9	0·60	25·8
0·20	18·4	1·00	30·2

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The observed values, according to Kapteyn and Adams, are as follows:

TABLE XVIII.

Kapteyn, Type K.

μ.	V_r.	No. of Stars.	μ.	V_r.	No. of Stars.
"	km.		"	km.	
0·013	10·9	27	0·11	20·7	12
0·032	12·1	37	0·13	16·5	24
0·05	14·9	21	0·175	15·6	24
0·07	10·7	24	0·25	16·5	18
0·09	15·7	26	>0·30	26·7	19

Adams.

Type.	μ.	V_r.	No. of Stars.	μ.	V_r.	No. of Stars.
	"	km.		"	km.	
B	0·016	8·2	61	0·041	9·6	52
A	0·019	10·0	55	0·067	10·7	104
F	0·011	10·1	20	0·530	24·6	45
G	0·013	10·6	63	0·670	24·9	69
M	0·015	12·6	27	0·170	17·6	12

The comparison between the observed and computed values shows a close agreement, considering the comparatively small number of stars in the individual groups. Hence we conclude that the observed velocities of Table XVIII. may be numerically represented by the equation:

$$\log V_r = 1{\cdot}53 + 0{\cdot}08M \quad . \quad . \quad . \quad (19)$$

of which equation (18) is a special form.

An essentially different form of this equation, however, is obtained if the radial velocities are collected in order of apparent magnitude without respect to proper motions. Table XV. shows that the mean absolute magnitudes, corresponding with specified apparent magnitudes, can be well represented by the simple relation:

$$\bar{M} = -7{\cdot}0 + 0{\cdot}4m.$$

Substituting into (19) we obtain therefore:

$$\log \bar{V}_r = 0{\cdot}97 + 0{\cdot}032m \quad . \quad . \quad . \quad (20)$$

where $\bar{V}_r$ now represents the mean radial velocity of stars of a given apparent magnitude m.

Perrine (*Astrophysical Journal*, 1915 June) has examined this particular relation between $\bar{V}_r$ and m. He subdivided the stars into two groups, brighter and fainter than $3^m{\cdot}0$. The mean magnitudes of these groups are not stated, but if calculated from Campbell's lists they are found to be approximately 2·5 and 4·5. For these magnitudes equation (20) gives:

$$\bar{V}_r = 11{\cdot}2 \text{ and } 13{\cdot}0,$$

as compared with Perrine's values 11·0 and 13·6.

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Equations (19) and (20) demonstrate that the usually accepted assumption of V = constant is not strictly correct. Hence if average parallaxes are determined by means of the *motus peculiares* on the assumption that $\bar{\pi}$ is proportional to $\bar{V}\bar{\pi} = \bar{\tau}$, the functional relation between such parallaxes and magnitudes must differ from that derived from the parallactic motions. The difference is seen at once if we combine equation (20) with Kapteyn's equation (11). We then obtain:

$$\log \bar{\tau} = \log (\bar{V}\bar{\pi}) = -0{\cdot}31 - 0{\cdot}076m \quad . \quad . \quad (21)$$

i.e. an appreciably smaller coefficient of m than that shown in equation (11).

In *M.N.*, vol. lxxvi., such a determination of the mean parallaxes in relation to magnitudes based on the cross components, τ, of the Boss and Carrington stars has been published by Unthank, who finds the magnitude factor:

$$0{\cdot}072 \pm 0{\cdot}005$$

in agreement with that of equation (21).

Summarising the results of this section, it is claimed that equation (19) represents a fundamental relation between motion and luminosity. The physical cause of such a relation, however cannot be a direct one. We must conclude that luminosity acts only as the interpreter of a different agency, this agency being in all probability the *mass*.

§ 6. *Analytical Expression for the Relation between* m *and* N. *A possible Physical Explanation of the Phenomenon of Extinction.*

For stars in general, the relation between the apparent photographic magnitude, m, and the number of stars brighter than this magnitude, as established by the preceding theory, may be expressed with a degree of precision sufficient for practical purposes by the formula:

$$m = \tfrac{5}{3} \log N + aN^{\beta} + b \quad . \quad . \quad . \quad (23)$$

where a, β, and b represent the constants:

$$a = 0{\cdot}1954\,;\ \beta = 0{\cdot}183\,;\ b = -0{\cdot}75.$$

The values of m computed by this formula differ from the theoretical values of Table IX. by the following residuals:—

m.	Δm.	m.	Δm.	m.	Δm.	m.	Δm.	m.	Δm.
1·5	0·00	5·5	0·00	9·5	+0·01	13·5	0·00	17·5	0·00
2·5	0·00	6·5	0·00	10·5	+0·01	14·5	0·00	18·5	+0·02
3·5	0·00	7·5	0·00	11·5	+0·01	15·5	−0·01	19·5	+0·05
4·5	0·00	8·5	+0·01	12·5	0·00	16·5	−0·01	20·5	+0·10

The agreement down to the 18th magnitude is almost perfect, but for fainter stars the residuals become more and more pro-

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nounced. Within the scope of Chapman and Melotte's investigation the formula (23) may therefore be considered a practically correct representation of the theoretical relation between m and N.

According to the seventh column of Table IX. the mean distance $\bar{r}$ is very nearly proportional to $N^{0.183}$, so that the second term on the right-hand side of (23) evidently is proportional to $\bar{r}$.

It will be noticed, however, that this relation between N and $\bar{r}$, and a similar relation between N and $\bar{\pi}$, cannot be extended much beyond the range of Table IX. The changes are more clearly shown when the ratios $\frac{\Delta \log N}{\Delta \log \bar{\pi}}$ and $\frac{\Delta \log N}{\Delta \log \bar{r}}$, given in the eighth and ninth columns, are inspected.

It can be shown, from general considerations, that for infinitely small as well as infinitely large distances these ratios approach asymptotically the value 3.

When equation (23) is applied to the *statistical* relation between m and N, shown in Table III., the following numerical form is obtained:

$$m = \tfrac{5}{3} \log N + 0.1946 N^{0.183} - 0.72 ;$$

and the residuals are:

m.	Δm.	m.	Δm.	m.	Δm.
5·0	−0·01	9·0	−0·02	13·0	−0·03
6·0	+0·02	10·0	+0·01	14·0	0·00
7·0	+0·02	11·0	−0·01	15·0	+0·04
8·0	0·00	12·0	−0·04	16·0	+0·02
				17·0	−0·02

Likewise, the visual magnitudes (Harvard), shown in table xiii. of Chapman and Melotte's paper, are represented as follows:

$$m = \tfrac{5}{3} \log N + 0.1592 N^{0.183} - 0.90 ;$$

m.	Δm.	m.	Δm.	m.	Δm.
4·0	−0·02	8·0	0·00	12·0	+0·01
5·0	+0·01	9·0	−0·03	13·0	+0·09
6·0	+0·02	10·0	−0·04		
7·0	+0·03	11·0	−0·05		

The ratio of the factors in the second terms, taken in the sense (Photographic − Visual), is 1·22. This ratio is much smaller than would be expected on the hypothesis that extinction is due to the "scattering" of light caused by cosmic "dust." On this assumption, according to Lord Rayleigh's law, the ratio should be about 2. It is worth noticing that a comparison between the coefficient $a = 0.00305$ (phot.), on which the preceding calculations were based, and the coefficient Δa (Photographic − Visual) found by Jones points likewise to a small ratio, viz. 1·19. Since the

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effective wave-lengths of photographic and visual light are approximately 0·43 μ and 0·52 μ, the assumption of the coefficient of absorption being inversely proportional to the effective wave-length would represent the observations.

With regard to a *physical* interpretation of extinction we are, however, by no means restricted to the acceptance of the theory of scattering. The following consideration may help in pointing out the possibility of an alternative explanation.

Let us take as an example the radiation of a black body of temperature T and at unit distance. The energy emitted, measured within a certain interval of wave-length, $d\lambda$, is given by Wien's equation:

$$dI = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} d\lambda \quad . \quad . \quad . \quad . \quad (24)$$

Hence the amount of energy contained between the limits $\begin{pmatrix} \lambda_1 = \lambda - \frac{1}{2}d\lambda \\ \lambda_2 = \lambda + \frac{1}{2}d\lambda \end{pmatrix}$ is

$$I_\lambda = \tfrac{1}{4}c_1(\lambda_2^{-4} - \lambda_1^{-4})e^{-\frac{c_2}{\lambda T}},$$

neglecting powers higher than the first in $d\lambda$. Writing this equation in logarithmic form and expressing I in terms of m_0, the apparent magnitude, we obtain:

$$m_0 = C - 2{\cdot}5 \log(\lambda_2^{-4} - \lambda_1^{-4}) + 2{\cdot}5\frac{c_2 \log e}{\lambda T} \quad . \quad . \quad (25)$$

If the body is placed at distance r, we have:

$$m_r = C + 5 \log r - 2{\cdot}5 \log(\lambda_2^{-4} - \lambda_1^{-4}) + 2{\cdot}5\frac{c_2 \log e}{\lambda T} \quad . \quad (26)$$

In these equations m_0 signifies the apparent magnitude of the body seen at unit distance, which is defined as its absolute magnitude, and denoted by the symbol M.

Provided that the range of wave-lengths, within which I has been measured, is the same, equations (25) and (26) lead to the well-known relation:

$$M = m - 5 \log r \quad . \quad . \quad . \quad . \quad (27)$$

But this conclusion applies only to the assumption that T has the same value. Supposing, however, that the radiation when leaving two bodies at different distances from the observer has exactly the same absolute temperature, are we justified in assuming that the energies on arrival are still of the same temperature? In dynamical processes the existence of dissipation of energy is recognised. Are we entitled to claim exemption for the processes involved in the propagation of light energy through space? If, on the other hand, the possibility of dissipation of radiant energy during transmission is admitted, then T must be looked upon as a function of the distance, and writing generally

$$1/T = f(r),$$

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the function $f(r)$ must be of such a character that its values *increase* with the distance. Accepting as a first approximation the simplest form:

$$f(r) = d + \delta \,.\, r,$$

equation (27) has to be replaced by

$$M = m - 5 \log r - 2\cdot 5 \log e \frac{c_2 \delta}{\lambda} \,.\, r \qquad . \quad . \quad (28)$$

which agrees with the equation on which the preceding theory is based if

$$a = 2\cdot 5 \log e \,.\, \frac{c_2 \delta}{\lambda}.$$

Hence, on the assumption that the phenomenon of extinction is due to dissipation of radiating energy, the coefficient a is found to vary inversely with the effective wave-length, in accordance with the above conclusions. At the same time, the assumption suggests that the adopted simple form of the law of extinction may not hold for very large distances where possibly higher powers of r may exert their influence.

Equation (23) is convenient when the star counts are given and the magnitudes are to be determined. Conversely, to find the star number N corresponding with a given magnitude, the equation

$$\log N = 0\cdot 219 + 0\cdot 6154 m - 0\cdot 0104 m^2 \qquad . \quad . \quad (29)$$

may be used, its adaptability to the theoretical curve between magnitudes 3 and 18 being practically as good as that of equation (23). (See footnote on p. 274.)

Equation (29) is the empirical form adopted by Chapman and Melotte. It has been extensively used in the investigations of Kapteyn, Schwarzschild, and others, and its theoretical importance will be pointed out in subsequent investigations of this paper.

§ 7. *Comparison with the Theoretical Results of Schwarzschild.*

In *Astr. Nach.* 4557 Schwarzschild has shown that the fundamental equations of stellar statistics can be derived from the assumption of three distribution functions for the density $D(r)$, the luminosity I, and the velocity V of the form:

$$\left.\begin{aligned} D(r) &= 10^{a_0 - a_1\rho - a_2\rho^2} & (\rho &= -5 \log r) \\ \phi(I) &= 10^{b_0 - b_1 M - b_2 M^2} & (M &= -2\cdot 5 \log I) \\ \psi(V) &= 10^{c_0 - c_1 G - c_2 G^2} & (G &= -5 \log V) \end{aligned}\right\} \qquad . \quad (30)$$

$$M = m - 5 \log r = m + \rho.$$

Of these the second is common with the present theory, and the third related to the second by means of the equation:

$$\log V = 1\cdot 53 + 0\cdot 08 M,$$

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which has been shown in § 5 to be in accordance with observed facts.

The present theory, therefore, differs principally from that of Schwarzschild only in the assumption regarding $D(r)$. It assumes $D(r) = \nu_0 =$ constant, but instead of

$$M = m - 5 \log r,$$

the relation

$$M = m - 5 \log r - ar$$

is introduced.

Let us define a parameter r_1 by the equation:

$$\left.\begin{aligned} \log r_1 &= \log r + \tfrac{1}{5} ar \\ \rho_1 &= -5 \log r_1 \end{aligned}\right\} \qquad (31)$$

The volume contained within a spherical shell between the radii $(r \pm \frac{1}{2} dr)$ is $4\pi r^2 dr$, and, according to the present theory, the total number of stars contained in this shell: $4\pi\nu_0 r^2 dr$. If a function $D(r_1)$ is defined by the equation:

$$D(r_1) = \nu_0 \frac{r^2 dr}{r_1^2 dr_1} \qquad (32)$$

the total number of stars in the shell may also be expressed by

$$4\pi r_1^2 dr_1 \,.\, D(r_1),$$

while the absolute magnitude is given by the relation:

$$M = m - 5 \log r_1 = m + \rho_1.$$

The form of $D(r_1)$ may be empirically determined from (32). It was found to be approximately represented by the expression:

$$D(r_1) = 10^{a_1 - a_1\rho_1 - a_2\rho_1^2}. \qquad (33)$$

which satisfies relation (32) well except for small distances. (See equation II. and Table XXI.)

Hence the present theory may be brought into formal agreement with Schwarzschild's assumptions (30), if r is replaced by r_1. This change alters considerably the results with regard to the distribution of stars of given magnitude in *distance*, but does not affect the distribution of *luminosities*.

Thus Schwarzschild found that the luminosities of stars of magnitude m are distributed about a mean magnitude $\bar{M}$ according to the law of errors, where

$$\bar{M} = -7{\cdot}7 + 0{\cdot}374m.$$

On the other hand, Table IX. (fifth column) shows the mean absolute magnitude of stars of given apparent magnitude as derived from the present theory. They are satisfactorily represented by

$$\bar{M} = -7{\cdot}17 + 0{\cdot}40m,$$

in good agreement with Schwarzschild's result. Moreover, from Table VIII. we may find the percentages of the total number

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of stars between magnitudes $(m \pm \frac{1}{2})$ enclosed within shells of specified bounding radii r. The stars enclosed must be intrinsically fainter than the limiting absolute magnitude:

$$M = m - 5 \log r - ar,$$

and the percentages derived from Table VIII. should conform with Schwarzschild's frequency law:

$$\int_x^M \phi_m(M)N = 100 \frac{h}{\sqrt{\pi}} \int_x^M e^{-h^2(M-\mathfrak{M})^2} dM \quad . \quad . \quad (34)$$

The comparison is shown in Table XIX. for two cases, viz. $m = 6{\cdot}0$ and $m = 10{\cdot}0$.

TABLE XIX.

$m=6{\cdot}0$; $\mathfrak{M}=-4{\cdot}67$; $h=0{\cdot}316$. Percentages.				$m=10{\cdot}0$; $\mathfrak{M}=-3{\cdot}07$; $h=0{\cdot}330$. Percentages.			
log *r*.	Table VIII.	(34.)	Diff.	log *r*.	Table VIII.	(34.)	Diff.
3·04	100·0	100·0	0·0	3·24	100·0	100·0	0·0
2·84	99·8	99·4	+0·4	3·04	99·5	99·5	0·0
2·64	96·6	95·8	+0·8	2·84	94·8	93·5	+1·3
2·44	86·3	85·6	+0·7	2·64	77·1	75·3	+1·8
2·24	67·9	68·2	−0·3	2·44	50·2	49·5	+0·7
2·04	46·5	47·5	−1·0	2·24	26·6	26·6	0·0
1·84	27·5	28·7	−1·2	2·04	11·7	11·8	−0·1
1·64	14·4	14·8	−0·4	1·84	4·4	4·4	0·0
1·44	6·6	6·5	+0·1	1·64	1·4	1·4	0·0
1·24	2·7	2·4	+0·3	1·44	0·4	0·4	0·0
1·04	0·9	0·8	+0·1	1·24	0·1	0·1	0·0
0·84	0·3	0·3	0·0				

According to the quantities h, the "mean error" of an absolute magnitude is $\pm 2{\cdot}24$ for stars of the 6th magnitude (phot.), and $\pm 2{\cdot}14$ for stars of the 10th magnitude.

The same considerations may be applied to cases where the distance law is known for stars brighter than a limiting apparent magnitude from other sources, as for instance for the stars of Boss's catalogue. The bulk of such stars are comprised within fairly narrow limits of magnitude, and may therefore be considered typical of the conditions of stars of the mean apparent magnitude of the catalogue. The distance law of the Boss Catalogue as found by Eddington from the proper motions is shown in Table XIII. Computing the limiting absolute magnitudes corresponding with the distances given in that table by the formula:

$$M = m - 5 \log r - ar,$$

and calculating the percentages of stars fainter than these limiting magnitudes from (34), the following comparison (Table XX.) between the percentages of Table XIII. and those computed from (34) was obtained.

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TABLE XX.

log r.	Percentage of Stars. Table XIII.	(34).	Diff.
1·221	2·1	1·2	+0·9
1·395	5·1	3·4	+1·7
1·569	9·5	8·7	+0·8
1·742	17·8	18·8	−1·0
1·916	32·7	34·6	−1·9
2·090	53·4	54·5	−1·1
2·264	74·6	74·4	+0·2
2·438	89·6	89·4	+0·2
2·611	96·0	97·1	−1·1
2·785	98·7	99·6	−0·9
2·959	100·0	100·0	0·0

$\bar{M} = -5·60$; $h = 0·363$.

The agreement is again quite satisfactory. The "mean error" is somewhat smaller, viz. ± 1·95, and the mean magnitude brighter than would result from theory, but they are certainly of the order to be expected.

Hence the distribution in distance of stars of given apparent magnitude may be derived from the luminosity law given in equation (34).

To show with what accuracy the relation

$$\log D(r_1) = a_0 - a_1\rho_1 - a_2\rho_1^2$$

adapts itself to the condition

$$\nu_0 r^2 dr = D(r_1) r_1^2 dr_1 \quad . \quad . \quad . \quad . \quad (I.)$$

the values $\log\left(\frac{r^2dr}{r_1^2dr_1}\right)$ were calculated for specified values of r_1, and from these the best fitting constants a_0, a_1, a_2 were derived. Thus the relation was found:

$$\log D(r_1) - \log \nu_0 = -0·75 - 0·232\rho_1 - 0·0174\rho_1^2 \quad . \quad (II.)$$

A comparison of (I.) and (II.) is shown in Table XXI.

TABLE XXI.

log r_1.	(log D(r_1) − log ν_0). I.	II.
1·0	−0·02	−0·02
1·5	−0·07	+0·01
2·0	−0·22	−0·17
2·5	−0·56	−0·57
3·0	−1·17	−1·18
3·5	−2·02	−2·02
4·0	−3·06	−3·07
4·5	−4·23	−4·34

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The values derived from equation (I.) show a progressive diminution of $D(r_1)/\nu_0$ from unity at zero distance. The values (II.), on the other hand, start from D = 0 at zero distance, reach a maximum at about $r_1 = 30$, and thence follow the course of (I.) within the limits of the table. The discrepancy at short distances, however, is not very serious on account of the fewness of stars contributing to the statistical equations.

The density law (II.) follows also immediately from the theoretical equations of Schwarzschild, developed in his paper "Ueber die Integralgleichungen der Stellarstatistik," *A.N.* 4422. Schwarzschild showed that if the relations

$$\log \phi(M) = b_0 - b_1 M - b_2 M^2$$
$$\log N = a_0 - a_1 m - a_2 m^2$$
$$M = m - 5 \log r$$

are satisfied, the density law must be of the form:

$$\log D(r) = a_0 - a_1 \log r - a_2 (\log r)^2 .$$

But in the preceding theory the first condition was assumed, and the second was found as result* if r_1 is substituted for r. Hence the density law, when the "fictitious" distances r_1 are used, is of the form :

$$\log D(r_1) = a_0 - a_1 \log r_1 - a_2 (\log r_1)^2 .$$

With the known quantities a_1, a_2 ; b_1, b_2 the coefficients a_1, a_2 can be calculated from Schwarzschild's theoretical equations. (The constant term is here omitted.)

The resulting equation is found to be :

$$\log D(r_1) = \text{const.} - 0{\cdot}244\rho_1 - 0{\cdot}0178\rho_1{}^2 , \quad (\rho_1 = -5 \log r_1)$$

in close agreement with (II.), and representing the condition (I.) equally well.

Adopting relation (II.) and the luminosity law of the present investigation, the constants of Schwarzschild's theory are :

$$a_1 = +0{\cdot}232 \qquad b_1 = -0{\cdot}615$$
$$a_2 = +0{\cdot}0174 \qquad b_2 = +0{\cdot}0269.$$

* A comparison of the theoretical values of log N (Table IX.) with the equation

$$\log N = 0{\cdot}219 + 0{\cdot}6154m - 0{\cdot}0104m^2$$

shows the following residuals Δ log N (Theory-Formula) :

m.	Δ log N.	*m.*	Δ log N.	*m.*	Δ log N.	*m.*	Δ log N.
1·5	+0·04	6·5	−0·005	11·5	+0·004	16·5	−0·001
2·5	+0·016	7·5	−0·004	12·5	+0·002	17·5	+0·005
3·5	+0·002	8·5	−0·001	13·5	+0·002	18·5	+0·008
4·5	−0·002	9·5	0·000	14·5	+0·001	19·5	+0·017
5·5	−0·006	10·5	+0·002	15·5	0·000	20·5	+0·028

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By means of the relations shown in the table on p. 371 of Schwarzschild's paper (*A.N.* 4557), the following equation between m and N is easily obtained, viz.

$$\log N = a_0 + 0{\cdot}590m - 0{\cdot}0106m^2,$$

in satisfactory agreement with the equation obtained from the present theory. Also, it is found that the absolute magnitudes of stars of a given apparent magnitude are distributed about a mean value

$$\bar{M} = -7{\cdot}0 + 0{\cdot}39m,$$

with a mean error $\pm 2^{m}{\cdot}21$, again in accordance with the previous result.

The relation between the mean parallax $\bar{\pi}_1$ and apparent magnitude m, according to Schwarzschild's theory, is derived from the (a, b) constants as follows:

$$\log \bar{\pi}_1 = -1{\cdot}17 - 0{\cdot}121m \quad . \quad . \quad . \quad (35)$$

and this relation should be strictly linear, *i.e.* terms of higher order than the first in m should be absent.

On the other hand, the values of $\bar{\pi}_1$ are easily derived from the present theory by consulting the star frequencies of Table VIII. In this way the following values were calculated, with which the actual mean parallaxes $\bar{\pi}$ may be compared:

m.	$\log \bar{\pi}_1$.	$\log \bar{\pi}$.
6·0	−1·922	−1·879
9·0	−2·303	−2·211
12·0	−2·654	−2·485
15·0	−3·001	−2·718
18·0	−3·366	−2·920

These values are closely represented by the formulæ:

$$\left.\begin{aligned}\log \bar{\pi}_1 &= -1{\cdot}209 - 0{\cdot}120m \\ \log \bar{\pi} &= -1{\cdot}097 - 0{\cdot}1445m + 0{\cdot}0024m^2\end{aligned}\right\} \quad . \quad (36)$$

the first of which is in good accordance with that derived from Schwarzschild's theory.

Hence, all the results of the present theory may be derived with sufficient approximation from the mathematical form of the density and luminosity law adopted by Schwarzschild, provided that the argument for distance is taken to be r_1, where

$$\log r_1 = \log r + \tfrac{1}{3}ar.$$

It will be noticed that the change from r to r_1 has comparatively little effect on the relation between the *mean* parallaxes and apparent magnitudes. In fact, within the narrow range of magnitudes for which parallactic motions are at present available, the observations can scarcely yet decide between the two relations given in (36), although the observed curve of fig. 1 adapts itself somewhat better to the second than the first of these relations.

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Judging from this criterion only, the results are therefore practically reconcilable with either the theory of extinction or that of decreasing star density.

A decisive distinction between these theories is, however, reached when the distribution of stars of a given magnitude derived from either theory is compared with corresponding results derived from the proper motions. Here Schwarzschild's theory agrees well with the distribution found by Eddington and Dyson when the argument r_1 is adopted, but fails when r is chosen. This fact must be considered as a proof in favour of the extinction theory.

§ 8. *Comparison with the Results of Kapteyn.*

The results of the preceding section have led to the conclusion that the statistical relations between m and N on the one hand, and m and $\bar{\pi}$ on the other, may be interpreted either on the assumption of uniform density and space extinction, or on the assumption of Schwarzschild's density law [equation (30)]. The sole difference lies in the interpretation of the distances. If r denotes the distance derived from the extinction theory, and r_1 that obtained from the alternative assumption, the relation between r and r_1 is expressed by

$$\log r_1 = \log r + \tfrac{1}{5}ar.$$

It is interesting to compare in this respect the distribution of stars of given magnitude derived from the present theory with that obtained by Kapteyn from the alternative assumption.

Table xxxv. of *Stellar Movements* shows the number of stars according to Kapteyn whose magnitudes are within specified limits, and which are found between specified limits of distance. Computing from these figures the percentages of stars brighter than $6^{m}\cdot5$, the numbers given in the second column of Table XXII. are found within the sphere whose radii r_1 (Kapteyn's distances) are shown in the first column. The third column contains the corresponding distances r which would result from the assumption of uniform density and space extinction.

Table XXII.

log r_1.	Percentages.	log r.	log r_1.	Percentages.	log r.
0·80	0·4	0·796	2·00	42·0	1·946
1·00	1·1	0·994	2·20	59·7	2·120
1·20	2·8	1·191	2·40	75·7	2·283
1·40	6·7	1·385	2·60	87·3	2·434
1·60	14·1	1·577	2·80	94·2	2·572
1·80	26·0	1·765	3·00	97·7	2·697

These results are graphically represented in fig. 4. The steeper drawn-out curve shows the relation between the number of stars and the distances r, whereas the more sloping dotted curve

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relates the same numbers to the distances r_1, the ordinates of Table XXII. being indicated in either case by crosses. The former curve should therefore be comparable with the results of this paper. To show that this is the case, the percentages of stars brighter than 6·5 taken from Table XIV. have been inserted, the ordinates being indicated by small circles. It is seen that all these circles are almost exactly on the steeper curve.

The figure shows that the distribution of stars in distance

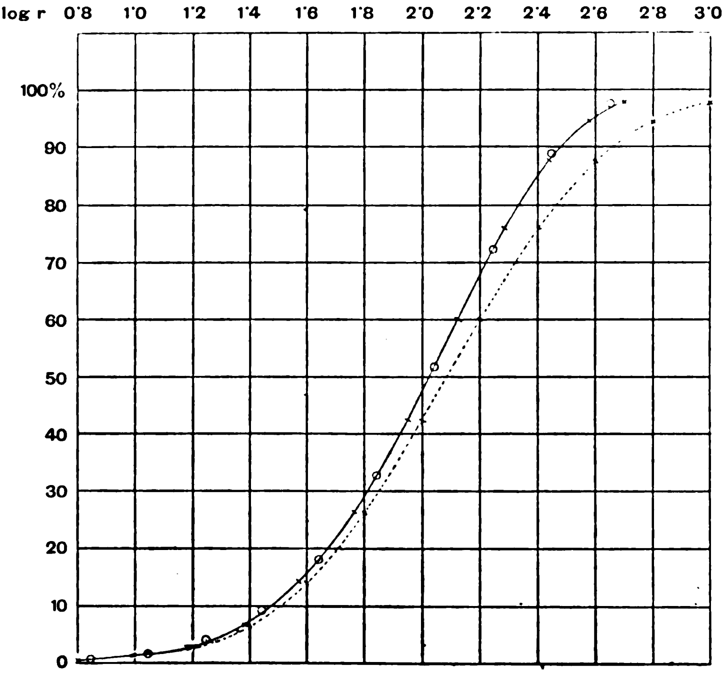

Fig. 4.

differs essentially for the two assumptions, the "spread" being less in the extinction theory. Eddington (*M.N.*, vol. lxxii. p. 384), in deducing the distance law for stars of a given magnitude (Boss's Catalogue) from the proper motions, had already come to the conclusion that the spread in distance must be *less* than that deduced from Kapteyn's data.

Kapteyn's results may be further examined from a somewhat different point of view. The figures given in the *vertical* columns of his table signify the number of stars of apparent magnitude ($m \pm \frac{1}{2}$) contained in the various shells whose limiting radii, r_1, are specified. If this number is denoted by n, and the *total* number of stars contained in the shell by N, then

$$\frac{h}{\sqrt{\pi}} \mathrm{N}\phi(\mathrm{M}) = n,$$

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where

$$M = m - 5 \log \bar{r}_1,$$

$\bar{r}_1$ denoting the average distance according to Kapteyn of the stars within the shell. With the known luminosity law, $\phi(M)$, the values of N may therefore be computed for each *horizontal* column of Kapteyn's table.

The luminosity law adopted by Kapteyn is of the form :

$$\phi(I)dM = \frac{\text{const.}}{I} \times e^{-a^2(\log I - T)^2} dM,$$

$$a^2 = 0{\cdot}385\,;\ T = 1{\cdot}400.$$

This expression may also be written in the form :

$$\phi(I)dM = \text{const. } e^{-0{\cdot}0616(M - 3{\cdot}98)^2} dM,$$

and is found to be practically identical with the expression adopted in the present work.

Denoting further by V_1 the volume of the shell, the density is :

$$D(r_1) = N/V_1\,.$$

The values of log N were computed for all the vertical columns of table XXXV. (*Stellar Movements*), except the first, which is unsuitable on account of the smallness of the numbers *n*. For each horizontal column these values should be constant, a condition which on the whole is remarkably well fulfilled. The final values of log N subsequently used were the arithmetical means derived from each horizontal column.

We proceed to calculate the densities $D(r_1)$ and $D(r)$ on the assumptions that either r_1 or r represent the correct distances, r_1 and r being connected by the relation $\log r_1 = \log r + \frac{1}{6}ar$. (For the values of r_1 and r see Table XXII.) The results of this simple calculation are shown in Table XXIII., which gives the limiting radii of the shells, the star numbers N, the volumes V_1 and V, and the resulting densities $D(r_1)$ and $D(r)$.

Table XXIII.

Limiting Radii.								
log r_1.		log r.		log N.	log V_1.	log V.	log $D(r_1)$	log $D(r)$.
3·00	2·80	2·697	2·572	7·405	9·497	8·475	−2·092	−1·070
2·80	2·60	2·572	2·434	7·033	8·897	8·126	−1·864	−1·093
2·60	2·40	2·434	2·283	6·613	8·297	7·736	−1·684	−1·123
2·40	2·20	2·283	2·120	6·212	7·697	7·301	−1·485	−1·089
2·20	2·00	2·120	1·947	5·735	7·097	6·826	−1·362	−1·091
2·00	1·80	1·947	1·765	5·241	6·497	6·318	−1·256	−1·077
1·80	1·60	1·765	1·577	4·708	5·897	5·779	−1·189	−1·071
1·60	1·40	1·577	1·385	4·142	5·297	5·219	−1·155	−1·077
1·40	1·20	1·385	1·191	3·552	4·697	4·645	−1·145	−1·093
1·20	1·00	1·191	0·994	2·958	4·097	4·066	−1·139	−1·108
1·00	0·80	0·994	0·796	2·356	3·497	3·477	−1·141	−1·121

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The assumption that r_1 represents the *real* distance leads to a density law of the form:

$$\log D(r_1) = -2{\cdot}08 - 0{\cdot}252\rho_1 - 0{\cdot}0173\rho_1^2, \quad (\rho_1 = -5 \log r_1)$$

which satisfies the tabular values remarkably well (except in the case of the nearest shell), and whose coefficients are in close agreement with those derived in the preceding section.

On the other hand, the assumption that the quantities r represent the real distances leads to the density law:

$$\log D(r) = -1{\cdot}092 = \text{constant}.$$

In the former assumption the absolute magnitudes are expressed by

$$M = m - 5 \log r_1,$$

and since r_1 is supposed to represent the real distance, extinction of light is not considered.

In the latter assumption:

$$M = m - 5 \log r - ar,$$

and hence extinction is supposed to exist.

The first assumption leads to a density law in agreement with Schwarzschild's theory; the second assumption to a distribution in accordance with the present theory.

Finally, the first assumption leads to a distribution in distance of stars of a given magnitude which cannot be reconciled with the results derived from the proper motions; whereas the second assumption leads to a distribution in satisfactory agreement with these results.

§ 9. *Collection of Formulæ.*

Assumptions:

(*a*) The total number of stars per unit volume is the same at all distances.

Total number of stars contained within a sphere of radius 10 parsecs:

$$307{\cdot}6.$$

(*b*) Frequency of stars whose absolute magnitudes lie between $(M \pm \frac{1}{2}dM)$:

$$\phi(M)dM = 0{\cdot}1405e^{-0{\cdot}0620(M-3{\cdot}95)^2}dM,$$

$$\int_{-\infty}^{+\infty}\phi(M)dM = 1.$$

$$M = m - 5 \log r_1,$$

$$\log r_1 = \log r + 0{\cdot}000610r.$$

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Results derived from these assumptions:

(1) Frequency of stars of apparent magnitude, m, whose absolute magnitudes lie between $(M \pm \frac{1}{2}dM)$:

$$\phi_m(M)dM = 0{\cdot}1813e^{-0{\cdot}1033(M-\bar{M})^2}dM,$$
$$\bar{M} = -7{\cdot}17 + 0{\cdot}40m,$$
$$\int_{-\infty}^{+\infty}\phi_m(M)dM = 1.$$

(2) Frequency of stars of apparent magnitude, m, whose distances are between $(r \pm \frac{1}{2}dr)$:

$$\psi_m(r)dr = 0{\cdot}906e^{-2{\cdot}582(\log r_1 - 1{\cdot}434 - 0{\cdot}12m)^2}d(\log r_1),$$
$$\log r_1 = \log r + 0{\cdot}000610r,$$
$$\int_0^{\infty}\psi_m(r)dr = 1.$$

(3) Mean parallax of stars of apparent magnitude m:

$$\log \bar{\pi} = -1{\cdot}101 - 0{\cdot}1445m + 0{\cdot}0024m^2,$$
$$\log \bar{\pi}_1 = -1{\cdot}209 - 0{\cdot}120m.$$

(4) Number of stars brighter than magnitude m:

$$\log N = 0{\cdot}219 + 0{\cdot}6154m - 0{\cdot}0104m^2.$$

The magnitudes are on the photographic scale.

To these relations may be added the provisional results of § 5, connecting proper motions with the absolute magnitudes and distances:—

(5) Mean radial velocity, $\bar{V}_r$, of stars of absolute (visual) magnitude M:

$$\log \bar{V}_r = 1{\cdot}53 + 0{\cdot}08M.$$

(6) Mean radial velocity of stars of apparent visual magnitude m:

$$\log \bar{V}_r = 0{\cdot}97 + 0{\cdot}032m.$$

(7) Proper motion, $\bar{\mu}$, of stars of apparent visual magnitude m and parallax $\bar{\pi}$:

$$\log \bar{\mu} = 1{\cdot}05 + 0{\cdot}06m + 1{\cdot}30 \log \bar{\pi}$$

(neglecting the small term due to extinction).

(8) Radial velocity, $\bar{V}_r$, of stars of apparent visual magnitude m and proper motion $\bar{\mu}$:

$$\log \bar{V}_r = 1{\cdot}20 + 0{\cdot}062m + 0{\cdot}308 \log \bar{\mu}.$$

Royal Observatory, Cape of Good Hope:
1916 *August* 30.

Erratum in Dr. Crommelin's paper, 1916, *Nov.*

Page 44, last line of footnote, *for* 0″·05 *read* 0″·10.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVII. FEBRUARY 9, 1917. No. 4

ANNUAL GENERAL MEETING.

Professor R. A. SAMPSON, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

The Report of the Auditors of the Treasurer's accounts for the year 1916 was read, and is given on p. 286.

The Annual Report of the Council was partly read.

The Address was delivered by the President, after which the Gold Medal was handed to the Foreign Secretary, for transmission to Mr. Walter Sydney Adams, to whom it had been awarded for his investigations in Stellar Spectroscopy, including the determination of Absolute Magnitudes.

The President having appointed the Scrutineers, the Society proceeded to the ballot for Officers and Council for the ensuing year. The names of those elected are given on p. 411.

The thanks of the Meeting were given to the retiring President, Vice-Presidents, Secretary, and other members of Council, and also to the Auditors of the Treasurer's accounts and to the Scrutineers of the ballot.

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Mrs. Louisa Petitot Freeman, 67 High Street, Oxford;
William Henry Pick, B.Sc. (Lond.), F.C.S., 141 Mare Street, Hackney, N.E.; and
J. G. E. G. Voûte, Royal Observatory, Cape of Good Hope,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

James Bootherstone, 36 St. Anne's Road, Chorlton-cum-Hardy, Manchester (proposed by W. C. Jenkins).

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REPORT OF THE COUNCIL TO THE NINETY-SEVENTH ANNUAL GENERAL MEETING OF THE SOCIETY.

The following table shows the progress and present state of the Society:—

	Patron	Honorary Members	Fellows: Compounders	Fellows: Annual Subscribers	Associates	Total
1915 December 31	I	I	242	468	48	760
Since elected	...	...	+ 3	+26	+2	...
Deceased	...	...	− 10	− 12	− 2	...
Resigned	...	...	...	− 8	...	...
Since compounded	...	...	...	...	...	...
Removed from list	...	...	...	− 8	...	...
1916 December 31	I	I	235	466	48	751

(Six Associates are also Fellows, and are therefore counted twice in the above table.)

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Mr. Knobel's Account as Treasurer of the Royal

RECEIVED.

				£	s.	d.	£	s.	d.
Balances, 1915 December 31 :—									
At Bankers', as per Pass-book				157	8	3			
Cheques not credited till 1916				6	6	3			
In hand of Assistant Secretary on Petty Cash Account				8	8	2			
							172	2	8
Dividends :—									
£1,250 Metropolitan 3-per-cent. Stock ...				28	16	9			
£2,000 Swansea Corporation 3½-per-cent. Stock				55	2	6			
£3,500 East Indian Railway 3-per-cent. Debenture Stock				80	14	4			
£3,500 London and North-Western Railway 3-per-cent. Debenture Stock				85	19	4			
£4,000 Midland Railway 2½-per-cent. Debenture Stock				81	17	6			
£1,000 Lancashire and Yorkshire Railway 3-per-cent. Preference Stock				24	11	3			
£2,400 Gas Light and Coke Co. 3-per-cent. Debenture Stock				58	19	0			
£2,000 Commercial Gas Co. 3-per-cent. Debenture Stock				46	15	0			
£1,000 War Loan 4½-per-cent. Stock				33	15	0			
							496	10	8
Received on account of Subscriptions :—									
Arrears				154	7	0			
Annual Contributions for 1916				690	18	0			
,, ,, in advance				12	12	0			
Admission Fees				63	0	0			
First Contributions				46	4	0			
							967	1	0
Composition Fees							94	10	0
Sales of Publications :—									
At Society's Rooms				51	8	0			
At W. Wesley & Son's	107	2	7						
Less discount, 10 per cent.	10	12	8						
				96	9	11			
Herschel's Works: half share, received from the Royal Society				3	0	0			
							150	17	11
Grant from Trinity College, Dublin, towards cost of replacing goods destroyed by fire ...							10	0	0
Sales of Photographs							24	10	6
Income Tax refunded by Commissioners of Inland Revenue							84	19	8
Cheque outstanding							3	0	0
							£2,003	12	5

Examined and found correct.

(Signed) G. J. Newbegin.
W. Shackleton.
J. Jackson.

1917 *January* 10.

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Astronomical Society, from 1916 *January* 1 *to December* 31.

PAID.

	£	s.	d.	£	s.	d.
Assistant Secretary: Salary ...	250	0	0			
,, ,, Editing Society's Publications	50	0	0			
Clerk; Salary ...	100	0	0			
Assistant Clerk: Salary ...	2	2	0			
				402	2	0
House Duty ...	2	12	6			
Fire Insurance ...	10	13	0			
Aircraft Insurance ...	10	8	0			
Insurance of stock at printers' ...	3	18	9			
Servants' Insurance ...	0	16	0			
				28	8	3
Printing, etc.:—						
Monthly Notices ...	525	19	11			
List of Fellows and Miscellaneous ...	67	4	3			
Photo-engraving, etc. ...	40	11	1			
				633	15	3
Reproduction of Photographs for sale:—						
Higgs' Atlas of the Solar Spectrum ...	11	1	10			
Other photographs ...	25	2	8			
Cases for Higgs' Spectra ...	1	2	6			
				37	7	0
Turnor and Horrox Fund: purchase of books for Library ...	13	13	2			
Binding books for Library ...	16	0	11			
				29	14	1
Expenses of Meetings ...	18	6	11			
Attendance to Lantern ...	3	10	0			
Time Signal: rental of wire ...	5	0	0			
				26	16	11
Subsn. to Bureau Internat. pour télégrammes astron.	2	6	0			
Telephone subscription ...	6	10	0			
Registration of telegraphic address ...	1	1	0			
Postage and telegrams ...	80	19	0			
Carriage of parcels ...	0	8	1			
Stationery and office expenses ...	15	15	0			
				106	19	1
House expenses; Allowance and sundry expenses	57	13	11			
Coal, Gas, and Electric Light ...	59	6	3			
Window blinds ...	8	19	6			
Sundry fittings and repairs ...	10	9	2			
Sundries ...	3	8	6			
				139	17	4
Expenses moving Society's telescope from Ashtead to Headley ...				20	0	0
Testimonial to Assistant Secretary ...				250	0	0
Lee and Janson Fund: Grants to Mrs. McDowell				12	0	0
Cheque-book and bankers' charges on cheques ...				0	12	8
Repayment to Assistant Secretary of amount due on Turnor and Horrox Fund Account ...				3	7	4
Balances, 1916 December 30:—						
At Bankers', as per Pass-book ...	298	12	6			
In hand of Assist. Sec. on Petty Cash Account	13	0	6			
,, ,, on Account of Turnor and Horrox Fund ...	0	19	6			
				312	12	6
				£2,003	12	5

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Report of the Auditors.

We have examined the Treasurer's accounts of receipts and expenditure for the year 1916, and have found and certified the same to be correct. The cash in hand on 1916 December 31, including the balance at the Bank, etc., amounted to £312, 12*s*. 6*d*.

The funded property of the Society is the same as at the end of the previous year.

A certain amount of the stock of the Society's publications, which was at the printers', having been destroyed by a fire at Messrs. Neill's works, an insurance has been effected upon such stock as may be at the printers'.

The books, instruments, and other effects appear to be in a satisfactory condition. As was mentioned in the Auditors' report of last year, some of the instruments in the list appear to be of no practical value.

A list has been laid on the table of the names of those Fellows who are in arrear for sums due at the last Annual General Meeting of the Society, with the amount due against each Fellow's name.

(Signed) J. Jackson.
G. J. Newbegin.
Wm. Shackleton.

1917 *January* 10.

Gifts to the General Funds of the Society.

In 1836 John Lee, Esq., LL.D., gave the Advowson of the living of Hartwell; in 1844 he gave the Advowson of the living of Stone, Bucks. In 1879 these Advowsons were repurchased by his heir, Edward Lee, Esq., for £700.

The Lawson Bequest (1856): Henry Lawson, Esq., bequeathed to the Society the sum of £200, free of legacy duty.

The Carrington Bequest (1876): R. C. Carrington, Esq., F.R.S., bequeathed to the Society £2,000 Consols.

The Lambert Bequest (1877): C. J. Lambert, Esq., presented to the Society the sum of £500, free of legacy duty, being part of a sum of money bequeathed by his father, Charles Lambert, Esq., for scientific purposes.

The McClean Bequest (1905): Frank McClean, Esq., LL.D., F.R.S., bequeathed to the Society the sum of £2,000, free of legacy duty.

The Farrar Bequest (1906): The Rev. A. S. Farrar, D.D., bequeathed to the Society the sum of £100, free of legacy duty.

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Trust Funds.

The Lee and Janson Fund: In 1834 John Lee, Esq., LL.D., gave £100, and in 1876 T. C. Janson, Esq., bequeathed £200, the interest to be given to the widow or orphan of a deceased Fellow. These amounts now stand in £334 10*s.* 9*d.* East Indian Railway 3-per-cent. Debenture Stock.

	£	*s.*	*d.*		£	*s.*	*d.*
1916 Jan. 1. Balance of Fund	47	0	1	1916. Grants to Mrs. McDowell	12	0	0
Dividends, East Indian Railway	10	0	9	1916 Dec. 31. Balance of Fund	45	0	10
	£57	0	10		£57	0	10

The Turnor Fund and the Horrox Memorial Fund: In 1853 the Rev. Charles Turnor, M.A., bequeathed a sum of £500, which now stands in £464 18*s.* East Indian Railway 3-per-cent. Debenture Stock. In 1876 the Society received a sum of £38 15*s.*, the unexpended balance of a fund collected for the purpose of erecting a memorial in Westminster Abbey to Jeremiah Horrox: to this the Society added the sum of £55 15*s.*, and the whole now stands in £103 6*s.* East Indian Railway 3-per-cent. Debenture Stock. The interest on these funds is to be used in the purchase of books for the Library.

	£	*s.*	*d.*		£	*s.*	*d.*
1916 Jan. 1. Balance of Fund	2	11	4	1916. Purchase of Books	13	13	2
1916. Dividends, East Indian Railway ...	17	1	0	1916 Dec. 31. Balance of Fund	5	19	2
	£19	12	4		£19	12	4

The Hannah Jackson (née Gwilt) Fund: In 1861 Mrs. Hannah Jackson (*née* Gwilt) gave £300 Consols, which now stands in £309 18*s.* 6*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given in medals or other awards in accordance with the terms of the Trust.

	£	*s.*	*d.*		£	*s.*	*d.*
1916 Jan. 1. Balance of Fund	35	2	5	1916 Dec. 31. Balance of Fund	44	8	5
1916. Dividends, East Indian Railway ...	9	6	0				
	£44	8	5		£44	8	5

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Assets and Present Property of the Society, 1917 *January* 1.

	£	s.	d.	£	s.	d.
Balances, 1916 December 30 :—						
At Bankers', as per Pass-book	298	12	6			
In hand of Assistant Secretary on Petty Cash Account	13	0	6			
In hand of Assistant Secretary on account of Turnor and Horrox Fund	0	19	6			
				312	12	6
Due on account of Subscriptions :—						
1 Contribution of 5 years' standing	10	10	0			
1 Contribution of 4 years' standing	8	8	0			
19 Contributions of 3 ,, ,, ...	119	14	0			
41 ,, 2 ,, ,, ...	172	4	0			
68 ,, 1 ,, ,, ...	142	16	0			
Other sums	2	11	0			
	456	3	0			
Less Contributions paid in advance	12	12	0			
				443	11	0
Due for Photographs and Publications supplied from Society's Rooms				9	9	0
Due from Wm. Wesley & Son for sales of Publications during the last quarter of 1916				14	9	2

£1,250 Metropolitan 3-per-cent. Stock.

£2,000 Swansea Corporation 3½-per-cent. Stock.

£3,500 East Indian Railway 3-per-cent. Debenture Stock.

£3,500 London and North-Western Railway 3-per-cent. Debenture Stock.

£4,000 Midland Railway 2½-per-cent. Debenture Stock.

£1,000 Lancashire and Yorkshire Railway 3-per-cent. Consolidated Preference Stock.

£2,400 Gas Light and Coke Co. 3-per-cent. Debenture Stock

£2,000 Commercial Gas Co. 3-per cent. Debenture Stock

£1,000 War Loan 4½-per-cent. Stock.

Astronomical and other Manuscripts, Books, Prints, Photographs, and Instruments.

Furniture, etc.

Stock of Publications and Photographs.

Five Gold Medals.

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Celestial Photographs.

The following has been added to the list of Celestial Photographs reproduced by the Society for sale to the Fellows :—

R.A.S.
Ref.
No.

275. Hydrogen (H_α) Flocculi. Mount Wilson Solar Observatory.

A catalogue of photographs, Nos. 1 to 234, was published in the Annual Report for 1912 (*M.N.*, lxxii. p. 225). Supplementary lists were given in the Reports for 1915 and 1916.

The Award of the Gold Medal.

The Council have awarded the Society's Gold Medal to Mr. Walter Sydney Adams for his investigations in Stellar Spectroscopy, including the determination of Absolute Magnitudes. The President will lay before the Society the grounds upon which the award has been founded.

Publications of the Society.

During the past year vol. lxxvi. of the *Monthly Notices* has been published.

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Obituary.

The Council regret that they have to record the loss by death of the following Fellows and Associates during the past year:—

Fellows:—Cleveland Abbe.
Walter Briscombe.
Arthur Brunel Chatwood.
John Francis Cole.
George Stickland Criswick.
William Ellis.
William Esson.
Benjamin Gregory.
William Hall.
Reinold John Jauget.
Thomas King.
Frederick William Levander.
Robert Cumming McFee.
John James Mellor.
John East Hunter Peyton.
David Powell Richards.
Thomas Davies Sewell.
Walter Spencer Stanhope.*
John Tebbutt.†
Samuel Veevers.
Arthur Mason Worthington.
Arthur Williams Wright.

Associates:—Johan Oskar Backlund.
Karl Schwarzschild.

Note.—Though not a Fellow of the Society at the time of his death, Professor Silvanus P. Thompson's resignation was received during the year, and it has been considered desirable that some account of his life and work should be given.

Professor Cleveland Abbe was born in New York City on 1838 December 3, and died at Chevy Chase, Md., on 1916 October 28, after a somewhat extended illness. He was a graduate of the College of the City of New York, receiving his A.B. in 1857 and A.M. in 1860. During this period the writings of William Ferrel, entitled *The Motions of Fluids and Solids on the Earth's Surface*, attracted his attention most especially, and seem to have helped to turn his attention more directly to the field in which he was

* Deceased in 1912, but not reported till 1916.
† Obituary deferred.

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afterwards so eminent. It seems, however, that even earlier in his life he had been impressed with the faulty knowledge of weather conditions and the lack of scientific accuracy in many of the meteorological papers then published.

In 1860 he enlisted in the Union Army, but his near-sightedness was so great that he was shortly afterwards discharged as being totally unfitted for military duty from this defect. He then turned to astronomy, which indeed he had already studied, and worked under Dr. Gould at Cambridge, Mass., in the U.S. Coast Survey. Next he went to the Pulkovo Observatory, Russia, where he remained as a guest from 1864 to 1866. On his return to America he served as aid in the U.S. Naval Observatory during 1867–8, until he was appointed to the directorship of the Cincinnati Observatory in Ohio, where he remained until 1873.

These many useful years in the service of astronomy did not, however, turn him from his earlier interest in meteorology, and indeed his appointment at Cincinnati opened to him the opportunity for his greater service to the latter science. This he accomplished by establishing a private weather service from his observatory, in co-operation with the Western Union Telegraph Company. This brought at once national attention to the undertaking as it never had been before. Urged on by the success of this experiment, Congress passed an Act in 1870 authorising the creation of a weather service, and placing it under the Signal Service Branch of the War Department. Professor Abbe was naturally called to take a prominent part in the new bureau, and began his duties in 1871 as Professor of Meteorology and Senior Scientific Assistant to the Chief Signal Officer. This post he filled for forty-five years, finally being forced to resign from ill-health only a few months before his death.

During this period of his life he found time to take part in many other activities. Thus we find him lecturing on meteorology at George Washington University from 1886 to 1909, and also at Johns Hopkins University in 1896. He edited with great success *The Monthly Weather Review* from 1872 to 1909, bringing that journal to its present high standard of efficiency. He took part in the expedition sent out by the Government in the U.S. cruiser *Pensacola* to West Africa in 1889 to observe the total eclipse in December of that year. While at sea on this expedition he also made most valuable cloud observations. Indeed his interest in astronomy never grew dim, and during the last years of his life he spent much study upon the invention of an instrument for automatically photographing and recording the velocity of meteors. A report on this was read at the meeting of the American Astronomical Society in 1916 August, in which he described the perfected form of this instrument. His interest in the subject of meteors was particularly great, because he realised their immense importance in giving us almost our only direct information on the currents in our upper atmosphere, both through study of their trains and other phenomena. Perhaps the very latest scientific act of his life was

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to approve of a report on the subject of meteors, drawn up by Dr. C. P. Olivier, and submitted to him for criticism and approval, only two weeks before his death. For many years he had been Chairman of the Meteor Committee of the American Astronomical Society.

Professor Abbe's influence upon the present status of meteorology in America is incalculable, and his contributions are far too numerous to begin even to mention. To write out the names of all the scientific societies at home and abroad of which he was a member would alone fill a full page. He was honoured both in Europe and America in nearly every way a scientist could be. Perhaps the honours he prized most came to him late in his life. These were the Symons Memorial Gold Medal, presented by the Royal Meteorological Society of Great Britain in 1912, and the Marcellus Hartley Medal, awarded in 1916 April by the National Academy of Sciences of the United States. This last medal was presented "for eminence in the application of science to public welfare."

Among his longer and best known scientific works are *Preliminary Studies in Storm and Weather Forecasting*, 1889; *The Mechanics of the Earth's Atmosphere*, vol. i. in 1891, and vol. ii. in 1909; and *Physical Basis of Long Range Forecasting* in 1902. Of more direct interest to astronomers would be several reports on total solar eclipses, not to mention his work contained in various observatory publications.

Professor Abbe was of a genial and sympathetic disposition, unobtrusive in his manner, and always willing and glad to help younger men forward by all means in his power. He had a remarkable memory and very comprehensive view, especially in his chosen fields, and a great linguistic ability. As was natural, occasional attacks upon the policies of the Weather Bureau partly fell upon him, but he bore these criticisms calmly, and these unworthy efforts always came to nothing. When at leisure he was most approachable and a very pleasant companion to anyone, old or young.

He was married twice, first in 1870 to Frances Martha Neal of Cincinnati, who died in 1908; secondly to Margaret Percival of St. Kitts, W.I., in 1909. One of his three sons, Cleveland Abbe, jun., has followed in his father's footsteps, and now occupies a very high position in the Weather Bureau as meteorologist and acting editor of *The Monthly Weather Review*.

Only a few of the chief points of interest have been touched upon in this totally inadequate sketch of Professor Abbe's career, but more complete accounts will appear in several American journals.

Professor Abbe was elected a Fellow of the Royal Astronomical Society on 1876 January 14.

[The Council are indebted for the above notice to Dr. C. P. Olivier, of the Leander McCormick Observatory, University of Virginia, U.S.A.]

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Walter Briscombe was born at White Lea, in the parish of Batley, Yorkshire, on 1836 March 30. After a few years in business, at the age of twenty-four he entered the Wesleyan Ministry, his first appointment being to the Ebenezer Chapel, Sheffield. In 1863 he was transferred to Grimsby, and subsequently was appointed to numerous circuits in various parts of England, in all of which his work was highly esteemed. After thirty-eight years of active circuit life Mr. Briscombe retired to Grimsby, but he still continued to preach and to carry on religious work. He took part in a missionary service the day before his death. He was keenly interested in astronomy, and obtained a practical acquaintance with the subject by the use of a 19-inch reflector and a 6-inch refractor which were placed at his disposal by a friend. He quietly disseminated a knowledge of astronomy in the various districts in which he was stationed.

Mr. Briscombe was twice married, and leaves a widow, a daughter, and a son. He died at Grimsby on 1916 September 29.

He was elected a Fellow of the Society on 1906 April 11.

John Francis Cole was born at Wandsworth on 1824 July 5. At the age of fourteen, his mother being a Swiss, he was sent to Switzerland to continue his education.

Mr. Cole was with Messrs. Baring Bros., Bankers, as confidential clerk, for fifty years. When he retired, at the age of seventy-nine, his faithful services were recognised in a very practical form by the continuance of his salary in full. He was an accomplished linguist, and spoke French, German, and Norwegian fluently. For four years in succession, after his retirement, he visited his old college at Lausanne, and delivered an oration in French to the students who had taken their degrees. He had a genuine interest in astronomy, dating from the time when, as a young man, he made some astronomical observations in Norway. Only a few years ago, notwithstanding his great age, he gave a lecture on Kepler's Laws to the Literary Society at Sutton, Surrey, where he resided during half a century.

Mr. Cole died from acute pneumonia, after one day's illness, on 1916 January 16, being then in his ninety-second year. He was twice married, but had no children. His widow survives him.

He was elected a Fellow of the Society on 1859 February 11, and it may be of interest to note that there now remain only two Fellows of the Society who were elected prior to 1860.

George Stickland Criswick.—During the past year the Society has lost two of its older Fellows who were intimate friends and colleagues for many years on the staff of the Royal Observatory, Greenwich. The younger of these, Mr. G. S. Criswick, died in the first month of the year, and forms the subject of this note. A similar note relating to the elder, Mr. William Ellis, is to be found next in this series.

George Stickland Criswick was a descendant of a Dorset family

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who formerly owned Langford Court in the Parish of Burrington. His mother belonged to the Stickland family of Somerset. He was born on 1836 January 31, and was educated at King's School, Sherborne, with which ancient foundation he remained in touch until the end of his life. His father died when he was fifteen years old, and a financial disaster to his family prevented him going to a university as was intended, but his mother was able to procure for her elder son George a nomination as computer in the Royal Observatory, and she herself removed to Greenwich with her family.

George Criswick joined the staff at Greenwich on 1852 August 25, and early in 1854, on Airy's recommendation, he was transferred to the Cambridge University Observatory, taking up the position of Junior Assistant to Challis on May 1 of that year. In the following October the well-known pendulum experiment at Harton Colliery took place, and for this Airy borrowed observers from various British observatories, Criswick being lent from Cambridge. His subsequent stay at the University Observatory was short, for in 1855 February the departure of Charles Todd for Adelaide caused a vacancy, and Criswick was offered, and accepted, the position of Junior Assistant at Greenwich. He entered on his new duties on 1855 March 26, and almost at once took his place among the four regular observers—it was the custom at Greenwich, as it is now, that four of the established staff should take turns at observing with the Transit-Circle and other principal instruments day by day—and he continued to so observe until the end of the year 1881, this twenty-six years of continuous work with the Transit-Circle being the longest period yet accomplished by any observer with that instrument. In 1862 he was chosen to accompany Mr. Dunkin to Valentia Island in the south-west of Ireland to make observations for the telegraphic determination of the longitude of Knightstown.

As to his work in the Observatory, besides observing, for some years he was associated with Mr. Ellis in the conduct of the transit reductions, and in the Time-Signal and chronometer work of the Observatory. Later on, at successive re-organisations, he was entrusted first with the reduction of the zenith distances, then with the care of the library and manuscripts, and when, in 1874, Mr. Ellis was transferred to the Magnetic and Meteorological Department, Criswick was made Superintendent of the Time Department, and so remained until the end of 1881, when he left off observing and became Superintendent of Computers, taking general charge of the preparation of the Greenwich volume. In 1890 the Astrographic Equatorial was installed at Greenwich, and this was put into his charge, the photographic and mechanical manipulation being both very much to his taste. He remained Superintendent of the Astrographic Department until his sixtieth birthday in 1896, when he retired from the Observatory service at his own desire.

The above is a categorical recital of Mr. Criswick's work, and it certainly may be said that he did all conscientiously and to the best

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of his ability. He was supple of hand and quick in perception, and took care to perfect himself in qualities and dexterities which go to make the skilled and useful observer, circumstances which may not have been sufficiently appreciated.

Apart from his official work, Mr. Criswick had several other interests. He supplied the astronomical information to *Whitaker's Almanack* from its first volume in 1869 until that for 1916. He was a great Freemason, and took a very active interest in the affairs of that brotherhood. He was of sociable and amiable disposition, and popular among a circle of many friends. He married in 1877 Miss Elise Tudor Hassall of Mount Sorrell, Leicestershire, who survives him. They had no children.* Mr. Criswick died on 1916 January 26 within a few days of his eightieth birthday.

He was elected a Fellow of the Society on 1868 February 14.

H. P. H.

William Ellis, who died on 1916 December 11, was the son of Thomas Ellis, an assistant to John Pond, Astronomer Royal in the early part of last century. Thomas Ellis was a native of York engaged in a commercial occupation, but evidently had a liking for and some knowledge of astronomy, for in 1825, at the instigation of William Richardson, who was also a north-country man, and already a member of the staff of the Royal Observatory, he came to Greenwich to take up a similar position. On 1828 February 20 his elder son William was born, and at the age of thirteen, after education at local schools, William Ellis was attached to the temporary computing staff then employed on the reduction of the observations of the Moon made at Greenwich between 1750 and 1830 which Airy had organised after the completion of the similar reduction of the ancient planetary observations. When these special computations came to an end in 1848, Ellis, with some of the others, one of whom was Charles Todd, was transferred to the Astronomical Department of the Observatory. In 1852 Richard Carrington, who had been until then the observer at the Durham University Observatory under the Rev. T. Chevallier, resigned that office, and on the recommendation of Airy the position was offered to William Ellis, who went to Durham in 1852 March.

The observer at Durham at that time seems to have been free to choose his own line of work to a great extent, and Ellis continued the series of observations of position of small planets begun by Carrington, and of comets. His observations of Westphal's Comet at its first apparition in 1852 will be found in vol. xxxvii. of the *Astronomische Nachrichten*, where also some of his planetary observations are published. Whilst Ellis was at Durham his father died (1852 May 29), the vacancy at Greenwich being filled by the promotion of J. C. Henderson, who did not hold his office for long, but resigned in 1854 April. Charles Todd, who had been

* Henry Criswick, who is mentioned in the Introduction to Carrington's Sun-Spot volume as having helped in that work, was the younger brother of G. S. Criswick, and was for a time a computer at Greenwich.

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transferred to the Cambridge University Observatory, was recalled to succeed Henderson in 1854 May. Another vacancy had occurred in the Greenwich permanent staff in 1853 April by the death of William Rogerson, who had been on Pond's staff, and W. Ellis returned to Greenwich to fill this in the following month, and at once took his place among the regular observers with the Transit-Circle and Altazimuth, the four being, when Todd left Greenwich, as he did shortly after, Henry, Dunkin, Ellis, Criswick. Henry * resigned and died in 1856, when W. T. Lynn joined the staff, and the four thus modified, with James Carpenter added, did the bulk of the routine work of the Observatory for nearly twenty years.

Todd left Greenwich in 1855, and his work, which had been the care of the newly introduced galvanic apparatus within the Observatory and the Time-Signals, was handed over to Ellis. When Henry resigned, the custody of the chronometers also passed into his hands, and thus what is now known as the Time Department was created. This remained in Ellis's charge until the end of the year 1874. During this period there was much extraneous work not in the immediate routine of the Royal Observatory that he was called upon to do, and even before that he had been with Mr. Dunkin one of the observers in the Harton Colliery pendulum experiment in the autumn of 1854. The galvanic determination of longitudes from Greenwich was work connected with his department, and in this way he took a share in the determination of the longitude of Edinburgh (1857–1858), Knightstown, Valentia (1862), and of Glasgow (1865). On these occasions Ellis made the arrangements for the signals and provided, with others, the error of the clock at Greenwich, but did not observe at the distant station. He helped similarly in the Trans-Atlantic longitudes made by American parties in 1866 and in 1873, and also in determining the longitude of Cairo in connection with the observation of the Transit of Venus in 1874. On this occasion he went to Porthcurno in Cornwall, the station of the Eastern Telegraph Company, to receive signals from Alexandria and Greenwich for comparison of the clocks at the two places.

At the end of the year 1874 Mr. James Glaisher having resigned the office of Superintendent of the Magnetic and Meteorological Department and left Greenwich, Ellis, at his own request, was transferred to that position, in which he earned considerable reputation. To estimate correctly the value of the contributions Ellis made to the science of earth magnetism and its connection with sun-spots, it is necessary to consider the history of the subject during the middle period of last century, which is briefly given in the following abstract from Mr. Ellis's writings :—

"It was said by Delambre that the observation of sun-spots was of little account, and that they were more curious than useful, but early in the nineteenth century (1826) Schwabe began his daily record of the solar surface, and

* This is Mr. John Henry Belville, whose name is known in connection with certain meteorological journals. He appears in the Observatory volumes as Mr. Henry.

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in 1843 made his announcement of a probable decennial period. In 1841 certain observatories, of which the Greenwich Magnetic Observatory was one, were established in different parts of the world where observations of the elements of earth magnetism should be made for a period of three years. Interest was aroused in this work shortly after their establishment by the occurrence of a considerable magnetic disturbance which was recorded at Greenwich, and was found to have taken place at other observatories at distant places, beginning, as it would appear, at about the same absolute time. Work at these observatories was continued beyond the three years originally intended, and the results were put into the hands of General (Sir Edward) Sabine for discussion. In a paper communicated by him to the Royal Society in 1852, 'On the Periodical Laws discoverable in the Mean Effects of the Larger Magnetic Disturbances,' he remarked that Schwabe's observations of sun-spots showed variations similar in character to those that he had pointed out in the magnetic elements, but added, that although there was this remarkable effect, it was necessary to wait for further observations to see if it was confirmed. Lamont at Munich, Wolf at Zurich, Alfred Gautier, and others found similar results, but astronomers in general seemed to be not very interested in this matter, some indeed disbelieved it. In England only Balfour Stewart wrote on the subject, and his papers were for some reasons not convincing. Whilst in the Astronomical Department my attention had been drawn to the question as one deserving investigation, and when I took up the position of Superintendent of the Magnetic and Meteorological Department and looked daily at the photographic registers of the magnets, I found that in character they were quite different to those of the immediately preceding years, and looking farther back the question seemed to be no longer doubtful, and in course of time I made a graphical representation on a considerable scale of the variations of diurnal range of Declination and Horizontal Force, adding thereto the representation of the variation of sun-spots from the years 1841–1877. On showing this to Sir George Airy, he was charmed to find that the Greenwich observations for so long a period should in so definite a manner confirm the existence of relation between sun-spots and magnetism. The paper that I then prepared, entitled 'On the Relation between the Diurnal Range of Magnetic Declination and Horizontal Force as observed at the Royal Observatory, Greenwich, during the Years 1841–1877 and the Period of Solar Spot Frequency,' was presented by Airy to the Royal Society, and published in the *Philosophical Transactions*, 1880, part 2."

Mr. Ellis's remark as to the attitude of the astronomical world towards a possible connection between sun-spots and magnetism, or at any rate to Sabine's suggestion of a decennial period in the latter, is supported by remarks made by Airy in two papers on Terrestrial Magnetism presented by him to the Royal Society in 1863 and 1868 respectively. In the first he says, speaking of great magnetic disturbances: "There is no appearance of decennial cycles in their recurrence," and in the second, "If they point to any cycle at all, it is one of 6 or 6½ years"; though, on the other hand, it is not to be denied that many persons must have had the possibility of such a connection in their minds, and that papers were written leading to this as a conclusion: but it seems that a definite relation between the phenomena of earth magnetism and sun-spots was not a household word in the world of science until after the publication of Mr. Ellis's Memoir of 1880. He extended the inquiry to include the period 1841–1896 in a paper in vol. lxii. of the *Proceedings of the Royal Society*, and in a third paper in the *Monthly Notices* for December 1899 the relation of magnetic disturbance to sun-spots was further considered.

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A paper "On the Simultaneity of Magnetic Variations at Different Places on the Occasions of Magnetic Disturbance, etc." to the *Proceedings of the Royal Society* in 1892, is a valuable contribution to the subject.

The other branch of Mr. Ellis's department, Meteorology, also received much of his attention during the latter part of his life. He was elected a Fellow of the Meteorological Society in December 1875, served on the Council from 1878 to 1908, was elected President in January 1886, and held the office for two years. He contributed many papers to the Quarterly Journal of the Society, these being generally of the nature of tabulation and discussions of the observations made at Greenwich. On the occasion of an exhibition of barometers organised by the Society in the first year of his presidency he delivered an address on the history of the instrument, comprising a list of nearly two hundred known forms, which must have cost much research. He wrote much on minor points connected with the subject in Symons's *Meteorological Magazine.*

As to his contributions to the literature of our own Society, two Memoirs are to be found in the list under his name, one being an astronomical interpretation of the inscriptions on four old tablets discovered in Egypt, published in 1856. The other is a discussion, or collection, of results showing the periodic movement of the Greenwich Transit-Circle, and it is known that the extensive computations for the memoir in vol. xxviii., "On the Apparent Projection of Stars on the Moon's Disk in Occultations," published in Airy's name, was done by Ellis in his spare time at Airy's request. In the volumes of the Royal Society's list of papers there are in all about fifty entries to Ellis's name, which include those above specified, with others contributed to the *Monthly Notices*, the *Horological Journal*, *Nature*, and other publications, dealing chiefly with matters concerning Time and its determination and distribution, or Meteorology. Mr. Ellis retired from the service of the Royal Observatory at the end of the year 1893. He attained the age of sixty-five, when resignation is usually requested by the rules of the Government Service, earlier in that year, but was retained in order that he might complete a Greenwich publication that he had in hand, this being a compilation in tabular form of the observations of temperature made at Greenwich in the fifty years 1841–1890 that he had arranged. This collection of a mass of figures, and its freedom from error, was one of the things of which Ellis was proud. He was clever as a statistician and a computer, and in the early part of his life did extraneous work in the way of reducing the observations made by travellers for latitude and longitude. It fell to him in 1857 to supervise the reduction of Dr. Livingstone's observations as part of his observatory work, and later, at the request of the Geographical Society, the observations of Cameron, also made in African travels, and those of several others came into his hands. Towards the end of his life he often spoke with pleasure of the

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confidence that Airy placed in him, and of his acquaintance with the leading men of his early days, Sheepshanks, De la Rue, and others, whom he felt honoured to have met on terms of friendship. Mr. Ellis was elected a Fellow of the Royal Society on 1893 June 1, soon after his paper "On the Simultaneity of Magnetic Variations, etc." He became a Fellow of our Society 1864 December 9, and was nominated an Honorary Member of the British Horological Institute in 1865. In 1873 he became a Member of the Institution of Electrical Engineers, and, as already said, a Fellow of the Royal Meteorological Society in 1875.

He was twice married, but had no children—on 1869 June 20 to Sarah Elizabeth Campion of Bellingham, Northumberland, who died in 1906, and secondly, on 1908 December 7, to a distant cousin, Margaret Anne Ellis of Settle, Yorkshire, who survives him. His younger brother died at sea in his youth, and his two sisters predeceased him. His portrait is to be found in the British Museum of Portraits—South Kensington Collection. Mr. Ellis died purely from the weakness of old age on 1916 December 11, being conscious and active in mind almost to the last.

H. P. H.

William Esson was born at Carnoustie, in Forfarshire, on 1838 May 17. He was educated at the Inverness Royal Academy and Cheltenham Grammar School. In 1855, before his seventeenth birthday, he matriculated at Oxford University on his election to a Bible Clerkship at St. John's College. In addition to all the possible honours in mathematics, he gained a First and a Second Class in the two Classical Honours Examination. In 1860 he gained a Fellowship at Merton College, and in 1897 he was appointed to the Savilian Chair of Geometry in succession to J. J. Sylvester.

He collaborated with Dr. A. V. Harcourt in investigations into the rate of chemical change, and for this work he gained the Fellowship of the Royal Society in 1869. After an interval of thirty years these researches were resumed, and a (third) paper on the subject appeared in the *Philosophical Transactions* in 1895. His other publications were neither numerous nor of great importance. He devoted his energies to lecturing and tuition as well as to administrative work, which grew more and more engrossing, and proved increasingly valuable to his College and to the University.

In 1875 June the Convocation of the University passed a statute establishing the University Observatory. Esson was elected a Visitor (at the same time with Bartholomew Price and Warren de la Rue), and at the first meeting of the Visitors he was elected their secretary. This office he held until his death, *i.e.* upwards of forty years.

He was elected a Fellow of the Society on 1863 February 13.

[The Council are indebted for the above notice to his friend Mr. H. T. Gerrans, M.A., Worcester College, Oxford.]

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Benjamin Gregory was the son of a contractor, and was born at Hanley on 1839 December 2. Owing to family financial losses he had few educational advantages, but by strenuous personal efforts he became qualified at the age of seventeen years to take up an appointment as assistant master in a school at Wolverhampton. He afterwards assisted for some time in his father's business, and subsequently served as cashier and confidential clerk to a firm of ironfounders at Wolverhampton. In 1866 he entered the firm of Messrs. Engall & Co., Auctioneers and Surveyors, Cheltenham, and remained with this firm, first as accountant and surveyor, and then as partner, until his retirement a few years ago.

Mr. Gregory was an extremely industrious and studious man. All his spare time was devoted to study, and he acquired a considerable knowledge of ancient and modern languages, as well as of a variety of scientific subjects. Astronomy, however, was his favourite study, and he pursued the subject in a practical way with the aid of telescopes which he constructed himself. He gave numerous instructive lectures at the various churches and institutions in Cheltenham, covering a wide range of subjects, among which astronomy took a prominent place in recent years. As a local preacher he rendered valued service to the Wesleyan Methodist Church, of which he was a greatly esteemed member.

He died on 1916 October 20, after a week's illness, from pneumonia, leaving a widow, three sons, and four daughters.

He was elected a Fellow of the Society on 1913 January 10.

William Hall was born at Birtley Vicarage, North Tyne, in 1867. He entered Rossall School with a mathematical scholarship in 1881, and distinguished himself by obtaining Honours in History and French as well as in his own special subject. He became captain of the school, and twice took the gymnasium championship. Having won an open Mathematical Scholarship in 1886, he proceeded to King's College, Cambridge, where he was awarded the Abbot University Scholarship, and came out as twenty-second Wrangler in the Tripos.

Mr. Hall returned to Rossall as a master in 1890, remaining there for three years, and finding full scope for his many gifts. He was truly an all-round man, for, apart from his scholastic attainments, he was an accomplished musician, and was skilled in carpentry and other crafts.

Having meanwhile married and taken Holy Orders, Mr. Hall left Rossall in 1894 to take up an appointment as chaplain and naval instructor at the Royal Naval College. In the following year he was appointed in the same capacity to H.M.S. *Endymion*, and subsequently served in other ships during a period of about ten years. He appears to have been strongly impressed with the possibility of improving the practice of navigation, and he devoted himself to the task of reform with characteristic energy and thoroughness. His invention of a nautical slide rule in 1902 was

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soon followed by his *Modern Navigation*, which, with his *Tables and Constants*, was adopted by the Admiralty for use in the Navy. He also revised and brought up to date *Inman's Nautical Tables*, in which were included many valuable aids to the rapid fixing of position. Shortly before his death he published a useful stereographic projection to facilitate the solution of problems met with in connection with night-marching and flying by the stars.

In 1905 Mr. Hall was recalled from the sea to the post of Chief Instructor at the Royal Naval College, Dartmouth, where he remained for five years. After a further two years afloat he was detached for service as Director of Studies at Geelong, and rendered important aid in the development of the Australian Navy. The leave to which he was so well entitled was sacrificed to the necessities of the war, and it was while serving on H.M.S. *Venerable* in the Mediterranean that he met with the immediate cause of his untimely death. An order for inoculation having been given, he insisted on taking his share of the discomforts, and, unfortunately, pyæmia set in. After several operations, and six weeks of suffering in hospital, he was sent home, but only survived his arrival by twenty-four hours. He died at the Royal Naval Hospital, Plymouth, on 1916 November 4.

He was elected a Fellow of the Society on 1910 January 14.

Reinold John Jauget was born at Libau, Russia, on 1870 August 18. He came to England about twenty years ago, and was naturalised in 1908. He followed the sea all his life, and at the time of his death was chief officer of H.M.S. *Ramazan*, which was torpedoed in the Mediterranean on 1915 September 19. The officers and crew had all escaped in the lifeboats, but were then fired upon, and the boat in which Captain Jauget had taken refuge was capsized, and he was never seen again.

Captain Jauget made his home in Sunderland, where he was married in 1911. He leaves a widow and a daughter, who was only eight months old when her father was drowned.

He was elected a Fellow of the Society on 1909 May 14.

Thomas King was born at Glasgow in 1858, and when two years old was taken out by his parents to New Zealand. He was educated at the Auckland Grammar School, and afterwards took up a business appointment at Wellington. Mr. King subsequently joined the staff of the observatory, under the directorship of Sir James Hector, and was responsible for the time service from 1887 to 1911, when he resigned. In 1903, in collaboration with Dr. Otto Klotz, he was occupied in a determination of the difference of longitude between Wellington and Doubtless Bay, as part of the programme for the determination of Trans-Pacific longitudes. He was the author of a paper "On New Zealand Mean Time, and on the Longitude of the Colonial Observatory, Wellington, with a Note on the Universal Time Question," published in the *Transactions of the New Zealand Institute*, vol. xxxv., 1902.

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Mr. King, who was unmarried, died at Wellington on 1916 March 16.

He was elected a Fellow of the Society on 1910 June 10.

Frederick William Levander was born at Exmouth in the year 1839. Like his brother, Henry Charles Levander—also a Fellow of this Society—he adopted the scholastic profession, and was for many years one of the assistant classical masters at University College School, then located at Gower Street, London, W.C. In connection with his profession he wrote several educational books, amongst them *Test Questions on the Latin Language*, *Memorabilia Latina*, and *Solutions of London University Questions on Electricity and Magnetism*. He also brought out in conjunction with a former Warden of the Standards a series of models for teaching weights and measures.

He early manifested a keen interest in astronomy, and in 1860 published some observations of the crêpe ring of Saturn and of markings on the surface of Mars, made with a telescope of only 1½ inches aperture. In 1871 he brought out a design for an equatorial mounting at a moderate price suitable for a refractor of 3 or 4 inches aperture, and seven years later a variable diaphragm for use in the observation of the Sun and stars.

His most important services to astronomy were, however, rendered rather in the field of literature than in that of observation. He compiled general indexes to the *Astronomical Register*, vols. i.–xx.; to the *Journal of the British Astronomical Association*, vols. i.–xviii.; and to the *Monthly Notices* of this Society, vols. lii.–lxx.

But it was the British Astronomical Association which was most deeply indebted to him. He was an original member of the Association, and became Librarian in 1895, to which, in 1900, he added the position of Editor of the *Journals* and *Memoirs*. To the *Journal* he contributed a great number of unsigned notes and reviews. He resigned the post of Librarian in 1906 on his election to the presidential chair of the Association, but he continued to fill that post until his death.

He was a prominent Freemason, being member of the Quatuor Coronati Lodge, for which he wrote several papers of an antiquarian character. Of this lodge he was Master at the time of his death.

Mr. Levander died at his residence, 30 North Villas, Camden Square, N.W., on 1916 December 20, after a very short illness, aged seventy-seven. He was interred at the Hampstead Cemetery, Fortune Green, on December 23.

His principal communication to this Society was a Discussion of the Colours of 4984 stars as described by various observers, with the special purpose of reducing such descriptions to a uniform scale. The paper was read at the meeting of 1889 November.

He was elected a Fellow of this Society on 1871 March 10.

B. W. M.

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Robert Cumming McFee was born at Saltcoats in 1847, and was educated at the local academy. Adopting the sea as his profession, he served his apprenticeship in the *City of Paris*; after obtaining his master's certificate he joined the Anchor Line, and was soon promoted to the *Sidonia* as master. After three years' service he was appointed assistant marine superintendent, and in course of time became marine superintendent, which appointment he held up to the time of his death. In the Royal Naval Reserve he attained the rank of Commander.

Captain McFee took a great interest in astronomical studies. He was a talented writer, and was the author of many poems and articles, notably *Random Rhymes* and *Norman*. He was also a clever composer, and published many songs.

He died at his home in Glasgow on 1915 December 6, and is survived by three sons and four daughters, his wife having died in 1914.

He was elected a Fellow of the Society on 1912 January 12.

John James Mellor, who died on 1916 January 12, was born at Oldham on 1830 August 12. He was educated by private tutors and at private schools, and entered upon a business career. For the greater part of his life he was connected with the cotton-spinning and manufacturing industries, and was also interested in the administration of railways. He was chairman of J. & J. J. Mellor, Ltd., Bury, and of the Brook Mills Co., Ltd., Heywood. He was also for some time a director of the South-Eastern Railway and of the Metropolitan Railway. For twenty-seven years he was an officer of Volunteers, and retired from that service as Hon. Colonel 1st Volunteer Battalion Lancashire Fusiliers.

Colonel Mellor represented the Radcliffe-cum-Farnworth division of Lancashire in the House of Commons as a Conservative during the years 1895–1900.

He took great interest in engineering and other scientific pursuits. He was a Fellow of our Society for nearly sixty years, having been elected on 1859 March 11.

J. E. Hunter Peyton was born at Wakehurst, Sussex, on 1841 March 28. He was the eldest son of J. Wakehurst Peyton, Esq., 2nd Life Guards, of Wakehurst, his mother being the eldest daughter of Sir East Clayton East, Bart., of Hall Place, Sussex. His parents having died while he was still a child, his early days were chiefly spent at Hall Place. He attended a school at Rottingdean, and passed into Sandhurst in 1855. Two years later he obtained a commission and was appointed to the 60th Rifles. In 1858 he sailed for India, arriving at Madras after a voyage lasting 136 days. He was promoted to a lieutenancy in 1860, and returned to England in the year following. After service at Aldershot and the Tower of London, he retired from the army in 1863 on account of ill-health.

Having disposed of Wakehurst in 1870, Mr. Peyton settled at

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St. Leonards and took a great interest in the local Philosophical Society, of which he was at one time President. He gave several lectures on astronomy and other scientific subjects, and on his experiences with the British Association in Canada. He also gave several papers on the water supply of Hastings, as determined by the local geological conditions. He had joined the Geological Society in 1871.

Mr. Peyton was married in 1866 to Sophia Henrietta, eldest daughter of the late Frederic Henry Lindsay, of the Commander-in-Chief's Office, Horse Guards. He had two sons and two daughters, of whom the latter survive him. He died on 1916 January 13.

He was elected a Fellow of the Society on 1870 January 14.

Thomas Davies Sewell was born in London on 1832 September 21, his father being Swordbearer of the City. He received his early education at the City of London School, and when thirteen years of age he entered the Chamberlain's department at the Guildhall. During fifty-six years, to his retirement in 1901, he rendered faithful service to the City Corporation, and became one of its best-known officials.

Colonel Sewell took a very active interest in military affairs. He was the first Volunteer to be enlisted in the London Rifle Brigade, in 1859, and was presented with a sword of honour in 1864, when he had attained the rank of captain. He joined the Royal London Militia as a lieutenant in 1866, and remained in association with this branch of the service until 1885, when he left as major and hon. lieutenant-colonel. After further military service he retired in 1892, but as senior proprietor of the *Broad Arrow* he maintained an interest in military matters to the end of his life.

Colonel Sewell was also closely associated with the work of various Livery Companies of the City of London, and rendered valuable services in the resuscitation of some of the minor Companies. His work in connection with the Spectacle Makers' Company will be of long-enduring value. As Clerk to this Company, Colonel Sewell helped to place on a firm basis the present scheme of examinations and awards of diplomas by which the qualifications of opticians to make and adjust spectacles are guaranteed and the confidence of the public secured.

Colonel Sewell died after a very brief illness on 1916 April 13, leaving a son and a daughter. Another son, Major S. D. Sewell, was killed in France in the early part of the year whilst serving with the Third London Field Company, Royal Engineers.

He was elected a Fellow of the Society on 1899 March 10.

Colonel Sir Walter Spencer Stanhope was born on 1827 December 21. He was the son of John Spencer Stanhope, of Cannon Hall, Barnsley, and Lady Elizabeth Wilhelmina, third daughter of the first Earl of Leicester. He was educated at Eton

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and at Christ Church, Oxford. From 1872 to 1880 he was Member of Parliament for the Southern Division of the West Riding of Yorkshire. He was made a K.C.B. in 1904. In 1856 he married Elizabeth Julia, daughter of Sir J. J. Buxton, Bart.; she died in 1880, leaving three sons and three daughters. He died in 1912, but his death was not reported to the Society until 1916. He was one of the oldest Fellows of the Society, having been elected on 1857 February 13.

Silvanus Phillips Thompson was born at York on 1851 June 19. He was the son of Silvanus Thompson, who for many years was a schoolmaster at Bootham, York, where his son Silvanus received his early education. His father was a botanist, and from this influence were sown the seeds for scientific study which the son was to develop in so eminent a degree in after life. In 1867 he decided to train himself for the teaching profession, and for this purpose entered the Flounders Institute at Ackworth, Pontefract. Soon after that he matriculated at the London University, and in 1869 took his B.A. degree. Upon this he became junior master at Bootham, and from 1870 to 1875 was science master. In the latter year he gained a scholarship at the Royal School of Mines, and proceeded to London to pursue his studies there under Huxley, Guthrie, and Frankland. In the same year he took the B.Sc. degree, being bracketed first in the honours section. In 1876 he went to Heidelberg, where he learnt German and attended lectures by Bunsen and Quincke. In 1878 he took his D.Sc. degree, and was appointed Professor of Experimental Physics at University College, Bristol, a position he occupied for many years. On the establishment of the City and Guilds of London Technical College at Finsbury in 1885 Silvanus Thompson was appointed Principal, and resigning his professorship at Bristol, he came to London to take up his duties at Finsbury, where he became Professor of Applied Physics and Electrical Engineering, appointments which he held for thirty years until the day of his death.

Professor Silvanus Thompson's scientific life was devoted mainly to optics and the comprehensive subject of electrical engineering, and the enumeration of his researches on these subjects would form a very lengthy list. Besides numerous contributions to the *Philosophical Magazine*, the Royal Society and other institutions, he published many books of great value and interest. His first book, *Elementary Lessons in Electricity and Magnetism*, published in 1881, has run through many editions, and its high value can be estimated from its having been translated into French, German, Italian, Polish, and Japanese, in addition to a circulation in this country of over 100,000 copies. Professor Thompson was a high authority on electrical engineering, and his Cantor Lectures in 1883 "were a revelation to the engineering world as to the construction, theory, and mode of working of dynamo-electric machinery. This was about the time that the boom in electric companies set in, and he was constantly consulted

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by financiers and others as to the value of the machines and processes which were being offered to the public." At this time he produced his manual on *Dynamo-Electric Machinery*, which has gone through eight editions, and has been translated into French and German. Later on he published his important work on *Polyphase Electric Currents and Alternate-Current Motors*, in which the subject of polyphase motors, transformers, and their application for factories and railways is treated with great fullness, and which has become a recognised text-book. He also published works on *The Electro-Magnet and Electro-Magnetic Mechanism*; on *The Dynamo-Electric Machine*; *Design of Dynamos*; *Electrical Tables and Memoranda*; also a translation from the Italian of Pacinotti's treatise on *The Electric Magnetic Machine*. In 1890 he again delivered the Cantor Lectures on the Electro-magnet, which embodied the results of six years' work. In 1901 he delivered the Boyle Lecture on "Magnetism in Growth," and in 1906 the Howard Lectures on "High Speed Electric Machinery," and in recent years contributed numerous lectures and papers on electricity and magnetism.

The eminent services which Professor Silvanus Thompson thus rendered to electrical engineering were recognised in 1899 by his election as President of the Institution of Electrical Engineers.

On the occasion of the three-hundredth anniversary of the death of William Gilbert of Colchester in 1903, Professor Thompson published a translation from the Latin of the chapter "On the Attraction of Amber, or more truly, on the Attracting of Bodies to Amber," from Gilbert's work *Tractatus, sive physiologia nova de Magnete*. He also published a translation of a letter of Peregrinus on the Magnet written in the thirteenth century.

To optics Professor Thompson also made many useful contributions. His Christmas Lectures at the Royal Institution in 1896 "Light Visible and Invisible" form an admirable elementary text-book.

Besides various lectures and papers on optics, he published a translation of Lummer's treatise on *Photographic Optics*, and also a very useful work on *Optical Tables and Data*. In 1912 he also translated from the French Huyghens's treatise on *Light*. In this connection he rendered important services to the Spectacle Makers' Company in conducting the examinations in optics for obtaining the diploma of that Company. This was productive of great advantage to the public in securing the efficient qualification of opticians in dealing with defective sight.

In other directions Professor Thompson's versatility was shown by a treatise on *Harmonic Analysis reduced to Simplicity*, by translations from Italian, and by some communications on rare works to "The Sette of Odd Volumes." As an artist he produced many clever water-colour drawings.

In his writings and lectures Professor Thompson displayed a power of very lucid exposition, with a command of excellent language and well-balanced sentences. This accomplishment enabled him

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to produce a lengthy and masterly account, in two volumes, of the life and career of that most distinguished philosopher, Lord Kelvin. He also contributed a biography of Lord Kelvin to the *Dictionary of National Biography.* In 1895 he published a book on the *Life and Work of Faraday.*

In his writings and lectures Professor Thompson devoted much consideration to the reform of education, and he enunciated his policy in the following principles:—

1. To remodel all education in all branches, in order to remove the disadvantages which arise from men devoid of any idea of scientific method growing up into civil and industrial life.

2. To reconstruct the universities as centres of earnest work, from which idlers and triflers are rigidly excluded.

3. To ensure that university degrees shall be a mark of thorough training under competent teachers, and not mere attestations of cleverness in passing examinations.

4. To associate in close and living alliance the industrial enterprise and the scientific laboratory, industry helping science, equipping its laboratories, and ensuring that they are directed by men of scientific worth and experience.

Professor Silvanus Thompson was distinctly a highly educated man, proficient in several languages, with developed tastes in many directions, and a wide comprehensive knowledge of the special branches of science to which he devoted himself. As a teacher he had the faculty of attracting large crowds to his lectures, which it is said "were always pure joy and marvels of lucidity"; and, what is important, his experiments were almost invariably successful. His influence at Finsbury College was of immense value to the students, and the loss of so rare and able a teacher is profoundly regretted.

Professor Thompson received many distinctions. He was elected a Fellow of the Royal Society in 1891; he was a member of the Royal Academy of Sciences of Stockholm, of the Academy of Sciences of Bologna, of the American Philosophical Society; LL.D. of Birmingham, D.Sc. of Bristol Universities, and M.D. of the University of Königsberg. He was Past-President of the Institution of Electrical Engineers, of the Physical Society, of the Institution of Junior Engineers, of the Optical Society, of the Illuminating Engineers Society, and of "The Sette of Odd Volumes."

"The parents of Professor Silvanus Thompson were members of the Society of Friends, and he adhered to that body all his life. He exhibited many of the virtues which have long distinguished the Society of Friends, and his memory will be cherished by many whom he assisted in an unostentatious fashion when they were in difficulties."

After a short illness Professor Thompson died on 1916 June 12, leaving a widow and four daughters.

He was elected a Fellow of this Society 1875 May 14, and resigned February 1916. E. B. K.

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Samuel Veevers was a well-known Liverpool cotton merchant. He started business on his own account in 1880, and throughout his life was held in high respect and esteem by his fellow-members of the Liverpool Cotton Association. Mr. Veevers devoted much of his leisure time to the study of astronomy, and wrote numerous articles on the subject for the local press. He died at his residence at Bootle, after a prolonged illness, on 1916 February 11, leaving a widow. He was sixty-two years of age.

He was elected a Fellow of the Society on 1907 February 8.

Arthur Mason Worthington was born at Manchester in 1852, and was educated at Rugby and at Trinity College, Oxford. After further work at Owens College, Manchester, and in the laboratory of Helmholtz at Berlin, he became headmaster of the Salt Schools, Shipley (1877–79), and went to Clifton College as an assistant master in 1880, remaining there until 1885. In 1887 he was appointed headmaster and professor of physics at H.M. Dockyard School at Portsmouth, and in the following year, when the school for naval engineers was transferred to the Royal Naval Engineering College at Keyham, Devonport, he went with it in the same position. Here he remained for twenty years, and was then transferred, as professor of physics, to the Royal Naval College at Greenwich. On account of ill-health, he retired in 1911.

Worthington's educational work was carried on with marked success throughout. He was a very able lecturer, and was especially successful in devising experiments with simple apparatus for the illustration of lectures and for purposes of instruction in the laboratory. He was a pioneer in the introduction of practical physics into schools, and his successful work at Clifton marks a definite epoch in the history of scientific education. While there, also, he did much to encourage interest in astronomy, and instructed a number of boys in the use of a 6-inch refractor erected by him in an observatory close to the College. His popular lectures on scientific subjects were always greatly appreciated, and it is recorded that in a lecture on astronomy in the prison at Princetown he succeeded in holding the attention of 1000 convicts from beginning to end.

Of the original investigations carried on by Worthington, his work on the stretching of liquids, and on splashes, is probably most widely known. His long series of observations of the latter phenomena are summarised in his well-known book, entitled *The Splash of a Drop.* He was also the author of works on *Physical Laboratory Practice* and the *Dynamics of Rotation.*

He was elected to the Fellowship of the Royal Society in 1893, and received the C.B. in 1902.

He died at Oxford, after a short illness, on 1916 December 5, leaving a widow, a son, and a daughter.

He was elected a Fellow of the Society on 1877 February 9.

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ARTHUR WILLIAMS WRIGHT was a descendant of Samuel Wright, who settled in Springfield, Mass., in 1639. He was born at Lebanon, Conn., on 1836 September 8, where his father, Jesse Wright, was at one time a member of the Connecticut House of Representatives. His whole life was practically identified with Yale University. His career as an undergraduate at Yale College was a distinguished one, the studies of his predilection being Latin, mathematics, and astronomy. He was also a very capable musician, both as a performer and as a critic.

After graduating in 1859 he continued his studies at Yale, specialising in science and mathematics, and in 1861 attained the degree of Ph.D., the first recipient of the degree in an American university.

For some years following he taught Latin and natural philosophy at Yale as a tutor. During this period he also studied law, and was admitted to the bar in 1868, though he never practised. In the years 1868–69 he studied at Berlin and Heidelberg, and from 1869 to 1872 held the professorship of physics and chemistry at Williams College. In 1872 he returned to Yale, and was appointed to the professorship of molecular physics and chemistry. This office he held until 1887, when he was relieved of the chemical professorship, and became the first professor of experimental physics.

To his initiative was due the construction of the first Sloane Laboratory of Yale College, the first laboratory for the study of experimental physics erected in America. In this laboratory too worked that distinguished physicist, Professor J. Willard Gibbs.

Not only did Professor Wright build a physical laboratory, but he for over thirty years conducted many important investigations, chiefly on electrical discharges. He was the first to experiment with X-rays in America, and to obtain photographs by their agency. His published papers appeared almost entirely in the *American Journal of Science* between the years 1870 and 1901. In particular, in the year 1877 Professor Wright published two important papers on the deposition of metallic films by the cathode discharge in vacuum tubes, and on a new process for the electrical deposition of metals, and for constructing metal-covered glass specula.

Professor Wright's astronomical work was chiefly concerned with polariscopic observations of the zodaical light, of Coggia's comet of 1874, and of comets *b* and *e* 1881, in which he successfully employed a polariscope of his own design. He also took part in an expedition in Colorado to observe the total solar eclipse of 1878 July 29, and observed the polarisation of the solar corona. As early as the year 1874 he had studied and determined the spectrum of the zodiacal light.

But probably the most important astronomical research which he undertook was his elaborate investigation of the gases of meteorites, contained in a series of five papers, published in the *American Journal of Science* during the years 1875 and 1876,

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which form the foundation of our knowledge of these substances. Professor Wright studied the gases given off from six stone and five iron meteorites, for the most part from the collection at Yale University. His chief conclusions may be thus summarised. In the first place, no one of the irons which he analysed gave any spectroscopic evidence of unknown elements. Next he determined the percentage of the various gases H, CO_2, CO, N_2, and CH_4 in the meteorites, and showed that CO_2 is more abundant in the stones, and H and CO in the irons. The relative abundance of oxides of carbon in meteorites was one salient result of this research, and Professor Wright did not fail to point out the bearing of these observations on the spectra of comets as tending to support their meteoritic origin.

Professor Wright's activities were not confined to the experimental work carried on in his laboratory. He served on several Government Commissions, and from 1881 to 1886 as an adviser to the Signal Service. He also acted as one of the revisers of the editions of Webster's Dictionary, published in 1864 and 1890, particularly in the matter of scientific and technical definitions in physics and chemistry.

Professor Wright was elected a member of the American National Academy of Sciences in 1881; he was also a member of the American Philosophical Society; of the Connecticut Academy of Sciences; the American Metrological Society; the New York Academy of Sciences, and other scientific societies.

His death took place at his home in New Haven, Connecticut, on 1915 December 19. He was married in New Haven, 1875 October 6, to Miss Susan Forbes, eldest daughter of Professor Benjamin Silliman, jun., who predeceased him on 1890 February 17. Two daughters and one son survive him.

He was elected a Fellow of the Royal Astronomical Society on 1879 January 10. A. L. C.

Johan Oskar Backlund (Oskar Andrejewitsch) was born 1846 April 28 in the Parish of Långhem, in Wermland, Sweden. At the age of twenty he was a student of mathematics and astronomy at Upsala. In 1873 he became assistant to Gyldén at the Observatory of Stockholm, writing there his dissertation on the planet Iphigenia. In 1875 he became Docent in Upsala, and from thence went as observer, in 1876, to the Observatory at Yuryev (Dorpat). In 1879 he was invited by O. Struve to become assistant astronomer at Pulkovo. His theoretical work, especially in regard to Encke's Comet, led to his election as member of the Academy of Sciences of Petrograd in 1883. About this time he became a naturalised Russian subject. From 1887 to 1895 he was resident in Petrograd. In 1895 he succeeded M. Bredikhine as Director of the Observatory of Pulkovo, where he remained twenty-one years, winning the devotion and affection of his colleagues and assistants by his genial and friendly ways. He died suddenly on 1916 August 29.

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His scientific activity falls into three parts: (1) the direction of the work of the observatories under his charge; (2) participation in international scientific projects; (3) the theory and the study of the movement of the bodies of our system, the comets, particularly Encke's, the small planets, and the earth. Under his care the observatory at Pulkovo has much increased, in personnel, in instruments, and in output. But the climate of this observatory being unfavourable for observation, he used his great personal influence to secure the foundation of two others in South Russia, one at Nicolaieff (seventy-five miles N.E. from Odessa), where, it is said, 300 days of the year may be good observing days, and another at Simeïs, in the Crimea. In the last few years the Russian Government had sanctioned his expenditure of large sums for the purpose of procuring astrophysical equipment for the new observatories. And his recent letters to English correspondents contained frequently reiterated expressions of the pleasure with which he looked forward to visits to England in connection with his engagements with Sir Howard Grubb.

Dr. Backlund represented Russia in the international commissions for geodesy, seismology, astrography, solar physics, etc. His death followed very closely on that of his friend Prince Boris Galitzine, who died on 1916 May 17, and thus observational science in Russia suffers a doubly severe blow in critical times. His unstinted activity in international scientific undertakings brought him into intimate friendly relations with colleagues in many lands. He was specially well known in this country, and was a warmly welcomed guest wherever he went. His tall, active form and his soldierly bearing made him a notable figure in any public gathering. He spoke English, French, German, Russian fluently, though he always maintained that in spite of his being able to speak many languages he was master of none, not even of his own; and the special turn that he gave to his English seemed only to emphasize the kindly geniality of his mind. Among his recent visits to England, we may recall his presence at the Cambridge Meeting of the British Association in 1904. He came again in the following year to travel to South Africa for the Meeting of the British Association, and for a visit to his friend Sir David Gill. In 1909 he made a special journey to England to receive the Gold Medal of the Society, and in September of the same year he came again for the Meeting of the International Geodetic Conference in London and Cambridge, under the presidency of his friend Sir George Darwin. Those who had the pleasure of visiting Pulkovo record not only the genial hospitality with which they were received and entertained by Dr. and Madam Backlund, but also the vivid interest with which he showed the treasures of the famous observatory.

His theoretical work appears to have been for the most part published in the Memoirs and in the Bulletin of the Academy of Petrograd (here referred to as *Petr. Mem.* and *Petr. Bull.* respectively), with occasional papers in the *Astr. Nachrichten* and

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elsewhere. He supervised the publication of the second volume of Gyldén's *Traité analytique des orbites absolues des huit planètes principales* (1908), of which only 192 of the 323 quarto pages were in the press when Gyldén died, though Backlund insists that the enormous calculations these contained are for the most part due to MM. Sundman and von Zeipel. Evidently he was throughout keenly interested in the fundamental problem of how to deal in a general theoretical way with the terms of long period, and his work in this regard, as coming from a practical astronomer, appears to have a high interest. He published papers on the development of the disturbing function: *Petr. Bull.*, xxiv., 1878, "Développement en séries convergentes des puissances négatives impairs des quarrées de la fonction $1 - 2\eta U + \eta^2$," and a paper with nearly the same title in *Petr. Bull.*, v., 1896. Also, *Astr. Nachr.*, 3223 (1894), "Sur l'application d'une formule de Tchebycheff au développement de la fonction perturbatrice," and *Petr. Mem.*, xxxii., 1884, "Zur Entwickelung der Störungsfunction." But particularly should be mentioned such papers as *Petr. Bull.*, xxvii., 1881, "Développement des perturbations absolues d'une comète"; *Petr. Bull.*, xxxi., 1887, "Report on Harzer's paper, 'Recherches sur un cas spécial des problème des trois corps'"; *Astr. Nachr.*, 2920–2921, 1889, "Ueber die kleinen Divisoren bei den elementären Gliedern in der Theorie der Planetenbewegungen"; then his two important papers, *Petr. Mem.*, xxxviii., 1892, and vi., 1898, on the planets of the Hecuba type, in connection with which may be mentioned his papers, *Petr. Bull.*, vi., 1897, "Ueber die Integration der Differentialgleichung der Radiusvector einer gewissen Gruppe der kleinen Planeten"; *Petr. Bull.*, xvi., 1902, "Ueber die Bestimmung der Glieder langer Perioden mit besonderer Rücksicht auf die kleinen Planeten der Hecuba Gruppe," and also, for convenience, "Ueber eine horistische Differentialgleichung Gyldén's," *Petr. Bull.*, xvi., 1902. Then also, with the same underlying problem, his paper, *Petr. Bull.*, viii., 1898, on the possible librations of the satellites of Saturn, and the discussion of the long-period terms in his paper, *Petr. Bull.*, xii., 1900, "Zur Theorie der Präcession und Nutation."

But though such papers are important, the work that took the eye, and had its price, is naturally the enormous task of securing the recalculation of the planetary perturbations of Encke's Comet, with the connected revision of the estimate of the mass of Mercury, which he finally places at $(9,700,000)^{-1}$. His final standard publications in regard to this matter are the "Calculs et recherches sur la comète d'Encke," *Petr. Mem.*, xxxviii.–vi., 1891–1898, six parts in folio, and "La Comète d'Encke," i.–iii., *Petr. Mem.*, xxiii., xxiv., xxx., 1909–1912. Of these he published summaries, *Bull. Astron.*, xi., Paris, 1894, "Sur la masse de la planète Mercure et sur l'accéleration du movement moyen de la comète d'Encke," and *Monthly Notices*, lxx., 1910, "Encke's Comet 1895–1908." This latter is subsequent to the valuable and interesting address given by Professor Newall on the occasion of the presentation

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of the Society's medal to Backlund, *Monthly Notices*, lxix., 1909. The reader is referred to these two papers in the *Monthly Notices*. Writing the mean daily sidereal motion of the comet in the form $n = n_0 + n_1\tau$, where n_0 is between 1070″ and 1080″ and τ is the time in days divided by 1200 (the period of the comet approximately), Backlund concluded that from 1819 to 1858 n_1 was 0″·1 ; in 1861, 1865, and 1868 n_1 had diminished to 0″·08 ; from 1871 to 1895 further to $n_1 = 0''\cdot062$, and that there were comparatively sudden changes in n_1 in 1858, 1868, 1895, and probably also in 1904. In regard to the mass of Mercury, it may be remarked that von Asten, in his two papers, *Petr. Mem.*, viii., 1872, "Berechnung eines wichtigen Theiles der absoluten Jupiterstörungen des Encke'schen Cometen," and *Petr. Mem.*, xxvi., 1878, "Resultate aus den Erscheinungen 1819–1875," beginning with the estimate $m_0(☿) = (4,360,000)^{-1}$ (p. 5 of the second memoir), finally concludes (p. 109) that $m_0 = (7,636,440 \pm 195,907)^{-1}$. Backlund, whose earlier papers* on Encke's Comet ("Untersuchung über die Bewegung des Encke'schen Cometen 1871–1881," *Petr. Mem.*, xxxii., 1884, "Comet Encke 1871–1885," *Petr. Bull.*, xxx., 1886, and "Comet Encke 1865–1885," *Petr. Mem.*, xxxiv., 1886) are intimately bound up with von Asten's papers, remarks (*Petr. Mem.*, xli., 1893) that for the period 1848–1865 he has recalculated the perturbations due to Venus, Earth, Mars, Jupiter, and Saturn, taking intervals of five, ten, and twenty days, the osculating elements being changed at least once in each revolution, and that then the terms of second order arising from the action of Jupiter become very sensible, and that this explains a divergence from von Asten's results, who had neglected these terms save for 1855–1858 and 1865–1868, and then used an incorrect formula. But Backlund pays a high compliment to von Asten's accuracy of computation, many of whose results are embodied in his own.

Some brief remarks seem to be called for as to the methods employed in these computations. Von Asten's earlier paper of 1872, referred to above, sets out with the intention of combining Gyldén's method for a general analytical theory with Hansen's suggestion of partial anomalies made in his Paris Prize Essay on Comets (1856). In his second paper of 1878, von Asten appears to abandon the possibility as being too laborious. Backlund, however, in his paper, "Zur Theorie des Encke'schen Cometen," *Petr. Mem.*, xxviii., 1881, returns to the attempt to formulate an analytical theory by improvement of von Asten's (see also *Petr. Mem.*, xxxii., 1884, "Zur Entwickelung der Störungsfunction"); and Part 6 of his standard papers (*Petr. Mem.*, vi., 1898), "Perturbations produites par la Terre, Vénus et Mercure dans la partie supérieure de l'orbite de la comète," is preceded by an exposition of the application of Hansen's Paris Essay to the problem in hand.

In the numerical work Backlund had sometimes used the Hansen formulæ for the computation of the mean anomaly, the

* Still earlier papers are, one written with Bonsdorff on the perturbations of Encke's Comet, Stockholm, 1876, and another published at Petrograd, 1881.

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logarithm of the radius vector, and the latitude in the orbit, and sometimes the older method, known as that of variation of the constants, in which the perturbations of the six elliptic elements are directly computed. Thus for the movement of the comet in the *lower* part of its orbit, in Part 5 of his standard papers, *Petr. Mem.*, xlii., 1894, he has used the method of variation of constants for 1819–1848, but for 1848–1871, 1871–1891 he has used Hansen's method (*cf.* also p. 6 of the paper, "Comet Encke 1865–1885," *Petr. Mem.*, xxxiv., 1886). In using Hansen's method he refers to the scheme of formulæ given by Powalky, *Astr. Nachr.*, 1438 (1863), which, however, he corrects in some points. He gives a *résumé* of the actual formulæ used, in Part 2 of his standard papers, *Petr. Mem.*, xli.* The formula for the mean anomaly

$$M = M_0 + n_0 t + k\tau^2 + m_1 \cos 2u + \ . \ . \ .$$

where $\tau = t/1200$, $k = 600 n_1$, $m_1 = -3''\cdot 66$, is deduced in Backlund's "Untersuchungen . . .", *Petr. Mem.*, xxxii., 1884, from the hypothesis of a resisting medium, by the introduction, after Gyldén, of an elliptic integral

$$u = \frac{\pi}{2K}\int_0^{\frac{1}{2}f} \frac{d\theta}{(1 - k^2 \sin^2\theta)^{\frac{1}{2}}},$$

with the help of the formulæ of elliptic functions originally due to Jacobi and Abel.

Attempt has been made in this note to give materials from which a competent account of Backlund's work in the theory of orbits could be begun. It may suffice to show that this would involve a wide and interesting study of an important phase in the history of theoretical astronomy. That he bore with such distinction his part in this development would be alone an enduring title to remembrance, apart from his contributions, many of them unmentioned here, to practical astronomy.

Dr. Backlund became an Associate of the Royal Astronomical Society 1898 December 9, and a Foreign Member of the Royal Society in 1911. H. F. B.

KARL SCHWARZSCHILD was born at Frankfort 1873 October 9. He was educated successively at the public school at Frankfort and at Strasburg and Munich universities. He gave early evidence of remarkable mathematical ability, and when barely sixteen contributed two excellent papers to the *Astronomische Nachrichten* on the problem of determining an orbit from three observations. At Munich he came under the inspiration of Seeliger's teaching, which had a very lasting influence on his mind. His first astronomical post was at the Von Kuffner Observatory, Vienna, where he was assistant from 1896 to 1899.

* *Cf.* Encke, *Astr. Nachr.*, 791–2 (1852); *Astr. Nachr.*, 814 (1852); Bond, *Mem. Amer. Acad.*, iv., May, 1849; Hansen, *Astr. Nachr.*, 799–801 (1852), and *Auseinandersetzung* . . ., Leipziger Abh. v., vi., vii. (1857).

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After a short interval as privatdocent at Munich, he was appointed Director of the Observatory of Göttingen in succession to Schur. Here he joined a brilliant circle of scholars who have made Göttingen world-famous. To a man of his wide interests in all branches of mathematics and physics the surroundings must have been very congenial; and to his growing reputation as an astronomer he added important achievements in other fields of research. Finally, in 1909 he was chosen at the age of thirty-six to succeed Vogel as Director of the Astrophysical Observatory, Potsdam, a position which he filled with distinguished success.

In reviewing his contributions to astronomy we may start with his work on photographic magnitudes, a long and sustained investigation, which required much perfection of detail as well as brilliance of conception. His first method, carried out at the Von Kuffner Observatory, was to measure the greyness of star-images taken out of focus. He set out to determine the variation of photographic effect with the length of exposure, as a step towards forming an absolute scale of magnitudes. It was already known that an increase of exposure-time has not the same effect as an increase of light-intensity—doubling the exposure is not so effective as doubling the brightness. Schwarzschild proposed the now well-known law that the photographic effect is proportional to It^p, where I is the intensity of the light, and t the exposure-time. For the constant p he found the value 0·675; but this appears to have been too low, and in the Göttingen Aktinometrie he redetermined the constant as 0·76—which means practically that an increase of exposure which might be expected to give a gain of four magnitudes in reality only gains three.

The fruit of this preliminary work was reaped when he came to Göttingen and, with the aid of his assistants, carried out the "Aktinometrie." This was a large undertaking, involving the determination of the magnitudes of all the stars brighter than $7^m{\cdot}5$ between 0° and +20° dec., and their reduction to an absolute scale. The great area of the sky covered by the survey necessitated the use of plates covering a large field 20° square. Naturally, in the outer parts of the field the extra-focal images were distorted and unsuitable for measurement, so Schwarzschild adopted the plan of displacing the image in a regular manner during the exposure so as to build up a square of uniform greyness; the device is known as the "Schraffierkassette." The immediate results of this work were of great interest; the visual magnitudes of all these stars had been measured at Potsdam, and the differences, photographic *minus* visual, gave the colour-indices of some 3500 stars. (The use of this quantity as a measure of colour seems to have been first suggested by him some years earlier.) In conjunction with similar work by King and Parkhurst, the Aktinometrie showed that spectral type is very accurately a function of the colour-index. It also brought out the fact that it is the stars of intermediate colour (yellow) which possess the largest proper motions, and that after a maximum about type G, the proper

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motions diminish very rapidly for the redder types. This feature of the red stars was just beginning to be recognised; but the change of proper motion appeared particularly striking when exhibited as a function of a continuous variable, colour-index, instead of the broad divisions of spectral class.

In 1895, acting on a suggestion by Michelson, Schwarzschild introduced the plan of placing a coarse grating of parallel strips in front of the object-glass of a telescope so as to give diffraction images at a small distance from the principal image of each star. This device has since been used extensively both for measuring the colour, or effective wave-length, of stellar light and for establishing an absolute scale of magnitudes. Schwarzschild's first application was not to these purposes but to the measurement of double stars, the arrangement acting as a kind of double-image micrometer. The separation of the diffraction-images could be varied in a known manner by tilting the grating at different angles to the line of vision. Successful results were obtained with a 10-inch refractor; but no doubt there would be difficulties in working the arrangement at the end of a long telescope.

The task of determining accurate data for large numbers of stars inevitably leads the mind to consider the great problems of the structure of the stellar universe. Somewhat in the way that the Cape Durchmusterung led Kapteyn to his cosmical researches, so the Göttingen Aktinometrie may have turned Schwarzschild's thoughts to this wide field of discovery. But in his case there was already the influence of his teacher Seeliger, the pioneer in the mathematical theory of the distribution of stars. Schwarzschild's best-known work in this subject is his ellipsoidal hypothesis of stellar motions. It had been shown by Kapteyn that the individual motions of stars are not haphazard, but are especially directed in two favoured directions—that there are, in fact, two streams of stars. The directions of these streams, when referred not to the sun but to the stars as a whole, automatically become opposite to one another. In an approximate mathematical representation it was perhaps natural to take the streams as though they were two independent systems of stars passing through one another; but the theory proposed by Schwarzschild showed clearly that this division into two systems was not an essential part of Kapteyn's discovery. The important point to preserve in the formulæ was the æolotropic distribution of the motions—a greater mobility to and fro along one axis in space than in perpendicular directions. He noticed that this property could be represented by a simple modification of Maxwell's law of velocities; by setting the frequency of a velocity (u, v, w) proportional to $e^{-a^2u^2-b^2v^2-c^2w^2}$, the distribution stands in the same kind of relation to Maxwell's random distribution as an ellipsoid does to a sphere. The apparent antagonism between the two-drift and ellipsoidal hypotheses disappears if we remember that the purpose of both is descriptive. In a letter communicating his results to the writer, Schwarzschild said, "The formal comparison of our two hypotheses of distribution shows very well how nearly

the same they are. . . . Within our visible system the gravitation of the whole is certainly much stronger than the attraction of single stars at casual approaches. We could represent the stellar system as a homogeneous ellipsoid, and assume that the initial velocities of the stars are so distributed that this homogeneous ellipsoid always remains similar; then the velocities parallel to the greatest axis must be on the average greater than those parallel to the least. In that way one can try to render my hypothesis more plausible. But I freely admit that it has difficulties like yours. . . . You will certainly continue your researches on the two-drift hypothesis, as I shall on the ellipsoidal; and by this competition I hope the solution of the whole marvellous problem may be accelerated." The ellipsoidal theory has great mathematical elegance both in its conception and its application. Whilst the two-drift theory has often been preferred in the ordinary proper-motion investigations, on account of an additional constant in the formulæ which gives it a somewhat greater flexibility, the ellipsoidal theory has been found the more suitable for discussions of radial velocities and the dynamical theory of the stellar system.

The study of proper motions leads naturally to the question of the distribution of the stars in space. In an elementary conception of this problem we may suppose that each part of space contains stars of varying degrees of luminosity in proportions determined by some standard law, and with various speeds also following a uniform law, but there will probably be a thinning out in the number of stars per unit volume as we recede from the centre of the stellar system. As knowledge has progressed, much complexity has been added to this simple scheme; we have had to step aside to investigate star-streaming or the varying behaviour of different spectral types. But the luminosity law and the density law, involving the specification of the shape and dimensions of the stellar system, must remain a fundamental aim of research. It is theoretically possible to deduce the two laws from counts of stars between different limits of magnitude together with the mean parallax (or parallactic motion) for each magnitude. The general solution of this problem was given by Schwarzschild; it involved the solution of certain integral equations by an original method of considerable mathematical interest. In practice it is usually more desirable to use a less general method, assuming particular forms of the laws which give a sufficient approximation; here Schwarzschild followed more closely the lines laid down by Seeliger, but by his keen insight he brought the mathematical discussion into an admirably clear form. His latest and fullest discussion of the observational data is given in the *Sitzungsberichte* of the Berlin Academy, 1914, where the different spectral types are treated separately. Workers in this kind of statistical research owe a grudge to the discovery of "giant" and "dwarf" stars, which has played havoc with some of the hoped-for simplicity of the problem, and seems to have placed definitive results for the moment out of reach; but step

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by step the knowledge of stellar distribution is improving, and Schwarzschild's mathematical treatment has notably contributed to the advance.

In solar physics Schwarzschild has left his mark by his theory of radiative equilibrium of the sun's atmosphere, published in 1906. Three possible modes of transfer are recognised by which the heat squandered into space from the outer layers may be replaced from the stores of energy within. Conduction can scarcely play a significant part, and it has usually been considered that convection currents are the main agents of transfer, as in the terrestrial atmosphere. But at the high temperature of the sun it is more likely that radiation outweighs all other causes—certainly in the interior, and probably also in the atmosphere proper. In this case the state of equilibrium must be determined by the condition that each element radiates just as much energy as it gains by absorption of the radiation passing through in all directions. In proposing this state of radiative equilibrium, Schwarzschild was unwittingly following the theory put forward by Sampson twelve years earlier; but by introducing Stefan's law, that the density of radiation varies as the fourth power of the temperature, he was able to give definite shape to the formulæ. He showed also that the radiative hypothesis appears to account satisfactorily for the observed degree of darkening of the sun's limb.

An account of Schwarzschild's achievements necessarily has a disjointed appearance, so varied were the subjects he pursued. We can only refer very briefly to some other scientific contributions. Among his theoretical researches with a definitely astronomical application may be noted his calculation of the light-pressure on small solid particles. He showed that the increase in the ratio of light-pressure to gravitation with diminishing size ceases to hold when the dimensions of the particle are comparable with a wave-length of light, so that there is a maximum value of this ratio (for solid particles), which in the case of the sun is about 20. Since the repulsion observed in comets' tails is believed frequently to exceed the limit of twenty times the gravitation, this conclusion is of considerable practical interest. Schwarzschild also made an important step in the discussion of the equilibrium of the pear-shaped figure of rotating fluid in connection with Poincaré's theory of exchange of stabilities. In geometrical optics he devised a reflecting telescope with two concave mirrors, which should give a large field of good definition (equivalent to that of the astrographic telescopes) for a focal ratio of 1 in 3. A preliminary memoir, in which, however, he expressly disclaims originality, contains what is probably the most concise and clear account to be found anywhere of the use of the characteristic function (or eikonal) in practical optics. Other branches of research were so diverse as the electron theory and the navigation of airships; for the latter he published suitable tables in 1909, and later devised a special form of sextant. The recent upheaval of the funda-

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mental ideas of physics interested him deeply; a paper by him on the relativity explanation of the motion of the perihelion of Mercury is not yet accessible in this country, and during his last illness he corrected the proofs of a paper on the quantum theory for the Berlin Academy.

The wide range of his contributions to knowledge suggests a comparison with Poincaré; but Schwarzschild's bent was more practical, and he delighted as much in the design of instrumental methods as in the triumphs of analysis. It is interesting, though sometimes misleading, to trace a unity in the varied researches of a man's lifetime, for it often happens that a thread of association binds the whole together. But Schwarzschild's versatility baffles any such summary; his joy was to range unrestricted over the pastures of knowledge, and, like a guerilla leader, his attacks fell where they were least expected. He sought no unity save the one bond which unites together all branches of scientific endeavour. "Mathematics, physics, chemistry, astronomy, march in one front. Whichever lags behind is drawn after. Whichever hastens ahead helps on the others. The closest solidarity exists between astronomy and the whole circle of exact science. . . . From this aspect I may count it well that my interest has never been limited to the things beyond the moon, but has followed the threads which spin themselves from there to our sublunar knowledge; I have often been untrue to the heavens. That is an impulse to the universal which was strengthened unwittingly by my teacher Seeliger, and afterwards was further nourished by Felix Klein and the whole scientific circle at Göttingen. There the motto runs that mathematics, physics, and astronomy constitute one knowledge, which, like the Greek culture, is only to be comprehended as a perfect whole." *

Soon after the outbreak of war Schwarzschild was placed in charge of a meteorological station for the German army at Namur. He was afterwards attached to the artillery staff, at first in France and later on the Eastern front. He received the order of Knight of the Iron Cross. As might be expected, the German army organisation took care to give scope to his scientific ability, and in 1915 November he contributed to the Berlin Academy a paper on "The Effect of Wind and Air-density on the Path of a Projectile": publication has, of course, been postponed. On military service he contracted a disease which ultimately proved fatal. After a long illness, attended with much suffering, he died on 1916 May 11. His lively and attractive character and his readiness to co-operate with others brought him into cordial relationship with astronomers in many parts of the world, and his early death is felt as a deep personal loss.

He was elected an Associate on 1909 June 11. A. S. E.

* Admission-speech before the Berlin Academy of Science, 1913.

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Proceedings of Observatories.

The following reports of the proceedings of Observatories during the past year have been received from the Directors of the several Observatories, who are alone responsible for the same.

Royal Observatory, Greenwich.
(*Director, Sir F. W. Dyson, Astronomer Royal.*)

As explained in last year's report, the activity of the Observatory has been necessarily circumscribed by the war. It has been possible, however, to keep up all observations where continuous record is required. Measurements and reductions are in a more forward state than might have been anticipated.

Transit-Circle.—6243 observations have been obtained. The transits are all reduced to apparent place, and the zenith distances as far as the end of October. The Sun has been observed on 125 days and the Moon on 89 days.

The catalogue of stars down to $9^{m}\cdot0$ in the Oxford Astrographic Zone (Dec. $+24°$ to $+32°$) has been completed. It contains 12,000 stars, with five observations each in most cases. Proper motions have been determined for all but 250 of the stars, for which the material was insufficient. Investigations have been made of parallactic motion, star-streaming, and some questions of stellar statistics.

Altazimuth.—The use of this instrument has been restricted to supplementing the observations of the Moon made with the transit-circle. The Moon was observed 10 times on the meridian and 25 times in other azimuths. With one or other of the two instruments the Moon was observed on 111 days. The mean correction in right ascension required by the *Nautical Almanac* is $+^{s}\cdot92$, and by the *Connaissance des Temps* (depending on the revised Delaunay tables) is $+^{s}\cdot28$.

Cookson Floating Telescope.—215 plates, each exposed on a group of 4 or 5 Talcott pairs, have been taken, and also 6 plates for the determination of scale. The measurement and reduction is well in hand, and it is hoped that the variation of latitude results for 1916 will be ready for publication in March.

Occultations.—8 disappearances and 2 reappearances of stars occulted by the Moon have been observed. The results have been communicated to the Society.

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28-inch Refractor.—529 double stars have been measured by M. Jonckheere. These include—

170	double stars with a separation	< 2″
145	,, ,, ,,	2″ to 3″
120	,, ,, ,,	3″ to 4″
56	,, ,, ,,	4″ to 5″
38	,, ,, ,,	> 5″.

During the year 70 new double stars were discovered, as well as a bright planetary nebula which had curiously escaped spectroscopic surveys.

The telescope was largely used for identification of stars in the preparation of a catalogue of double stars discovered since 1905. This catalogue of M. Jonckheere's was in the printers' hands a year ago, but a delay in publication was caused by fire. It has been set up again, and will be finished very shortly.

Thompson Equatorial.—173 plates have been exposed for parallax determinations and put away without being developed to await a second exposure; 202 plates have been re-exposed at the end of the half cycle.

24 plates of Jupiter's satellites were taken on 14 nights near Aug. 20, and 18 on 13 nights near Dec. 20. These have been taken at the request of Dr. de Sitter, and are in continuation of a series taken for him at the Cape when Jupiter was further South. The first batch has been forwarded to Dr. de Sitter.

6 photographs of Jupiter have been taken for place in connection with Mr. Knox-Shaw's observations of the eighth satellite.

16 photometry plates have been taken of Kapteyn's areas, mostly at declination +15°.

During the year 45 parallax plates have been measured, and provisional values of the parallaxes of 21 stars determined.

Astrographic Equatorial.—The instrument has been out of use most of the year. 54 fields have been taken with short exposures for comparison with earlier photographs. The places of the stars in vol. iii. of the *Astrographic Catalogue* have been compared with earlier catalogues. Also a direct comparison of plates about twenty years apart is being made in the duplex micrometer, with the view of examining all the stars in the *Bonn Durchmusterung* in the Greenwich zone for proper motion. At the request of Mr. Innes, a number of pairs of plates have been sent to him at Johannesburg for the determination of proper motions by means of a blink micrometer.

Photoheliograph.—Photographs of the Sun were obtained with this instrument on 227 days during the year 1916, of which about 20 were days of poor definition. At the Cape Observatory photographs were obtained on no fewer than 332 days during the year ending 1916 October 31; leaving 17 days unrepresented by satisfactory photographs from the combined record of the two observatories. To fill these gaps 11 photographs from Dehra Dûn and 2 from Srinagar have already been received, and it is hoped that

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one or other of the Indian observatories will be able to supply plates for the four remaining days.

The solar activity has continued to increase during 1916; March, June, and October being months specially prolific in Sun-spots, and the year closed with a fine succession of large spots in the N.E. quadrant of the disc. The mean daily spotted area for the whole year may be roughly estimated as not far short of 1000 millionths of the Sun's visible hemisphere, as compared with about 750 for 1915, 152 for 1914, and 7 for 1913.

The great increase in the solar activity and the reduction in the staff of the department has inevitably thrown the measurement and reduction of the solar photographs into arrear. Nevertheless by the close of 1916 all photographs had been measured in duplicate up to and including 1916 May 6, and the reductions of the areas and positions of the spots and faculæ had been completed, and the copy for press of the Daily Results prepared up to the end of October 1915.

Personal.—Three of the junior assistants and nineteen computers are absent on Military Service. Mr. Jones is lent to the Inspection Department of the Woolwich Arsenal, and Mr. Edney to the Hydrographic Office.

Royal Observatory, Cape of Good Hope.
(Director, Mr. S. S. Hough, H.M. Astronomer.)

Transit-Circles.—The programme of observations of the Backlund-Hough stars in conjunction with a selected list of stars from Boss's fundamental catalogue was brought to completion early in November. In connection with this programme the following observations were secured during the year:—

Observations of Right Ascension:—

Number of meridian transits . . .	4557
Determinations of collimation by collimators	84
,, ,, ,, by reflexion and reversal	25
Determination of level	711
,, ,, azimuth	127
Observations of meridian marks . . .	1223

Observations of Zenith Distance:—

Determinations of Z.D.	4981
,, ,, runs	324
,, ,, nadir	698
,, ,, flexure	60

Included with these observations are:—

Observations of the Sun on 139 days.
,, ,, Mercury ,, 53 ,,
,, ,, Venus ,, 93 ,,

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On completion of this programme the instrument was subjected to a complete overhaul, and a new programme of observations of close circumpolar stars, arranged with special reference to the determination of the constant of aberration, was commenced. A sketch of the method of arrangement, which it is anticipated is capable of leading to a determination of the constant referred to, with the minimum of liability to systematic error, and with an accidental error not exceeding $\pm 0''\cdot 01$ as the result of a single year's observations in either element, has been communicated to the Society.

In connection with this new programme the following observations were obtained during the year:—

Observations of Right Ascension:—

Number of meridian transits	159
Determinations of collimation by collimators	3
,, ,, level	27
Observations of meridian marks . . .	44

Observations of Zenith Distances:—

Determinations of Z.D.	152
,, ,, runs	10
,, ,, nadir	26

The programme of observations by Mr. J. Voûte for parallax determinations with the non-reversible transit-circle was brought to completion on August 16.

The observations during the year consist of:—

3039 evening transits } of parallax and
4731 morning transits } comparison stars.
12 transits of azimuth stars.

This instrument was also used for occasional time determinations during the time that the new instrument was undergoing overhaul.

Heliometer.—Observations of the major planets at the time of opposition have been continued as in previous years, and the following obtained:—

Opposition of Saturn * .	17 obs. in distance	on 3 nights.
,, ,, Neptune .	48 ,, ,, ,,	on 6 ,,
,, ,, Mars .	44 ,, ,, ,,	on 4 ,,
	4 ,, ,, position-angle	on 1 night.
,, ,, Jupiter .	12 ,, ,, distance	on 6 nights.
	12 ,, ,, position-angle	
with standard pair of	12 ,, ,, distance	
stars	12 ,, ,, position-angle	

* In addition to 32 observations in distance on 4 nights in the year 1915.

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Victoria Telescope and Astrophysical Laboratory.—The telescope was in use for the photography of stellar spectra on 151 nights during the year.

171 plates, chiefly of long exposure on faint stars, for the determination of radial velocities, were secured, in addition to 19 daylight plates and 16 focus plates. The programme of observations with the 4-prism spectrograph and long camera, commenced in 1908 July, was completed before the end of the year.

Experiments have since been made with the use of a shorter camera with a view to the extension of the programme to fainter stars than those included in the previous list.

In the laboratory 420 plates have been measured with the Hartmann Comparator, inclusive of repetition measures of a considerable number that had been previously measured.

With the "wave-length" measuring machine 23 plates have been measured by Mr. Voûte for the determination of possible displacements of lines in spectra of different star types.

Papers by Dr. Lunt "On the Orbit of the Spectroscopic Binary σ Puppis" and "On the Orbits of the Spectroscopic Binaries α Orionis and α Scorpii" have been communicated to the *Astrophysical Journal.*

The Astrographic Telescope.—With the astrographic telescope the following plates for the determination of stellar magnitude have been obtained:—

	No. of Plates.
Exposures on a single region . . .	109 (218 exposures)
,, on two regions . . .	4 (16 ,,)
Photographs of Pleiades . . .	2 (16 ,,)

Photographs of 20 star regions for the determination of stellar parallax work were obtained, including 51 plates with exposure at a first epoch, 28 with exposure at the second epoch, and 7 with one epoch only.

5 "chart" plates have been repeated for the determination of proper motion.

Of the parallax plates, all which have been developed have been measured by means of the measuring apparatus of the Zeiss Stereo-Comparator. These plates have been obtained and measured by Mr. Voûte in connection with his general programme of observations for stellar parallax. He has also used the same apparatus for the measurements of some 37 photographs of double stars.

Photoheliograph.—Daily photographs of the Sun to supplement the Greenwich series have been taken on 333 days. In all 691 photographs were taken and forwarded to Greenwich. At the request of the Astronomer Royal the duplicate photographs are taken when possible at an interval of not less than 15 minutes after the originals. In this way the existence of small sun-spots, of short duration, has been detected.

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Occultations.—Twenty-six separate phenomena of occultations of stars by the Moon have been observed during the year, viz.:—

Disappearances at dark limb . .	15
Reappearances " " " . .	11

Reductions of Meridian Observations.—The reductions of meridian observations to apparent place have been completed to 1916 December 18 in both elements. The reductions to mean place have been computed for all observations to 1916 February 8, and the mean places formed to 1915 November 30. The results for the years 1912–14 have been entered in ledgers with a view to the formation of a catalogue which will be based on the observations of the years 1912–16.

The detailed results of the observations of the years 1909–11 have been passed through the press. The printers have since reported that the bulk of the type set up has been destroyed by fire. Duplicate copies in proof have since been furnished to them.

The results, together with a discussion, of the meridian observations of the Sun, Mercury, and Venus during the years 1907–11 have been issued as vol. viii. part iv. of the *Cape Annals.*

"*Carte du Ciel*" *Reductions.*—Revision work, consisting of comparisons with the C.P.D. and comparisons of overlapping areas, has been continued.

During the year the discordances between overlapping areas have been examined for 222 pairs of overlapping quadrants, and the identification of the C.P.D. stars for 360 plates. The final copy for press has been completed for 297 plates.

As no overlap was available for the extreme zone ($-51°$), the results for this zone have been verified by a rough re-measurement of all stars down to the 11th magnitude.

Vol. iv. of the *Cape Astrographic Zones*, containing rectangular co-ordinates and measures of diameters of stars contained on the astrographic plates in zone $-44°$, has been passed through the press, but the sheets have been destroyed by fire at the printers. The printing of results has been discontinued during the war.

Counts of stars, grouped according to magnitude, have been made for 184 plates.

In connection with the investigations of stellar magnitude arising out of the conversion of the diameters of the astrographic zones into photographic magnitudes, a statistical paper on "The Question of the Extinction of Light in Space and on the Relations between Stellar Magnitudes, Distances, and Proper Motions" has been communicated to the Society.

Miscellaneous.—The time-signal service, meteorological observations, and seismograph records have been maintained as in previous years.

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Royal Observatory, Edinburgh. Director, Professor R. A. Sampson, Astronomer Royal for Scotland.

General.—Astronomical work has been much restricted during the year owing to occupation of the staff upon war work. Mr. Storey and Mr. Wrigley have been absent throughout the year. For the latter part Miss E. A. Falconer has been absent also.

Transit-Circle and Clocks.—Close attention has been given to time service and the going of the clocks. Two experimental clocks have been added, and are housed in the clock-cellar under circumstances of constant temperature and pressure. One of these is by the Synchronome Company, and is made with Mr. W. H. Shortt's gravity escapement. The other is by Mr. E. T. Cottingham, F.R.A.S., with a version of Gill's escapement. Both have been fitted with scales and reading microscopes, by which the arc can be read to about 1″. These two clocks and Riefler have all shown a high degree of accuracy. Riefler in particular, which was cleaned by Mr. Cottingham in March, has gone admirably. The Synchronome clock was set up in March and Mr. Cottingham's in September. Both promise to be on an equal level with Riefler, if time is allowed for working off certain initial difficulties—for example, the Synchronome clock has shown for several months a daily increase of gaining rate of $0^s.02$, due presumably to shrinkage of the pendulum rod, which is a fully aged and certificated piece of invar. A large amount of information has been acquired as to the barometric equations and temperature equations of these and the other clocks, of the variation of arc with pressure, of rate of decline of arc when the impulse is withdrawn, and other features. Continuous readings of arc and simultaneous comparisons of the clocks upon the chronograph are kept up. These, when analysed, will give further information as to systematic variations of arc and rate, and will serve to isolate anomalous variations of rate, which indeed appear to be very small. One difficulty in connection with precision time-work is the chronograph error. The moving pieces of a chronograph are as a rule fairly massive, and must be started from rest for each record, besides being operated somewhat indirectly. When anomalies occur in a comparison it is usually impossible to say how much may be due to the chronograph up to, say, $\pm^s.02$ or more. To meet this difficulty experiments have been made to record the actual current as it leaves the clock itself; the general principle is that of the oscillograph or Einthoven galvanometer, the weak clock current being carried between the poles of a powerful electro-magnet and deviation recorded photographically. As is known, such a method can be worked so as to be entirely free from relative lag. Encouraging results have been obtained, and it is hoped to develop a practical system that will permit reliable comparisons to be made much within the above limit.

Wireless Time-Signals.—The rhythmic time-signals from Paris have been taken on 246 nights and the comparisons telegraphed to

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Paris Observatory. The mean difference found is *Paris–Edinburgh* = $+0^s \cdot 02$, compared with $+0^s \cdot 05$ last year.

Solar Spectroscope.—Six plates of good quality have been obtained during 1916, each with about 10 exposures on East and West limbs at the Sun's Equator. The region is the neighbourhood of λ6300.

Equatorials.—No work has been done with these during the year, but preparations have continued with the purpose of taking up photoelectric determination of magnitude. Mr. Baker has made and filled some experimental cells. Great trouble has been found in securing seals for the platinum electrodes which should not crack. Several seals made by professional glass-blowers have, without exception, broken down.

Astrographic Chart. — Calculation of plate constants and standard co-ordinates for certain positions remains to be done, and has not been advanced during the year. Counts have been made of the stars of different magnitudes, and the results are now under examination.

Seismograph.—Observations were made regularly, the records read, and the results communicated to Shide.

Meteorological Observations were made regularly as heretofore.

Armagh Observatory.

The work was continued on the same lines as during the last few years until the end of September. Dr. Dreyer then resigned the office of Astronomer, which he had held for 34 years, and removed to Oxford in order to devote his whole time to the new edition of Tycho Brahe's works.

Mr. J. A. Hardcastle has since been appointed to the vacant post, and will assume charge of the Observatory in the Spring. In the meantime the meteorological observations only are being continued.

Cambridge Observatory. (Director, Professor A. S. Eddington.)

Both the assistants have been absent on war service throughout the year, Mr. Hartley as Naval Instructor, and Mr. Entwistle as Second Lieutenant. There is consequently very little progress to report.

Some photographs of short-period variable stars have been taken with the Sheepshanks telescope, and a series consisting of 129 ten-minute exposures on SU Aurigæ has been completed. Preliminary trials of various methods of comparing the diameters of

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the images have been made, and the systematic measurement of the series is now in progress.

Demonstrations in practical astronomy have been given to students by the Director on evenings during the two winter terms.

By the kind assistance of Mr. J. A. Aldis and Mr. C. P. Butler it has been possible to continue the Saturday evening exhibitions of celestial objects with the Northumberland Equatorial. The attendance in the Lent term was often large, but during the present winter the weather has been persistently unfavourable.

Solar Physics Observatory, Cambridge University.
(Director, Professor H. F. Newall.)

Spectroheliograph.—Photographs of the Sun's disc in K_{2-3} light have been obtained on 108 days, and photographs of the prominences at the limb on 92 days.

The solar activity continues to increase in general, but several intervals of comparative quiescence with respect to spots and flocculi have occurred during the year.

The 18-inch and 6-inch mirrors were dismounted on April 12, and were replaced on April 20 after being resilvered.

The 12-inch photovisual objective was dismounted on October 26, and after the deposit which had gradually accumulated during recent years on the surfaces of the component lenses had been cleared away, it was remounted on the same day.

For the period 1915 July 2–1916 June 30, the Director of the Kodaikánal Observatory has sent 315 spectroheliograms showing the solar disc in K_{2-3} light. Of the 51 days missed in the Kodaikánal records during this period, the Cambridge record has photographs for 22 days.

Photoheliograms.—Daily photographs taken with the photoheliograph at Dehra Dûn have been received, and the negatives are stored at the Science Museum, South Kensington. Positive paper prints have also been received from Dehra Dûn.

Solar Spectrum.—A revision of the origins assigned in Rowland's Tables of Solar Wave-lengths has been made in the region of the spectrum ranging from λ3900 to λ5900. Since Rowland's Solar Tables were published in 1895–98, many records of laboratory wave-lengths have been published, and reference to these has shown that although the vast majority of Rowland's identifications are confirmed, there is a fair proportion which, on the available evidence, must be revised. Many lines left unidentified by Rowland have been traced to their chemical origins by reference to the new work.

Stellar Spectra.—A long series of spectra of various carbon compounds has been investigated, chiefly with respect to their relation to celestial spectra of various types.

Work in the Laboratory.—The work in connection with the

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hydrocarbon spectrum has been continued. The photographs taken with the high dispersion spectrograph which required very long exposures, from eleven to fifty hours, suffered in definition in consequence of the shifting of the spectrum during exposure. This has been in large measure remedied by providing round the prism-box a felt jacket and an electric heater arranged in coils in the cover.

A preliminary note announcing the identification of Fraunhofer's Group G in the solar spectrum with the hydrocarbon band λ4314 was published in the *Monthly Notices* (vol. lxxvi. p. 640).

For an investigation dealing with the determination of the origin and conditions of occurrence of the various types of carbon spectra, a suitable spectrograph of small dispersion, capable of recording the spectrum from λ6600 to λ3600, has been successfully elaborated, mainly from portions of the physical apparatus presented by the late Lady Huggins. It now gives satisfactory definition throughout this long range of spectrum.

Investigations in Atmospheric Electricity.—Investigations relating to atmospheric electricity have been continued. In a paper "On Some Determinations of the Sign and Magnitude of Electric Discharges in Lightning Flashes," published in the *Proceedings of the Royal Society* (vol. xcii. A, pp. 555–574), Mr. C. T. R. Wilson discussed the results of about 100 determinations of the sudden changes produced in the vertical electric field by lightning discharges. In the case of about 50 of these discharges it was found possible to get measurement of the distance of the lightning from the place of observation by timing the arrival of the sound of the thunder. From the data obtained, an estimate was deduced of the average quantity of electricity discharged in lightning flashes; this amounts to about 30 coulombs per flash. The paper contains an account of the apparatus and methods used at the Observatory for the investigation of atmospheric electricity.

The apparatus needed for securing automatic photographic records of the electrical effects has now been installed and is found to work well.

Personal.—The Assistant Director, Mr. Stratton, has been with the British Expeditionary Force in France since 1915 July. He was gazetted (1916 April 8) Major in the Royal Engineers (20th Signal Company), was mentioned (second time) in Sir Douglas Haig's despatches, and was awarded the D.S.O. on 1917 January 1.

The First Junior Observer, Mr. W. E. Rolston, who received his commission and left the Observatory in 1915 June to join the 2/5th Weald of Kent Battalion (The Buffs), was gazetted Captain, 1916 April 12.

The Second Junior Observer, Mr. W. Moss, took up duties on 1916 March 29, in the Inspection Department at Woolwich Arsenal.

The Resident Attendant, W. H. Manning, has been engaged since 1915 May in munition work.

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Dunsink Observatory.
(*Director, Prof. H. C. Plummer, Royal Astronomer of Ireland.*)

It was mentioned in last year's report that the weather of 1915 was considered more unfavourable for observations than in previous years, though in the absence of comparative records this was a matter of general inference only. It can now be stated definitely that the conditions in 1916 were distinctly worse than in 1915, and gave a total of 49 fine nights as against 80 in the previous year. As a "fine" night merely implies a clear interval of some hours' duration, with no regard to the quality of the seeing, and a considerable deduction must be made for bright moonlight and for the very short nights of midsummer, it will easily be understood that the photographic work on variable stars has been carried on under the greatest difficulty. A total of 193 plates have been taken on 57 nights, and 170 plates have been measured.

During the year papers on three variables in one region of Lacerta, including V and X Lacertæ, on the eclipsing binary TT Aurigæ, and on the irregular variable RS Camelopardalis, have been prepared and communicated to the Society. Other miscellaneous work also published includes a statistical investigation of the Minor Planets.

The time-ball in Dublin has been re-erected in a position at the end of the South Wall, where it is visible to the shipping in the river. Determinations of time were made with the Pistor and Martins meridian circle on 76 nights. The legal adoption of standard time in Ireland on 1916 August 23 is an event which deserves mention.

The staff, consisting of the Director and Mr. Martin as assistant, remains unchanged.

Glasgow Observatory.
(*Director, Professor L. Becker, Assistant, James Connell.*)

The Director has been on leave of absence since the end of March. The usual routine work connected with the time service and meteorological observations has been carried on by the assistant.

At the beginning of June a number of chronometers and watches were deposited at the Observatory by the Admiralty; these are compared daily with the Standard Mean Time Clock and their errors and rates derived.

The Astrophotographic lens belonging to the large spectrograph has been lent to the War Office.

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Liverpool Observatory, Bidston.
(Director, Mr. W. E. Plummer.)

The work of this Observatory has been both hampered and curtailed on account of the severity of the police regulations mentioned in the last report. These inconveniences continue, and the observations have been confined to those which are necessary to ensure the accurate determination of time-signals and to fulfil other services of a routine character which this Observatory has been accustomed to render to the port and the mercantile marine.

In this connection it may be mentioned that the number of chronometers deposited for test has been less than at any time within the last twenty-five years—an index of the effect of the war in decreasing the tonnage using the port.

The meteorological and seismological work, not being disarranged by the untoward conditions that have affected the astronomical, has been pursued without interruption, but that does not call for any particular mention here.

The diminished activity of the Observatory in its several departments of usefulness is entirely attributable to war conditions.

Radcliffe Observatory, Oxford.
(Director, Dr. Rambaut, Radcliffe Observer.)

The programme of work with the 24-inch refractor on the proper motion of stars in Professor Kapteyn's "Selected Areas" has been carried on during 1916 on the same lines as in recent years—so far at least as the reduction of the staff and bad weather would permit; but, owing to difficulties arising from these two causes, the year has not been a successful one.

Most of the areas of which photographs were wanting in order to complete the programme come into position in the autumn and early winter months, and during the recent autumn the state of the sky has been peculiarly unfavourable, so that, although special early morning watches were again arranged during the months of September, October, November, and December, with a view to overtaking arrears, and although no effort was spared to utilise every possible opportunity, the number of these outstanding fields, though slightly diminished, is still considerable. During this period only 5 plates could be secured, and of these two were considered unsatisfactory. The exacting character of this work renders it peculiarly difficult in a climate like that of Oxford. During the year 25 proper motion photographs were secured —each plate requiring two exposures of 30 minutes. Unfortunately, 5 of these plates are incomplete or imperfect, leaving only 20 plates which are considered good enough for the purpose in hand. Twelve plates, exposed but not developed, were packed in an air-tight box, to be exposed to the same region of the

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sky when ten years have elapsed. This makes twelve dozen proper motion plates which have now been stored in this way.

Observations were made on June 2 and 6, by the aid of the 18-inch refractor, for the purpose of determining the position of the polar axis of this instrument. These observations show that the errors in azimuth and altitude have changed very slightly since the axis was last adjusted on March 17 of the previous year, and are still less than $5''\cdot5$ in either co-ordinate. This result indicates a very satisfactory degree of stability in the mounting of the instrument.

The 10-inch Barclay equatorial has been used by Mr. Barrett as in former years for observing a selected list of variable stars. Four stars have chiefly engaged his attention during the year, viz. ST Draconis; VV Aurigæ; 4, 1913 Tauri; and 5, 1913 Arietis. Of these stars 310 observations (involving 985 comparisons of brightness) on twelve nights have been secured.

The work of reducing and preparing for press the results deduced from parallax plates taken in recent years with the 24-inch refractor has proceeded but slowly, chiefly in consequence of the reduction of the personnel of the Observatory. No astronomical computations whatever could be effected during the two holiday months, July and August, as the meteorological duties and the reduction of the meteorological observations occupied the whole attention of the available members of the staff. The reading of proof-sheets of a new volume of observations also occupied a good deal of time. With these interruptions, Mr. Robinson has been engaged chiefly upon the astronomical reductions. Part of his work consisted in determining the photographic magnitude of each star observed for parallax by comparison with a polar plate. In this way 5650 separate determinations of magnitude for 2390 stars have been made, their values carried into the card-catalogue, and the means formed. The weighted means of the separate results for the parallax of each star have also been taken, and their probable errors computed. An investigation has also been made into the possible cause of errors depending on magnitude which affect some of the parallax results.

Volume li. of the *Radcliffe Observations* was printed and distributed early in the year. This volume, containing the usual meteorological results for the years 1911–1915, includes also the observations of underground temperature at the Radcliffe Observatory made daily, by means of platinum-resistance thermometers, during the twelve years 1898–1910, with a discussion of the principal results to be deduced from them.

University Observatory, Oxford.
(*Director, Professor H. H. Turner.*)

The reductions for the Vatican plates of the Astrographic Catalogue have gone on steadily, being only limited by the supply of measures from Rome. Zone $+59°$ was completed, and the first

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twelve hours of zone $+58^\circ$ are fully reduced, which is as far as measures are available. Standard co-ordinates have been computed for $+57^\circ$, $+56^\circ$, and as far as 17^h in $+55^\circ$, leaving only seven hours' standard co-ordinates to complete the whole Vatican portion ($+64^\circ$ to $+55^\circ$) in this respect. The second printed volume of measures (zone $+63^\circ$) was issued in April last; and the first third of the volume for $+62^\circ$ is in type. The war has naturally delayed the printing and interfered with the work generally, but the amount accomplished during the year in spite of various delays may be regarded with satisfaction.

The analysis of meteorological observations has been continued during the year, the chief results being summarised in two papers to the Royal Meteorological Society (*Q.J.*, xlii., 1916, p. 163, and xliii. p. 43). The first paper dealt with Sunderland rainfalls, and Paris rainfalls and temperatures, showing the confirmation of the "chapters" in all these series. The second paper was an analysis of rainfall at twenty-eight stations scattered over Europe for the years 1850–1905, showing confirmation of the "chapters," giving a closer specification of the separating dates, and showing also confirmation of the five-month period which had been found in earlier papers. Many other series have also been reduced and are nearly ready for publication. It seems clear that no analysis of meteorological records over a long series of years can be effective unless account is taken of this division into separate chapters, and, further, that the study of the chapters may be fruitful in other directions. Mr. Bellamy has nearly completed a laborious analysis of the *daily* readings of barometer at the Radcliffe Observatory, Oxford, from 1853–1915; and by the courtesy of Dr. Dreyer and the Rev. Charles Faris has collected some of the Armagh records (mainly unpublished) for the eighty-four years from 1833 to 1916.

The Director has continued his superintendence of the seismological work at Shide. A number of Milne-Shaw machines are now being made for replacing the early Milne pendulums, the expense being partly defrayed by a generous private benefaction. The best records for 1914 have been fully discussed, and derived corrections to the tables for P. and S. are nearly ready for adoption. They indicate a curious anomaly near the epicentre, probably due to the anomalous structure of the earth's crust; and the corrections carry with them the possibility of double reflection of waves, which seems to afford an explanation of certain observed discordances at distant stations. The discussion is given fully in the 21st Report of the Seismological Committee to the British Association.

The editing of Baxendell's Variable Star Observations, in collaboration with Miss Blagg, is approaching completion. T Herculis (revised), R Persei, R Sagittæ, R and T Serpentis, T Tauri, and R and S Ursæ Majoris have been dealt with during the year. The conclusion that the Argelander-Chandler terms are not justified by observation has been steadily confirmed, and was adopted by the Rev. T. E. R. Phillips in his able address to the British Astronomical Association.

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Only one further note (dealing with the single Melbourne zone −65°) was published during the year on the astrographic magnitude scales; but a number of counts were made and are nearly ready for publication.

Three more notes on stars with large P.M. have been added to the series: the ninth and eleventh by Mr. R. J. Pocock of Hyderabad, who measured some Oxford plates while his other work was brought to a standstill (owing to loss of plates in the *Persia*); the tenth note by Mr. Bellamy and Mr. J. D. M'Neile of Wellington College.

The number of students in Oxford at present is of course small, but there have been various calls for popular lectures to troops, which have been promptly met.

The Observatory has to record with sorrow the death of Mr. John Mullis, who was in its service forty years, and performed a variety of duties with great efficiency.

The Coats Observatory, Paisley. (Convener, Dr. Ferguson.)

During 1916 the work of the Observatory—astronomical, meteorological, and seismological—has been carried on as usual, without interruption. Climatalogical observations are reported regularly to the Meteorological Office, and the recording instruments have been kept in good order.

Temple Observatory, Rugby School.
(Director, Mr. G. M. Seabroke.)

This Observatory has been used for educational purposes as usual during the year, and the attendance of boys has been good. No one has had time during the year to do more than carry on school work.

The Hill Observatory, Salcombe Regis, Sidmouth.

The work on the spectra of the fainter stars has been continued, the working list containing all the stars of the revised Harvard photometry between magnitude 5·0 and 5·59. In Bulletin No. 5, a new catalogue of 287 stars was published in November.

In September the third assistant, Mr. H. E. Goodson, left for war service, and as he was the only assistant remaining, the others having joined the colours, the Observatory was closed. The services of Mr. Lake, as caretaker, were retained to keep the instruments in proper condition and to set going the electric heaters in inclement and specially damp weather.

The buildings and instruments are all in order.

The Hill Observatory was incorporated by the Board of Trade in 1916 July 25. A copy of the Memorandum and Articles of Association has been sent for the information of the Society.

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Imperial College, South Kensington. (*Professor A. Fowler.*)

The astronomical instruments were chiefly used for the instruction of students, but occasionally also for spectroscopic observations of the Sun.

Research work in the spectroscopic laboratory has been mainly directed to the enhanced lines of silicon, carbon, and oxygen in relation to their occurrence in stellar spectra, and with a view to the identification of series lines. The region covered by the investigation is from λ1900 to λ7200, and the spectra have been photographed under a great variety of experimental conditions. In the case of silicon, attention has already been drawn to the identification of four additional lines of the fourth group in B stars (*M.N.*, lxxvi. p. 196). Other results of considerable interest have also been obtained, and will be published when the determination of wave-lengths is completed. Mr. J. S. Clark and Mr. J. Brooksbank, of the Physics Department, have collaborated in this work.

The demonstrator, Mr. W. E. Curtis, is still absent on military service, having received a commission in the Royal Engineers on his return from Gallipoli. The acting demonstrator, Mr. F. S. Phillips, has been almost entirely occupied in the X-ray department of the War Office. Other work connected with the war has also absorbed a considerable amount of time.

Stonyhurst College Observatory.
(*Director, Rev. W. Sidgreaves, S.J.*)

In our last report we had the satisfaction of acknowledging the kind permission of the late Postmaster-General to re-erect our radiotelegraphic apparatus. Now we have to express our regret that the Military Authorities have requested the suppression of the installation. We have pressed our claim to an exception in our favour from the general policy against private wireless installations, but without avail.

The meteorological, magnetic, and seismological observations have been carried on as usual.

The solar surface has been observed on 215 days, and 211 drawings on 208 days have been added to our collection. Of these 183 are complete, as showing both the spots and the faculæ, and 25 are complete for all spots, but wanting the faculæ. On 4 days notes with rough sketches of the spots were made. On 2 days only the Sun's surface was found to be free from spots. The mean disc-area of the spots (in units 1/500th of the visible surface) for the year was 4·52, which is almost identical with the value 4·51 for 1915. The drawings during the summer months were made by Mr. W. M'Keon, who paid particular attention to the accurate delineation of the faculæ. Twenty-nine of such draw-

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ings, when the weather conditions were particularly favourable, are of excellent quality. They range between the dates May 28 and September 24, and it would be interesting to compare them with spectro-heliograms in H_a and K_3 light. A new light and rigid metal projection apparatus, constructed of an aluminium collar and bicycle rods, designed and made by Mr. G. J. Gibbs, was fitted to the equatorial at the beginning of the year.

The spectra of a few spots were photographed, in the region C – D, with the large grating spectrograph, and a few visual observations were also obtained. These observations confirm the essential permanence of the sun-spot spectrum since the year 1882.

Owing to want of assistance, due to the war, work with the Whitelow prismatic camera has been in abeyance; but, with the Thorp prism equatorial, spectra have been secured of *o* Ceti at two maxima, *a* Orionis, *a* Canis Minoris, *a* Leonis, the Nebula in Orion, and of the planets Mars and Saturn.

Our Belgian guest Dr. Henroteau left us at the beginning of the year to take up a position at the Ann Arbor Observatory of the University of Michigan.

Wolsingham Observatory (Rev. T. E. Espin's).

The examination of stars in the selected zones has been continued during the year with the 24-inch Calver reflector, and particulars of the new double stars detected have been communicated to the Society, together with a list of micrometrical measures of miscellaneous double stars already known, but of which there were few or no measures.

Mr. A. M. Newbegin's Observatory, Wallington, Surrey.

Observations of the Sun during 1916 were hindered in the early part of the year by war work and throughout the whole of the year by very unfavourable weather. The results are given in tabular form :—

Number of observations	= 100.
,, ,, prominences	= 1255.
Daily average	= 12·55.

Distribution :—

Prominences on N. hemisphere	= 641.
,, S. ,,	= 614.
,, E. limb	= 661.
,, W. ,,	= 594.
Number of prominences under 30″ in height	= 281.

Metallic prominences have been observed on 51 days, and at Prof. A. Fowler's suggestion an attempt has been made to classify them under three heads. It is too early to discuss this idea at

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present, but there is no doubt of the differences in these metallic outbursts.

The prominences as a whole have shown greater strength than in 1915, not only in density, but in profile area; this, together with the slightly increased daily average, shows the upward tendency of activity very clearly. Apparently the activity has advanced to the northern hemisphere once more, in accordance with previous experience.

Many very fine outbursts have been observed, one on Nov. 17 attaining a height of 360″. It was, however, of quite short duration.

The 5-inch Cooke astrographic lens lent by Mr. Percy Morris has been in use on many nights during the year.

Mr. G. J. Newbegin's Observatory, Lyndale, Sutton, Surrey.

Owing to the very cloudy conditions of the year 1916 only 63 solar observations for prominences were made. By these observations 522 prominences were observed and noted down. A mean daily average of 8·3 was thus obtained.

The position of prominences has now become more northerly though not excessively so, but indicating a movement in that direction.

The planet Venus was a very prominent object in the spring months, and was well observed on many occasions. Later in the year came Jupiter and Saturn, the rings of the latter having appreciably commenced closing again.

Rousdon Observatory, Devon.
(Sir Wilfred Peek's; Mr. C. Grover, Observer in charge.)

The building is kept in good repair and the exterior was repainted in the spring. The equatorial is in perfect working order. Weather has been very changeable. April, May, and November were the best observing months, at other times conditions were so bad as seriously to interfere with the continuity so desirable in Variable Star work. Observations were made on 131 nights. Observations of the long-period variable stars have been continued on the same lines as in former years, and 402 magnitudes were determined, making a total of 13,269 magnitudes during the 31 years this work has been in progress. 485 maxima and 348 minima have been observed. The light curves of many of these stars show large and well-marked irregularities, with a difference in light at maxima and minima of fully two magnitudes; the periods also show wide differences, and the dates of maxima and minima very rarely agree with the predictions.

Observations of stars with the transit instrument are frequently taken for regulating the sidereal clock. The usual meteorological observations have also been regularly continued.

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The Rev. T. E. R. Phillips's Observatory,
Headley, Epsom, Surrey.

At the beginning of the year the Observatory was undergoing removal from Ashtead to Headley Rectory. The Coleman refractor belonging to the Society was dismounted and re-erected by Messrs. Cooke & Sons, the original observatory from Dover being again utilised, with the addition of a small transit house. A new observatory with 17 ft. 6 in. dome was built to cover the 12½-inch Calver equatorial. This is a good deal larger than is required for this instrument, but it is hoped that a reflector of greater aperture may eventually be installed. The meteorological instruments were also re-established and were in use by the end of the first week in January. The telescopes were not in full working order till the end of February.

The sidereal clock in the Coleman observatory at first gave a good deal of trouble. It was found that when the cord carrying the weight attained the same length as the pendulum, the weight took up the vibration, and the rate of the clock was altered by a considerable amount. A much firmer support of masonry was therefore erected and the trouble largely removed, but it may be found necessary to carry the weight outside the clock to eliminate it entirely.

The Observatory is on high ground about 575 ft. above sea level, and the increased altitude appears to make a perceptible difference in the general character of the seeing. Mist is sometimes experienced in the winter months from low-lying clouds when the valleys are free from fog, but it seems that a larger proportion of the clear nights give good definition. At times the seeing was astonishingly good, and on the whole the experience of the first year is most encouraging.

As regards the programme of work, the Coleman equatorial was again employed in the measurement of double stars, and both instruments in the observation of Mars, Jupiter, and comets. An investigation with the reflector of the colour and visual magnitude of a number of stars within 25° of the pole which show a considerable discrepancy between the B.D. and photographic magnitudes is also in progress.

The main work of the year, however, has been the study of the light curves of 80 long-period variables. These have been analysed harmonically, and a discussion of the results has been communicated to the British Astronomical Association in a Presidential Address at the Annual Meeting in October. It was found on reducing the phases of the first three harmonics to the maximum of the first as reference point, and plotting those of the second and third as abscissæ and ordinates, that with very few exceptions the stars fall into two separate groups. These groups are distinguished not only by the peculiar relationships of their harmonics, but also in some degree by differences in range, brightness, period, and coefficients.

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A study has also been made of the oscillations of the maximum of R. Ursæ Majoris, and the results are set forth in the same address.

Mr Reynolds' Observatory, Harborne, Birmingham.

Owing to causes connected with the war, very little work has been done with the 28-inch reflector during the year. A good series of photographs of Jupiter were, however, secured in the autumn of 1916, which show considerable detail. The methods adopted were the same as at the previous opposition, and transparencies of the photographs will be presented to the Society.

Mr. F. Sargent's Observatory, Eastville, Bristol.

The usual planetary observations have been continued, the chief object of study being Jupiter. For the present opposition, observation was commenced during the early hours of July 8, since when the planet has been observed on every favourable occasion, and numerous transits of surface markings timed.

Adverse weather conditions have reduced the number of observing nights, nevertheless the transit total of the previous opposition has been already far exceeded, and interesting results may be anticipated.

An excellent filar micrometer by Cooke of York has been added to the 5-inch refractor, and, within the limits of the equipment, a beginning has been made at double-star measuring.

Sweeping for comets has not been neglected, and will be continued.

Early in the year the mirror of the 10½-inch reflector was refigured, with excellent results.

Mr. W. H. Steavenson's Observatory, West Norwood.

During the earlier part of the year the three comets, Taylor, Mellish, and Neujmin were regularly observed. The last named exhibited a marked fluctuation in brightness at the end of April.

The examination of Struve's pairs of equal magnitude, begun in 1915, was continued with the 15-inch reflector in 1916. Several cases were met with in which the magnitudes are now considerably unequal.

Several multiple stars were regularly observed for variability, notably the system of ε Lyræ, and a faint quadruplet in Pegasus, which, falling in the Oxford astrographic zone, was suggested by Prof. Turner for examination.

During April estimates of the magnitudes of 97 of the brightest

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stars in the cluster N.G.C. 2632 (Præsepe) were made with the 15-inch. The results have been communicated to the B.A.A.

Barnard's proper motion star was observed several times during the summer with a view to determining its visual magnitude.

Observations of Nova Persei, made in the autumn, showed that the star was still variable, though less so than in 1915. The total range of brightness was 0·6 mag., the average magnitude being about 13·4. To averted vision the Nova appeared to be surrounded by a nebulous glow, 10″ or 15″ in diameter. This was not noticed in 1915.

Helwân Observatory, Ministry of Public Works, Egypt.
(Superintendent, Mr. H. Knox-Shaw.)

Time-Service.—The time-service has been maintained as usual.

Reynolds Reflector.—The telescope was used on 93 nights during the year, and 100 plates were exposed.

30 plates on the variable nebula, N.G.C. 6729.
44 ,, ,, 40 other nebula.
8 ,, ,, J VIII.
12 ,, ,, stars of large proper motion.
5 miscellaneous plates.

In addition, four photographs of type A stars were taken with the prismatic camera.

A set of 20 lantern slides of the variable nebula has been presented to the Society, and they were shown at the June meeting of the British Astronomical Association. The plates taken of this nebula in 1916 show nothing strikingly new. They confirm the complexity of the character of the variation of the nebula and R Coronæ.

Solar Radiation.—The programme for 1915 was continued, and complete observations were taken on 143 days during the year. It has not been possible to discuss the 1915 observations yet, but they seem to confirm the results obtained from those of 1914.

General.—Bulletins 17 and 18 were published during the year, the former containing a discussion of the observations of solar radiation made during 1914, and the latter positions of J VIII from photographs taken at the oppositions of 1913 and 1914. A short note was communicated to the Society on the variable nebula N.G.C. 6729. During the second half of the year the Superintendent was working a large part of his time at the Physical Service, and thus little work other than that of a routine character has been done. While he was on leave in the summer, his place was taken by Mr. E. W. Bliss. The same meteorological, seismological, and magnetic observations were made as in 1915.

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Nizamiah Observatory, Hyderabad, Deccan.
(Director, Mr. R. J. Pocock.)

At the beginning of the year the astrographic work was temporarily in abeyance owing to the fact that plates ordered from England some time previously were lost with the S.S. *Persia*. A further batch of plates was lost with the S.S. *Maloja*. The first two months of the year were therefore mainly devoted to measuring plates taken at the University Observatory, Oxford, with a view to the determination of proper motions; the results of this investigation have already been communicated to the Society.

Apart from this delay, astrographic work has continued steadily throughout the year.

The total number of plates measured during the year is 211, containing 70,278 stars, which with those measured previously makes a total of 322 plates, containing 115,233 stars.

Zone $-17°$ is now complete, and the copy for press of the whole is in the hands of the printers. The 180 plates in this zone contain 63,439 stars, an average of 362 stars per plate. The printing of this zone was seriously interfered with by the fire at Messrs. Neill & Co.'s works in Edinburgh, in which about 80 pages of corrected proofs were destroyed; by the end of the year, however, 166 pages of first proof had been read and returned to the printers, and in the absence of any further delays this volume should be ready for distribution during 1917.

In Zone $-18°$, 153 satisfactory plates have been secured, of which 142, containing 51,794 stars, have been measured, reduced, and copied for press; it is hoped that the printing of this zone may be started very shortly.

In Zone $-19°$ the whole of the necessary tables and standard co-ordinates have been completed, and 53 satisfactory plates secured.

In Zone $-20°$, 14 satisfactory plates have been secured, but no measures had been made of plates in Zones $-19°$, $-20°$ up to the end of 1916.

The four lenses lent to the Kodaikánal Observatory are still retained at Kodaikánal on loan.

Kodaikánal and Madras Observatories.
(Director, Mr. John Evershed.)

Dr. Royds was in charge of the Observatory from the beginning of the year until November 23, the Director being on special duty in Kashmir.

The year has been a favourable one for solar work, and visual observations of the Sun's limb with the 6-inch telescope and large grating spectroscope were made on 315 days, while sun-spots were recorded on 342 days.

Cambridge Spectroheliograph.—Very satisfactory photographs

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were obtained throughout the year whenever the definition was good; that is, usually before 8.30 a.m. Disc spectroheliograms were obtained on 329 days and prominence plates on 346 days, including plates taken at Srinagar. All the prominence plates were measured.

Grating Spectroheliograph.—Photographs of the Sun in Hα light were obtained on 258 days. The plates for this work are now sensitised at the Observatory by the photographic assistant, and are superior to commercial red sensitive plates. A special apparatus has been constructed for drying the plates after sensitising. The number of absorption markings due to dense prominences on the Sun's disc has increased largely, and some very remarkable forms were photographed in April.

Photoheliograph.—This was dismantled in 1915 for work in Kashmir, and no direct solar photographs were obtained at Kodaikánal in 1916. The series of 8-inch photographs will be resumed in 1917. At Srinagar a series of 8-inch photographs was obtained during the earlier months of 1916. In May the instrument was modified to give a larger scale, and during the summer sun-spot regions were photographed on a scale of 15 inches to the diameter.

Grating Spectrograph.—Dr. Royds has employed this instrument for studying the solar displacements at the centre of the disc, and at the limb, of the nickel and titanium lines, using these metals in the electric arc for comparison with the solar lines. The plates obtained have all been measured and reduced, and the results were ready for publication at the close of the year. In general these results confirm those obtained with iron, and indicate a low pressure in the reversing layer, and a descending movement of the gases at the centre of the disc. Spectrum plates were also obtained in continuation of the research on the displacements in the Sun of lines which are greatly shifted at the negative pole of the arc.

During December the spectrograph was modified for the purpose of photographing the spectrum of Venus. In this work the 15-inch Hyderabad lens, after its return from Kashmir, was used to great advantage, and spectra were secured of the planet having a dispersion of 0·7 A. per millimetre. Measures of the plates by the positive on negative method will probably yield a fair value of the solar parallax, but the main purpose is the determination of the wave-lengths of some of the solar lines on the side of the Sun turned 90 degrees or more from the direction of the earth.

The research on the change of wave-length of the iron lines between the centre of the Sun's disc and the limb has been completed and published (Kodaikánal Observatory Bulletin No. xlix.), and the sun-spot radial motion plates obtained in 1915 have been measured and the results published in Kodaikánal Observatory Bulletin No. xli.

A number of measures of solar and arc spectra has also been accomplished for the purpose of testing the anomalous dispersion theory. The results show fairly conclusively that anomalous

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dispersion is not an effective agent in displacing solar lines (*Observatory*, vol. xxxix. 432).

Kashmir Expedition.—The purpose of this expedition not having been fully accomplished by March 1916 owing to very abnormal weather conditions, the Government of India sanctioned an extension of the work for a further period of seven months at the request of Dr. Walker. The two assistants, Messrs. Sitarama Aiyar and Krishna Aiyar, who had rendered such excellent service during the earlier period, returned to Kodaikánal in March, and the Director and Mrs. Evershed continued the work at Srinagar until November 1st.

The results obtained during the summer of 1916 amply confirmed the original estimates of the excellence of the climate for solar work. During the winter and early spring, as also in late autumn, the conditions do not differ very appreciably from those in other localities, the definition being good in the morning and evening and poor at midday. In May a marked improvement occurs in the midday seeing, which appears to coincide with the flooding of the paddy fields. In the summer months good definition throughout the day is the rule, and superlative definition is of quite frequent occurrence. Very beautiful solar photographs were secured in July and in August under temperature conditions ranging from 80° to over 90° in the shade.

The two principal factors which it is believed conduce to the good definition are the absence of disturbing winds excluded by the surrounding wall of high mountains, and the very large areas of wet cultivation which in summer greatly reduce the heating effect of the Sun on the soil.

Publications.—Four Bulletins were issued during the year, Nos. xlix. to lii.

At Madras the time-service was maintained as in former years.

The Dominion Observatory, Ottawa.

Geophysics.—This branch of the activities of the Observatory comprises seismology, terrestrial magnetism, and gravity. The equipment of the three divisions consists of the same instruments as reported in the preceding year.

Twenty-two bulletins of earthquakes recorded at the Observatory were issued during the calendar year 1916, representing 104 seismic disturbances. During the preceding year only 81 earthquakes were recorded, so that 1916 shows an increase of 23. There was issued early in the year in the *Publications of the Dominion Observatory*, No. 2, vol. iii., Seismological Tables of sixty-one pages and two diagrams, by means of which seismograms can be more readily interpreted and the location of the epicentre is facilitated. The study of microseisms, which are often a disturbing factor in reading seismic records, was continued in connection with

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the records of the undagraph exposed to the action of the waves of the Atlantic at Chebucto Head, near Halifax, Nova Scotia.

The magnetic survey—declination, inclination, and intensity—of Canada was continued and 55 stations were occupied, making a total now of 479 stations distributed across Canada from the Atlantic to the Pacific. Quite a number are "repeat" stations.

The gravity survey of Canada was suspended for the year. There are now 45 gravity stations in Canada between the Atlantic and Pacific. The observations are all standardised with those of Washington, and through the latter with the European work, by observations with the Observatory pendulums on several occasions in Washington.

The reduction of the observations follows closely the method adopted by the U.S. Coast and Geodetic Survey.

Meridian Observations.—Observations with the meridian circle were obtained on 103 nights: they were devoted in part to determinations of clock error for longitude work and the purposes of the time-service, and in part to a continuation of work on the list of latitude stars mentioned in previous reports. Two observers were engaged throughout the year, and two others for shorter periods. During the last eight months of the year a modification of Hough's printing chronograph was employed for the registration of transits, and has worked very satisfactorily.

Four Laplace stations were established, two in British Columbia, one in north-western Ontario, and one in New Brunswick. The longitudes of three of these were determined by the ordinary telegraphic method, that of the fourth by wireless telegraphy: at three of the stations the azimuth of one of the lines of the geodetic triangulation was measured with an alt-azimuth, at the fourth a meridian mark was established and its azimuth determined by observations with the astronomical transit used for the longitude.

A new list of 519 stars has been prepared for use in field longitude operations. It includes stars between 10° north declination and the pole, and is somewhat more extended than those found in the national ephemerides, especially in the case of stars of comparatively high declination culminating during the summer months. The stars were selected from Boss's Preliminary General Catalogue, and the right ascensions and proper motions of the weaker ones strengthened by the inclusion of available data from observations at Ottawa and at Greenwich, after the application of suitable systematic corrections to reduce the observations to the system of Boss.

The time-service has been maintained as in previous years Nearly 300 electrically driven clocks, including two tower clocks, are operated in the various Government buildings by means of several secondary master-clocks continuously synchronised from the Observatory. In addition, relays beating seconds are maintained in several offices in the city, time-signals are sent out by telegraph and telephone, and the time is recorded on the various seismographs at the Observatory.

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Astrophysics.—During the past year the 15-inch refractor has been employed, as hitherto, almost entirely in determining the radial velocities of spectroscopic binary systems with a view to the determination of the elements of their orbits.

During the calendar year 1916 a total of 506 plates was obtained on 162 nights, the average photographic magnitude of the stars observed being 5·6. This somewhat smaller number of spectra obtained than previously is due partly to the absence of one of the observers on military duty and partly to the scarcity and faintness of suitable binaries for observation. The orbits of six binaries—14 Aurigæ, *x* Aurigæ, 125 Tauri, Boss 2484, Boss 6142, and 2 Sagittæ were completed during the year. Of these *x* Aurigæ is of special interest as having an unusually long period, 655 days, for a B type star, and because of the peculiar behaviour of the H and K lines of calcium.

A determination of the solar motion from the radial velocities of the spiral nebulæ giving a value of about 600 kms. per second is of interest as indicating the possibility of an enormous velocity of our stellar system.

A table for the conversion of right ascensions and declinations into longitudes and latitudes has been constructed and is ready for publication. The interval in R.A. is four minutes of time and in Dec. 1 degree, while the longitudes and latitudes are given to the nearest minute. This table will facilitate the reduction of stellar radial velocities to the Sun and will doubtless find other applications.

Extra-focal photometric observations are being carried on with the six-inch doublet, testing the spectroscopic binaries whose orbits have been determined here for variation in magnitude, but as yet without positive result. Photometric work by the same method for the purpose of improving the light curve of δ Libræ is also being pursued, and the methods for testing and developing the plates have been improved.

Observations with the cœlostat telescope on the solar rotation and allied problems have been continued during the year, and four papers bearing on this work have been published. The first on a variation in the solar rotation showed strong evidences of a cyclic change in the rate of about eight per cent. taking place in about a month, while the last, on the effect of haze on the solar rotation, showed that some of the observed changes found here and elsewhere might be due to the presence of a varying amount of haze in the terrestrial atmosphere. Work on this latter phase of the question is being continued to determine to how great an extent haze can effect solar rotation measures, and to what degree past measures have been influenced.

The 72-inch reflecting telescope, the condition of which was reported upon last year, has made satisfactory progress and is now nearly completed.

The structural work and operating mechanism of the dome arrived at Victoria in April, was erected during May and June,

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covered by its double sheet-metal casing during August and September, and placed in complete operating order in October. The mounting of the telescope was completed in April, but shipment was delayed until the dome was sufficiently advanced in July, and it arrived at Victoria about the middle of August. Erection began early in September and was completed in about six weeks. Both telescope and dome are splendid examples of engineering skill, operate to perfection, and reflect great credit on the makers, the Warner & Swasey Co., of Cleveland.

The main mirror of the telescope is now in the final stages of parabolizing, and is expected to be completed early in 1917. As the observatory building and telescope are all ready for the installation of the optical parts, actual observing with the instrument should be possible in the Spring.

Adelaide Observatory, South Australia.
(Director, Mr. G. F. Dodwell, Government Astronomer.)

Meridian and Time-Service.—The transit-circle telescope was used throughout the year for the observation of the intermediate standard stars of the Sydney zone ($-52°$ to $-64°$). The observations were reduced from day to day by the aid of graphic reductions. The construction of R.A. graphs was greatly facilitated by the use of new differential tables due to Professor W. E. Cooke of Sydney Observatory. Fundamental stars in the zone were used for the daily time-service. The time-ball at the semaphore was dropped at 1 p.m. daily, except on Sundays. On occasions when telegraphic communication was temporarily interrupted, the time-ball was dropped by hand by the signalman on a signal given through the telephone from the Observatory. For this purpose a telephone and push-button were installed at the Time-Ball Station close to the releasing trigger. A return signal apparatus which had also been installed showed that the internal lag of the mechanism was 0·3 second. The ordinary system of distributing the time to the country telegraph stations and various Government Departments was continued. The transmission of time-signals to the Adelaide Radiotelegraphic Station and thence by wireless to shipping has been continued, the signals being those adopted by the International Convention. They were emitted throughout the year at midday and midnight, Adelaide standard time.

Field Work. — Latitude, Longitude, and Magnetic Stations were established at Bordertown, Naracoorte, Wilmington, Melrose, Booleroo Centre, Port Pirie, and Mount Brown. For the magnetic work a magnetometer outfit, lent by the Carnegie Institution, and dip circle lent by the Sydney Observatory, were used. Professor Kerr Grant of the Adelaide University, and one of his research assistants, Mr. R. S. Burdon, co-operated in the work at Bordertown and Naracoorte. For latitude and longitude work the al-

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mucantar method of observing was used, and longitude signals from the Adelaide Observatory *via* the Radio Station were received from a portable wireless receiving outfit. A permanent mark at the site of observations and concrete block to indicate the meridian are being established at each of these stations.

Seismology.—The Milne Seismograph was continuously at work during the year, and the tabulated records were sent as usual to the Shide Observatory, Isle of Wight.

Miscellaneous.—The Board of Visitors held their second Annual Meeting at the Observatory on November 17.

Mr. C. A. Maddern, a member of the Staff, has been at the front with the Australian Imperial Forces since the beginning of the year. His work at the Observatory is being carried out *pro tem.* by Mr. W. R. Blackman.

The annular eclipse of the Sun on July 30 was observed under ideal weather conditions. A series of photographs was taken with a photographic attachment fixed at the eye end of the 8-inch Cooke equatorial telescope. These photographs were used by Professor Grant in connection with actinic measurements made at the Adelaide University during the eclipse. A report of this work, with copies of photographs, etc., was sent to the Royal Astronomical Society. The equatorial telescope was used principally in connection with the regular Friday night visits by the public, and also on certain evenings by members of the S.A. Astronomical Society for variable star and other observations.

Perth Observatory, Western Australia.
(Director, Mr. H. B. Curlewis, Acting Government Astronomer.)

Meridian Observations.—Throughout the year work on the Transit-Circle was confined almost solely to time determinations, 197 nights being thus employed.

Astrographic Catalogue.—The work is proceeding steadily, and only zone 33° and a portion of 35° remain to be finished. With regard to the printing, progress is slow, but, so far as can be foreseen, no stoppage is likely to occur. The four volumes of zone 36° and those of 37°, which should be out of the printer's hands shortly, will not be distributed till after the war, as at present there is no guarantee of their safe transmission.

Seismology.—With the exception of a few hours lost by the failure of the light, the record of the Milne Seismograph was continuous. Nineteen distinct earthquakes were recorded, of which seven had semi-amplitudes of 10 or more millimetres, while the remaining thirteen were small, the semi-amplitudes being under five. Distinct tremors numbered nine, while faint tremors or microseisms were innumerable. A very fine record of the Rabaul earthquake of January 1 was received, and also one on 13th of the same month.

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During the latter half of the year local earth tremors, of some severity, were reported from various parts of the State.

Tidal.—During the year the predicted tide curves for Port Hedland for 1917, based on the constants derived from the analysis of the 1913 hourly records, were received from the Superintendent of the United States Geodetic and Survey Department. The figures extracted from these curves were published and distributed towards the end of the year. Beyond this the tidal work remains in abeyance, which is to be regretted. It is to be hoped that the work will be re-started before long, for a more exact knowledge of the tides, especially on the N.W. and N. coasts of Australia, is very desirable.

Time-Service.—The time-ball at Fremantle only failed on three occasions. This is a very decided improvement on all previous years, and speaks well for the attention bestowed upon the line by the Telegraph Department. The automatic check upon the time of ball drop is still maintained, and with few exceptions the error was within one-tenth of a second. On June 30 the ball was two seconds slow, owing to the current for regulating the Mean Time Clock being switched on in the wrong direction. The method of disseminating the time throughout the State remains the same as in the past, and was carried out satisfactorily.*

The Observatory is not yet connected with the Wireless Station at Applecross, and therefore time-signals cannot be sent from there. Repeated efforts have been made to bring this about.

Popular, Educational, and Miscellaneous.—Day and night visitors were even more numerous than usual, nearly every Tuesday afternoon being engaged and 109 evenings. The total number of people visiting the institution on these 109 nights was 1270, which constitutes a record, and proves that public interest in the Observatory and in Astronomy is increasing.

A number of lectures were given during the year, and the usual information supplied to the Press and private correspondents.

The Western Australian Astronomical Society, under the ægis of the Observatory, still flourishes, and at the end of the year published a useful and instructive journal.

Union Observatory, Johannesburg.
(Union Astronomer, Mr. R. T. A. Innes.)

Instrumental Equipment.—This remains unchanged.

9-*inch Refractor.*—Measurements of double stars have been continued upon the usual lines, and the phenomena of Jupiter's Galilean Satellites have been regularly observed. Miscellaneous observations were also made.

* An apparatus was installed during the year, by means of which the second-beats of the Mean Time Clock can be transmitted automatically by telephone.

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$2\frac{5}{8}$-inch Talcott Telescope.—This has been used for the Time-Service. The observations for variation of latitude 1910 March–1914 December have now all been reduced and the final results have been sent to press.

Franklin-Adams Star Camera and Twin-Telescope.—These two instruments have been in partial use for the observation of minor planets, variable stars, etc.

Time-Service, Meteorological and Seismological Observations.—These have been continued without change. The shocks due to past mining operations have decreased from 893 in 1915 to 414 in 1916, and both public anxiety and interest in them have abated, although several shocks have been strong enough to rattle the whole town.

Staff.—Mr. Van der Spuy, who has left the observatory, has been awarded the Military Cross. Mr. Wood volunteered and was accepted in April, and Mr. Johnson in January, for service in German East Africa, and they have not yet returned. Mr. Worssell volunteered both in England and here, but was rejected on account of his myopia. Mr. Worssell and the two lady assistants have maintained the routine work, and all reductions are up to date. The state of the staff was such that in 1916 little besides routine work could be undertaken. Mr. Worssell has, however, photographed some minor planets, and observed southern variable stars and some occultations.

Observatory Circulars.—Nos. 33 to 35 were actually distributed in 1916, and No. 36 is in the hands of the Government printers. No. 33 contains the fifth series of Double Star measures, lunar occultations of stars in 1915, and some miscellaneous observations. No. 34 contains the observations of Jupiter's Galilean Satellites during the opposition of 1915, and here it might be added that a complete comparison with Sampson's Tables is now available; this number also included observations of minor planets and miscellaneous matter. No. 35 contains the result of a blink examination of pairs of plates loaned by the Astronomer Royal for the determination of proper motions, and marks the beginning of a systematic piece of work which has been continued throughout the year and is still in progress. The Astronomer Royal has proposed that we should examine here the whole zone +66° as far as any suitable pairs of plates are available, and other pairs to the North Pole of any very good pairs of plates. The zone +66° is now completed to 18h. 36m. Three further excellent pairs of plates from H.M. Astronomer at the Cape have been examined, and the results are ready for publication: one of the regions of these plates had previously been measured by Professor Kapteyn, and the comparison of the two results is instructive. The Director of the Melbourne Observatory has already sent some selected pairs of plates, and will be sending more. It has been agreed with the Director of the Sydney Observatory that the whole of the Sydney zone shall be blinked here, and a start has already been made. Hence there is already on hand a number of unpublished proper motions. A list of those

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exceeding half a second a year and previously unpublished is given here:—

	m	h	m	°	′	Centennial Proper Motion.
B. D. + 66° 34′						178″·6
Anon.	13	13	50	+ 66	7	57·5
,,	10·4	0	36	− 53	2	88·6
,,	11	6	11	− 65	9	82·0
,,	10	11	39	− 64	8	269·0
C.P.D. 65° 1714 :						55·0 : = Double star.

A good deal of work with the blink microscope has been done on variables near η Argûs and near Corona Australis: and in the latter region over 150 new variables have been found, and the hard work of looking them up on our 28 plates of the region is half completed.

Miscellaneous.—Some experimental work on the proposed charts of the southern sky was done, and the Government printer has imported a finer screen. The original idea was to reproduce on the scale of $1' = 1^{mm}$ (carte du ciel scale), but at present a scale of $1° = 36^{mm}$ is in favour: this latter is practically the scale of the Wolf Palisa charts.

Desk work has included an algebraical or literal development of the perturbative functions required in the theory of Jupiter and Saturn, and some attempts to produce eclipse tables for Jupiter's Satellites on the lines of Damoiseau's Tables, but with modern constants derived from Sampson's and de Sitter's work.

Lovedale Observatory, South Africa.
(*Dr. Alexander W. Roberts.*)

During the year 1916 the work done at this observatory has been mainly the reduction of past observations.

In the light curves of a great majority of stars, both of long-period and of short-period variation, a definite secular change has been made manifest, and this has caused the work of reduction to be more onerous than was at first anticipated.

In addition to the reduction of past observations, there was also carried on the regular observation of stars whose variation indicated secular change of period. The number of observations made were:

Short-period stars	1120
Long-period stars	1200
	2320

Further, there was carried on during the year a systematic

comparison of southern fields with northern fields. The photometer used was one of very simple construction, viz. two mirrors capable of being inclined at any angle, and moving on universal bearings.

Eight settings, in which the position of the mirrors and of the observer were so allied as to eliminate personal and instrumental errors, gave a relative error of $\pm 0^{m} \cdot 03$ in the comparison of any two fields.

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Notes on some Points connected with the Recent Progress of Astronomy.

Discovery of Minor Planets in 1916.

Sixty-four minor planets were discovered or first announced in 1916, as follows:—

Letter and Number.	Date.	Discoverer.	Letter and Number.	Date.	Discoverer.	Letter and Number.	Date.	Discoverer.
	1904.			1916.			1916.	
a	July 7	W	ZH 824	Apr. 1	W	AG	Sept. 23	W
	1915.		ZJ	2	,,	AH	24	,,
a	Feb. 16	T	ZL 825	3	,,	AJ	24	,,
b	Sept. 12	,,	ZM	Mar. 31	,,	AK	25	,,
c	14	,,	ZO 826	Apr. 28	,,	AL	Oct. 1	,,
d	Dec. 8	,,	ZP	28	,,	AM	1	,,
e	Sept. 10	Pe	ZQ	30	,,	AN	Sept. 30	T
	1916.		ZR	May 6	,,	AO	30	,,
YT 814	Jan. 2	N	ZS	July 28	Pe	AP	Oct. 1	R
YU 815	Feb. 2	W	ZT	Aug. 1	,,	AQ	18	T
YV 816	8	,,	ZU	1	,,	AR	17	,,
YW 817	6	,,	ZV	9	,,	AS	Nov. 16	W
YX	1	T	ZW	29	Pa	AT	26	G
YY	19	,,	ZX	29	,,	AU	29	,,
YZ 818	21	W	ZY	31	W	AV	29	T
ZA 819	Mar. 3	,,	ZZ	Sept. 3	,,	AW	Dec. 24	W
ZB 820	30	,,	AA	20	,,	AX	27	,,
ZC 821	31	,,	AB	20	,,	AY	27	,,
ZD 822	31	,,	AC	20	,,	AZ	27	,,
ZE	31	,,	AD	20	,,	*a*	May 26	S
ZF	31	,,	AE	23	,,	*b*	Oct. 2	,,
ZG 823	31	,,	AF	23	,,	*c*	Dec. 16	M

Another minor planet is announced as discovered last year by Peters at Washington, 1916 April 10.

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The letters stand for:—

W = Professor M. Wolf, Königstuhl, Heidelberg.
T = Dr. Thiele, Bergedorf, Hamburg.
G = Herr Gyllenberg, Bergedorf, Hamburg.
Pa = Dr. Palisa, Vienna.
R = Herr Rheden, Vienna.
N = Mr. G. Neujmin, Simeis, Crimea.
Pe = Mr. G. H. Peters, Washington.
S = M. Sy, Algiers.
M = Dr. Metcalf, Winchester, Mass.

Of the planets in last year's Report, the identity of XU with 193 Ambrosia is confirmed; YC = 239 Adrastea, YQ = 1911 LG = 1894 AW = 604 Tecmessa. The following have received permanent numbers: 1901 GY (seen again 1908, 1915) 808, 1915 XP 809, XQ 810, XR 811, XV 812 (= 1902 KC), YR 813. 1915 YL was probably a comet.

Planets 753, 797 have been named Tiflis, Montana, respectively. Planet 814 is identical with 1907 ZR, 817 with 1908 CP, 823 with 1913 *a*, 824 with 1913 TB. An object (ZK) found by Wolf on 1916 Apr. 3 proved to be a comet (see Comet Report). ZN, found on Apr. 28, is 271 Penthesilea.

The motion of planet 1916 *e* (Metcalf) indicated that it was near the Earth, but the orbit is not yet to hand. The magnitude was 13½, so it must be an exceedingly minute body.

It will be seen that the second double alphabet for denoting new discoveries has now been completed, and the third commenced. Each of them contains 625 combinations, the letter I being omitted. The first extended from 1894 to 1907, the second from 1907 to 1916, indicating a somewhat higher rate of discovery in the latter period. It is necessary to prefix the year of discovery to the letters to avoid confusion.

Besides the 826 numbered planets, 19 more have been observed sufficiently for the determination of elliptical orbits, and 51 more for circular orbits, so that orbits of some sort have been found for 896 planets.

Of the first 520 planets there are only 13 that must be regarded as lost, having been seen at no opposition since that of discovery. The lost planets are 132, 155, 220, 285, 323, 330, 392, 396, 400, 452, 463, 473, 515.

Mr. Dinsmore Alter gives a search ephemeris for 132 Æthra, on the assumption that it is identical with an object photographed at Flagstaff 1913 June 10. On this assumption the magnitude in 1917 April would be 9·9, the declination 44° south (Lick Observatory Bulletin, 285).

433 Eros was in perihelion early in December 1916, and fairly bright, though it was several months before opposition. Mr. Leon Campbell, using the 15-inch equatorial at Harvard Observatory, noted a light variation of 0·4 magnitude on December 29: minimum light occurred 29^d 21^h G.M.T.

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Professor H. C. Plummer (*M.N.*, vol. lxxvi. 5) and Mr. S. G. Barton (*Astr. Journ.*, 702) have examined the statistics of the orbits of minor planets. The most striking feature is the grouping of perihelia near that of Jupiter. There are three times as many perihelia within 10° of Jupiter's perihelion as within 10° of its aphelion. The average eccentricity is 0·12. Professor Plummer suggests that when material is insufficient for determining an elliptical orbit it would be better to assume an ellipse with perihelion in the direction of Jupiter's and eccentricity 0·12, than to assume the orbit a circle.

Mr. Barton finds that g (the stellar magnitude of a planet at a certain fixed distance from Sun and Earth) averages $8^{m}\cdot 38$ for the first 400 planets, and $9^{m}\cdot 32$ for the second 400. The corresponding average values of μ (the mean daily motion) are 793″, 765″. Since g measures the size of the planet on the assumption of uniform albedo, these figures indicate that the later discoveries are both smaller and more distant than the earlier ones. He also finds a correlation between i and e, indicating that large eccentricities usually go with large inclinations. A. C. D. C.

Discovery of Comets in 1916.

1916 *a*. The first cometary discovery of the year was made by M. Neujmin at Simeis on February 21. The comet was faint throughout, not becoming much brighter than the 11th magnitude. The last observation was made on June 3 at Johannesburg. It proved to be a short-period comet of Jupiter family, with period $5\frac{1}{2}$ years. Its orbit and that of Encke's Comet are almost in the same plane, and have both their foci coincident, which may indicate a former connection between them. Neujmin's Comet appears to have been near Jupiter in 1903.

1916 *b*. This comet was discovered by photography by Professor Wolf at Königstuhl, Heidelberg, on April 3. It was at first taken for a minor planet, and designated ZK. It was subsequently found to have a nebulous envelope, and to be moving in an approximately parabolic orbit. Perihelion will not be passed till 1917 June 16, $14\frac{1}{2}$ months after discovery, an even longer interval than in the case of Delavan's Comet, 1913 *f*. This comet will be well placed for observation next summer, and will probably become a naked-eye object, but its perihelion distance, $1\frac{2}{3}$ units, is too great to give expectation of a very brilliant display. It was followed till the beginning of July, and recovered as a morning star on December 23 by M. Gallo in Mexico, the positions agreeing very closely with those of Crawford's ephemeris, based on the elements printed below.

1916 *c*. This was a photographic observation of Encke's Comet by Professor Wolf at Königstuhl on September 22, one month after aphelion passage. He found it with the aid of an ephemeris prepared by Mr. F. E. Seagrave; the motion between two exposures agreed with that given by the ephemeris. It was of magnitude $16\frac{1}{2}$, and appeared as a very faint nebulous envelope with a small well-

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defined central condensation. It was also photographed by Thiele at Bergedorf on November 15, 26, 27 (*Astr. Nach.*, 4870). It will not reach perihelion till 1918 March 25.

1916 *d*. Dr. J. Metcalf announced the discovery of a comet on November 21 on a zodiacal plate taken in the course of his search for minor planets. It was almost exactly in opposition to the Sun. Photographic observations on November 22, November 26 were published, but I understand that they were subsequently withdrawn as erroneous. Professor Wolf published an observation on November 28 of an object which he took to be the comet, but it proved to be the minor planet 368 Haidea. Messrs. Braae and Vinterhansen give the following very uncertain orbit of Metcalf's Comet: T = 1916 Sept. 18·185 G. M. T., $\omega = 94° 56'.2$, $\Omega = 237° 49'.0$, $i = 18° 57'.9$, $\log q = 9.87672$. It distantly resembles the orbit of Biela's Comet.

Besides the above comets, a bright cometary object, with very rapid motion, was seen by Professor Perrine and his assistants at Cordoba on May 4. Dr. Estelle Glancy deduced an approximate orbit, on the assumption that it was a comet. This indicated a very close approach to the Earth (much nearer than the Moon). As the object was not seen again, its character must remain somewhat doubtful.

Mellish's Comet, 1915 *a*, was followed by photography by Professor Thiele at Bergedorf, Hamburg, till 1916 September 25, when it was considerably outside the orbit of Jupiter. Delavan's Comet, 1913 *f*, was followed till nearly the end of 1915. The orbit appears to be hyperbolic, as was pointed out by M. Van Biesbroeck.

Taylor's Comet, 1915 *e*, was observed till 1916 May 2. This comet belongs to the Jupiter family, with a period of 6½ years. Like Mellish's Comet, 1915 *a*, it exhibited the phenomenon of partition of the nucleus. Professor Barnard was the first to detect this on February 9, when the two nuclei were 10″ apart, in P.A. 21°. Calling the northern and southern components N and S, N was at first the fainter. Both had short tails. For a time N continued to get fainter, S became brighter and more condensed; then N brightened and showed strong condensation, while S became diffused and faint; finally it disappeared altogether, leaving N a strongly condensed comet. Professor Barnard gives the following cases of partition of comets:—Biela's Comet, Comet 1860 III., Great Comet of 1882, Brooks 1889 V., Swift 1899, Kopff 1906, Halley's Comet in 1910 May.

The following are the elements of Taylor's, Neujmin's, and Wolf's Comets, referred to the Equinox of 1916·0:—

Comet Computer.	1915 *e* (Taylor) Jeffers, Neubauer.	1916 *a* (Neujmin) Braae.	1916 *b* (Wolf) Crawford, Alter.
T in G. M. T.	1916 Jan. 30·9403	1916 Mar. 11·2350	1917 June 16·4806
ω	354° 49′ 1″·6	193° 43′ 17″·7	120° 37′ 7″·9
Ω	113° 54′ 5″·1	327° 30′ 59″·6	183° 16′ 58″·8
i	15° 31′ 43″·5	10° 39′ 53″·0	25° 40′ 6″·4
log q	0·192540	0·127494	0·226855
Period (years)	6·3662	5·49969	Parabola

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The elements of Taylor's Comet show a slight resemblance to those of Daniel's Comet 1909 IV. (period 6·4 years). This comet made a near approach to Jupiter in February 1912, when the elements must have undergone considerable alteration.

Perrine's Periodic Comet (1896 VII.–1909 III.) escaped detection at its return in 1916.

MM. A. Andoyer and B. Jekhowsky in the course of developing a new method of computing perturbations for planets and comets (*Bulletin Astronomique*, 1916 March), incidentally suggest an interesting method of computing preliminary orbits for new comets. It is well known that three complete observations give one equation too many for computing a parabolic orbit. They suggest treating the Sun's mass as a sixth unknown; the equations can then be satisfied exactly, and there is only one real solution. If the value found for the Sun's mass is near the truth, it indicates that the true orbit is practically parabolic, while an erroneous value of the Sun's mass would generally indicate an elliptical orbit. Even in the latter case the authors demonstrate, from the case of Tuttle's Comet, that the parabola found will suffice to give a tolerable ephemeris for a month or two (assuming that the observations used are good), so that the labour of computing an ephemeris from poorly determined elliptical elements is avoided. They further indicate that by combining their method with that of Leuschner, a cubic equation can be formed, with only one real root, for determining the comet's distance from the Earth.

A. C. D. C.

Solar Activity in 1916.

Sun-spots.—The year 1916 was one of full, but not of exceptional, activity so far as sun-spots are concerned. Quite a considerable number of groups were easily visible to the naked eye, and the complexity of the structure of many groups, as seen in the telescope or on photographs, was a striking feature of the year.

January opened with much the same moderate average of spot activity as shown by 1915 December, but early in February a very fine stream developed in the Southern Hemisphere, and another, slightly smaller, in the Northern. In March the Sun's surface was greatly disturbed, no fewer than four groups being large enough to be seen by the naked eye. This activity continued into April, but somewhat declined in the middle of the month. May was distinguished by several fine groups, much the largest of which formed in the visible hemisphere, and developed into a continuous stream of 14° of solar longitude in extent. June was remarkable for the long successions of spots forming about latitude 15° on both sides of the Equator, giving towards the end of the month a "belted" appearance to the solar disc. This action was much more conspicuous in the Northern Hemisphere, where it was repeated on a minor scale in July, while the southern disturbances died down.

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August and September were in general months of solar quiescence, though each had one considerable disturbance. But in October a marked revival set in, and a fine succession of active spots were seen in the Northern Hemisphere from October 4 to 21, followed towards the end of the month by a fine stream in the Southern Hemisphere. November was distinguished rather by widespread and diffuse disturbances than by large concentrated groups, and December, so far as the weather conditions permitted the Sun to be observed, seems to have been a month of moderate activity. On the whole, 1916 seems to have shown a distinct but not a great increase on the sun-spot activity of 1915. E. W. M.

The Prominences.—The mean daily areas of prominences in square minutes of arc, derived from photographic records made at Kodaikánal and at Srinagar, are as follows:—

	North.	South.	Total
1916 January to June . .	2·06	1·77	3·83
July to December .	1·98	1·65	3·58

The corresponding totals for the year 1915 were, for the first six months 5·27, and for the second six months 5·29. A reduction of area amounting to about 30 per cent. is thus shown.

The mean daily number of prominences recorded during the year is 18·9, a reduction compared with 1915 of under 1 per cent.

The distribution in latitude shows that the diminution of area affects practically the whole Sun, but is especially marked in the south between 20° and 40°. There has been a progressive change during the year in the position of the high latitude zones of activity, which are now approaching the poles. They occupied almost the same position as in the year 1905, and will probably close over the poles during 1917. The northern zone is, however, about 5 degrees ahead of the southern, and this is clearly shown also by the prominences projected on the disc as absorption markings.

The northern hemisphere has shown a greater activity than the southern, not only in mean area but also with regard to metallic prominences and displacements of the hydrogen lines. The northern prominences were also denser than the southern, as indicated by the preponderance of Hα absorption markings in the north.

The distribution east and west of the Sun's axis is interesting, as indicating a return to the condition of eastern preponderance. There is, however, only a slight excess of east over west in prominence areas and numbers, the percentage east being 50·6 and 50·5 respectively, derived from a total of 6129 prominences. Prominences projected on the disc as absorption markings give percentages east of the central meridian as 52·2 for areas and 51·5 for numbers, derived from 2618 prominence markings. D_3 darkenings also preponderate east of the central meridian, and of 489 bright reversals of Hα on the disc, 54·3 per cent. were east. Only fifty-eight metallic prominences were recorded during the year, and

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these were more frequent west of the central meridian. 438 displacements of Hα were observed in the chromosphere and prominences, and of these 55 per cent. were on the east limb.

As is usually the case, there was an excess of displacements towards red, although less marked than in former years. The largest displacements were observed on April 7th 4 A. to red, April 11th 5 A. to violet, and on November 6th 4 A. to red.

On May 26 a very complete record was obtained at Kodaikánal and at Srinagar of an eruptive prominence which rose to the extraordinary height of over 18′, or about half a million miles above the chromosphere. Measures of the photographs show the usual accelerating velocity of ascent, and a very remarkable outward motion attaining over 300 km./sec., apparently radiating from a centre situated in the chromosphere. Motion in the line of sight in this prominence did not exceed 140 km./sec.

It is interesting to note that on May 28 a very similar eruption was observed by Buss in the same solar quadrant, and described by him in the *Observatory*, vol. xxxix. p. 351. This showed large movements in the line of sight as well as across the line of sight.

On June 29 a brilliant disc eruption was photographed at Srinagar, lasting for one hour between 9^h and 10^h I.S.T. This appeared suddenly near the following end of an extensive group of spots and flocculi. The bright calcium clouds extended from the central meridian to longitude 8° west, and between north latitude 7° and 20°. It is very remarkable that simultaneously smaller clouds appeared in a distant group of flocculi nearing the west limb, and in addition a large eruptive prominence was also developing on the east limb. J. E.

Solar Research in 1916.

The Solar Spectrum.—Fraunhofer's G group of the solar spectrum has been identified by Newall, Baxandall, and Butler, who have satisfactorily established that the main features of the group are produced by the well-known hydrocarbon band beginning about λ4314 (*M.N.*, lxxvi. 640). The identification is an important one, and will doubtless aid also in the classification of stars which immediately precede the Sun in type.

Sun-spot Spectra.—At the Mount Wilson Laboratory, A. S. King has investigated the origins of the band spectra which are of special interest in connection with sun-spots (*Ap. J.*, xliii. 341). The outcome of his experiments on titanium, which were made with the electric furnace, is to confirm Fowler's conclusion that the bands in question represent the actual compound titanium oxide. It was also found that the bands attributed to magnesium hydride and calcium hydride were not produced in the absence of hydrogen. The upper limit of temperature for maximum intensity

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of the bands was found to be about 2300° C., which accords well with previous deductions regarding the relatively low temperature in spots.

In another paper (*Ap. J.*, xliv. 172) Dr. King gives an account of the remarkable variations in the apparent structure of the red line of lithium depending on the amount of vapour in the source. It may now be considered certain that the strong line at λ6708 in the spectra of sun-spots has its origin in lithium, and it is noted that this is the only direct proof of the presence of lithium in the Sun, since there are no lithium lines in the ordinary Fraunhofer spectrum.

From further observations in the region C–D, Father Cortie finds that there has been no substantial change in the spectra of sun-spots since the year 1882 (*Stonyhurst Obs. Rep.*, 1915, p. 12).

Radial Motion in Sun-spots.—The phenomena of radial motion in sun-spots have been further investigated by Evershed with improved instruments (*Kod. Bull.*, li.). The radial motion displacement was found to be somewhat greater on the limb side of the spot than on the side towards the centre. Usually there is an acceleration from the umbra to the outer edges of the penumbra, then a sudden fall to zero, or to a lower velocity which diminishes to zero at some distance outside the spot. The variations of indicated velocity depending upon the intensities of the lines, attributed by St. John to differences of effective level, were confirmed. Rotational movement does not appear to be a constant feature.

Solar Rotation.—During the summer of 1915 a series of spectroscopic photographs was made at Ottawa by H. H. Plaskett for the special purpose of investigating possible rapid variations in the Sun's rotation (*Ap. J.*, xliii. 145). The plates were taken in the region of λ5900, and of the lines measured, five were of solar and three of telluric origin. The deduced equatorial velocities seem to show a short-period cyclic variation with a range of 0·15 km., but the observations were not sufficiently extended to indicate any definite period. The mean of all the measures gave the equatorial velocity 0·04 km. lower than that derived in 1913. Mr. Plaskett concludes that since 1906 the rotation value has been slowly falling from 2·050 km. to 1·950 km.

The results for rotation obtained at Mount Wilson also suggest a progressive diminution of the rate since 1906. From 2·064 km. in 1906, the equatorial velocity fell to 2·053 km. in 1908, and to 1·949 km. in 1915 (*Mount Wilson Rep.*, 1915).

An investigation of the effects of haze on solar rotation measures has been made at Ottawa by R. E. de Lury (*Jour. R.A.S., Canada*, x. 345, and *Ap. J.*, xliv. 177, 198). A superposed sky spectrum, coming mainly from the central parts of the Sun's disc, would clearly tend to reduce the rotational displacements at the limb, and the measurable effects would doubtless depend in part on the characters of the lines. It is concluded that variations hitherto attributed to the Sun, including the short-period change

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noted above, can be accounted for by variations in the scattered light.

In regard to the Mount Wilson observations, however, St. John and Adams have drawn attention to some further tests of the sky conditions, from which they infer that scattered radiation, whether from sky, haze, or instruments, is a negligible quantity at Mount Wilson under the usual working conditions, and is therefore not a probable source of error in the observations on solar rotation (*Jour. R.A.S. Canada*, x., 553).

Displacement of Solar Lines.—A minute investigation of the changes of wave-length of selected lines of iron in passing from the centre of the Sun's disc to the limb has been made by Evershed and Royds (*Kod. Bull.*, 49). The measures appear to show that the displacements at the centre are not due to a radial circulation on the Sun, but are probably part of a general displacement increasing from the centre to the limb. The relative shifts between the limb and centre cannot be explained by a difference of pressure in the effective regions of absorption, since the shift continues for a long distance within the limb, while a pressure effect would only be appreciable quite close to the limb. It is suggested tentatively that both limb shift and centre shift are due to a general movement of the solar gases all over the disc in a direction away from the Earth, and a method of testing this supposition by observations of Venus is indicated.

Anomalous Dispersion in the Sun.—A further reply to some of the objections which have been brought against the view that anomalous dispersion is an effective agent in solar phenomena has been given by Julius (*Ap. J.*, xliii. 43). He finds no reason to modify the principal conclusions at which he has previously arrived.

In view of Dr. Albrecht's conclusion that Rowland's measurements of close pairs of lines gave substantial evidence of mutual repulsion, such as might be explained by anomalous dispersion, further inquiry on this point has been made by Evershed (*The Observatory*, xxxix. 432). In agreement with Dr. St. John and Miss Ware (*Ap. J.*, xliv. 15), Evershed finds that Rowland's separations were almost invariably over-estimated, the errors of measurement being such as to simulate the effects of anomalous dispersion. Dr. St. John has also made a further study of the whole question, and finds that the separations of pairs are the same both in terrestrial sources and the Sun; he concludes that in so far as mutual influence of close lines is a necessary corollary of anomalous dispersion in the Sun, evidence for anomalous dispersion is entirely wanting (*Ap. J.*, xliv. 311). An experimental investigation by A. S. King (*Proc. N.A. Sci.*, ii. 461) has also given no evidence of a mutual repulsion between close lines when anomalous dispersion is active.

Valuable theoretical contributions to the discussion of anomalous dispersion phenomena have been made by Sir J. Larmor (*The Observatory*, xxxix. 103, 230; *Ap. J.*, xliv. 265).

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The Spectrum of the Corona.—The discovery of a new red line, λ6374, in the coronal spectrum during the eclipse of 1914 August 21 has led to an important extension of Nicholson's theory of the coronal spectrum. It was first noted by Carrasco (*C. R.*, clxi. 631) that the new line was a member of the same cube-root series as the green line λ5303, and Nicholson (*M.N.*, lxxvi. 415) has deduced therefrom that the *coronium* atom is a simple-ring system with nucleus 7*e*. When it has eight electrons, or a single negative charge, it emits the lines 6374, 5303, 4566, 4359, 3642, 3534. The neutral and positively charged atoms yield lines which are outside the range of observation. All the remaining known lines of the corona have previously been assigned by Nicholson to protofluorine.

Photographic Study of Sun-spots.—An important contribution to our knowledge of the structure of sun-spots has been made by Father S. Chevalier, S.J., in vol. ix. of the *Annales de l'Observatoire de Zô Sè*, which includes a great number of beautiful reproductions of large-scale photographs. Among other results the photographs confirm the reality of the previously observed increase of light at the interior border of the penumbra of a spot, and throw considerable light on the structure of the penumbral filaments and their connection with the granulations of the photosphere. The bright bridges which often cross the umbra do not appear to be elevated portions of faculæ or photospheric clouds, but broad lanes of photospheric matter on the floor of the spot; the term "bridge" is therefore considered to be a misnomer. Spot outbursts appear to be preceded by faculæ, the spots first appearing among the faculæ as a few dark pores.

Solar Radiation.—In the course of an interesting paper on this subject (*Ap. J.*, xliii. 197) F. Biscoe, of the University of Warsaw, deduces from the observational data of Abbot an absolute temperature of the solar surface of 7300° ± 100° C. He also gives an account of a series of observations made in the Caucasus with an Ångström compensation pyrheliometer provided with red, green, and violet filters. He is led to express the opinion that the cause of the variations is to be sought, not in cosmical, but in terrestrial meteorological phenomena and in the methods of reduction.

This paper has since been criticised by Messrs. Abbot, Fowle, and Aldrich (*Ap. J.*, xliv. 39), who again object to the assumption of a sharply defined "black body" photosphere radiating through a thin gaseous envelope. They maintain their confidence in the evidences of solar variability which is provided by (*a*) observations of the solar constant at Mount Wilson, (*b*) comparison of Mount Wilson and Bassour observations, (*c*) comparison of Mount Wilson and Arequipa observations, (*d*) comparison of Mount Wilson and magnetic observations, (*e*) comparison of solar-constant work with solar-contrast work. Details of the solar-contrast work are given in the Smithsonian Miscellaneous Collections, vol. lxvi. No. 5, and a general summary of the year's work on solar radiation in the

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Annual Report of the Astrophysical Observatory of the Smithsonian Institution.

A valuable account of the various instruments which have been employed for the measurement of solar radiation has been given by R. S. Whipple (*Trans. Optical Soc.*, 1915 March).

Solar Activity and Planetary Phenomena.—T. Köhl has found reason to believe that the northern cloud belts of Jupiter are relatively weak at times of sun-spot maximum. He also suggests that the luminosity which has occasionally been observed on the unilluminated part of Venus may occur at the same time as terrestrial auroras (*Astr. Nach.*, 4821).

An examination of the records of the planet Mars between 1862 and 1914 has led M. Antoniadi to conclude that, in general, the polar caps melt most rapidly near times of sun-spot maxima (*M.N.*, lxxvi. 413). This accords well with the view that solar radiation is greatest when spots are most numerous.

The Sun and Terrestrial Magnetism.—An interesting account of the changes in terrestrial magnetism as related to the Sun was given by Dr. Chree in a lecture to the Institution of Electrical Engineers on February 17 (*Nature*, xcvii. 509). The range of the mean diurnal variation from year to year not only varies almost exactly with the sun-spot frequency, but also shows the 27-day period. As regards disturbances, in some years there appears to be a clear connection with sun-spots, but in other years little, if any, relation is shown.

Dr. L. A. Bauer has discussed the relation between changes in solar activity and the Earth's magnetic activity, as manifested in the years 1905–1914 (*Nature*, xcvii. 493). He finds that there is a close relation between the variations in the solar constant as determined by Abbot and the changes in the Earth's magnetic constant. Also, a closer correspondence is found between these two sets of data than between either set and that of sun-spot frequencies.

From a discussion of the mean daily range of the declination for the period 1900–12, and a comparison of the curves of the mean latitudes of the sun-spots, Father Cortie adduces further evidence in support of his view that the efficiency of sun-spots in relation to terrestrial magnetic phenomena is a function of the heliographic latitudes of the spots (*M.N.*, lxxvi. 631). The efficiency of sun-spots in relation to the mean daily range in magnetic declination is greatest when the sun-spots are falling towards the equator. For magnetic disturbances the efficiency is approximately as 2 to 1 to 0·7 for latitudes 10°, 15°, 20°.

Miscellaneous.—A mathematical investigation of solar vortices has been made by Professor Störmer (*Ap. J.*, xliii. 347). It is shown that if the Zeeman effect be considered to be due to a convection current of electricity, an extremely large electric field would result. The apparent absence of the corresponding Stark effect is entirely against this, and the possibility that galvanic

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currents may play an important part in sun-spots must be given serious consideration.

A theoretical discussion of the possible velocities which might result from the expansion of prominence gases has led R. J. Strutt (*M.N.*, lxxvii. 59) to conclude that the observed velocities are far too great to be attributed to the action of the same kind of forces which produce the movements in our own atmosphere. An explanation may possibly be found in repulsion by electrostatic forces.

Dr. Henroteau has drawn attention to several cases of elliptical currents in groups of sun-spots at their first appearance, as observed and drawn by him at Stonyhurst in 1915 (*M.N.*, lxxvi. 404).

A detailed account of the spectroscopic and other observations made at Theodosia by the Italian expedition during the total solar eclipse of 1914 August 21 has been published by A. Ricco (*Mem. Soc. Spett. Ital.*, series 2, vol. v.).

A preliminary report on the total solar eclipse of 1916 February 3, as observed in Venezuela, has been given by C. D. Perrine (*M.N.*, lxxvii. 65). A. F.

Double Stars.

New Pairs.—Fewer discoveries of new double stars than usual of late years have been announced. R. G. Aitken concluded his great double star survey in 1915, and the pairs found lately by R. Jonckheere are reserved for volume lxi. of the *R.A.S. Memoirs.*

T. E. Espin in *M.N.*, vol. lxxvi. p. 210, gives 74 new pairs, 45 of which are under 5″. Philip Fox, in *Annals of the Dearborn Observatory*, vol. i., publishes his first list, Fox 1 to 52; of these, 47 are new double stars under 5″.

E. D. Roe, jun., in *A.N.*, 4830, has 13 pairs, of which 5 are less than 5″. In *Popular Astronomy*, vol. xxiv. p. 613, F. C. Leonard has a note on a new double star. There are two new pairs found by F. L. Brown in *A.J.*, No. 703, one by C. P. Olivier in *A.J.*, No. 704, and three by E. E. Barnard in *M.N.*, vol. lxxvi. p. 603.

In *The Observatory*, No. 498, R. Jonckheere announces that the pair J 320, found at Lille six years ago, is not a real double star, although, with a low power, it can be measured as such. It appears with the 28-inch of the Royal Observatory, Greenwich, to be an extremely small, bright, elongated nebula, which has escaped the spectrographic surveys.

Measures.—The largest contribution of measures is this year found in the *Annals of the Dearborn Observatory*, vol. i., which contains double star observations made by Philip Fox from 1908 to 1915. This is a handsome volume of 229 quarto pages, of which the first twenty are devoted to an introduction dealing with the history of the Dearborn Observatory, and an investigation of the 18½-inch objective with which most of the observations were made. The measures are tabulated in four parts: a complete

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re-measurement of the 171 Holden pairs; a similarly complete series of the 66 Küstner stars; measures of about 1200 miscellaneous stars; and a short list of pairs discovered by the author. The lists of Holden and Küstner double stars being complete are useful for reference. Each star has been measured on three or four nights, so that the volume must contain about 5000 independent observations. In Professor P. Fox, double star astronomy has found a new and energetic observer. Reviews and errata to this work are given by R. A. Aitken in *Pub. Ast. Soc. of the Pacific*, No. 165, and by E. Doolittle in *Popular Astronomy*, 1916 October.

G. Abetti in *Memorie ed osservazioni, R. osservatorio astronomico al Collegio Romano*, serie iii. vol. vi. part ii., gives his observations of double stars made from 1910 to 1913 with a 15-inch refractor. This contains measures of 387 pairs, and there is a very careful investigation of the micrometer screw, and of the accidental and systematic errors. Each star has been measured at least three times.

A second contribution from the Dearborn Observatory is a paper by another new double star observer, F. L. Brown, who, in the *Astronomical Journal*, No. 703, gives measures of 234 miscellaneous pairs obtained with the 18½-inch refractor. In most cases the stars have been measured on at least three nights.

T. E. R. Phillips has measures of 44 stars in *M.N.*, vol. lxxvi. p. 655. These are mostly of well-known Struve pairs, and some of the stars have been observed on many nights. A few of the measures were made by H. F. Acocks, the instrument used being the Coleman 8-inch equatorial of the Society.

In *A.J.*, No. 699, E. E. Barnard continues his most interesting record of measures of Sirius which he started sixteen years ago. The star seems now to be near apastron :—

Barnard	75°·1	10″·66	1915·99	40-inch
Jonckheere	74°·2	10″·57	1916·07	28-inch

From T. J. J. See's elements, R. Jonckheere derives the following positions :—

79°·8	10″·28	1916·20
75°·4	10″·65	1918·20

from which it appears that on See's orbit the observations are at present two years ahead of the calculated place.

A.J., No. 704, contains measures of 100 double stars made by Charles P. Olivier with the 26-inch refractor of the Leander McCormick Observatory. In this list, 8 pairs only are wider than 3″ separation, and many stars are of recent discovery. These inclusions from later lists are greatly needed in the present observing programmes, as most of the close pairs have not yet been re-measured. We learn that in future double star work will probably receive more attention at the Leander McCormick Observatory.

T. P. Bhaskaran in *A.J.*, No. 700, gives position angles and

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distances of 65 double stars measured in the ordinary course of the Hyderabad astrographic work. As is usual in such determinations when the same pair is measured on different plates, large discrepancies are found. The magnitudes given are taken from β.G.C. or the A.G.

In *A.N.*, 4842, C. Luplau-Janssen and Chr. Hansen publish measures of 40 pairs, mostly Struve stars, observed at the Urania Observatory, Copenhagen.

E. E. Barnard gives a most extensive study of the double star Krueger 60 in *M.N.*, vol. lxxvi. p. 592. This follows the paper published eight years ago in *M.N.*, vol. lxviii. p. 629, where the parallax of the star was given as $\pi = +0''\cdot249 \pm 0''\cdot010$, a value confirmed photographically by Schlesinger and Russell. This interesting binary was first discovered by Burnham as one of the components of Krueger 60, but being a pair of magnitudes 9·6–12·0, it was evidently not thought important enough to receive a number. Since Burnham's measure in 1890 the faint star of this little pair has shown a motion of 242°. The measures of the binary and of surrounding stars will be invaluable for the independent determination of the masses. The paper is made comprehensible by the diagrams. Two interesting photographs of the triple star, taken with the 40-inch at an interval of seven years, show most plainly the motion of the system AB with respect to C, and also the quick orbital motion of AB.

R. G. Aitken in *Pub. Astr. Soc. of the Pacific*, vol. xxvii. p. 230, has a note on the re-measurement of the Aitken double stars. We learn that the author, before publishing a complete catalogue of the Aitken stars, has planned to re-measure them. Of the 420 pairs so far re-observed, 62 show angular change of more than 5°, and of these 37 give differences as great as 10°. There is a list of 16 stars with angular change greater than 40°.

Orbits and Masses of Binaries.—Only two new orbits have been published in the year. R. G. Aitken in *Pub. Astr. Soc. of the Pacific*, vol. xxviii. p. 121, gives a new investigation of β 80, and finds that orbits with periods of 95 years and of 156 years will satisfy the observed position angles with equal accuracy. Two sets of elements are given.

The second orbit is by G. Van Biesbroeck, in *A.J.*, No. 692, and is for the pair A.G.C. 11 = ζ Sagittæ. This new discussion of the observations shows that it is a quick binary with a period of only 25·20 years. The elements are followed by an ephemeris giving the positions up to 1938.

G. Van Biesbroeck has also in *A.J.*, No. 694, a study of the mass-ratios of visual binary stars. From photographic determinations Van Biesbroeck obtains the relative masses of four pairs: η Cassiopeiæ, 70 Ophiuchi, τ Cygni, and 85 Pegasi. For one only, 85 Pegasi, the fainter component is found to have the greater mass, but the case is extremely interesting, as there is a difference of five magnitudes between the components, and the brighter is only 0·36 of the whole mass. Although not accepted by Boss, this

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had been found by various methods by Lewis, Furner, and Bowyer. The paper is completed by a list of 40 binaries suitable for similar determinations.

Miscellaneous Papers.—T. Lewis in *The Observatory*, No. 500, continues his instructive notes on double star astronomy. The present paper relates to eye-pieces, magnifying powers, etc., and is useful to those who wish to know how best to determine the power of the different eye-pieces they employ.

In *A.J.*, No. 700, A. Hall publishes observations of pairs of stars used for determining values of equatorial micrometer screws at the Naval Observatory, Washington.

In *Popular Astronomy*, No. 232, it is stated that the spectrum of the companion of Sirius has been investigated at the Mount Wilson Observatory, and that the line spectrum is practically identical with that of Sirius.

A.N., 4849, contains a paper by C. Luplau-Janssen on the Sun's motion and the apparent orbit of double stars.

R. T. A. Innes in *South African Journal of Science*, June, 1916, has remarks on the masses of visual binary stars. Two of his conclusions are that for solar type stars, double stars cannot exist below a separation of five times the Earth's distance from the Sun; that very faint binary stars are only found in proximity to bright stars.

The *Companion to the Observatory* for 1917 includes recent measures of well-known double stars and calculated places for 1917–1918 for pairs with separation equal to or greater than 0″·4 at the time.

Eric Doolittle has in *M.N.*, vol. lxxvi. p. 204, a paper of corrections to recent lists of new double stars.

[The Council are indebted to Mr. R. Jonckheere for this note.]

Variable Stars.

The last note on Variable Stars in the Council's Report was published in 1914. Since then the study of variables of all types has proceeded with much energy along the lines both of observation and theory.

The photo-electric cell introduced in 1913 has now established itself as an instrument of remarkable sensitiveness and accuracy in the investigation of small changes of brightness. In *Observatory*, No. 501, J. Stebbins emphasizes its great advantages as compared with the earlier selenium cell, which by reason of its irregularities presented so much difficulty in practical working.

Cepheid Variables.—Stars of this class have received a great deal of attention in recent years, and numerous important papers have been published by Wendell, Russell, Shapley, Adams, Duncan, Perrine, Guthnic, Prager, and others. Perrine, in a paper on Cepheid-Geminid variables (*Ap. J.*, xli.), has put forward the idea that the changes might be caused by an evolution of light in a secondary body as it approaches periastron, analogous

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to the development shown by comets in the neighbourhood of perihelion. It seems generally agreed, however, that the available evidence on the whole weighs heavily against the supposed binary character of these stars, and perhaps Shapley's suggestion of pulsations in a single body receives as much support as any other.

It is now clear that in many stars of this class, and probably in all, spectral changes occur simultaneously with the observed variations of light and velocity. Special mention may be made in this connection of a paper on the spectrum of the typical star—δ *Cephei*—by W. S. Adams and H. Shapley (Contributions of Mount Wilson Observatory to National Academy of Sciences, No. 22), and of another by H. Shapley (*Ap. J.*, Dec. 1916), in which the variations in spectral type of 20 Cepheid variables are clearly given in diagrammatic form, as well as in figures.

Another point which has been brought to light is that the return of the maximum in Cepheid variables is not so regular as has commonly been supposed. An exhaustive investigation of the light curve of XX Cygni as a contribution to the study of Cepheid variation has been made by H. and M. R. Shapley. They based their inquiry on 300 Mount Wilson photographic determinations of the changes in light of different wave-lengths, and the analysis of 2700 visual measures by European astronomers, and conclude that the periodic recurrence of maximum so far from being clock-like in its precision, is distinctly irregular in phase and other details (Contributions of Mount Wilson Observatory, *Ap. J.*, No. 104).

Cluster Variables.—In *H.C.O. Circular*, 193, S. I. Bailey gives a preliminary account of an inquiry into the character of certain variables in the cluster *M* 5. The majority of the stars in this cluster conform to the usual type designated sub-class *a* in the discussion of the ω *Centauri* variables in *H. A.*, xxxviii. 132, but there are eight notable exceptions. The mean period of these 8 stars is only a little over ½ day, and it is suggested that they may consist of pairs of the usual type and period, but with alternating maxima. In illustration of this hypothesis the light-curve of No. 62 is resolved into two contributory curves, each of the ordinary cluster type, but of which the combined light expressed in magnitudes is equal to the magnitude of the star at each phase. Reference is also made to the probable case of a double variable in *M* 3, and to the fact that in the cluster ω *Centauri* the variables of sub-class *c* have periods about half those of sub-class *b*. Attention is further drawn to the similarity in the mean periods of these cluster variables, the values for the stars in ω *Centauri*, *M* 3, and *M* 5 being $0^{d}\cdot549$, $0^{d}\cdot544$, and $0^{d}\cdot547$ respectively. In many cases the periods seem to be unchanged even after 10,000 returns of the maximum, but in a few instances there is a variation which can be fairly represented by a single sine term.

Eclipsing Variables.—Several papers have also been published on eclipsing binary systems. Attention may be drawn to a discussion of the elements of the systems of *TV*, *TW*, *TX Cassiopeiæ* and

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T Leonis Minoris, which has been carried out by R. J. McDiarmid under the direction of H. N. Russell (*Ap. J.*, xlii.).

In *A.N.* 4833, P. Guthnic gives an account of a method of treating eclipsing binaries which show remarkable eccentricities and movements of the line of apsides.

But the most important contribution to this line of research is the considerable Memoir by H. Shapley, which forms No. 3 of the *Princeton Univ. Obs. Contributions*. In it 90 orbits are fully treated and discussed, and several general conclusions are reached. The light curves, when sufficiently well observed, are always smooth and symmetrical: the fainter component is always luminous and not dark: both components are probably darkened at the limb: the brighter star is seldom the larger: and so on. A dozen problems suggested by the research are given at the end: and it is noteworthy that the last sentences in the Memoir proper record a despair of getting direct parallaxes for eclipsing binaries. Since the Memoir was written, the Medallist of this year has revived these evanescent hopes.

Long-period Variables.—Observations of stars of this type have been continued with undiminished energy. Amongst the large mass of material now accumulating for discussion, reference may be made to the memoir of the *Variable Star Section of the B.A.A.*, communicated to the Association by C. L. Brook at the last annual meeting, but not yet published. It contains over 16,000 separate observations made by members of the section in the years 1910 to 1914 inclusive.

H. H. Turner and Miss M. A. Blagg have continued their editing and discussion of Baxendell's observations. The seventh instalment has already appeared, and the series is now nearing completion.

In *A.N.*, 4856, H. Ludendorff gives an account of *R Hydræ*, and points out the progressive shortening of the period from about 500 days near the end of the seventeenth century to a little over 400 days at the present time.

Some peculiarities in the behaviour of o *Ceti* have been discussed in papers by A. Bemporad and F. de Roy in *Soc. degli Spettroscopisti Italiani*, vols. iv. and v. Bemporad drew attention to the fact that the feeble first maximum of 1915 was the climax of progressive changes following 1910 in brightness, amplitude, and the value of $M - m$. F. de Roy has inquired into the earlier cases of faint maxima, and gets similar and accordant results, though there does not as yet appear to be any evidence of these changes being periodically recurrent.

Amongst theoretical problems relating to long-period variables which have lately come to the front is that presented by the inequalities of period so often shown by these stars. Evidence has for some time been accumulating that the attempt to represent the oscillations of the epoch of maximum by the addition of a sine term to the simple mean period is unsatisfactory, and no longer suits the facts. In some cases a combination of terms may for a

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time be found to give fair agreement between theory and observation, and in a paper on the irregular variable *RS Camelopardalis* (*M.N.*, lxxvi. 7), C. Martin and H. C. Plummer infer from their discussion of that star the probability that in sufficiently extended and accurate series of observations the inequalities of long-period variables also might be satisfactorily represented by the superposition of a number of periodic terms. This view will probably be disputed, and H. H. Turner and Miss Blagg in their discussion of Baxendell's observations of *R Leonis*, *T Herculis*, and *R Persei* (*M.N.*, lxxvi. 2 and 6) have suggested *discontinuity* or sudden change of period as an alternative hypothesis to the simple sine term expedient. This seems in some cases to fit the deduced dates of maximum much better than the smoothness and regularity of a sine term, but there are many stars for which it appears equally unsuitable. In his presidential address to the B.A.A., the Rev. T. E. R. Phillips has also made reference to this question. He discussed the cases of *R Ursæ Majoris* and other stars which show considerable irregularities, and proposed as a further hypothesis continuous change of period of a non-repeating character (*B.A.A. Journal*, vol. xxvii. 1).

Another question of which the importance has been felt for some years is that of classification. In *R.A.S. Memoirs*, vol. lv., H. H. Turner had shown that the Rousdon long-period variables might be arranged in a definite sequence according to the harmonics of their light-curves. T. E. R. Phillips in the address above mentioned has followed up this matter. Making use of a much larger amount of material (the light-curves of eighty stars), and reducing the phases of the first three harmonics to the maximum of the first as reference point, and then plotting the phases of the second and third as abscissæ and ordinates, he found that with very few exceptions the stars are grouped into two distinct classes. The relationship of the harmonics is peculiar. In Class I. the phases of the first and third are almost constant, while that of the second ranges over more than 200°. In Class II. the phases of the second and third advance sympathetically, though at different rates. The following linear equations were found by calculation to represent these phases (ϕ_2 and ϕ_3) in the two classes :—

I. . . $\phi_3 = 202^\circ{\cdot}1 - 0{\cdot}04\phi_2$

II. . . $\phi_3 = 1{\cdot}67\phi_2 - 126{\cdot}8$

Owing to the smallness of the coefficients of these harmonics, especially the third, the individual stars naturally show somewhat large departures from the equations, but they are not larger than might be expected under the circumstances. Moreover, the distinction between the two classes appear to be supported by differences in the ranges, brightnesses, coefficients, and periods. Class I. consists of stars with smaller range, superior brightness, and shorter period on the whole, while the mean relative coefficients of the second and third harmonics are $0^{m}{\cdot}32$, $0^{m}{\cdot}15$,

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and $0^m \cdot 52$, $0^m \cdot 21$ respectively, the coefficients of the first in each case being $2^m \cdot 84$. The two classes of stars may be broadly distinguished by the appearance of the light curves. In Class I. the values of $m - M$ and $M - m$ are nearly equal, with a strongly marked tendency for the stars to show a pause on the rise. In Class II. the value of $M - m$ is small, the rise to maximum being short and sharp, while the decline is slow and gradual.

Irregular Variables.—In *M.N.*, lxxvi. 7, C. Martin and H. C. Plummer discuss the Dunsink photographic magnitudes of *RS Camelopardalis.* They find that it must be classed as irregular, but that the light changes can apparently be represented by the superposition of terms of about 960, 158, 96, 89, and 82 days.

SS Cygni has shown unusual activity in 1916, there having been as many as ten maxima during the year. The minima seem to have been on the average about three-fourths of a magnitude brighter than usual, and very erratic.

Reference must be made to the publication of *An Introduction to the Study of Variable Stars*, by Miss C. E. Furness, Director of the Vassar College Observatory. It contains an admirable account and summary of the historical, instrumental, and observational development of all departments of variable star astronomy, and will be found a valuable book of reference to workers in this field. T. E. R. P.

Stellar Distribution and Motions.

Many notes have been published on faint stars with large proper motions, giving data useful both for statistical purposes and for indicating the stars likely to have measurable parallaxes. It is interesting to notice the different methods that are used. The results of F. A. Bellamy and R. J. Pocock (*M.N.*, *R.A.S.*, 76, pp. 102, 428, 538, 657) were obtained by comparing measures of astrographic plates; those of R. T. A. Innes (*Union Obs. Circ.*, No. 35) by the blink method; of M. Wolf (*Astr. Nach.*, passim) by the stereo-comparator; and the Royal Observatory, Greenwich, lists of stars with centennial proper motion greater than 20″ (*M.N.*, *R.A.S.*, 76, p. 585; 77, No. 3) were found by comparison of meridian observations. The outstanding discovery of the year is a star with the largest proper motion yet known, 10″·3 per annum, found by E. E. Barnard (*Harv. Bull.*, No. 613) by the blink microscope. The visual magnitude is about 10, and the parallax 0″·5 or possibly larger, so that the absolute brightness is not more than $\frac{1}{1000}$ of the Sun. Other stars with luminosity less than $\frac{1}{1000}$ of the Sun are Groombridge 34 *comes*, and a companion to Pi 2^h 123 found by A. van Maanen (*Pub. Ast. Soc. Pac.*, 27, p. 240). Like other feebly luminous stars Barnard's star is of type M*b*, and has a very large linear velocity in space. Another interesting star with a proper motion of 5″ per annum, visual magnitude 10, has been found by R. T. A. Innes near *a*

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Centauri (*Union Obs. Circ.*, No. 30); two eleventh-magnitude stars with annual motions of 2″·6 and 1″·4 have been announced by M. Wolf.

Some short lists of parallaxes have been published, but they do not include any particularly near stars. Especial interest is now centred on a new method of determining stellar distances developed by W. S. Adams (*Proc. N.A. Wash.*, **2**, p. 147). It was found by Adams and Kohlschütter (*Astrophysical Jour.*, **40**, p. 385) that the relative intensity of certain spectral lines varies according to the absolute brightness of the star, and the correspondence is so close that the spectroscopic observations can be used to give a good determination of the absolute magnitude, and hence of the distance. The comparison of results found by Adams in this way with the measured parallaxes shows a remarkably close agreement; and although it is too early to judge of the exact scope and reliability of the new method, it seems likely to prove a powerful means of extending our knowledge of stellar distances and luminosities. In particular this spectroscopic criterion seems to discriminate between the giants and dwarfs of the later types with certainty.

Two determinations of the mean parallaxes of stars for different magnitudes have been published. The investigation by H. W. Unthank (*M.N.*, *R.A.S.*, **76**, p. 529) is based mainly on the Groombridge proper motions, and employs a theoretical analysis which tries to avoid giving the nearer stars an undue influence on the result—a drawback of the usual method of taking the mean parallactic motion. Sir F. W. Dyson and W. G. Thackeray's discussion (*M.N.*, *R.A.S.*, **77**, p. 7) is remarkable for the extensive data employed—upwards of 26,000 proper motions; their range of magnitude extended from $5^{m}\cdot5$ to $10^{m}\cdot3$. The chief point to determine is the factor by which the parallax diminishes for a step of one magnitude. Unthank gives the value 0·85, and Dyson and Thackeray 0·76, the latter agreeing closely with Kapteyn's result 0·78, published in 1901. Certain statistical investigations had seemed to indicate that Kapteyn's value was considerably too low, but it now appears probable that the recently found correlation between speed and absolute luminosity had caused the divergence; in any case, the confirmation by such an unprecedented wealth of material must reinspire confidence in Kapteyn's formulæ. It may often be very misleading to derive the mean distances of groups of stars by taking the reciprocals of the mean parallaxes, and there is a similar uncertainty in deriving the mean luminosities; C. V. L. Charlier (*K. Svenska Vetensk.*, **10**, No. 28) calls attention to this, and shows the importance of taking into account the range or dispersion of the absolute magnitudes of the stars considered.

It appears to be well established that the stars of type B differ very little from one another in absolute luminosity, so that the distance can be judged with fair accuracy from the apparent magnitude; this is especially the case when small differences between the different subdivisions of type B are taken into account. On this basis C. V. L. Charlier (*Lund Obs. Meddelande*, Ser. 2,

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No. 14) has calculated rectangular co-ordinates for the stars of this type, 804 in number, and has discussed the shape of the galaxy which they form. The centre of the cluster lies 90 parsecs from the Sun in the direction of Carina; the cluster is of oblate form, the extension in the plane of the Milky Way being three times as great as in the perpendicular direction. The extreme diameter in the plane of the Milky Way is about 1000 parsecs, and in the direction of the galactic poles about 300 parsecs; but the stars thin out as we recede from the centre. The motions are somewhat greater in the galactic plane than at right angles to it, so that the velocity surface is an oblate-spheroid. This appears to be also the conclusion of B. Boss (*Astron. Jour.*, No. 691); and it is known that the motions of stars of type A 0 are similarly distributed (*M.N.*, *R.A.S.*, **75**, p. 521), although in the later divisions of type A a different arrangement holds and star-streaming is strongly manifested.

The differences in the behaviour of different spectral types, the distinction of giant and dwarf stars, and the presence of a third drift, have complicated discussions of star-streaming. S. C. Meijering (dissertation published by Hoitsema, Groningen) has taken the largest homogeneous group of stars available, the K stars, and discussed Boss's proper motions of 1420 of these stars according to the two-drift theory. The results differ very little from those previously obtained from this catalogue when all the stars were taken together. In particular the division of the stars between Drifts I and II in the ratio 1·43 : 1 agrees closely with the value 1·5 : 1 for all types together. The third drift is shown very conspicuously in the diagrams which exhibit the statistics.

If any doubt remained as to the complete separation of the stars of later type into two groups of high and low luminosity (giants and dwarfs), a discussion by W. S. Adams, using the new spectroscopic method of finding absolute magnitudes, would appear to be conclusive. In *Proc. N.A. Wash.*, **2**, p. 157, he has discussed 58 stars of type M; these fall into groups of high and low luminosity, with *a clear gap* of 6·4 magnitudes between them. Moreover, the same division (with a clear interval) is shown when the stars are classed according to the intensities of the hydrogen lines. This evidence supplements the results obtained by Russell from measured parallaxes in an important respect; because it might be expected that the connecting link between the two groups, if it exists, would be found among stars of moderate proper motion and brightness which do not give measurable parallaxes; these are included in Adams's discussion, but the gap remains unbridged. A discussion of the theoretical relations between the effective temperatures and densities of the stars by A. S. Eddington (*M.N.*, *R.A.S.*, **77**, p. 16) gives results which seem to be in numerical agreement with Russell's theory. It is found that for gaseous stars of the same mass the effective temperature should vary as the sixth root of the density. This makes the total radiation of a giant star independent of its stage

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of evolution, and accounts for the comparatively uniform brightness of the giants of all the different types.

Considerable attention has been given during the past year to the problems of globular clusters, both practical and theoretical. An important study of the colours and magnitudes of stars in globular clusters has been made by H. Shapley (*Contrib. Mt. Wilson Obs.*, Nos. 115–17; *Observatory*, **39**, p. 452; *Pub. Ast. Soc. Pac.*, **28**, p. 171), including a colour classification of 1300 stars in the Hercules cluster. One striking result is that the colour-index increases with brightness—that is to say, the red stars are brighter than the blue stars. The stars measured extend to about $15^{m}\cdot5$, and it is evident that the counts, except in the case of the reddest stars, have been seriously truncated by this limit. Consequently it is difficult to state precise results; but the tables apparently indicate that the average magnitude of the B stars in the cluster must be about 4 magnitudes fainter than the K stars. It is evident that only the giant stars have come under consideration; but outside the clusters the giant stars have nearly the same brightness whatever the type, the redder types being if anything the fainter. This difference in character between the cluster stars and those elsewhere seems very curious; it is confirmed by Shapley's studies of some other clusters. Assuming that the bluest stars of the cluster are comparable in magnitude with those of the same colour elsewhere (it is probable that a star cannot have the temperature giving a negative colour-index unless it has a certain minimum mass and brightness), the distance of the Hercules cluster is found to be 30,000 parsecs, and its diameter about 350 parsecs. It thus seems to be an extra-galactic object with dimensions not greatly inferior to our own system.

A. H. Joy (*Astron. Jour.*, No. 685) has determined the proper motions of 292 stars in and near the cluster M 37 Aurigæ by comparing plates taken with the Yerkes 40-inch refractor eleven years apart. The individual motions are not much greater than their probable errors. The motion deduced for the cluster as a whole is probably trustworthy; it amounts to $0''\cdot66$ per century, relative to the reference stars.

The problem of the dynamics of a globular cluster has been discussed with special reference to the law of density put forward by H. C. Plummer from the observed counts. E. Strömgren (*Astr. Nach.*, No. 4850) has discussed the orbits of stars in such a cluster. J. H. Jeans (*M.N.*, *R.A.S.*, **76**, p. 567) and A. S. Eddington (*ibid.*, p. 572) have investigated the conditions for a steady state. The former suggests that a falling off of density in the outer parts according to the law r^{-4} (instead of r^{-5}) would fit the observations at least as well, and it would be a more natural law to expect dynamically. Neither writer finds any sufficient dynamical reason for Plummer's law, if indeed it is the general law of distribution of stars in clusters. A. S. Eddington (*M.N.*, *R.A.S.*, **76**, p. 527) shows that there must be a simple relation between the extension of a cluster and the average velocities of

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the stars within it; for a steady state the internal kinetic energy must in all cases be one-half the exhaustion of potential energy.

J. H. Jeans (*M.N., R.A.S.*, **76**, p. 552) has investigated the effects of encounters of two clusters, showing that a simple encounter would lead to a spreading out in the form of a flat disc, as in the Ursa Major cluster. More complicated encounters suggest explanations of some general features of the stellar system.

The Ursa Major "moving cluster" has been examined by L. Courvoisier (*Astr. Nach.*, No. 4832) with a view to detecting its orbital motion under the general attraction of the stellar system. If the stars are following one another round an orbit, those in front will be moving in a rather different direction from those behind. It is possible to estimate the curvature of the orbit by comparing the foremost and rearmost stars. The plane of the orbit seems to be parallel to the galaxy, and the centre of curvature is found to be distant 930 parsecs towards Cygnus. If the orbit is circular the period would be about 180 million years, which agrees very well with the order of magnitude calculated from our knowledge of the general density of stellar distribution; but the direction and distance of the centre disagree altogether with Charlier's conclusions from the type B stars, already quoted.

According to Einstein's theory of gravitation, the frequency of atomic vibrations depends on the gravitational potential at the point where they occur. It should thus be possible to determine the decrease of potential at distant points of the stellar system by observing relative displacements (towards the violet) of the spectral lines of distant stars compared with those of nearer stars, and hence to find the total mass within a given radius. There are many complications, and it seems certain that the systematic K term in the radial velocities is due mainly to other than gravitational effects; but W. de Sitter (*M.N., R.A.S.*, **77**, No. 2) has obtained in this way an *upper limit* to the density of matter in space, including dark stars.

In the work of Shapley on the colours of stars in clusters (*loc. cit.*), it was found that negative (*i.e.* blue) colour-indices were abundant, and the range in colour-index from $-0{\cdot}52$ to $+1{\cdot}96$ agreed with the range manifested by the stars known to be near to us. This is strong evidence against any appreciable scattering or absorption of the light in traversing the great distance (supposed to be 30,000 parsecs) separating us from the cluster; because the effect of absorption would be to redden all the stars systematically. Shapley calculates that the absorption must be less than $0^{m}{\cdot}01$ in a thousand parsecs. A difference in average colour-index between the inner and outer parts of the cluster may possibly point to sensible absorption in the interior of the cluster. With regard to local absorption in special regions of the sky, H. H. Turner has found evidence of obscured regions by comparing counts of bright and faint stars made on astrographic plates in a number of zones (*M.N., R.A.S.*, **76**, p. 149, etc.). The obscured patch can be represented approximately by the spiral $\alpha + 3{\cdot}66\delta = 247^{\circ}$. A. S. E.

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The Motions of Spiral Nebulæ.

An accurate knowledge of the motions of spiral nebulæ, both radial and angular, would go far towards settling the status of these bodies in relation to the system of the stars. The spectroscopic linear velocities alone may indicate important dynamical relations (or absence of relation); and by comparing the radial and cross motions we could estimate the distance, and hence the size, of these bodies. The relative motions of the different parts, and particularly the rate of rotation, will also give evidence as to their nature, and perhaps determine the total mass involved. The interesting question has arisen whether the spiral nebulæ are comparatively small bodies scattered among the stars and belonging to the stellar system, or whether they are great sub-systems on the fringe of the stellar system, or, thirdly, whether they are distant galaxies coequal with our own. For the last alternative the distances must be not merely great from the ordinary standpoint, but of the order of 100,000 parsecs at least; the proper motions would have to be absolutely insensible, since a motion of $0''\cdot1$ per century at 100,000 parsecs corresponds to a linear velocity of 474 km. per sec.

Our object in this note is to survey the observational material that has been collected rather than to discuss theoretical consequences. It will be realised that the observations are of a very difficult character. The spectroscopic work involves very long exposures, during which the apparatus must be protected from changes, and the dispersion used is necessarily small. The measurements of position are made on diffuse knots which will not allow of high accuracy; and since the displacements are generally small, the unknown motions of the comparison stars may sometimes have an important influence.

The first determination of the radial velocity of a spiral nebula was made in 1913 by V. M. Slipher (*Lowell Obs. Bull.*, No. 58), who found for the Andromeda nebula a velocity of 300 km. per sec. approaching. Other investigators have found values in excellent agreement with this, as may be seen from the following:—

Wright . . .	−304 km. per sec.
Pease . . .	−329 ,, ,,
Slipher . . .	−300 ,, ,,

M. Wolf has also made several determinations, his latest result being −450 km. per sec. (*Vierteljahrschrift*, 51, p. 115), but his results have not been very consistent among themselves.

For fainter objects the values may be more uncertain. Thus for N.G.C. 1068 the velocities +1100, +765, and +910 km. per sec. have been given by Slipher, Pease, and Moore respectively (*Pub. Ast. Soc. Pac.*, 27, p. 192); but even here there is evidently some measure of agreement.

Slipher has given radial velocities for fifteen spirals, but except

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in these two cases there is no check on the results.* The average speed from his measures is between 300 and 400 km. per sec. In addition, R. E. Wilson (*Proc. N.A. Wash.*, **1**, p. 183) has determined the velocity of a number of nebulæ in the Magellanic Clouds; it is true that these were all gaseous nebulæ, but presumably the results give the velocities of the clouds as a whole, and these may perhaps be regarded as objects akin to the spirals. The values for the Greater and Lesser Clouds are +275 and +158 km. per sec. The whole material is evidently too scanty as yet to derive a value of the motion of the stellar system relative to a possible super-system of galaxies, and not much stress need be laid on the provisional determinations by Truman (*Pop. Ast.*, **24**, p. 111), and by Young and Harper (*J.R.A.S. Canada*, **10**, p. 134)—about 600 km. per sec. towards an apex at R.A. 20^h, Dec. $-20°$. It may, however, be recalled that Sir W. Herschel's solar apex derived from only seven stars turned out to be quite near the truth, and history may repeat itself in this case.

It seems to be very doubtful whether or not spiral nebulæ show any detectable proper motions. H. D. Curtis (*Pub. Ast. Soc. Pac.*, **27**, p. 216) has measured the motions of sixty-six spiral nebulæ relative to comparison stars of the twelfth to fifteenth magnitude. The average was $3''\cdot3$ per century, but this includes a number of large motions, to which the author attaches small weight. He considered, however, a motion of 8″ per century for N.G.C. 253 to be worthy of confidence. The results have not yet been published in detail. A determination for a single nebula, Messier 101, by A. van Maanen (*Astrophysical Jour.*, **44** p. 210) deserves attention, since it depends on measures of a very large number of points. The motion is $1''\cdot2$ per century relative to thirty-two comparison stars, but since the parallactic motion of these stars would probably be of about this amount and in the appropriate direction, the result does not necessarily indicate a motion of the nebula. An attempt to determine the systematic proper motions of a large number of spirals has been made by C. Wirtz (*Astr. Nach.*, Nos. 4861, 4866), who compared recent measures with those made by Schultz at Upsala forty to fifty years earlier. He found that they were drifting towards R.A. 245°, Dec. $-3°$ (a direction almost opposite to the motion of the stars), whereas the gaseous nebulæ were moving in a similar direction to the stars. These results have been criticised by Kobold and Seeliger, the latter showing that following another method of reduction the resulting apex was nearly the same as that of the stars.

Turning now to the internal motions, spectroscopic observations have shown in some spiral nebulæ a curvature of the spectral lines, which points to a rapid velocity of rotation. Slipher detected a rotation in N.G.C. 4594 (Virgo), which is a spiral seen edgewise (*Lowell Obs. Bull.*, No. 62); Wolf found a rotation in M 81 (Ursa Major), which amounted to 100 km. per sec. not far from the nucleus; Pease found a difference between the velocities of the

* Since this was written another of Slipher's results, +1100 km. per sec. for N.G.C. 4594, has been confirmed by Pease (*Pop. Ast.*, **25**, p. 26).

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nucleus and a knot in M 33 (Triangulum) amounting to 200 km. per sec.; and in N.G.C. 4594 he found a linear velocity of rotation of 300 km. per sec. at 2′ from the nucleus (*Pop. Ast.*, **25**, p. 27).

Some very interesting results with regard to internal motions have recently been obtained by the comparison of early and recent photographs. A. van Maanen (*loc. cit.*) has studied the spiral M 101. From measures of a large number of points he has found that the flow is outwards along the spiral arms. Considering the rotation, it appears that the motion of the arms is concave side foremost—opposite to that of a Catherine-wheel; thus there would be a tendency to uncurl. The motion is surprisingly large, so that at 5′ from the centre the rotation-period would be 85,000 years. For the nebula M 51 (Canes Venatici) S. Kostinsky has found indications of internal motion with the stereo-comparator (*Monthly Notices*, **77**, No. 3). His conclusions appear to agree in the main with van Maanen's. It has been pointed out by J. H. Jeans (*Observatory*, **40**, p. 60) that these results will fit in with a possible dynamical theory.

It would, we think, be premature to draw any general conclusions from these observations; but they show that a new field of astronomical investigation is rapidly being opened up, and we may hope to acquire a more definite knowledge in the near future.

A. S. E.

Einstein's Theory of Gravitation.

A Council Note in 1910 begins with the sentence, "Celestial mechanics, which has hitherto been based on the Newtonian laws of motion, is profoundly affected by the discoveries which have been made in recent years regarding measurements of space, time, and force." * These words were of the character of a prediction, for the new ideas had at that time scarcely reached the principal force with which astronomy is concerned—gravitation. But remarkable progress has been made, especially in the last two years, and there now emerges a complete and self-consistent new system of mechanics, which can be applied without ambiguity to the problems of gravitational astronomy.

Certain conclusions of modern physics have strongly suggested that there is more to be learnt about the effects of gravitation than is to be found in Newton's simple law. It is now believed that the mass of ordinary matter is due to the electromagnetic energy of the ultimate particles, and that electromagnetic energy wherever found must possess mass (*i.e.* inertia); hence the question arises whether all kinds of energy may not, in virtue of their mass, exert and be subject to gravitation. In particular the energy projected in a ray of light may be subject to gravitation and have a curved trajectory in a sufficiently intense gravitational

* *Monthly Notices*, vol. lxx. p. 363.

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field—so that rays of light would be deflected by gravity. Again, it has been supposed that the velocity of light is a fundamental relation between space and time so that no effect of any kind could conceivably be propagated faster; if so, it would seem that the attraction between two bodies must be modified to some extent when their speeds become comparable with that of light.

The new mechanics, which seeks to give a precise answer to these and other questions, rests on an hypothesis of the relativity of space and time. A *restricted* principle of relativity is described in the Council Note already quoted: experimental confirmation has been ample, and no serious doubt of its truth is entertained, criticism being confined to questions of its exact scope and philosophical implications. But we are now concerned with a far wider generalisation, which must be subjected to new experimental tests. So far the consequences that can be tested observationally all lie in the domain of astronomy.

We are accustomed to locate all phenomena in a framework of space and time, to which we assign definite (*e.g.* Euclidean) properties. According to the generalised principle of relativity this framework is a mere creation of our own minds. The variables x, y, z, t which occur in our mathematical calculations have no more direct physical significance than any other of the parameters which we sometimes introduce for convenience as independent variables. *They do not indicate the location of the phenomenon in an absolute space and time.* Probably it is hopeless to try to divest our minds of all idea of location, so the theory can perhaps best be realised if we say that any four independent functions of x, y, z, t might equally well have been taken as the rectangular co-ordinates specifying the location. A little reflection will show that any physical measurement is ultimately an observation of coincidences either of material particles or more usually of light-waves. The whole progress of nature, so far as it is revealed by observation, can be resolved into an aggregate of coincidences of the values of x, y, z, and t for various entities, and it is clear that these coincidences will be unaltered if we substitute other functions of x, y, z, t to indicate the location, *i.e.* if we make any mathematical transformation of our space-time framework.

It is perhaps difficult to realise that there is no absolute framework, because with our measuring-rods and our clocks (or their substitutes) we appear to be measuring an absolute space and time of nature. But this rests on the assumption that our standards remain constant; and the fallacy of believing that our measures are absolute was revealed by the Michelson-Morley experiment, which showed that any measuring-rod, perfectly rigid in the ordinary sense, alters its length when set in motion longitudinally. The failure of the constancy of our standards leads to a reinterpretation of the meaning of our measures. We say that a length AB is one metre, but that does not mean that the distance is one metre of absolute space: it means that if the standard metre were brought and set along AB it would experience an unknown contraction or

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expansion which would result in its extremities exactly coinciding with A and B. We can observe coincidences, but we cannot make absolute measures.

According to this line of argument it would appear that the truth of the principle of relativity is a matter for *a priori* discussion. But for practical applications something more is involved; for us space is filled with the luminiferous æther, and our co-ordinates x, y, z, t, which have no significance as regards location in absolute space, may be quite properly regarded as expressing definite relations to the æther. Other systems of co-ordinates would express other relations to the æther, and we should expect the form of physical laws necessarily to depend on the co-ordinates chosen. The new theory, however, makes the bold hypothesis that these relations to the æther have no effect on the phenomena and can be disregarded—a step which appears to divest the æther of the last remnants of substantiality. The hypothesis is the more acceptable because we have already become familiar with the result that uniform motion relative to the æther has no influence on phenomena; and there is a logical satisfaction in this wider generalisation. (It should be said, however, that there is matter of controversy how far the co-ordinate relations to the æther have really been eliminated by Einstein's theory; they have disappeared completely from the differential equations, but they seem to reappear in the boundary conditions at great distances.)

In finding a new law of gravitation Einstein set himself to fulfil two conditions;—(1) in accordance with the principle of relativity the equations expressing the course of nature must be the same for all transformations of the co-ordinates, since no one system of co-ordinates is more fundamental than any other, and (2) the old laws established by observation must be satisfied as approximations under ordinary conditions. We shall not attempt to give here any account of the mathematics,* and shall only state that these two conditions appear to determine the new law of gravitation uniquely. The fundamental part played by gravitation in the generalised theory of relativity is due to the impossibility of discriminating between a gravitational field and an acceleration of the framework of reference—a fact which is illustrated by our sensations of increased weight when a lift starts to move upwards.

Astronomical Consequences.—Certain results were ascertained before the theory was put in a complete form. Of these the most striking was the refraction of a ray of light by an intense field of gravitation—a result which follows, as already stated, from attributing gravitation to all forms of electromagnetic energy. The energy in a light-ray travels with the velocity of light c; if we treat it like a stream of material particles, and calculate the total angular deviation produced in passing a mass M at a distance r from

* For a general outline of the mathematical methods, see *Observatory*, vol. xxxix. p. 412, or *Nature*, vol. xcviii. p. 328. The full theory is given by W. de Sitter, *M.N.*, vol. lxxvi. p. 699, vol. lxxvii. p. 155, where references to Einstein's and other papers will be found.

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its centre, the result is $2\gamma M/c^2r$. This mode of treating the problem is, however, too crude, and the actual deflection given by the complete theory is twice as great, viz.

$$\frac{4\gamma M}{c^2r}$$

γ is the constant of gravitation, $6{\cdot}66 \times 10^{-8}$ in C.G.S. units. For a star seen close to the limb of the Sun the apparent displacement would be $1''{\cdot}75$. The amount seems considerable, but the difficult conditions of observations of this kind have so far prevented any observational test. For Jupiter the displacement of a star at the moment of occultation would be only $0''{\cdot}017$.

Another result is the displacement of all spectral lines to the red in an intense field of gravitation—that is to say, atomic vibrations are executed more slowly. This is sometimes expressed by saying that time progresses more slowly at points in the gravitational field, since the retardation affects all processes which we could use for measuring time. At the surface of the Sun the displacement is equivalent to the Doppler displacement produced by a radial velocity of 0·634 km. per sec. For a star of mass M and density ρ in terms of the Sun, the displacement is

$$0{\cdot}634 M^{\frac{2}{3}}\rho^{\frac{1}{3}} \text{ km. per sec.}$$

This velocity is in fact v^2/c, where v is the velocity in a circular orbit through the point considered. It is probable that the effect, if it exists, is masked by other uncertainties of the same order of magnitude.

Electromagnetic theory and experiments have shown that the momentum of a body moving with velocity v is proportional, not to v, but to $v(1 - v^2/c^2)^{-\frac{1}{2}}$; this leads to certain corrections in the theory of planetary motions which might be sensible, at least in the case of Mercury. If this were all, the corrections could easily have been made, but it was clear that other deviations of the same order must exist which could not be formulated so definitely. In particular it was not certain how gravitation would be affected by large planetary velocities. These difficulties have been surmounted by Einstein's theory. The complete calculation introduces in planetary theory one very important correction. An elliptic orbit will rotate in the direction of motion with an angular velocity

$$\frac{24\pi^3}{c^2} \cdot \frac{a^2}{T^2(1 - e^2)} \text{ radians per revolution of the planet,}$$

where a, T, and e are the semi-axis, period, and eccentricity of the orbit. Practically the ratio of the angular velocity of the orbit to the angular velocity of the planet is $3v^2/c^2$. For Mercury, this gives a rotation of 43″ per century—just the amount of the well-known discordance in the motion of the perihelion.

The removal of this discordance furnishes the only observational test of the new theory which has yet been applied; the effects on other planets and on the Moon are too small to be detected. To estimate the importance of this result we must consider it in

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connection with the other secular variations of the orbits. There are sixteen secular variations for the four inner planets which are usually considered together. On the old theory the perihelion of Mercury was discordant by thirty times its probable error. On Einstein's theory the largest residual, that of the node of Venus, is $4\frac{1}{2}$ times its probable error—a little larger than the theory of probability predicts,* but scarcely to be considered a very serious discordance. The other residuals are all within the limits expected.

The theory has also been applied by Professor de Sitter to determine corrections to the motions of the lunar node and perigee. The corrections are not quite large enough to be checked at present; but a comparatively small increase in observational accuracy, which may be expected in the future, will render them of practical importance.

Nature of the New Law of Gravitation.—It is natural to inquire what are the chief points of difference between Einstein's and Newton's laws of gravitation. Is the force † between two masses at rest exactly according to the inverse-square? And for bodies in motion is it directed along the line of centres? Unfortunately we cannot give a plain answer. Before we can say how the force varies with the distance, we must define how the distance is to be measured. If we were to take the circular orbit of a planet, and with a measuring-rod measure first the diameter and then the circumference, we should find (if Einstein's theory is true) that the ratio is not equal to π; there would be a discrepancy of some kilometres. We have to suppose that the rod changes length systematically according as it is placed radially or transversely, so that one or other of the measures needs correction—it is impossible to say which.

For illustration we shall suppose that the transverse measures are correct and that the radius of an orbit is deduced by measuring its circumference. In making the comparison with the old law we must use the language of Euclidean geometry and Newtonian mechanics. With this convention the attraction is not exactly according to the inverse-square law even for bodies at rest; there is a small additional term—a repulsion varying as the inverse-cube. There are other forces depending on the velocity, partly radial and partly transverse. If u and v are the radial and transverse velocities, the components of force are approximately

$$R = -\frac{\gamma m}{r^2}\left(1 - \frac{2\gamma m}{c^2 r} - \frac{3u^2 - 2v^2}{c^2}\right)$$

$$\Theta = \frac{\gamma m}{r^2} \cdot \frac{2uv}{c^2}$$

* We should expect the largest of 16 residuals to be about 3 times its probable error.

† The new theory does not use the term "force" at all; we are here attempting to express the practical results of the new theory in the old terminology for the purpose of comparison.

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But it must be emphasized that this is only one way of looking at the matter. It is, of course, possible to choose a system of measurement according to which the inverse-cube term would disappear. Also in accordance with Newtonian mechanics we have taken the mass to be constant, and have therefore included among the gravitational effects terms which on the electromagnetic theory would be attributed to varying mass. The relativity theory draws no such sharp distinction between the different causes which produce change of motion; we have to pass straight to the observable results without the intermediate resting-place which is supplied by the conception of force. A. S. E.

Stellar Spectroscopy in 1916.

Nebulæ.—In spectrographic observations for signs of rotation in the Ring Nebula (M 57) and the Dumb-bell Nebula (M 27), Max Wolf has found (*Sitzber. Heid. Akad.*, 1915) unsymmetrical tilt of certain spectral lines; but he has, after careful experimenting, concluded that the appearance is due to some untraced effect in the spectrograph used. On the other hand, he finds incontrovertible evidence of peculiarities in the distribution of luminosity of different vapours, as judged by the length of the spectral lines of different elements, both in the Ring Nebula and in the Dumb-bell Nebula. Similar effects have been discovered by him also in the nebula N.G.C., 2438 (*Sitzber. Heid. Akad.*, 1916).

With a spectrograph giving dispersion of 1 mm. per 20 A, Campbell and Moore have detected evidence of rotation in the two planetary nebulæ N.G.C. 7009 and 6543 (*L.O.B.*, **9**, 1). The results of two independent methods of measurement are in accord, and indicate that the rotational speed of matter in the first-named nebula at a distance of 9″ from the nucleus is of the order of 6 km./sec. In the case of N.G.C. 6543, the rotational speed is of the order of 5 km./sec. at a distance of about 3″·3. Considerations are adduced that make it not improbable that the masses of these nebulæ are several times as great as that of the Sun.

The wave-lengths of the green nebular lines have been redetermined by Campbell and Moore (*L.O.B.*, **9**, 6) from careful observations of the three nebulæ N.G.C. 6572, 7027, and the Orion nebula by comparison with titanium lines, H_β, and helium lines. The resulting values are—

	Rowland System.	International System.
N_1	5007·023	5006·847
N_2	4959·080	4958·902

By corrections of the wave-lengths adopted for the comparison lines in earlier determinations, Wright (*L.O.B.*, **9**, 9) has recalculated the wave-lengths given in his observations, and finds values in good accord with those of Campbell and Moore.

The spectra of the nuclei of the nebulæ N.G.C. 6826, 418_2, and

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40 have been studied by Paddock (*L.O.B.*, **9**, 29) on photographs taken by Professor Campbell for radial velocity. They disclose the characteristics of Wolf-Rayet stars, and thus tend to generalise the evidence already found by W. H. Wright (*Ap. J.*, **40**, 466). These results would appear to suggest that the Wolf-Rayet spectrum is a first step from the nebular state to a photosphere.

The spectrum of the spiral nebula M 33 has been photographed at Mount Wilson by Pease (*A.S.P.*, **28**, 33). This nebula is the faint and extended spiral in *Triangulum.* The slit of the spectrograph was set upon a bright knot, 10′ from the nucleus. The exposure was 34½ hours. The spectrum showed only three bright lines H_γ, H_β, and 5007. Adams has deduced a radial velocity of − 278 km./sec., from the photograph of which the scale was 9 mm. from H_γ to K. A second photograph, taken with a camera only 3 inches long and of aperture a little more than 1½ inches, gave a very strong spectrum of the nucleus with an exposure of 32 hours. The scale is described as 0·9 mm. from H_γ to K, and the Mount Wilson observers have been able to discern not only that the spectrum is of the type Fo, but also that a velocity of − 70 km./sec. is indicated.

The question of internal motion in the spiral nebula Messier 101 (N.G.C. 5457) has been successfully attacked by van Maanen (*Proc. N.A.W.*, **2**, 386, and more fully in *Ap. J.*, **44**, 210). It is referred to in a special Council note, page 377 *supra.*

In studying the rotation and radial velocity of the nebula N.G.C. 4594, Pease (*Proc. N.A.W.*, **2**, 517) has employed an ingenious device of multiple slits cut in a silver film on glass in suitable positions corresponding to the bright points shown in a direct photograph of the nebula. With an accumulative exposure of 80 hours he has obtained spectra from which he has deduced the velocity at five points in the nebula. The radial velocity is found to be + 1180 km./sec., which compares well with the velocity + 1190 km./sec. determined by Slipher.

The faint nebulosity in the neighbourhood of *ρ Ophiuchi* has been examined spectroscopically by Slipher (*Lowell Obs. Bull.*, **2**, 155), who finds that the spectrum is continuous, with absorption characteristics similar to those of the star near the nebulosity. It thus appears possible that the nebulosity is visible by reason of reflected star light. A somewhat similar case has been made known to us by Slipher's earlier study of the nebulosity in the *Pleiades* (*Lowell Obs. Bull.*, **2**, 26). The light of the nebula round the star Merope was found to give a distinctive spectrum similar to that of the neighbouring star.

Nova.—The spectrum of *Nova Geminorum* No. 2 was photographed with an exposure of 9 hours in 1916 February by Adams and Pease (*A.S.P.*, **28**, 80). The spectrum is still of the definite Wolf-Rayet type, with bright hydrogen lines and an exceedingly prominent bright band at λ 4686. The continuous spectrum is very strong.

Special Studies.—Photometric observations of *β Lyræ* by means of the photoelectric method are given by J. Stebbins (*L.O.B.*, **8**,

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186). The observations were made at the Lick Observatory by means of a sensitive rubidium cell perfected by Professor Stebbins and Kunz in the physics laboratory in the University of Illinois. They extend over three cycles of the 12·92 day period, and while in general features they confirm the visual observations, unexplained discordances occur in successive cycles. There is a marked want of symmetry about the primary minimum.

The spectrum of γ *Argus* has been studied by G. F. Paddock (*L.O.B.*, **9**, 40) with special reference to the nature and wave-length of the Wolf-Rayet band at λ 4652. The spectrum has also been measured by Professor Fowler on photographs obtained at Johannesburg by W. M. Worssell (*M.N.*, **76**, 418).

The spectrum of v_4 *Eridani* is found to contain the enhanced lines of manganese such as Baxandall had discovered in α *Andromedæ*. These lines are noted by Paddock (*L.O.B.*, **8**, 172, and **9**, 47).

The spectrum of Barnard's star of large proper motion has been photographed by Campbell and Moore (*A.S.P.*, **28**, 207), and is found to be of type Mb similar to that of α *Herculis* (giant). W. S. Adams (*A.S.P.*, **28**, 278) finds that it has the characteristics of a dwarf star of class Mb. Seares (*A.S.P.*, **28**, 280) has determined the photographic magnitude 11·4 and colour index + 2·00.

The intensity of the continuous spectrum of stars and its relation to absolute magnitude has been studied by G. S. Monck (*Ap. J.*, **44**, 45).

The origin of the absorption band in the solar spectrum near Fraunhofer's group G, with head approximately at wave-length 4314, has been traced by Newall, Baxandall, and Butler (*M.N.*, **76**, 640), who have identified it with the band usually attributed to hydrocarbon, a feature in the spectrum of the blue flame at the base of a candle-flame. The presence or absence of this band is one of the marked characteristics whereby discrimination can be made between stellar spectra of types G and F.

The interpretation of the spectral characteristics of Cepheid variables is still a matter of difficulty. One of the most remarkable features of the case is the change of spectral type first noted by Albrecht and Duncan (*L.O.B.*, **5**, 93) in the photographs of η *Aquilæ* taken by Wright. Pease's continuous observations in 1914 of *RS Boötis* showed a change of type from B8 at maximum to F0 at minimum in the period of light variation, $9^{h}\cdot 1$. In a note (*Ap. J.*, **44**, 273) H. Shapley gives an account of the variations in spectral type of twenty Cepheid variables. The type is deduced (with an uncertainty stated to be one- or two-tenths of a type interval) from photographs, in which the scale of dispersion is $H_\beta - H_\epsilon = 5\cdot 2$ or 12 mm., according as one or two prisms are used. (In a short note in *Proc. N.A.W.*, **2**, 208, Shapley gives a photographic reproduction under fivefold enlargement of the original spectra in the case of δ *Cephei*.) In many cases the results are illustrated by graphical representations of the light curves with the spectral type assigned at frequent epochs. It would seem as

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if this remarkable piece of work carried out at Mount Wilson must throw light not only on the nature of the causes of light variation in Cepheid variables, but also on the meaning of spectral type as an indication of stages of stellar evolution.

A further paper by Adams and Shapley (*Proc. N.A.W.*, **2**, 136) deals with the change of spectral type at different epochs in the periodic variation of δ *Cephei.*

Classification.—It is announced in the 70th annual report on the Harvard College Observatory that the first step in the classification of stellar spectra for the *New Draper Catalogue* has been completed. The number of spectra classified is 233,050, covering the whole sky from north pole to south.

A catalogue of 287 stars classified according to their chemistry and temperature has been issued from the Hill Observatory, Sidmouth, as Bulletin No. V. It is a continuation of the earlier catalogue issued as Bulletin No. III., and relates mainly to stars between magnitudes 5·0 and 5·59.

The work of Adams on a quantitative method of classifying stellar spectra (*Proc. N.A.W.*, **2**, 143) and on the spectroscopic evidence for the existence of two classes of M type stars (*Proc. N.A.W.*, **2**, 157) is dealt with in the President's Address (page 395 *infra*).

Radial Velocities.—The radial velocity of β *Ursæ Majoris* has been further investigated by Professor Campbell and Miss Hobe at Mount Hamilton (*L.O.B.*, **9**, 22), with results that cannot be brought into agreement with Ludendorff's observations of 1907 (*Ast. Nach.*, **177**, 235), which indicate a period of $27^{d}\cdot16$ and a total amplitude of 15 km./sec.

The radial velocity of Barnard's star of large proper motion has been determined by Campbell, − 128 km./sec., and by Adams, − 94 km./sec. (*A.S.P.*, **28**, 207 and 278); and by Slipher, − 40 km./sec. (*Lowell Obs. Bull.*, **2**, 157).

An interesting case of two stars of common motion is recorded by Adams (*A.S.P.*, **28**, 81). The stars are A Oe 14318 and 14320, of similar magnitudes 9·6, 9·2, of similar spectrum G5 and G4, of similar proper motion $3''\cdot76$, and similar radial velocity + 300 km./sec. Russell has determined the parallax of the stars $0''\cdot035$. Hence their distance from one another is 8600 times the radius of the Earth's orbit, or about one-thirtieth of the distance to α *Centauri*. The stars are separated by 5′ in declination, and are moving in precisely the same direction at a rate of about 580 km./sec.

Variable Radial Velocities.—Observations made by the D. O. Mills Expedition disclose variable velocities in six stars, a list of which is given by J. H. Moore (*L.O.B.*, ix. 37). The stars are of classes A5, F2, B5, B4, B2, F5.

A further list of four stars is given by R. E. Wilson (*L.O.B.*, ix. 38) from the work of the same expedition at Santiago. The stars are of classes B3, B2, A, B8.

The Algol variable star *RS Sagittarii* was observed at Santiago

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in 1909 for the purpose of finding variable velocity. G. F. Paddock (*L.O.B.*, ix. 39) announces that from nine photographs a range of velocity of 160 km./sec. is deduced, in good agreement with R. E. Wilson's observations in 1915.

The brighter component of the double star *σ Coronæ* has been found to be a spectroscopic binary in which the spectra of both components appear at times, both being of type F8p. W. S. Adams, who notes this fact (*A.S.P.*, 28, 80), also announces that the star Boss 5996 (A2) has variable velocity.

Observations, old and new, of the variable velocities of the stars *α Orionis* and *α Scorpii* are summarised by J. Lunt (*Ap. J.*, **44**, 250), who calls attention to the need for further observations for further revision of the orbits deduced by Bottlinger (*Ast. Nach.*, **187**, 33) and Halm (*Ann. Cape O.*, **10**, pt. 3).

Spectroscopic Binary Stars.—Orbits are given for the following stars :—

					P.	e.
h_4	Centauri (B5)	Paddock	*L.O.B.*,	9, 42	6^d·927	0·23
B.A.C.	5890 (F)	Parker	*Ottawa Obs.*,	2, 331	26·2742	0·49
μ	Persei (G)	Cannon	,,	2, 353	284	0·06
Boss	3323 (A5p)	,,	,,	2, 369	38·3	0·07
12	Lacertæ (B2)	Young	,,	3, 65	0·1931	0·0
A	Boötis (G5)	,,	,,	3, 95	211·95	0·54
α	Trianguli (F5)	Harper	,,	3, 114	1·7365	0·12
14	Aurigæ (A2)	,,	,,	3, 221	3·789	0·03
B.D.	78° 412 (A5)	Lee	*Ap. J.*,	**43**, 320	1·271	0·0
σ	Puppis (K5)	Lunt	,,	**44**, 261	258	0·2
σ	Aquilæ (B8)	Jordan	*Alleg. Obs.*,	3, 189	1·9502	0·0
T	Vulpeculæ (F)	Beal	,,	3, 197	4·4358	0·44

Work in the Laboratory.—In new observations of the spectrum of silicon, Fowler (*M.N.*, **76**, 196) has found four lines which from their behaviour under suitable conditions must be classed in Lockyer's group IV., developed when condensed discharges are utilised. Lines of this group were found by Lockyer to be characteristic of the earliest varieties of the B stars. Fowler now finds that his four lines occur in the spectrum of *β Crucis*, a star of type B1.

In a further note on the nature of the coronium atom (*M.N.*, **76**, 415), Nicholson has been led (by the discovery by Deslandres and by Carrasco of a new strong line in the red part of the spectrum of the solar corona observed in 1914 August) to a remarkable completion of his analysis of the spectrum of the corona. His earlier researches enabled him to assign all the observed lines except five to a hypothetical element protofluorine, with a positive core or nucleus with charge 5*e*. He now assigns

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the remaining five lines, including the well-known line at 5303, to an element which he consents to allow the name coronium, with a positive nucleus with charge $7e$ and a simple ring system. When the ring contains eight electrons—thus having a single negative change—it emits the five lines unaccounted for in his earlier work.

The banded spectra ascribed to titanium oxide, magnesium hydride, and calcium hydride have been studied in the electric furnace by King (*Ap. J.*, **43**, 341), who finds clear evidence that the bands connected with titanium appear in the presence of oxygen, and that those connected with magnesium and calcium appear in the presence of hydrogen, and that the temperature of about 2300° C. seems to be the upper limit for the greatest strength of the bands.

The spectrum of iron in the oxyhydrogen flame has been studied by Sir Norman Lockyer and H. E. Goodson (*Proc. R.S.*, **92**, 260).

The structure of broadened spectrum lines has been investigated by Merton (*Proc. R.S.*, **92**, 322) by means of a Fabry and Perot interferometer.

A note on the pole effect in the spectrum of calcium has been published by Gale and Whitney (*Ap. J.*, **43**, 161). It has special reference to the effect in triplets of the first and second subordinate series. In a second paper (*Ap. J.*, **44**, 65) Whitney gives the pole effect for all the principal lines of calcium spectrum.

The decomposition of spectral lines in an electric field was discovered by Stark and by Lo Surdo in 1913. The effect has been studied by Janet Howell at Mount Wilson (*Ap. J.*, **44**, 87).

Instruments.—The 72-inch reflector which is being erected on the summit of Saanich Hill, Victoria, B.C., is described by Plaskett (*Jour. R.A.S. of Can.*, 1916, p. 276) with striking illustrations of the pier, dome, and instrument. A description of the tests made to ascertain where the conditions were most suitable for this instrument is given by W. E. Harper (*Ott. Publ.*, **2**, 275).

A site has been chosen for the 1½-metre reflecting telescope of the National Observatory of the Argentine Republic. It is described by C. D. Perrine (*A.S.P.*, **28**, 253) as being situated near the southern extremity of the Sierra Chica, about 40 kilometres south-west of Cordoba, at a height of 4000 feet above sea-level.

H. F. N.

"*Summer*" *Time.*

Though astronomy and navigation are specifically excluded from the operation of the Act, of which the above is the short title, an alteration of public time is sufficiently akin to our subject to demand notice here.

About the year 1907 Mr. William Willett, the head of a firm of London builders and estate agents of reputation, and a Fellow of our Society, conceived the notion that it would be for the good of

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the community if business, and life generally, began earlier in the solar day than they do in the summer months of the year, in order to have more daylight in waking hours. He proposed to effect this by an alteration of the clocks of the country, which should be set forward by some desired amount so that mean noon should be earlier than twelve by the clock; and he advocated this scheme by circular and by notices in the press, under the heading "The Waste of Daylight." In 1908 Mr. Robert Pearce, member of the House of Commons for Leek, Staffordshire, and also a Fellow of our Society, presented a Bill to promote the earlier use of daylight in certain months of the year. In this Bill, known as the Daylight Saving Bill, it was proposed that twenty minutes should be dropped from the hour between two and three o'clock in the morning of each of the first four Sundays in April, so that during the summer months the clock would be eighty minutes in advance of Greenwich Mean Time, and that twenty minutes should be added to each of these four defined hours in September. The time thus established was to be known as British Time. The Bill passed its second reading in the House of Commons on 1908 March 26—a report in *The Times* says "amid laughter and ironical cheers," and a note in the same paper says that the passing was due to some parliamentary accident; but the Bill was referred to a Select Committee, and the subject was then fully discussed. Sir William Christie, Sir David Gill, Sir Robert Ball, Dr. Rambaut, and Dr. W. N. Shaw among others stated their views. The first two and Dr. Shaw were opposed to the Bill, Sir Robert Ball and Dr. Rambaut spoke in its favour. The Committee amended the Bill by proposing that the change should be made by putting the clock forward one hour on the third Sunday in April, and one hour backward on the third Sunday in September of each year, and reported to the House in this sense. The Bill thus amended was re-introduced by Mr. T. W. Dobson, member for Plymouth, on 1909 March 5, and the second reading was carried by 130 to 94. The Bill was again referred to a Committee, who examined twenty-four witnesses, and the Chairman drafted a Report in its favour, but the Report was rejected by a majority of one, and the Committee recommended that the Bill be not further proceeded with owing to great diversity of opinion and to "doubts which have been expressed as to whether the objects of the measure can be attained by legislation without giving rise, in cases involving important interests, to serious inconvenience." It was suggested that the Bill would be again brought forward by Mr. Pearce in 1911, but this was not done, and there is good reason for saying that the Government did not intend that the measure should become law. However, last year, owing to the necessity for economy in lighting, the Government adopted the scheme, and a resolution approving the principle of altering the clock passed the House of Commons on 1916 May 8 by a majority of 170 to 2. On the next day Mr. Herbert Samuel, President of the Local Government Board, introduced a Bill, which quickly passed through all stages, and was made law on 1916 May 17. Clause 1

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of the Act, which is known as the Summer Time Act (1916), runs as follows :—

(1) During the prescribed period in each year in which this Act is in force, the time for general purposes in Great Britain shall be one hour in advance of Greenwich Mean Time.

The next clause explains that this was to be in force in the year 1916, and that the prescribed period during that year was from May 21 until October 1 two o'clock in the morning of each day, but that in any other year during this present war the Act may be brought into force by an Order in Council for any specified period. The Act also applied to Ireland, Dublin Time being substituted for Greenwich Time.

Proclamation was made in the press and by placard, and on the night of Saturday, May 20, clocks of all kinds were put forward one hour, and the community lived by this altered time, apparently without inconvenience, though there were a few expressions of dissatisfaction with the scheme. In September proclamation was again made in the same way, and on the night of Sunday, September 30, the clocks were put back one hour, and so Great Britain reverted to Greenwich Time.

The scheme was adopted, and we believe also carried into effect, in Austria, Denmark, France, Germany, Holland, Italy, Norway, Sweden, and Portugal.*

It may be noted that the Ordinances above mentioned are the first which specify Greenwich Time or any other time, as the time to be used for general purposes. The Statutes (Definition of Time) Act, 1880, which is the first British enactment dealing with time, removes doubts as to expressions of time in Acts, deeds, and legal instruments, but does not ordain any system for general use.

H. P. H.

Zone Standard Time.

Following the above legislation, the Time (Ireland) Act became law after very little opposition on 1916 August 23, and thus after many years' agitation of the question the time adopted for use in Ireland was assimilated to that adopted for use in Great Britain. The Act came into force on 1916 October 1, and is to operate when the Summer Time Act is in force as well as at other times.

Opportunity may be taken to say here that since the last note on the subject in these pages there have been several additions to

* Since this note has been written a Parliamentary Committee appointed to inquire into the working of the scheme in Great Britain has reported that they found a strong feeling in favour of the reform throughout the kingdom, and it is announced that the Act will be brought into force in these islands this year (1917) from April 8 until Sept. 23. It is learned from announcements in the daily press that the clocks will be advanced in France and Algeria on March 4, and in Germany on April 1.

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the list of nations and Governments that have adopted the Zone Standard Time system. The most important perhaps of these is Brazil, which is divided into three zones, the time of these being respectively three, four, or five hours slow on Greenwich. In certain Brazilian islands in the Atlantic the time is two hours slow. In some West Indian islands the time four or five hours slow has been adopted. The time in Fiji is now twelve hours fast, that of Samoa 11½ hours slow, in the Sandwich Islands 10½ hours slow, and there are other additions which are given in detail in other places. H. P. H.

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Harvard College Astronomical Observatory.
Helsingfors, Finska Vetenskaps Societet.
Helwân Observatory.
Hobart, Royal Society of Tasmania.
Hyderabad, Nizamiah Observatory.
India, Survey Department.
India, United Service Institution.
Japan, Astronomical Society.
Johannesburg, Union Observatory.
Kodaikánal and Madras Observatories.
Lick Observatory.
Lisbon, Observatorio Astronomico.
Lowell Observatory.
Lund, Astronomiska Observatorium.
Madrid, Observatorio Astronomico.
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Mauritius, Royal Alfred Observatory.
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Milan, Reale Osservatorio di Brera.
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Mount Wilson, Solar Observatory.
Neuchatel, Observatoire Cantonal.
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Paris, Université.
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Perth, Western Australian Astronomical Society.
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ADDRESS

Delivered by the President, Professor R. A. Sampson, on presenting the Gold Medal of the Society to Mr. Walter Sydney Adams.

THE Council has decided to award the Gold Medal to our Fellow and Associate, Mr. Walter Sydney Adams, of Mount Wilson Observatory, California, for his investigations in Stellar Spectroscopy, including the determination of Absolute Magnitude. It is my duty to lay before the Society to-night the grounds for this award.

The outline of Mr. Adams's scientific activity is a simple one. In 1898 he was attached to the staff of Yerkes Observatory as a Fellow in Astronomy. He remained there till 1904, when Mr. Ellerman, Mr. Ritchey, and he accompanied Professor Hale to form a nucleus of the staff at Mount Wilson. Latterly he has been Assistant Director of the Observatory. During these seventeen or eighteen years of his work he has produced a series of notes and memoirs engaged with solar and stellar spectroscopy, of which the number appears to be about fifty, and many of which embody prolonged labour, both in observing and in collating.

It is to my judgment a sign of a rightly managed observatory when it is not possible always to separate clearly the contributions of the various members of the staff. This proposition, like the H and K lines of the Solar Spectrum, can be made to appear also under the form of a double reversal, or rather, as one of several: as, that an observatory is wrongly managed where each man's credit is distinct; or that it is rightly managed where it is possible to confuse the parts contributed; and so on. I would guard myself formally against seeming to affirm any one of these, though I suppose I cannot logically escape from affirming some one among them all. Be that as it may. It is a fact that some minds work best alone, and some more naturally in association. We find a substantial proportion of Mr. Adams's papers written in collaboration now with one colleague, now with another; Adams with Frost, Adams with Hale, with Gale, with Kapteyn, with Kohlschütter, with Shapley, with Pease, with Miss Lasby and with Miss Burwell. We can admit, as ought to be, that there was mutual assistance in all these cases, which the authors themselves did not desire to divide. In a long series of this kind an experienced reader will not find difficulty in recognising the merit he is in search of, and I imagine it is specifically the duty of your Council, using their knowledge of the circumstances of observatory work, to admit such collaboration in making their award, and not to seem to put a premium upon half-effective isolated work or selfish credit-hunting.

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There are times and positions, there are directions, in the history of science, where no real advance can be made except by an act of genius. Naturally there is no time when genius does not find its field and opportunity. But there are times too when the greatest, the indispensable need is well-directed spade-work.

Mr. Adams has set himself with undivided purpose, along with his colleagues, in connection with a great institution, to do the spade-work of spectroscopy; the work is well and truly done, and at the end I shall have to tell you how the spade has turned up a veritable pot of gold. I recall a passage of Roscoe's, in which he anticipates the advent of a Newton some day to interpret the spectrum to us. No Newton has arrived so far; to wait for one would be ignoble; bit by bit we seem to be winning through without him. And as we go on we realise that early investigators little knew the delicacy and riches of their own material and its almost infinite possibilities for interpretation of the structure of the atom or molecule that yields the spectrum, of the circumstances under which it originated, and of its history on its passage to us.

The distinct spectra produced by arc and spark, peculiarities of enhanced lines, the Doppler effect, pressure shift, the Zeeman effect, wings, broadening, density, diffuseness, bright, reversed and doubly reversed lines, rhythmic series, bands, give indications obviously full of meaning, and as yet imperfectly explored, of states of temperature, pressure, atmosphere, combination of elements, motion, magnetic field, and other circumstances of the genesis of the spectra. In the same spectrum different lines and different collections of lines behave in distinct ways under the same stimulus. I may take in illustration two papers by Gale and Adams in collaboration * on the pressure shift of lines in the spectra of iron and titanium. The subject was not new; but in the spectrum first-hand knowledge of the phenomena is essential, and it was necessary to repeat the experiments with special view to the ranges of pressure that might be expected in the reversing layer of the Sun and stars of like class. Both spectra contain very many lines, and they behave with extraordinary variety under pressure; some are reversed symmetrically, some unsymmetrically; some remain bright and fairly narrow under pressure, some are bright and symmetrically broadened, some broadened very unsymmetrically towards the red; yet another class, as if to defy any facile theory, shows a displacement towards the violet. Again, there is the distinction of behaviour of arc, flame, and enhanced lines. These are the circumstances; it is not within my purpose to summarise the conclusions, which, generally speaking, are a confirmation of the work of Duffield, Humphreys, Hale, and St. John. But I would remark that it seems as if before the end we should have to schedule the behaviour of each individual line,

* *Astrophysical Journal*, 35 (1912), p. 10; 37 (1913), p. 391.

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just as we catalogue the proper motion of every star. Certainly the researches of which I have to speak further to-night, and which form the field of Adams's labour, chiefly depend for their success upon the detection of the idiosyncrasies of particular lines in the solar and stellar spectra, and it would be obtuse to suppose that they are anything but the opening of a door. I propose to give you a summary of his researches, as far as time permits and as far as they are suited to summary, but it is less my intention to give an adequate list of the contents of all his papers than to convey a picture of sedulous examination of every feature of his subject, never, it would seem, with a sensational intention or declared hope, but by a practical process of exhaustion to make sure not to miss anything that was within the scope of his resources.

I think it will be clearest if I arrange these writings, not chronologically, but in groups according to their subject. Without difficulty we may group them under the heads, (i) instrumental and laboratory studies; (ii) papers on sun-spots and other features of the solar surface, including papers on the solar rotation; (iii) radial motion and spectroscopic binaries; (iv) spectra of particular stars; and (v) general considerations relating to spectra, leading up to the momentous discovery—of which he is careful to share the credit with another—of the possibility of predicting parallax.

A description of the spectrograph employed at Mount Wilson should not be passed over,* for instrumental points are quite as important as general ideas. I would ask your indulgence, however, here and in what follows. I cannot but feel that I may miss some critical feature, and for your sake I wish the duty had fallen to one who was a regular student of the spectroscope. It was with the 60-inch telescope that most of the work was done. This telescope of Professor Ritchey's, massive as it is, is certainly the most flexible instrument that was ever constructed. It is capable of use as a Newtonian, and in three different ways as a Cassegrain. Similarly it has three spectroscopes. One, for the spectra of faint stars, is of low dispersion, and is fitted at the Newtonian focus with a short camera. One is a very powerful instrument, used only for the brightest stars; it is attached in fixed position when the telescope is used as a Cassegrain with equivalent focal length of 150 feet, the beam passing by reflection down the polar axis; the spectroscope is of the auto-collimating type, with 18 feet camera, and gives a scale which runs from 6·2 A. to the millimetre at $H\alpha$ up to 1·4 A. at $H\gamma$. The third instrument, the one most frequently used, is fitted with one, two, or three prisms and cameras from 40 inches to 7 inches in length, varying the scale according to the star from 5 A. to the millimetre down to 96 A. Several thousand spectra have been taken with one or other of these combinations, all with slit and comparison spectra, and are available for classification, radial motions, and

* *Astrophysical Journal,* 35 (1912), p. 163.

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other detailed work. The following particulars of exposure times are of technical interest.* The definition is on a scale of 5.

Star.	Mag.	Spectrum.	Exposure Time.	Slit-Width.	Definition.
			min.	mm.	
Lal. 27742	6·8	G2	27	0·038	3
W.B. 15^h720	6·8	G9	30	·038	3
Groom. 1855	7·4	G8	61	·038	3–4
OΣ 298	7·9	Ko	75	·038	3–4
Lal. 5761	8·0	A3*p*	77	·038	3
Groom. 34	8·2	Ma	122	·038	2–3
Lal. 21185	7·6	Ma	92	·038	4

Let me next pass in rapid review the papers on the solar surface. They are a good example of the joint work of the staff at Mount Wilson, and of the way long-standing problems of the Sun's surface are dealt with there, defined, isolated, and followed to their inmost lairs, and, we must say to a large extent, solved. One does not fail to see Hale's hand. Under his direction research always seems to find its point. Instrumental and laboratory resources, great to begin with, grow to meet the requirements of the problems as they arise. Questions are divided or handed on from one member of the staff to another. One knows that it is one of Professor Hale's anxieties to encourage the amateur, and keep the small man alive. I should agree that in that view he shows as much wisdom and foresight as in his great organisation. The everlasting paradox of things is not going to fail, at this period of the world, to humble the mighty and exalt them of low estate—from time to time. Still, we shall not pass this organisation under our view without offering afresh our admiration, not dazzled or discouraged or envious, for a growth that I believe has had no parallel in astronomy since Tycho built Uraniborg.

Three of the papers I have to refer to are in collaboration with Professor Hale, one of them with Mr. Gale as well.† They establish the fact that the differences shown between sun-spots and the disc are in fact just temperature differences, the sun-spot being at a lower temperature. It is not a recondite result. It had been implied before by Lockyer, perhaps not very clearly. It might be said and unsaid half a dozen times in speculation without attracting attention or possessing importance. It is therefore worth while to point out how the papers in question handle it.

Taking the metals titanium, chromium, vanadium, iron, and manganese as touchstones, their spectra were produced for comparison with the spark, a high current arc, a low current arc, a synchronous alternating arc devised by Professor Crew, and the electric furnace, and differences were shown which with little doubt must be

* *Ap. J.*, 39 (1914), p. 342.
† *Ap. J.*, 23 (1906), p. 11; 24 (1906), p. 185; 25 (1907), p. 75.

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ascribed to temperature merely. The relation of these results to the spot spectrum may be illustrated by the table :—

Element.	No. of Lines Strengthened in Spots.	No. of Lines Strengthened in Weak Arc.	Unchanged in Weak Arc.
Ti	88	83	5
Cr	46	42	4
Fe	19	18	1
V	56	52	4
Mn	11	9	2
	220	204	16

It is clear that the lines strengthened in spots are those strengthened in the weak arc, pointing to the conclusion that the spot temperature is below that of the general solar surface.

The conclusion is certified by the identification of a vast number of lines with the fluted spectrum of titanium oxide. I cannot here pursue the matter as it deserves, nor do more than mention the confirmation afforded by Fowler's subsequent detection of the lines of magnesium hydride in spot spectra, and of calcium hydride by Olmsted, nor need I say that the matter is not ended with one conclusive stroke. It is the subject of a further elaborate review under Adams's name in 1909,* and Mr. Evershed has also dealt with it in several of the Kodaikánal Bulletins.

While still at Kenwood, Hale had photographed certain bright chromospheric lines without an eclipse by setting his slit strictly tangential to the Sun's limb. He carried on this work at Yerkes, and one of his joint papers with our medallist is upon the same subject. Adams, with Miss Burwell, took it up again in much greater fulness, showing among other things that he was able by this method to equal the best eclipse results — for example, Mitchell's—and also to reach a lower level of the chromosphere, that the heavier metals show up more strongly as the level falls, and that an absorption spectrum that presents itself at the same time differs from the normal spectrum by the same characteristics as Hale and Adams found in comparing the disc with the limb. These results remove an important group of questions from their sole dependence upon the precarious region of eclipse observations.

Returning to the question of the disc and the limb, let me describe at somewhat greater length a paper published under Adams's sole name in 1910 † upon that subject. The matter originated in a remark of Halm's made while determining the Sun's rotation with the heliometer at Edinburgh, namely, that the comparison of the limb and centre showed that generally the lines at the former were displaced to the red, as would be the case under pressure, to which in fact he ascribed it, supposing that the lower levels of greater pressure contributed a greater proportion of the

* *Ap. J.*, 30 (1909), p. 86. † *Ap. J.*, 31 (1910), p. 30.

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appearance at the limb than they do at the centre. An earlier investigation of Mr. Hale and our medallist upon the subject* showed three clear features in the limb spectrum, which up to then had been supposed singularly little modified except by general absorption, viz. a weakening and disappearance of the wings shown by many of the strongest lines at the centre of the Sun; second, a slight widening in the case of nearly all lines; and third, a strengthening or weakening of certain lines which in general closely corresponds to the strengthening or weakening of the same lines shown in sun-spots. At the same time it was clear that a more searching investigation was wanted, and this Mr. Adams undertook with the 30-foot spectrograph of the tower telescope, upon very much the same plan as is used for the rotation of the Sun, except that he observed only upon the Sun's equator, and compared the east and west limbs with the centre of the disc instead of directly with one another. The half sum of the displacements shown would then give the solar equatorial rotation, while the half difference gave the displacement of the line which was characteristic of its origin at the limb. No plate was included which did not give a reasonably accordant value of the rotation, for it is a baffling feature of this work that isolated plates to which by inspection no exception can be taken, may give outstanding results for this constant. Adams's value for the resolved component in the line of sight for 1908 was 1·86 km./sec.; the mean of the present series is 1·87 km./sec.; the range included among plates retained is from 1·70 km./sec. to 1·98 km./sec., a range which is comparatively small beside the other shifts under consideration. Forty-two plates were obtained, covering the spectrum from λ3818 to λ6420, or from below K up to C. As may be supposed, some difficulty was experienced in keeping a research begun upon so comprehensive a plan within reasonable limits. Eventually it was confined to the following programme:—

(1) all prominent enhanced lines;
(2) all lines of calcium, magnesium, sodium;
(3) the measurable lines in the cyanogen flutings;
(4) the α and γ lines of hydrogen;
(5) lines of elements like lanthanum and cerium of very high atomic weight;
(6) a large number of lines of iron, titanium, vanadium, chromium, and other elements distributed as uniformly as possible throughout the spectrum;
(7) lines specially strengthened or weakened in the spectrum of the Sun's limb, making 470 lines in all.

On making the measures a problem arose which is characteristic. It is accepted that the fluted spectrum of cyanogen is not displaced by any agency except motion in the line of sight. Now a small but distinct displacement appeared which could not be ascribed

* *Ap. J.*, 25 (1907), p. 300.

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to line-of-sight motions at the limb, for these were verified by the values found for the rotation, and must then be ascribed to line-of-sight motions at the centre. Assuming that there is upon the average a displacement due to rising gases which at the centre of the disc are approaching us, its value was determined on the supposition that the cyanogen bands were undisturbed, and the resulting allowance subtracted from the whole of the other readings. The modification is not very great, but it forces Mr. Adams to bring under review a double set of results, the modified and the unmodified ones, neither of which can be dismissed, though neither is entirely satisfactory. I do not propose to take you over the conclusions of this discussion, which would require to be very carefully stated with all their qualifications, and some of which are still in debate; I shall only point to one arresting exception that declares itself. It is that while we conclude that the cause of the changes are seated in the lower strata from the absence of displacement effect in hydrogen, calcium, and other high-level lines, yet we find that the deep-seated heavy elements lanthanum and cerium show very little displacement too. Such a fact proves the necessity for so careful a survey, and produces, in me at least, a confirmed scepticism of ready generalisations that are not based upon such scrutinies.

Mr. Adams's general conclusion is that Halm's surmise is correct, and that the features shown upon the whole are evidence of a higher pressure in the strata which the lines of the limb stand to represent, and that they cannot be explained as line-of-sight motions. As to some of these conclusions I would mention that Mr. Evershed, certainly with good ground, takes a critical position.* Nor do I wish to magnify Adams's work here or elsewhere into an oracle. That would misrepresent equally it and him. The problems require unravelling. When one or another is unravelled there may be so much of it that it is almost as puzzling as before. My aim is merely that you should understand the nature of the task and the manner in which Adams has gone to work.

The rate of rotation and the law of change with latitude of the solar photosphere may be called the first step in the spectroscopy of displacement of spectral lines. The comparisons for the Doppler effect are here all to hand, either by matching the approaching and receding limbs of the Sun directly with one another, or by the mediation of unshifted atmospheric lines. Dunér at Upsala first measured the effect in 1890, working visually. Halm at Edinburgh, working with Gill's heliometer, also visually, distinctly improved the accuracy, and in the course of it found the significant alteration of the whole of the limb which led to comprehensive researches later on the same matter by our medallist and others. Sir Frank Dyson, while at Edinburgh, again improved upon Halm's methods, introducing photography and a large-scale image of the Sun, and at Cambridge and at Ottawa also the same

* *Kodaikánal Bulletin*, No. 39 (1914).

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problem has been handled by modern methods. It was among the original proposals to engage the Mount Wilson Observatory. In 1906–7 Mr. Adams made a determination with the Snow telescope, a horizontal beam and an 18-foot camera.* In 1908 he made a second with the tower telescope and 30-foot camera.† In 1909 the same apparatus was employed to compare the two limbs separately with the centre of the disc, and led to a third determination, at any rate, of the rotation at the equator—this I have already described; while a fourth study, made also with the tower telescope and 30-foot spectrograph, was occupied specially with the behaviour of hydrogen lines, particularly Hα.‡ It is superfluous to say that very good results were got. The feature that marks them in comparison with what was done elsewhere is in my judgment less than the advantage given by greater instrumental power or better atmospheric conditions (it is easily possible to exaggerate these) than the plan to examine methodically the behaviour of high levels such as give genesis to the hydrogen lines as well as low levels marked by lanthanum and cyanogen, alongside the mean results. The investigation becomes more than an inquiry into surface drift; it is a sounding of the Sun's atmosphere. To derive trustworthy measures from the line Hα is no small feat, no matter what resources you have, but Adams's conclusion seems abundantly proved that the hydrogen strata rotate everywhere faster than the strata below, and that Carrington's decrease of motion from equator towards the poles has at their level largely disappeared. Comprehensive theories of the Sun are at present, I suppose, in the lumber room. When the time comes to bring them out again, this fact will be a very awkward feature to find a place for.

It will not, I hope, be written down to partiality if I express the view that the advantages of climate and equipment possessed by Mount Wilson by no means rob their full value from the other parallel researches. Debate is the healthy state for scientific questions. Where there is agreement it means, usually, somnolence. But we may say this, no one would now produce measures of the solar rotation without taking into careful review the way they stand in comparison with Adams's results.

Referring now to the group of papers which deal with stellar spectra, and first with those that deal with individual spectra of general or of particular interest, I shall not say much more than that they represent the notes of points of special interest which arise to vary the round of regular work. Many of them are in collaboration, first with Frost at Yerkes, afterwards with various members of the staff at Mount Wilson. Three of them relate to Novæ, namely, Nova Persei (1901) and Nova Geminorum No. 2, (1912) at the time of their discovery, and the third, which will perhaps be felt more characteristic of Mount Wilson work, is a

* *Ap. J.*, 26 (1907), p. 203.

† Publications of the Carnegie Institution of Washington, No. 138 (1911).

‡ *Ap. J.*, 27 (1908), p. 213.

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review in 1914 of the spectra of four Novæ,* viz. Nova Aurigæ (1891), Nova Persei (1910), Nova Lacertæ (1910), and Nova Geminorum No. 2 (1912). At this date their magnitudes were respectively 14, 12·4, 12·5, and 10. With a single-prism spectrograph of 16 cm. length, at the Newtonian focus of the 60-inch telescope and a slit-width of ·062 mm., an exposure of sixteen hours was required to secure the first. It is a *tour de force* worth noting.

Several other papers are notes of individual stars or short lists of stars of variable radial velocity. In the early days of radial work the investigator did not do badly if his results succeeded in not flatly contradicting one another. We are now very far from that. In Adams's lists of 500 stars the probable error is generally less than 1 km./sec. Hence a discrepancy of a very few km. between the determination from plates at different dates becomes a strong presumption in favour of real variation of velocity, and such stars are marked for investigation as spectroscopic binaries, and possess an interest of their own as being a very large proportion—say one-third or more—of the whole, of shorter period than can usually be established with the micrometer, and when they also happen to be eclipsing systems, as offering very valuable information as to the *densities* of the stars.

One of the papers upon individual spectra should be noted, not so much from its contents as that it gives a foretaste of what we may look for in the future. This is the paper on the spectra of Sirius, Procyon, and Arcturus.† Among the original plans of Mount Wilson observation, large-scale spectra of the brightest stars had a place. This illustrates what was intended. The high-power spectroscope was employed, giving a spectrum up to Hα of scale approaching that of Higgs's solar spectrum. The purpose of so minute a study is to obtain knowledge of the conditions of atmosphere as to pressure, temperature, and so forth by examining the modifications undergone by the spectral lines. It may be said to start from an observation of Professor Hale's of the presence of sun-spot lines in the red stars.‡ The quantity of information contained in these spectra must be enormous. Naturally the treatment given in the first discussion is far from exhaustive. Some of the characteristic results may be mentioned. The spectrum of Sirius is of class A, that of Procyon F5 or approaching a solar type, while that of Arcturus is K. The three stars, therefore, would form a progression, according to the current view. It is found that in the spectrum of Sirius the enhanced lines which are very prominent are shifted distinctly towards the red relative to the arc lines, that the same feature in less pronounced degree is shown by Procyon, but for Arcturus it has almost or quite disappeared. Discussing Arcturus separately, Hale's remark of the similarity of its spectrum to a sun-spot is confirmed by the correspondence in the relative strengthening and weakening

* With F. G. Pease, *Ap. J.*, 40 (1914), p. 294.
† *Ap. J.*, 33 (1911), p. 64.
‡ *Publ. Yerkes Obs.*, No. 2, p. 129; *Ap. J.*, 23 (1906), p. 400.

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of many different classes of lines; and it is shown that the elements hydrogen, calcium and magnesium, vanadium titanium and nickel, and iron fall into four classes, the lines of each being shifted measurably towards the red relative to the preceding. As to the explicit interpretation of these results, variety or reserve of opinion may be allowed. One can say that they appear to fit in quite well with views that are widely held as to the states of these stars; no one would hesitate to admit them as facts of the first importance and very difficult to arrive at.

I come now to a very important memoir on the radial velocity of 500 stars.*

The plan of this investigation included—

1. A and B type stars, mainly between magnitudes 5 and 6·5, a knowledge of whose motions is of particular interest as aiding in the determination of the elements of the two principal star-streams.

2. A, F, G, K, and M stars of magnitudes 5·5 to 6·5, which have very small proper motions. These may be regarded as very distant stars of high luminosity.

3. Stars with measured parallaxes, most of which have very high proper motions, of magnitudes from 5·5 to 8·5.†

Results require to be accumulated before they can be interpreted; this investigation belongs to the period of accumulation, and does not lend itself to summary description. I may, however, mention the following contribution to a question which is usually quoted as settled, namely, that of increasing radial, and presumably absolute, velocity with advancing stellar type. Adams gives the following comparison:—

Spectral Type.	Campbell.		Mount Wilson.	
	Proper Motion.	v' km./sec.	Proper Motion.	v' km./sec.
O and B	0″·031	8·99	0″·016	8·23
A	·094	9·94	·019	10·04
F	·234	13·90	·011	10·14
G and K	·202	15·15	·014	11·03
M	·074	16·55	·015	12·56

It will be observed that Campbell's progression is very much reduced when we pass to Adams's catalogue of more distant stars. A consideration of the way in which stellar drift would affect this table rather emphasizes its point, and Adams remarks that it seems to indicate that with very distant stars there is but slight change of velocity with spectral type.

Reviewing this great mass of work that I have imperfectly

* *Ap. J.*, 42 (1915), p. 172.

† One hundred stars belonging to the last heading had been published earlier in collaboration with Kohlschütter, *Ap. J.*, 39 (1914), p. 341.

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described, it might seem that there was nothing to signalise it beyond technical excellence combined with great opportunities of equipment, and in the matter of technical excellence Adams to-day by no means stands alone. One might look for some single clue or master idea binding the whole together and fail to find it, for in his writings Adams is strictly an observer and recorder, and in the one or two cases where he offers an interpretation, he accompanies it with an apology for its speculative character. For that matter it is easy to attach too much importance to a master idea; too often it turns out upon examination to be a mere shell, capable of all kinds of meanings according to the contents that are put into it.

An investigator must make a flexible as well as a lucid plan, for he must always be prepared to change or discard it. We search for what we do not know, and we search essentially by instinct.

One might say generally that Adams's intention was to elucidate the stellar atmospheres. Such was the purpose of the paper on Sirius, Procyon, and Arcturus. Stellar atmosphere may seem a good long way from contributing to the parallax question to which I now come. But we must remember where the real strength of the spectrum, for astronomy, is found. In spite of the endless qualifications of which I have spoken so much, it is, in a certain sense, absolute. The distance-factor, where it affects stellar spectra at all, affects them in utterly different degree and law from the old geometrical astronomy. We may, therefore, succeed in confronting the geometrical distance-effect with an alternative measure, and so exchange the one for the other. Such an idea, in a more or less fluid state, is nothing new. Kapteyn, in a recent paper "On the Change of Spectrum and Colour Index with Distance and Absolute Brightness: Present State of the Question,"* enumerates from the unique magisterial position that all the world accords him the investigations required, and says, "A well-determined influence of absolute brightness, besides being important in a study of the physics of stellar atmospheres, would furnish the means of determining parallaxes where the other methods break down." Adams had already written a paper "On the Relative Intensity at Different Wave-Lengths of the Spectra of Some Stars having Large and Small Proper Motions,"† that is to say, allowing freely for individual exceptions, of stars at small or great distances. For distant stars, generally speaking, he finds that the strength of the spectrum is more in the red than in the blue. Whether this means absorption in space, or is a consequence of the much greater absolute magnitude of the distant star, does not at once appear. The matter is dealt with further in the pregnant paper to which I now come, namely, Adams and Kohlschütter, "Some Spectral Criteria for the Determination of Absolute Stellar Magnitudes."‡ One sees in this paper that though the general purpose is, as I said, not new, the way has still to be felt for a reliable method. The paper begins with a reconsideration of the continuous spectrum and

* *Ap. J.*, 40 (1914).

† *Ap. J.*, 39 (1914), p. 89.

‡ *Ap. J.*, 40 (1914), p. 385.

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its increasing intensity towards the red for distant stars. It is shown for a large number of stars that this effect increases as the type of star advances, from F to K. Hence at least a portion, possibly all, of this reddening of the very distant stars, proved in the previous paper of Adams, must be ascribed to the star's own atmosphere, and not exclusively to scattering of light in space. This is by the way. The kernel of the communication follows, in the relation of the intensity of spectral lines to class and to absolute magnitude. Systematic differences of intensity for certain lines between stars of large and stars of small proper motion had become evident in the course of classifying spectra. In order to secure an accurate system of classification, as well as to investigate these differences, pairs of lines were selected not far from one another in the spectrum, and as nearly as possible of the same intensity and character, and estimations were made of their relative intensities. A line decreasing in intensity with advancing type, such as one of the hydrogen lines, was compared with one which increases in intensity, such as an ordinary metallic line. Thus $H\gamma$ or $H\beta$ was compared with five suitable metallic lines, numerical estimates made, and no systematic differences appearing, the mean of these numbers was accepted as the *measure* of the star's spectral type. The internal agreement of the method was found to be good and to yield a consistent numerical index of spectral class.

With regard to absolute magnitude, a number of other lines were taken which are strong or weak respectively in stars of high luminosity and show a change in the opposite sense as the luminosity is less. The relative intensities of the five most marked pairs were estimated and combined in a mean as before. This gave a numerical index of absolute magnitude, and using known absolute magnitudes to work from—defined as the apparent magnitude which the star would show at a distance of 10 parsecs—formulæ were derived, by which the index could be interpreted in any case. From these formulæ 162 stars were calculated. The average difference between these estimates and absolute magnitudes derived from measured distance was about 1·5 magnitudes, a result that was taken as showing that there appeared to be considerable promise in the application of spectrum-line criteria to the determination both of absolute magnitudes and of parallaxes.

This was the first stage, and I suppose it represents the degree of utility that the best friends of such a method would have anticipated for it—a method something on a par with the derivation of parallaxes from proper motions, reliable on the average, available when other methods failed, but in individual application honeycombed with large exceptions. That the writers themselves looked for nothing more is shown by the fact that they based their formula and made their comparisons indifferently with measured parallaxes and with theoretical parallaxes inferred from proper motions. The method, as Adams tells us, was devised in large measure by Kohlschütter. Kohlschütter, I understand, is a German officer, and would, I presume, at the present moment be

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fighting against us in Germany's infamous ranks if we had not captured him on his way from Mount Wilson. Let me not, for that reason, seem to deny any prestige that may be his due—the uncertain, poor, treasured reward of the scientific worker.

Since his departure the method has been applied at Mount Wilson with little modification, but the work has been greatly extended with the aid of additional material available. The result is to put it upon an entirely different level.

It is worth while to pause a moment and consider what is the essence of the change introduced. It is seen best in the matter of the classification. The Harvard classification, which the new one does not displace, was itself intended to be as arithmetical as it was wise to make it, the transition from class to class being measured in tenths, estimated from the intensities of certain lines. But it was with the same lines in other spectra, not with other lines in the same spectrum, that the comparison was made. It would be difficult to overrate the value of this small change. It makes the circumstances of comparison identical. It turns a descriptive classification into a physical arithmetical measure. Now arithmetic is the final stage of science. One is justified in anticipating for it an almost incalculable increase of power in the future, as is illustrated already by the rapid progress made since it was devised.

In place of a general adjunct to other resources by which we could infer distance, the method has been shown by Adams to be as precise as first-class direct measures. Where an agreement in magnitude of about 1·5 magnitude was at first regarded as satisfactory, it now predicts individual parallaxes with an average deviation from the independent measures of one-fortieth of a second of arc.* This great advance in effectiveness may be ascribed largely to a more thorough treatment. The material available for discussion consists of several thousand spectra. One difficulty of the first discussion, and in part of the second one, was the want of parallax determinations of distant stars. It was natural for parallax observers to desire that their results should not be, predominantly, null, and therefore they have confined themselves to stars which there was reason to believe were not too remote. This want was partly filled by observations by another of the Mount Wilson observers, Mr. Adrian van Maanen, of parallaxes of certain stars of small proper motion, and by the Yale observation of the very bright stars. Another change made is to confine attention to two varying lines alone, one of which increased while the other diminished in intensity with progressive type. Looking back, these lines seem now a fairly obvious choice. One is the line λ4216 *Sr*, an enhanced line, that is, probably, a high temperature line, prominent in the solar "flash" spectrum and upper chromosphere. This line recedes in intensity as the type of the star advances. It was compared for intensity with an iron line that is closely adjacent. The other

* *National Academy of Science, U.S.A., Proceedings*, vol. ii. (1916), pp. 147, 152.

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is the line λ4455 *Ca*, which in the solar spectrum is greatly strengthened in sun-spots, and increases in intensity with decreasing temperature. This was compared twice over, first with an adjacent line λ4462, which is a blend of *Fe* and *Mn*, and as a corrective with the line λ4495 *Fe*. All the stars of measured parallax were then collected for which spectral observations were available. They were divided into two blocks. From the first the formulæ connecting absolute magnitude with difference of intensity of line was derived, for the second the same formulæ were used to predict the absolute magnitude and thence the parallax. Agreement in the former group between calculated and observed parallax shows only the adequacy of the simple formulæ employed to represent the data. The mean discrepancy is ± 0″·024. The second group is pure prediction. The mean discrepancy is ± 0″·026. There are 58 stars in the first group, and 63 in the second. The spectral types range from F0 to Ma, and the magnitudes from 0·2 to 9·3.

To illustrate further the flexibility of the method and its application to other researches, it is shown * how clearly these criteria separate the stars of class M into two groups of high and low intrinsic brightness, supplementing most admirably the indications of "giant" and "dwarf" stars derived from measured parallaxes.

Writing in 1912, Gill quotes from Schlesinger's account † of his observations for parallax with the Yerkes telescope:—"The number of stellar parallaxes that can be determined per annum with an average probable error ± 0″·013 will in the long run be about equal to the number of clear nights available for the work"; and adds, "How great this advance is, only those who have determined parallaxes of like accuracy with inferior means can fully realise." This is a standard that is still a high one. To the majority of the parallaxes used in constructing and testing Adams's formulæ, the *whole* of the apparent error of a fortieth of a second of arc might be attributed without violence. We have, then, a method which will turn a star's parallax out with the same accuracy as the best direct measures, as a bye-product of its spectrum, by mere comparison of the relative intensity of a few of the lines and at the cost of a negligible calculation. I suppose it is not too much to expect that in a very short time the Mount Wilson observers may give us the absolute magnitudes and spectral parallaxes of some thousands of stars whose distances have never been measured. Nor in a very short time shall we hesitate to believe as confidently in these as in direct measures. For I think one is not less surprised that the measured parallaxes are reliable enough and so free from systematic errors as to stand such a recalculation, than that it should be possible to find a method of recalculating them in detail.

A few further general remarks upon the method, its limits

* *Nat. Acad. Proc.*, vol. ii. (1916), p. 157.
† *Monthly Notices*, lxxii. (1912), p. 326.

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and prospects, may not be out of place. I have described the method in terms of the familiar notion of parallax, but parallax has essentially nothing to do with the matter, except to supply attested examples of absolute magnitude from which the coefficients of the formulæ may be determined. Once the formulæ are established, the examination of the spectrum gives the absolute magnitude of a newly observed star directly and without mediation of geometry. If the distance is what interests us, we deduce it by help of the apparent magnitude. Briefly put, we look at a star and say how big it really is, irrespective of distance. We can rebut Shakespeare's scoff of

> the star . . . .
> Whose worth's unknown although his height be taken.

It has been described to me as a new door into the universe, and the language does not seem immoderate.*

At present it applies only to spectra from classes F to M. This is a limitation which one will look confidently to see removed when sufficient study has been given to reveal corresponding features in the stars of earlier type. As to the range of distance that can be explored there is, strictly speaking, no limit, provided only the star is bright enough to record its spectrum.

One may reflect how dependent the method is upon a sound, well-determined scale of apparent magnitude. The subject of magnitude has been isolated, and has sometimes been regarded dubiously as barely scientific in conception.† Great pains have been spent nevertheless upon rectifying it, and it now comes into its own as the indispensable link uniting geometrical and spectroscopic astronomy.

One remark I would throw out at hazard for what it may be worth. Many of the spectra used were of very moderate dispersion. No comparison spectra are required. It is not apparent that the power of the 60-inch telescope, or even a narrow slit with its great loss of light, are in any way essential for success. If this is so, the method brings a vast quantity of work of the first importance within the reach of very moderate equipments.

I have now completed my description of our medallist's work, and I think the Society will rest satisfied that our award is well bestowed. Brilliance and patience, the provision of adequate means and the adaptation of means to the end of producing ordered knowledge,—by one or all of these qualities not a few American astronomers have given the world in our own generation an unsurpassed model of research. The direction of such research to really necessary objects will, as a rule, preclude a man from seeking primarily striking results for himself, not least so when he is part of an organisation whose mere healthy working produces almost automatically more problems than any individual can cope with.

* In the address as delivered there was an error at this point which I am indebted to Professor Turner for pointing out.

† Cf. *Carte Phot. du Ciel, Réunion de* 1909, Procès-verbeaux, p. 43.

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He works, and he gets and gives ideas. It would be a mistake to say that their value lies in their genealogy, or in their momentary shape, or in their working out. The three are inseparable. Without dwelling upon this, the right man finds a contented reward for his work in its conscious thoroughness and worthiness. Not less do we rejoice when it leads in direct line to some signal advance. Our medal bears two mottoes, both of good ancestry: *Quicquid nitet notandum*, which we take from Herschel, and *Nubem pellente mathesi*, from Halley's encomium of Newton. I think you will agree that both of them are particularly appropriate to the comparatively short but full record of our medallist that I have endeavoured to describe to you.

Addressing the Foreign Secretary, the President said:—I ask you to convey this Medal to Mr. Adams with the good wishes of the Society for his health and continued activity. Many here are his personal friends, still more know and value him only from his writings. All unite in their appreciation for what he has done.

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The Meeting then proceeded to the election of the Officers and Council for the ensuing year, when the following Fellows were elected:—

President.

Major P. A. MacMahon, D.Sc., Sc.D., LL.D., F.R.S.

Vice-Presidents.

Sir F. W. Dyson, M.A., LL.D., F.R.S., Astronomer Royal.
A. S. Eddington, Esq., M.A., M.Sc., F.R.S., Plumian Professor of Astronomy, Cambridge.
H. F. Newall, Esq., M.A., D.Sc., F.R.S., Professor of Astrophysics, Cambridge.
R. A. Sampson, Esq., M.A., D.Sc., F.R.S., Astronomer Royal for Scotland.

Treasurer.

E. B. Knobel, Esq.

Secretaries.

A. C. D. Crommelin, Esq., B.A., D.Sc.
Alfred Fowler, Esq., F.R.S., Professor of Astrophysics, Imperial College of Science and Technology.

Foreign Secretary.

Arthur Schuster, Esq., Sc.D., Ph.D., Sec. R.S.

Council.

J. L. E. Dreyer, Esq., Ph.D.
J. W. L. Glaisher, Esq., Sc.D., F.R.S.
Col. E. H. Hills, C.M.G., D.Sc., F.R.S.
J. H. Jeans, Esq., M.A., F.R.S.
H. S. Jones, Esq., M.A., B.Sc.
E. W. Maunder, Esq.
W. H. Maw, Esq., LL.D.
J. W. Nicholson, Esq., M.A., D.Sc., Professor of Mathematics, King's College.
Rev. T. E. R. Phillips, M.A.
A. A. Rambaut, Esq., M.A., D.Sc., F.R.S., Radcliffe Observer.
J. H. Reynolds, Esq.
H. H. Turner, Esq., D.Sc., D.C.L., F.R.S., Savilian Professor of Astronomy, Oxford.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LXXVII. March 9, 1917. No. 5

Major P. A. MacMahon, D.Sc., Sc.D., LL.D., F.R.S., President, in the Chair.

John Beebe, "La Rocca," East Bendigo, Victoria, Australia;
George Rusby Kaye, Curator of the Bureau of Education of the Government of India, Simla, India; and
John Stewart, 45 Raleigh Street, Essendon, Melbourne, Australia,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Arthur Brinley Mayne, B.A. (Oxon.), Mathematical Master at Trinity College, Glenalmond, Perthshire, and 3 Bryn-y-mor Crescent, Swansea (proposed by H. H. Turner);
Wilfred C. Parkinson, Observer, Department of Terrestrial Magnetism of the Carnegie Institution of Washington, The Observatory, Perth, Western Australia (proposed by A. D. Ross); and
Thomas James Richmond, B.Sc. (Lond.), Cartographer, Hydrographic Department, Admiralty, "Eastdene," St. James' Road, Sutton, Surrey (proposed by David J. R. Edney).

Sixty-two presents were announced as having been received since the last ordinary Meeting, including, amongst others:—

Tacubaya, Observatorio Astronomico, Catalogo Astrofotographico, $-9°$ a $-17°$, tomo 1 (presented by the Observatory).

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Photographs of Jupiter, taken Oct.–Dec. 1916 (presented by J. H. Reynolds).

The Star Identifier, and diagram for the graphical solution of problems in nautical Astronomy (Planisphere, with explanation and examples). Constructed and presented by J. E. McGegan.

On Frederick de Houtman's Catalogue of Southern Stars, and the Origin of the Southern Constellations. By E. B. Knobel.

The origin of the constellations surrounding the South Pole is involved in much obscurity. They have all been attributed to various Dutch, Spanish, Portuguese, and Italian navigators, but the original records of the observation and formation of the different constellations appear to be lost.

The earliest existing map of the southern heavens is found in the map of the world made by Peter Plancius in 1594, entitled "Orbis terrarum typus de integro multis in locis emendatus Petro Plancio, 1594," given by Linschoten (*Navigatio ac Itinerarium*, 1599). In this map, besides the Ptolemy constellations, are shown Columba (the stars of which are in Ptolemy), Crux as a separate constellation (also in Ptolemy), Eridanus continued from Ptolemy's 34th star to α Eridani, Triangulum Australe, and a large constellation in the figure of a man called Polophilax, consisting of seven stars which I have not identified. The two Nubeculæ are shown, but nothing else. It is tolerably clear that at this date no other southern constellations were known. Thomas Hood (*Celestial Map*, 1590) states that no more stars than those in Ptolemy have been observed.

The earliest publications containing the constellations surrounding the South Pole as we know them, are in Frederick de Houtman's *Catalogue of Stars*, Bayer's *Uranometria*, and Blaeu's *Celestial Globe*, which all appeared in the same year, 1603.

In the 49th map of Bayer's *Uranometria*, published at Augsburg, 1603 September, are shown the constellations Phœnix, Hydrus, Tucana, Grus, Indus, Pavo, Apus, Triangulum Australe, Musca, Chamæleon, Volans, and Doradus. His map of Eridanus shows the continuation beyond the last of Ptolemy's stars to α Eridani; it also gives some stars in Phœnix and Doradus; that of Canis Major gives Columba, "recentioribus Columba"; Argo shows some stars in Volans; Centaurus includes Crux as a figure, and Triangulum Australe; Ara shows some stars in Triangulum, Grus, and Pavo; Piscis Austrinus includes stars in Grus. Bayer states that these constellations were observed partly by Amerigo Vespucci, partly by Andrea Corsali and Pedro de Medina, but their places were determined by that "most learned" seaman Petrus Theodorus.

Blaeu in his *Celestial Globe*, published in 1603, ascribes all the above constellations to Frederick de Houtman. Merula (*Cosmo-*

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graphia Generalis, 1605) attributes all the observations of the "longitudes, latitudes, declinations, etc.," of the stars in question to Petrus Theodorus. Julius Schiller (1627) quotes Bayer, and mentions that these constellations are found on globes by "Petrus Plancius, seu Petrus Karif, Jansonius (Blaeu), and Houtman, etc." Sherburne (1675) says that these constellations were "first found out and denominated by some eminent navigators sayling beyond the line, as particularly by Americus Vespuccius, Andreas Corsalius, Petrus Medina, but principally by Fredericus Houthman, who, during his abode in the island of Sumatra, made exact observations of them, being by Petrus Theodorus and Jacobus Bartschius reduced into order." Weidler gives merely the statements of Bayer and Blaeu. Ideler gives equal merit to Petrus Theodorus and Frederick de Houtman.

I have explored the resources of the British Museum as far as possible, to throw light upon the claims of the above persons.

Of Amerigo Vespucci it is quite clear that he contributed little or nothing to our knowledge of the southern heavens. He states that "the stars of the South Pole are much larger and brighter than those of our pole"; though, in a work published two years before his death (*Sensuyt le nouveau monde et navigations faictes par Emeric de Vespuce*, 1510), he says correctly, "There is no bright star near the South Pole." In his third voyage he says he has "noted as many as twenty stars as bright as we sometimes see Venus and Jupiter." He gives a sketch of a few stars which cannot be identified. He concludes thus: "I have known many other very beautiful stars which I have diligently noted down, and have described very well in a certain little book describing this my navigation, which at present is in the possession of that most serene King (of Portugal), and I hope he will restore it to me." Vespucci mentions no constellation, and gives no reliable observation.*

Coignet (*Instruction nouvelle touchant l'art de naviguer*, Anvers, 1581) says: "Albericus Vespucius escrit de trois etoilles qui ont leur mouvement à l'entour du Pole Antarctique, faisans ensemble un triangle rectangle, dont le milieu est $9\frac{2}{3}$ degrez du Pole Antarctique." These are possibly the stars β Hydri, β Octantis, and ν Octantis.

Of Andrea Corsali (1517) all we know is contained in two letters given by Ramusio (*Navigationi e viaggi*, 1563–74), in which he exclaims on the wonderful sight of the southern heavens, and describes the two Nubeculæ and the Southern Cross of five stars. He gives a sketch showing Crux and the Nubeculæ with thirteen other stars which cannot be identified. He mentions no other constellation, nor is there any further astronomical reference in his letters.†

* Vespucci states that he observed the new moon on the same day that it was in conjunction with the sun!

† There is much confusion in Houzeau (*Annals of Brussels Observatory*, tome 1), who says that the first grouping of the southern stars was made by the seaman Peter Theodore, and *published* in 1516 by Andrea Corsali.

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The Spanish navigator Pedro de Medina (born about 1510) is known in this connection from his work *Arte de navegar*, 1545, of which several editions were published in Spanish, Italian, and French. All in the British Museum have been examined, and the only reference to southern stars is his instruction for determining the latitude in the southern hemisphere by observations of α Crucis, which he says has a "declination" of 30°, by which he means south polar distance. This value for his period is in error about half a degree. No other constellation is mentioned, or any other stars than those in Crux. Other Spanish works on navigation by Rodrigo Zamorano (1581), and Garcia de Palacio (1587), and Clavius (*Sacrobosco Commentarius*, 1585), give similar instructions.

Before considering the question of Petrus Theodorus, it will be convenient to enumerate the southern constellations which were known prior to the first Dutch expedition to the Indies:—(1) Crux, first mentioned by Pigafetta in his journal of Magellan's voyage, 1519, in which he also mentions the two Nubeculæ. (2) The continuation of Eridanus beyond Ptolemy's 34th star to α Eridani, first mentioned by Robert Hues in his *Tractatus de globis*, 1592, in which he says: "There are but three stars of the first magnitude that I could perceive in all those (southern) parts which are never seen in England. The first of them is that bright star in the stern of Argo, which they call Canobus; the second is in the end of Eridanus; the third is in the right foot of the Centaur; to which, if you will add a fourth, the which is fixed on the Centaur's left knee, I shall not much stand against it, but other stars of the first magnitude than those which I have named, that part of the world cannot show us." (3) Columba, a constellation probably made by Plancius out of the unformed stars of Ptolemy's Canis Major. (4) Triangulum Australe, which first appears in Plancius's map of 1594, but the origin of which has not been traced. (5) Polophilax, a large constellation of seven stars given by Plancius (not mentioned by Ideler, *Sternnamen*). This is clearly a group of seven stars which Andrea Corsali shows in the drawing in his letters given by Ramusio. Blundeville (*His Exercises*, 1594), speaking of his celestial globe, says: "Least the South part of this hemisphere or halfe globe should remain void and emptie, I have taken these Southerne Starres of the observation of Andreas Corsalius Florentine, and haue diligently compared the same with the writings of Americus Vesputius and of Petrus Medina, and haue reduced the said starres into this forme or shape. But for so much as I haue seen nothing as yet to my satisfaction or contentment touching the longitude, latitude, magnitude or nature of the said starres." Further on he says: "There are lately found out by the Portugales and others that have sailed in the East and West Indies four other images towards the South Pole, as the Crosse or Croster, the South Triangle, Noah's Dove or Pigeon, and another image made like a philosopher, called Polophilax, all which are set down in the celestial globe lately set forth by M. E. Mullineaux." He adds that the longitude and

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latitude of Polophilax "hath not as yet been rightly set down by any that I have read." This shows that Columba, Triangulum Australe, and Polophilax were known before the publication of Plancius's map of 1594. Jodocus Hondius (*Construction and use of the Globes*, 1597, in Dutch) mentions only the remarks of Andrea Corsali.

So far the evidence does not warrant Bayer's attributing any of the southern constellations to Vespucci, Corsali, or Pedro de Medina. He probably made the statement on the authority of the above quotation from Blundeville.

Much uncertainty has existed as to the identity of Petrus Theodorus, and of the extent of his contributions to our knowledge of the southern constellations. His name is given in Latin as Petrus Theodorus F. Embdanus, and in German, Peter Theodors Sohn; but in 1825 Moll, in his work on the early naval expeditions of the Dutch (*Verhandeling over eenige vroegere zeetogten der Nederlanders*), identified him as Pieter Dircksz, or (as the final z stands for zoon) as Pieter Dirckszoon; but he was unable to find any information about him, and he inquires whether this is the same person mentioned by Blaeu as Frederick de Houtman. I find in a Dutch dictionary that the English equivalent of the Dutch name "Dirk" is "Theodoric"; hence, in translating the Dutch name Pieter Dircksz(oon) into Latin, it became Petrus Theodorus F(ilius). From Olbers (*Schumacher Jahrbuch für* 1840) and De Jonge (*De opkomst van het Nederlandsch Gezag in Oost-Indie*, 1862) we learn that his full name was Pieter Dircksz Keyzer. This man was instructed in mathematics and astronomy by Petrus Plancius, who was celebrated as a geographer and for his knowledge of astronomy and nautical matters, and who was one of the principal promoters of the Dutch expeditions to the Indies.

The first expedition of the Dutch to the East Indies sailed from the Texel 1595 April 2. It consisted of four ships, the second of which, the *Hollandia*, commanded by Jan Dignumz, carried Pieter Dircksz Keyzer as chief pilot and head of the steersmen, and Frederick de Houtman as sub-commissioner in the mercantile part of the expedition, with whom we here first become acquainted. From the journals and log of this ship (*Linschoten Vereenigen*, Cornelis Houtman's voyage) and De Jonge (*Opkomst, etc.*) we learn that the *Hollandia* arrived at Madagascar about 1595 September 3,* and stayed there some time. It was here that Pieter Dircksz Keyzer "sought comfort in science, and enriched his knowledge of astronomy by improving the position of old and the observation of new constellations." It must have been at this time that Frederick de Houtman prepared that portion of his vocabulary, referred to below, which deals with the dialect of Madagascar. The ship went on to the Malay Archipelago, and though it is difficult to be certain about dates, it apparently anchored at Achin in Sumatra about 1596 June 5. After three months there it went on to the Straits of Sunda, where on 1596 September 13

* September 25 in Purchas's *Pilgrims*.

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Pieter Dircksz Keyzer died. Frederick de Houtman returned in the *Hollandia*, arriving in Holland about 1597 August 10. In the following year, on 1598 March 15, he went out again with a fleet, in command of the ship *De Leeuwin*. The chief pilot of the fleet was John Davis (not to be confounded with John Davis of Dartmouth, the inventor of the Davis Quadrant), and from Davis's journal we learn that the fleet arrived at Achin in Sumatra 1599 June 21. Tiele (*Mémoires sur les voyages Néerlandais*) says that Frederick de Houtman was taken prisoner by the King of Achin in 1598, and kept so for twenty-six months. This can hardly be correct, as he did not arrive there till sometime in 1599. He was, however, for a long time prisoner, and only escaped on 1600 December 31, when he went on board a ship commanded by van Caerden, which was then at Achin. He then returned to Holland. To conclude his career, in 1603 he went out on another voyage as mercantile commissioner, and two years after became governor of Amboyna. I have thus endeavoured to account for all Frederick de Houtman's time, which bears upon the authenticity of his *Catalogue of Stars*.

In 1603 * Frederick de Houtman published at Amsterdam a Malay and Madagascan vocabulary, entitled *Spraeckende woordboeck Inde Maleysche ende Madagaskarche Talen met vele Arabische ende Turksche woorden*. At the end of this work he gives a catalogue of southern stars, the title-page of which is thus translated: "Here follow several fixed stars (observed), with efficient instruments, by Frederick de Houtman, in the island of Sumatra, (their positions) corrected and their numbers increased. For the use and service of those who navigate south of the equinoctial line, also for all amateurs and those who have occasion for the best. These stars are arranged according to their Right Ascension; that is, the degree and minute which a star in the south or north has from where the equinoctial line cuts through (*sic*). Declination is the number of degrees and minutes a star is distant from the equinoctial line towards the South or North Pole. Magnitude is the size of the stars: often a star is of the first size or greatest light: thus there are seven (*sic*) degrees of size and light." In a dedicatory letter to the vocabulary Houtman says: "There will be found at the end the declinations of several fixed stars in the region of the South Pole which I had observed on my first voyage, and which on my second voyage I revised and corrected with more care and brought up to the number of 300, as may be seen on the *Celestial Globe* published by William Jansen (Blaeu)."

This work is extremely rare; every effort that I have made for many years to obtain a copy in Germany, France, Italy, and elsewhere, and by advertising, has failed. Four copies are certainly known, at the British Museum, the Bodleian, the Bibliothèque Nationale, and at Leiden. Thanks, however, to the enterprise of

* Gould is in error in stating that Bayer's *Uranometria* antedated Houtman's *Catalogue*. They were both published in the same year.

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Professor Turner, he has had the copy in the Bodleian photographed, and thus this interesting catalogue—the first catalogue of southern stars ever made—is available.*

The catalogue consists of the right ascensions, declinations, and magnitudes of 303 stars, of which 107 stars are found in Ptolemy, leaving 196 newly discovered. The right ascensions are too rough to allow of identification, but by plotting the whole catalogue in a map, and comparing with the maps of the *Uranometria Argentina*, it has been easy to identify nearly every star. The object of the catalogue was to determine declinations for the use of seamen; accordingly, a comparison of the declinations of some fifty stars, with modern observations reduced, is appended.

The new stars are thus distributed:—

Phœnix	13	Volans	5
Corona Australis	7	Chamæleon	9
Eridanus	6	Lupus	14
Hydrus	15	Triangulum Australe	4
Dorado	4	Apus	9
Columba	1	Ara	5
Argo Navis	34	Pavo	19
Centaurus	18	Indus	11
Crux	1	Grus	11
Musca	4	Tucana	6
			196

Was this catalogue made by Frederick de Houtman at Sumatra on the occasions of his two voyages there, as he states?

On his first voyage, then twenty-four years of age, he was in a purely commercial capacity, and there is no record of his being an astronomer or of his having any mathematical ability. Most of his time at Sumatra on both occasions was at Achin, which is in latitude 5° 34′ North; and, as we infer from the ship's journals of the first voyage, he was there on that occasion only some three or four months. The journal of his second voyage is not published, but we have enough information to infer that while at Achin he had but a limited time at his disposal, as he was early taken prisoner by the king and kept in confinement for many months; and unless we assume that during his imprisonment he was able to make astronomical observations, there would have been no sufficient time for an inexperienced man to make the catalogue in question.† How did he determine the right ascensions, rough as they are? In his catalogue he gives the positions of twenty-four

* I had commenced the investigation of this catalogue in 1882, and only after translating some portions of it did I become aware that a translation of the work into French by M. A. Marre had appeared in the *Bulletin des Sciences Mathématiques*, tome v., 1881. I acknowledge several quotations from M. Marre's memoir.

† The dates given by De Jonge (*Opkomst*) for the second voyage do not agree with those in the "Account of Frederick de Houtman at Achin" (*Cort verhael, etc., Gouda*, 1880).

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stars, 7 of the 4th, 15 of the 5th, and 2 of the 6th magnitude, whose zenith distances at Achin ranged from 80° to 89° 15′; *obviously he could not have seen them at all.*

It is practically certain that the catalogue used by Bayer was identical with most of Houtman's catalogue, but it contained some additional stars. Blaeu's *Globe*, of which there are two somewhat mutilated examples in the British Museum, is certainly limited to the stars published by Houtman in his vocabulary. According to different authors, the total number of stars in the twelve constellations, Phœnix, Hydrus, Doradus, Volans, Chamæleon, Musca, Apus, Triangulum Australe, Pavo, Indus, Grus, and Tucana are—

Houtman, 1603	112
Bayer, 1603	121
Bartsch, 1624, Text	121
,, ,, Catalogue	131

Kepler, *Tab. Rudolph*, 1627, as Bartsch's catalogue.

Houtman, who was at the time very ill, escaped from prison on 1600 December 31, and he could hardly have got back to Holland before the end of 1601. The preparation of Bayer's *Uranometria* would take considerable time, and it is highly probable that he possessed the materials for his maps some time before Houtman's return. Whence did he obtain those materials?

Pieter Dircksz Keyzer, who was versed in mathematics and astronomy, and was held in high estimation by the Dutch East India Company, would have a favourable opportunity during the long stay of the *Hollandia* at Madagascar, from 1595 September to April or May of 1596, to make observations and to chart the new constellations. As Merula says: "sedens ibi in corbe, vel ut vocant nonnulli, galea." In that latitude he could quite well observe 5th and 6th magnitude stars near the South Pole. For the right ascensions we may surmise that Plancius would furnish him with some of Tycho Brahe's observations, from which they might be roughly derived; certainly there was no other material available for the purpose. I have failed to find any information as to relations between Tycho Brahe and Plancius.

Frederick de Houtman was in Pieter Dircksz's company all the time, but as commissioner he would be engaged in the development of trade, and, moreover, he was then collecting materials for his Malay and Madagascan vocabulary. Though all the evidence is circumstantial, the following inferences are irresistible: that the whole catalogue and the formation of the new twelve constellations must be attributed to Pieter Dircksz Keyzer, and not in any way to Frederick de Houtman: that the catalogue was sent back by the *Hollandia*, and after the return of that ship on 1597 August 10 it would go to Plancius, who probably communicated it to Bayer: and that Frederick de Houtman obtained an imperfect copy of it which, as Pieter Dircksz was dead, he published as his own work in the above-mentioned vocabulary.

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Since coming to this conclusion I learn, a few weeks ago, from a high authority in Holland, that the opinion there at the present time is to the same effect, and that the Linschoten Society contemplate publishing a full account of Pieter Dircksz Keyzer.

In either case the whole merit of the discovery and formation of the constellations must be attributed to the Dutch.

The catalogue gives the descriptions of the stars, which I have translated from the Dutch, and compared with the translation of M. Marre. The modern designations are from the *Uranometria Argentina*, and the magnitudes from that work are appended.

In the table which follows, the declinations of fifty stars are compared with Neugebauer's *Sterntafeln*. No information can be found as to the instruments used, which were probably limited to the astrolabe and quadrant. With such instruments the comparison shows that some of the observations were very good.

Catalogue of Stars given in Frederick de Houtman's Malay and Madagascan Vocabulary.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A.	Decl.
The bird Phœnix has 13 stars.					
Phœnix.				° ′	° ′
1. The most northern star in the flame	16 Sculptoris β*	5	4·8	347 20	40 50
2. The middle one in the flame	11 Phœnicis	5	4·5	348 35	44 15
3. The most southern star in the flame	9 ,, ι	4	4·4 v	349 45	42 45
4. One in the right wing	39 ,, ε	4	3·8	357 10	47 45
5. A star in the neck of the bird Phœnix	48 ,, α	2	2·4	1 0	44 5
6. One in the neck	46 ,, κ	4	3·9	1 30	45 34
7. One in the body of Phœnix	54 ,, λ^1	4	4·6	2 40	50 15
8. One in the breast	64 ,, μ	4	4·7	4 30	47 30
9. One below the right foot, in the wood	68 ,, η	4	4·5	7 20	59 30
10. One below the left wing, in the flame	85 ,, β	3	3·3	11 20	49 20
11. One below the left foot, in the wood	89 ,, ζ	4	4·2	13	57 0
12. At the end of the left wing	106 ,, γ	3	3·4	18	44 45
13. One below the same wing, in the fire	109 ,, δ	3	4·0	19 30	51 25
The Southern Crown has 16 stars.					
Corona Australis.					
1. The most western star in the crown	15 θ	4	5·1	269 18	41 45
2. The second following it	13	5	5·6	270	40 15
3. The third ,,	16 κ	5	5·4	270 10	39 18
4. The fourth ,,	18	5	5·8	271 26	43 16
5. The fifth ,,	25 η^1	5	5·7	273 28	44 10
6. The sixth ,,	32	5	6·6	275 36	45 20
7. The seventh ,,	30	6	5·8	276 30	43 30

* The positions of 1, 2, and 3 Phœnicis are erroneous; Bayer gives correctly the identified stars.

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R A.	Decl.
				° ′	° ′
8. The eighth following it	39 ζ	5	5·2	277 35	43
9. The ninth ,,	43 δ	4	5·0	278 40	42 2
10. The tenth ,,	46 β	4	4·1	279 45	40 35
11. The eleventh ,,	44 α	4	4·2	280 30	38 50
12. The twelfth ,,	41 γ	5	4·6	280	37 30
13. The thirteenth ,,	34 ε	5	5·5	277 37	37 40
14. The fourteenth ,,	19 λ	5	5·4	274	38 20
15. The fifteenth ,,	20	5	5·6	274 25	40
16. The last star in the crown	24 μ	5	5·5	274 50	41
The southern end of the Nile, with corrections and additions. 7 *stars.*					
Eridanus.					
1. Acarnar, the end of the Nile	2 α	1	1·0	20 50	59 20
2. The first star following	7 χ	4	3·9	25 20	54 30
3. The second ,,	14 φ	4	3·5	31 25	53 45
4. The third ,,	16 κ	4	4·2	33 50	49 30
5. The fourth ,,	18 S	4	5·0	35 40	44 24
6. The fifth ,,	19 ι	4	4·2	37 24	41 28
7. The sixth following in the Nile	48 θ	3	2·6	41 20	41 50
The Water-snake has 15 *stars.*					
Hydrus.					
1. The head of the water-snake	24 α	2	2·9	26	63 30
2. One star in the neck	32 Horologii β	5	5·2	38 23	65 0
3. The second following	47 Hydri ζ	5	5·2	41 38	68 25
4. The third ,,	44 ,, ε	5	4·2	39 50	69 18
5. The fourth ,,	38 ,, δ	5	4·1	34 45	70
6. The fifth is above the small nebula (Nubecula minor)	70 Tucanæ λ^2	5	5·5	16 50	71 45
7. The sixth following	50 ,, π	5	5·7	10 27	72 15
8. The seventh ,,	47 ,,	5	6·8	4 30	75 10
9. A star above the preceding one.	45 ,,	5	5·9	356 42	74 6
The small nebula	Nubecula minor	...	...	14 25	75
10. The ninth following	5 Hydri β	4	2·7	4 30	80 12
11. The tenth ,,	88 Octantis θ	6	5·4	359	79 30
12. The eleventh situated in the tail	75 ,, β	4	4·4	332 45	83 40
13. The end of the tail	60 ,, ν	4	3·8	312 34	78 46
14. One before the breast of the water-snake	14 Reticuli β	4	3·9	56	65 45
15. One situated below the last	62 Hydri γ	4	3·2	62 30	74

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A.	Decl.
Doradus has 4 stars.				° ′	° ′
1. The extremity of the tail	8 α	3	3·1	62	53 20
2. One below the fin, near the tail	20 ζ	3	4·8	67 50	56 26
3. One above the body	29 β	3	3·9	85 20	61 54
4. One below, near the mouth	33 δ	3	4·5	87 33	65 12
The Dove with the Olive Branch.					
Columba.					
1. The right (? left) wing	22 ε	5	4·1	79 40	36 18
2. The bright star in the body	38 α	2	2·5	82 0	34 25
3. One under the left wing	41	5	6·4	82 40	31 54
4. The right shoulder	53 β	2	2·9	84 0	36 10
5. The right wing	57 λ	5	5·2	84 45	34 15
6. One in the neck	65 γ	5	4·5	86 15	36 0
7. One in the beak	79 θ	5	5·3	90 40	37 50
8. The southern end of the olive branch	66 η	5	5·8 v	86 5	42 30
9. One above the head, in the olive branch	84 κ	5	4·8	91 40	35 0
10. The second one above this	91 δ	5	3·9	92 50	33 20
11. The most northern of these	35 Canis Maj. λ	5	4·7	93 42	32 15
Argo Navis. The Ship.					
1. The sounding line	32 Pictoris β	5	3·9	84 32	51 42
2. One in the water, behind the rudder	35 ,, γ	3	4·7	87 20	56 10
3. One below the rudder	48 ,, δ	4	5·2	91 10	54 12
4. Canopus, the brilliant star in the rudder of the ship	7 Carinæ α	1	0·4	93 40	52 15
5. One in the hand of the steersman	20 Puppis ν	4	3·5	95 45	42 14
6. One behind this, in the stern gallery	39 ,, τ	4	3·2	99 25	49 55
7. One in the stern of the ship	18 Carinæ A	3	4·8	99 30	52 35
8. One behind this, below the ship	66 Pictoris α	4	3·5	100 15	61
9. One in the mizen-mast	99 Puppis σ	5	3·5	110 12	42 20
10. One above this, in the mizen-shrouds	162 ,,	6	5·7	111 40	37 0
11. One in the mizen-yard	213 ,, *a*	4	4·0	113 45	34 30
12. One above, in the mizen-top	214 ,, *b*	5	4·9	115 30	30 25
13. One at the end of the main drift,* against the main shrouds	65 Carinæ κ	3	3·7	115 40	52 20

* 13. Argo. The Dutch word here is "fortuning," which in modern Dutch is "vertuining," and means the old shipbuilders' term "drift," where the sheer is raised and the rail cut off: here it is the main-drift. Argo in the catalogue is a three-masted ship.

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*			Magnitude. H.	Magnitude. U.A.	R.A. °	R.A. ′	Decl. °	Decl. ′
14. One at the ridge of the main yard	248	Puppis	ζ	3	2·5	116	30	*32	0
15. One in the bolt-rope of the main-sail	267	,,	h^1	5	4·8	117	10	38	15
16. One other near this	279	,,	h^2	5	4·8	117	20	39	40
17. One in the main-rigging	9	Velorum	γ	3	3	119	30	46	25
18. One in the fore-shroud	56	,,	*o*	4	4·0	124	50	51	40
19. One in the main-mast	48	,,	*e*	5	4·6	125	40	42	12
20. A small star below this	58	,,	*n*	6	5·2	125	45	47	0
21. Another small star near this	53	,,	*b*	5	4·1	126	35	45	30
22. One in the main-sail	66	,,	*a*	5	4·1	129	0	45	0
23. One above this	64	,,	*d*	5	4·4	129	30	41	26
24. Another one before this	97	,,	*c*	4	4·6	132	12	45	
25. One above in the sail	100	,,	λ	3	2·5	135	18	42	2
26. One in the main-top	22	Pyxis	β	5	4·4	128	20	34	5
27. One at the top of the main-mast	24	,,	α	5	3·8	129	15	31	34
28. One in the main-top mast	36	,,	γ	5	4·4	131	0	26	
29. One in the ship, before the main-mast	89	Carinæ	ε	3	2·1	123	10	58	20
30. One below the main-sail, before the mast	65	Velorum	δ	3	2·2	127		53	45
31. One in the rail below the main-sail	127	Carinæ	ι	3	2·5	136	20	57	30
32. Another one near the preceding	117	,,	*a*	4	3·8	134	36	57	28
33. One in the bolt-rope of the sail	129	Velorum	κ	3	2·7	137		53	45
34. One below the belly of the sail	144	,,	N	5	3·2	139	25	55	31
35. One below this, in the forecastle	147	Carinæ	*h*	5	4·9	139	50	57	46
36. Another below this, near the shrouds	150	,,	*m*	5	5·1	140	30	59	10
37. Another below the shrouds	157	,,	*l*	5	3½–5 *v*	142		60	35
38. One below the ship	123	,,	β	3	2·0	138		67	30
39. One on the bend-wale, below the anchor	160	,,	υ	3	3·3	143	45	63	
40. One in front, below the keel of the ship	185	,,	ω	3	3·6	150	46	68	2
41. One below this	193	,,	I	4	4·4	153	30	71	50
42. One in the stern of the ship	223	,,	θ	3	2·9	155	40	62	20
43. One in the anchor	187	,,	*q*	5	3·3–4·5*v*	149	30	59	30
44. One above the anchor-stock	196	,,	*s*	4	4·6	151	53	56	50
45. Another in the anchor-stock	203	,,	*p*	4	3·6	152	54	59	46
46. One above the foresail	204	Velorum	*r*	6	5·3	151		41	23
47. One above, in the foremast	191	,,	*q*	4	4·0	149	50	41	26
48. One below this, in the shrouds		?		5	...	148	35	42	34
49. Another one below this	193	Velorum		6	6·2	149		44	

* 14. Argo. The declination is probably a misprint for 38° 0′.

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A.	Decl.
				° ′	° ′
50. One in the middle of the foresail	216 Velorum *s*	5	5·4	153 10	45 20
51. One other near this	218 ,,	6	6·4	152 12	45 36
52. One more near this	219 ,, *t*	5	5·6	155 36	45 50
53. One in the fore-sheet	222 ,, *p*	4	4·1	157 30	46 16
54. One below this	229 ,, *μ*	4	2·9	158 40	48
55. One below, in the foresail	215 ,, *x*	5	5·6	155 46	52 30
56. One below the bowsprit	246 Carinæ *u*	5	4·1	159 30	56 50
Centaurus.					
1. One below, in the tail of Centaurus	260 Carinæ *x*	5	4·6	163 15	57 10
2. The bright star in the tail	24 Centauri *π*	4	4·3	166 55	52 26
3. One other next to this	42 ,, A	5	5·2	170 0	52 15
4. One near the hind-foot of Centaurus	46 ,, λ	4	3·4	170 25	61
5. One below, near the foot	15 Muscæ λ	4	3·8	172 30	64
6. Another one near this	16 ,, *μ*	6	5·3	173 0	64 30
7. Another one following	69 Centauri *j*	5	4·9	172 35	61 45
8. Another one following	18 Muscæ	5	5·5	174	63
9. Another one in the hind-foot	6 Crucis *θ*	5	4·7	176 40	61
10. One below this	10 ,, *η*	5	4·7	177 35	62
11. One in the foot of the Cross	19 ,, *ζ*	5	4·6	179 34	61 35
12. One above, in the leg of Centaurus	94 Centauri *δ*	3	2·8	178 15	48 18
13. One below this	101 ,, *ρ*	5	4·5	179	50
14. One in the belly of Centaurus	121 ,, *σ*	5	4·3	182 24	47 36
15. A small star near this	119 ,, G	6	5·7	182	48 30
16. One in the body of Centaurus	134 ,, *γ*	3	2·4	186 12	46 48
17. One above this, in the body	131 ,, *τ*	5	4·4	185	46 25
18. Another one in the body	149 ,, *e*	5	5·0	188 50	46 15
19. Another one below, in the body	142 ,,	5	6·2	190	49 10
20. One above the preceding	156 ,, H	5	5·8	190 30	48
21. Another one there above	165 ,, ξ^1	5	5·8	190 45	46 12
22. The uppermost in the body	171 ,, *f*	5	5·3	191 10	45
23. One below this in the body	173 ,, ξ^2	5	4·8	192 43	46 20
24. The left shoulder	204 ,, *ι*	3	3·0	194 40	34 0
25. One below this	227 ,, *d*	4	4·5	196 55	37 20
26. One in the head of Centaurus	265 ,, *i*	5	4·5	200 30	30 40
27. The most northern star in the head	284 ,, *h*	5	5·2	202	29 35
28. One other below this	280 ,, *k*	5	4·7	201 28	30 32
29. Also another below this	274 ,, *g*	5	4·6	201 20	32 10

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	U.A.	R.A. °	′	Decl. °	′
30. One above in the right leg	245 Centauri ε	3	2·6	199	50	51	15
31. One on the back of this (constellation)	273 ,, μ	4	3·4	200	20	41	8
32. One above this, on the back	272 ,, ν	4	3·7	200	35	39	58
33. Another one near this	296 ,, φ	4	4·1	203	8	40	30
34. Another one following	311 ,, χ	4	4·8	205	16	39	40
35. One in the belly	289 ,, ζ	3	2·7	202	18	45	20
36. One above in the right side	297 ,, υ^1	4	4·2	203	15	43	
37. Another one near this	303 ,, υ^2	5	5·0	204	10	44	
38. One in the left leg of Centaurus	304 ,, β	2	1·2	204	20	58	8
39. One in the right foot of Centaurus	363 ,, α	1	0·7	213	48	58	48
40. One below this, near the foot	17 Circini α	3	3·5	210	50	62	30
41. The right shoulder	314 Centauri θ	3	2·2	206	15	35	
42. One in the flag (of the lance)	338 ,, ψ	4	4·4	208	40	36	0
43. One below this, in the lance	342 ,, a	4	4·9	209	10	37	35
44. One above, in the flag	371 ,, c^1	4	4·3	214	30	33	20
45. Another one below this	368 ,, b	4	4·2	215	35	35	30
46. One in the right arm	356 ,, η	4	2·5	213	50	41	20
47. One in front, in the same arm	41 Lupi β	4	2·8	217	40	41	18
48. One below this, in the arm	385 Centauri κ	4	3·3	217	45	40	38
Crux.							
1. The western arm of the Cross	18 δ	3	3·4	179	35	56	26
2. One below this	22 ε	5	4·0	180	20	58	25
3. The lowest star, or foot of the Cross	26 α	2	1·3	182	25	60	40
4. The head of the Cross	34 γ	3	2·0	183		54	48
5. The eastern arm of the Cross	46 β	3	1·7	186		57	8
Musca.							
1. One below, in the body	44 γ	4	4·0	182	55	69	50
2. One above, in the left wing	45 α	4	2·9	184	10	66	30
3. The head	51 β	4	3·4	186	8	65	20
4. The right wing	54 δ	4	3·7	189	25	69	10
The Flying Fish. Volans.							
1. One in the tail of the Flying Fish	10 δ	4	4·1	111	16	67	30
2. Another one below this	9 γ^2	4	3·8	111	18	68	32
3. The right wing	16 ζ	4	4·3	119		71	15
4. The left wing	22 ε	4	4·5	122	46	67	33
5. One in the neck	25 κ^1	4	4·7	127	10	70	

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A.	Decl.
Chamæleon.				° ′	° ′
1. The end of the tail	5 θ	5	4·7	130 30	74 30
2. One above this, the second in the tail	4 α	5	4·2	128 26	73 40
3. The third in the tail	8 η	5	5·6	140	76 20
4. The fourth in the tail	13 ι	5	5·8	148	77
5. The fifth in the tail	14 ζ	5	5·5	150 34	77 15
6. The most northern star in the back	23 γ	5	4·4	158	75 30
7. One below this, in the body	26 δ^2	5	4·9	161 30	77 25
8. Before the shoulder	37 ε	5	5·0	176	74 50
9. One below this, before the body	40 β	5	4·6	179 40	76 20
Lupus.					
1. One in the tail	332 Centauri	5	6·7	208 30	44 25
2. One other above this	2 Lupi	5	6·3	208 35	43 26
3. Another near this	3 ,,	5	5·7	209 40	44 10
4. One at the end of the tail	9 ,, τ^1	6	5·3	210 34	43
5. Another near this	10 ,, τ^2	6	4·9	211	43 15
6. One in the right foot	27 ,, α	4	2·6	213 35	46
7. One other below this	20 ,, ρ	5	4·5	212 45	47 30
8. Another below the preceding	15 ,, σ	5	5·2	*211 15	48 32
9. One in the left buttock	30 ,, b	5	5·8	215 30	50 52
10. One other near this	42 ,,	5	6·7	218 33	50 45
11. One above, in the left thigh	64 ,, ζ	4	3·6	221 45	50 12
12. One behind, on the hip	44 Circini β	5	4·7	221 42	56 45
13. Another one near this	47 ,, γ	5	5·2	223 32	57 15
14. One below this	42 ,, δ	5	5·6	221 28	59 10
15. Another one, a small star, near this	45 ,,	6	5·9	222	58 30
16. One in the belly	47 Lupi π	4	4·3	219 25	45
17. One below this, in the belly	62 ,, κ	4	4·1	221	46 50
18. One above this, in the belly	57 ,, λ	4	4·8	221 30	44
19. Another one in the belly	75 ,, μ	4	5	222 55	46 30
20. Another one near this	86 ,, ν^1	6	5·8	223 5	47
21. One at the front of the left hip	83 ,, δ	4	3·7	223 30	39 55
22. The heart of the wolf	91 ,, ε	4	3·7	224 20	43 40
23. Another one in the body	114 ,, d	5	5·1	227	43 45
24. One at the front of the shoulder	113 ,, γ	4	3·2	227 30	40 16
25. One below this	117 ,, ω	5	4·7	227 56	41 25

* 8. Lupi, right ascension given erroneously by Marre as 211° 11′.

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A. °	R.A. ′	Decl. °	Decl. ′
26. Another one below this	121 Lupi *g*	5	5·2	228	15	43	30
27. Another one near this	115 ,,	5	6·2	227	25	43	25
28. One in the neck	146 ,, η	4	3·7	233	12	37	20
29. Another one near there	157 ,, θ	6	4·9	235		35	30
Triangulum Australe.							
1. The southern angle	5 γ	3	3·1	219	20	66	48
2. One above this	11 ϵ	5	4·6	222	50	64	32
3. The northern angle	17 β	3	3·1	226	18	62	10
4. The apex of the triangle	42 α	2	2·2	238	45	68	
The Bird of Paradise. *Apus.*							
1. One in the foot, by the tail	19 Octantis δ	4	4·7	206	50	80	35
2. One in the tail	6 Apodis η	4	5·3	207	10	77	40
3. Another one near this	9 ,, ϵ	6	5·5	209	8	76	45
4. Another one above this	14 ,, α	4	4·0	212	45	76	26
5. One at the end of the body	40 ,, δ^1	5	5·2	228	40	76	10
6. One below this	44 ,, γ	5	3·9	229	30	77	
7. Another one near this	47 ,, β	5	4·5	234		76	20
8. One in the neck	33 Octantis ϕ	5	5·8	261	30	75	25
9. One in the beak	41 Pavonis	5	6·2	271		72	12
Ara.							
1. The west foot of the altar	13 η	4	3·8	241	10	57	30
2. The western part of the upper edge	23 ζ	4	3·2	244		54	40
3. The western part of the flame	25 ϵ^1	4	4·2	245	46	52	
4. The eastern part below, in the altar	60 δ	4	3·7	248	32	60	30
5. The eastern part above, in the altar	50 γ	4	3·6	249	42	56	32
6. The eastern part below, in the flame	51 β	4	2·8	250		55	
7. One there following, in the flame	62 α	4	2·9	255		49	38
8. The fourth star in the flame	84 θ	4	3·9	262	30	49	35
9. The fifth star in the flame	3 Telescopii ϵ	4	5·2	265	8	46	10
10. The sixth star in the flame	14 ,, ζ	4	4·5	269	20	49	
11. The seventh star in this flame	13 ,, α	4	3·5	269	25	46	20
12. The last in this (constellation)	16 ,, δ^1	6	5·7	270	15	46	30
The Tail of Scorpius.							
1. The first, in the tail	98 μ^1	3	3·2	246	30	37	10
2. The second, in the tail	103 ζ	3	5·8	247		41	

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A.	Decl.
				° ′	° ′
3. The third, in the same	104 ζ^2	3	3·6	247 10	41 36
4. The fourth in this	126 η	3	3·6	252 18	42 32
5. The fifth in this	160 θ	3	2·1	257 30	42 38
6. The sixth, in the tail	169 ι^1	3	3·3	260	39 50
7. The seventh in this	165 κ	3	2·6	258 35	38 45
8. The eighth of this	156 λ	3	2·0	256 34	37
9. The last of the tail of Scorpius					
Pavo.					
1. A star at the extremity of the tail	2 η	4	3·8	252 50	64 40
2. One following, in the same	16 π	5	4·6	259 48	64 10
3. The uppermost, in the same	26 ξ	5	4·4	265 5	62 30
4. One below this	32 ν	6	6·0	266 20	63 50
5. One below this	39	5	5·3	267 22	65
6. One other in the same tail.	45 λ	5	4·3	271 45	63 28
7. One below this	46 κ	5	4·0–5·6*v*	270 30	66 5
8. One other below the preceding	43 θ	6	6·1	270 32	67
9. One at the extremity of the body	56	5	5·7	274 30	69
10. One in the right foot of Pavo	78 ϵ	4	4·0	280 50	73 45
11. One above, in the body	88 δ	3	3·5	290 30	67 30
12. One below this	83 μ^2	5	5·6	289 20	68 15
13. The head	99 α	2	2·1	297 15	58 14
14. One in the neck	104 ϕ^1	5	4·9	299 30	62 30
15. One below this	107 ρ	5	4·9	300 40	64
16. Another one in the neck	109 ϕ^2	5	5·5	301 50	62 33
17. The heart	6 Tucanæ δ	3	4·8	330 36	68
18. One below this	110 υ	5	5·6	*309 45	68 40
19. The breast of Pavo	111 β	3	3·3	311 52	68 30
Indus					
1. The fore-end of his spear	78 Telescopii ξ	5	5·5	291 15	55 16
2. One in the right shoulder	2 Indi η	5	4·7	304 20	54 20
3. One near this, in the shoulder	5 ,, ι	5	5·6	305 30	54 16
4. Another one near this	8 ,,	5	6·0	306 28	54 15
5. One below the belly	6 ,, β	4	3·7	305 30	60 20
6. One in the chest	12 ,, μ	5	5·8	306 40	56 45
7. One in the head	?	4		310 28	52 35

* 18 Pavonis. Misprint of 209° for 309° in the Right Ascension.

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Catalogue of Stars given in Frederick de Houtman's Malay and Madagascar Vocabulary—continued.

No. and Description.	No. and Name. *Uranometria Argentina.*	Magnitude. H.	Magnitude. U.A.	R.A. ° ′	Decl. ° ′
8. The left shoulder	?	4		313 40	54 26
9. One in the upper part of the spear	?	4		319 45	55 22
10. One in the middle part	60 ,, δ	4	4·8	323 30	56 12
11. One in the lower part	64 ,, ε	4	5·2	323 37	57 25
The Heron.					
Grus.					
1. The head of the heron	18 γ	2	3·0	322 44	39
2. One in the neck	27 λ	4	4·7	325	41 5
3. The left wing	28 α	2	1·9	326 25	49
4. The second, in the neck	35 μ^1	4	5·0	327	43 25
5. The third, in the neck	43 δ^2	4	4·4	329 45	45 40
6. One near this	42 δ^1	5	4·2	329 15	45 35
7. The heart	57 β	2	2·2	335 12	49
8. One in the left leg	61 η	4	5·1	335 40	55 45
9. One above this, in the leg	68 ε	4	3·5	336 30	53
10. One in the tail	77 ζ	4	4·0	340 40	54 10
11. One in the right wing	84 θ	4	4·2	341 20	46 12
12. One other below this, in the wing	90 ι	4	3·9	341 50	48
The Indian Magpie, named by the Indians Lang.					
Tucana.					
1. One above, on the beak	26 γ	4	4·0	344	60 15
2. One in the body	?	5		351 20	65 30
3. Another one in this	43 η	5	5·9	353 10	66 36
4. One behind, in the left leg.	44 ε	5	4·3	355 35	68
5. One at the extremity of the body	49 ζ	5	4·1	0	67 24
6. One in the tail *	52 β	5	4·3	2 50	64 34

Comparison of Houtman's Declinations for Epoch A.D. 1600.

Star.	Houtman. ° ′	Neugebauer. ° ′	N.-H. ′
α Phœnicis	−44 5	−44 30	−25
α Eridani	59 20	59 17	+ 3
θ ,,	41 50	41 56	− 6
α Columbæ	34 25	34 20	+ 5

* The position of 6 Tucanæ is correctly identified as 52 β, but it is not the star in the tail.

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Comparison of Houtman's Declinations for Epoch A.D. 1600—continued.

Star.	Houtman.		Neugebauer.		N.-H.
	°	′	°	′	′
α Argus	52	15	52	30	−15
τ Puppis	49	55	50	11	−16
σ ,,	42	20	42	32	−12
ζ ,,	32	0?	38	55	
γ Velorum	46	25	46	11	+14
ε Carinæ	58	20	58	15	+ 5
δ Velorum	53	45	53	17	+28
λ ,,	42	2	41	52	+10
β Carinæ	67	30	68	5	−35
ι ,,	57	30	57	38	− 8
κ Velorum	53	45	53	20	+25
N ,,	55	31	55	17	+14
υ Carinæ	63		63	14	−14
θ ,,	62	20	62	19	+ 1
μ Velorum	48		47	19	+41
α Crucis	60	40	60	52	−12
γ ,,	54	48	54	52	− 4
β ,,	57	8	57	29	−21
δ Centauri	48	18	48	29	−11
γ ,,	46	48	46	45	+ 3
ι ,,	34	0	34	35	−35
ε ,,	51	15	51	24	− 9
ζ ,,	45	20	45	17	+ 3
β ,,	58	8	58	23	−15
θ ,,	35		34	22	+38
α Lupi	46		45	37	+23
α Centauri	58	48	59	7	−19
α Muscæ	66	30	66	55	−25
β Lupi	41	18	41	26	− 8
γ ,,	40	16	39	45	+31
η ,,	37	20	37	10	+10
γ Trianguli Austr.	66	48	67	3	−15
α ,,	68		68	9	− 9
μ^1 Scorpii	37	10	37	17	− 7
ζ^2 ,,	41	36	41	35	+ 1
η ,,	42	32	42	37	− 5
λ ,,	37		36	43	+17
θ ,,	42	38	42	38	0
κ ,,	38	45	38	44	+ 1

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Comparison of Houtman's Declinations for Epoch A.D. 1600—continued.

Star.	Houtman.		Neugebauer.		N.-H.
	°	′	°	′	′
ι^1 Scorpii	39	50	39	53	− 3
ζ Aræ	54	40	55	14	−34
β ,,	55		55	2	− 2
α ,,	49	38	49	26	+12
α Pavonis	58	14	57	55	+19
α Gruis	49		48	51	+ 9
β ,,	−49		−48	57	+ 3

The Number of Stars of Different Magnitudes in the Hyderabad Astrographic Catalogue, Zone −17°. By R. J. Pocock, B.A., B.Sc.

1. The first volume of the *Hyderabad Astrographic Catalogue*, containing the diameters and measured co-ordinates of over 63,000 stars on plates with centres approximately in Decl. − 17°, is now completed. The numbers of stars recorded under different diameters have been counted, and the results are discussed in the present note. As the printing of the volume is not yet complete, it may be best to begin with a brief description of the Hyderabad practice in determining diameters.

2. As each star is measured an estimate of the diameter is made and entered on the measure sheets. Since each star is measured twice in different positions of the plate; two estimates of the diameter are made, and these are added together and the sum printed in the catalogue under the heading d (thus following the Greenwich plan). In order to convert these numbers into magnitudes a formula of the form $m = a - b\sqrt{d}$ has been adopted, since other observatories have found such a formula suitable. The numbers of stars of each tenth of a magnitude among the reference stars used for this zone, taken from the *Washington A.G. Catalogue*, were counted, and by means of these counts the Washington magnitudes were converted to a scale comparable with that of Chapman and Melotte.* These modified magnitudes (m) were then compared with the diameters (d) separately for each plate, and hence the constants a, b determined. Several different groups of plates gave values of b all close to 1·09, which was therefore adopted for the whole zone, and the values of a determined separately for each plate. The resulting formulæ, however, very often result in giving much too faint a magnitude for the fainter stars on the plate. This is due to the fact that most of the measurers used in reality two distinct scales, one for the bright stars and another for the faint stars, estimating all the stars as over 20 or under 10 say, with very few between, so that a formula determined

* See *Astronomical Journal*, vol. xxix., No. 688.

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from the bright stars alone necessarily makes the faint stars too faint, and *vice versa*. We shall have evidence of this fact later. Another peculiarity is that some of the measurers estimate a very large proportion of their stars as 4 or 5 (printed as 8 or 9); on the other hand, other measurers go to the other extreme, and never use these low numbers at all. Since there were five different measurers working throughout, and six towards the end of the work, and the work of the different measurers is well mixed up in each part of the zone, this can have little effect on the final counts.

3. Since one of the objects of this paper is to look for discontinuities in the ratio of faint to bright stars, it is well to consider first what artificial discontinuities may exist, owing to seasonal changes, etc. The zone here considered is particularly favourable for such a consideration; the whole work occupied just two years (1914 Dec.–1916 Dec.), and only two different batches of plates were used. In the earlier part of the work (1914 Dec.–1915 May) the plates used belonged to an old batch that had been in India for some considerable time, and had lost much of their sensitiveness. The exposures given were at first fifteen minutes, and towards the end of this period were increased to eighteen and in a few cases to twenty minutes. All the plates from three hours to ten hours belong to this group, as also a few plates outside these limits of R.A. Since this region was taken within a comparatively short period of time and with similar plates, there is little likelihood of the artificial introduction of any discontinuity in the series. With a few exceptions (mostly in $11^h - 13^h$) all the plates from 10^h to 3^h were taken between 1916 March and 1916 December with a different batch of plates. A few of these newer plates were given an exposure of fifteen minutes, but as they were found to be much more rapid than the older ones, the exposure was soon diminished to twelve minutes. The great majority of these plates were exposed during the period 1916 March 1–1916 May 7, while of the plates exposed later in the year, most were exposed in October–November. Hence here again there is very little chance of any discontinuity. It is true that in the period December–March there is a very great rise in temperature, but this occurs slowly and should cause no sudden break. There is, on the other hand, a sudden change about the middle of June, when the monsoon sets in, but in June, July, or August it is very rarely possible to secure plates, and there are very few in this volume taken in these months.

4. Turning to the numbers themselves, and denoting by N_x the number of stars whose recorded diameters (*i.e.* the sum of two estimations) are not less than x, I have tabulated in Table I. the totals for each hour of R.A. of N_{50}, N_{30}, N_{20}, N_{12}, N_8, the latter being equal to the total number of stars measured. Since the hours of R.A. contain alternately eight and seven plates, the numbers have first been multiplied by a factor to make them equivalent to ten plates in each hour of R.A.

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TABLE I.

Cumulative Totals for Ten Plates in each Hour of R.A., Zone – 17°.

R.A. h	N_{50}	N_{30}	N_{20}	N_{12}	N_{8}
0	93	553	909	1413	1940
1	60	379	656	1075	1620
2	53	366	726	1313	1777
3	59	308	729	1272	1665
4	94	353	832	1500	2310
5	248	1121	1841	2694	3270
6	236	1016	2059	3383	4620
7	164	1108	3100	6401	10720
8	123	724	1554	3067	5220
9	79	378	871	1610	2750
10	51	259	554	1116	1966
11	51	252	581	1182	2341
12	69	316	613	1249	1974
13	100	434	799	1520	2799
14	107	637	1180	2044	3990
15	106	514	1151	2220	4236
16	119	494	821	1490	2917
17	182	739	1356	2720	5910
18	110	611	1214	2490	6167
19	91	330	1046	2189	5282
20	89	587	1206	2273	4034
21	108	708	1146	1864	2620
22	87	429	820	1376	2203
23	64	446	789	1261	1861
C. and M. Mag.	9·14	10·77	11·55	12·25	12·94
Calcd.	9·16	10·62	11·51	12·40	12·95
Diff.	– ·02	+ ·15	+ ·04	– ·15	– ·01

5. The most noticeable feature about the table is the sudden very large increase in the star density between 6^h and 7^h, which, moreover, the table shows is entirely due to the fainter stars. At the foot of each column is given the corresponding magnitude on the C. and M. scale, determined from the table in *Monthly Notices*, vol. lxxv. p. 145. Below these are given the magnitudes calculated from the formula $m = 15{\cdot}44 - 0{\cdot}88\sqrt{d}$. Now the value of b, viz. 0·88 in this formula, is sensibly different from that mentioned above, viz. 1·09, adopted for the whole zone. It will be seen that the formula just given suits very well both for faint and bright stars, whereas if we use the value 1·09 and only take account of the bright stars we obtain the formula $16{\cdot}92 - 1{\cdot}09\sqrt{d}$, which will

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make the faint stars 13·84 instead of 12·95, nearly a whole magnitude too faint; this is an instance of the fact mentioned above, that the bright stars are estimated too bright relatively to the faint stars. The formulæ given in the printed volume must therefore be considered as only suitable for the bright stars; at about mag. 11·0 they will make the stars 0·5 mag. too faint, and below 12·0, 0·9 mag. too faint.

6. Next, taking the logarithms of the numbers in Table I. and treating them in the same way as Professor Turner has done in several papers (see *Monthly Notices*, lxxv. p. 63), that is, subtracting the mean of each column from each number in the column and then subtracting one column from the remainder, we form Table II. Since the column N_{30} has been taken as standard and subtracted from the remainder, this column is omitted, and the signs of the column headed N_{50} are changed so as to be comparable with the other columns.

Table II.

Values of log N' to log N'_{30} for Selected Magnitudes in Units of 0·01.

R.A. h	N_{50}	N_{20}	N_{12}	N_8	Gal. Lat.
0	− 6	− 9	− 16	− 25	− 77
1	− 9	− 7	− 11	− 16	− 75
2	− 13	− 1	− 1	− 11	− 64
3	0	+ 7	+ 4	− 6	− 51
4	+ 14	+ 6	+ 6	+ 2	− 38
5	+ 6	− 9	− 19	− 33	− 24
6	+ 8	0	− 4	− 14	− 11
7	− 11	+ 14	+ 20	+ 19	+ 1
8	− 6	+ 2	+ 6	+ 6	+ 14
9	+ 3	+ 6	+ 6	+ 7	+ 25
10	+ 1	+ 2	+ 7	+ 9	+ 35
11	+ 2	+ 6	+ 10	+ 17	+ 42
12	+ 5	− 2	+ 3	0	+ 45
13	+ 8	− 4	− 2	+ 2	+ 44
14	− 6	− 4	− 6	0	+ 39
15	+ 3	+ 4	+ 7	+ 12	+ 31
16	+ 10	− 8	− 9	− 2	+ 20
17	+ 10	− 4	0	+ 11	+ 8
18	− 3	− 1	+ 4	+ 21	− 4
19	+ 16	+ 20	+ 26	+ 41	− 17
20	− 11	0	+ 2	+ 4	− 30
21	− 10	− 10	− 15	− 23	− 43
22	+ 2	− 2	− 6	− 8	− 57
23	− 13	− 6	− 12	− 17	− 69

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Looking at the column headed N_8, it is at once evident why the lines have been drawn separating the region 7^h to 20^h from the rest. We have here a discontinuity similar to those found by Professor Turner in other zones. Following Professor Turner, I shall speak of the region 7^h to 20^h as unobscured, and of the region 21^h to 6^h as obscured, simply for convenience, without intending to imply that the difference is necessarily due to any actual obscuring matter in space. It may be noted that the obscured region from 21^h to 6^h fits very well into the diagram given by Professor Turner in *Monthly Notices*, lxxv. p. 480.

7. Now, taking the numbers given in Table I. and multiplying by such factors as to reduce the total number of stars in each hour of R.A. to about 2000, we can determine the amount of the obscuration without giving undue weight to the galactic regions. Without giving the details, we arrive at the results shown in Table III., where I., II. are the obscured and unobscured regions respectively. Table III. is similar in form to Tables V., VII., XIII., XIV. in *Monthly Notices*, lxxv. p. 470. In this case log N_{50} has been subtracted from the other logarithms.

TABLE III.

Amount of Obscuration, Zone – 17°. *Logarithms in Units of* 0·01.

Mag.	9·14.	10·77.	11·55.	12·25.	12·94.
I.	867	3788	3878	5645	5774
II.	742	2924	4020	7288	13318
log I.	0	64	65	82	83
log II.	0	60	73	99	125
log I.-II.	0	+4	–8	–17	–42

8. It will be seen that the final figure comes out –42 in exact agreement with the number found in other zones; and, comparing the amount of obscuration with the two adjacent zones as given in *Monthly Notices*, lxxv. p. 480, we get—

Mag.	10·0.	11·5.	13·0.
Algiers –1°	–10	–18	–42
Hyderabad –17°	0	–7	–42
Perth –32°	0	0	–42

So that the Hyderabad results fall naturally between the other two. In the Hyderabad zones the obscuration begins about 11·0.

9. Now, arranging the numbers of Table II. according to galactic latitude, we obtain the results shown in Table IV.

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TABLE IV.

Amount of Galactic Condensation.

Obscured Region.						Unobscured Region.					
Mean Gal. Lat.	No. in Group.	9·14.	11·55.	12·25.	12·94.	Mean Gal. Lat.	No. in Group.	9·14.	11·55.	12·25.	12·34.
−71°	4	−10	−6	−10	−17	+41°	5	+2	0	+2	+6
−50°	3	−3	−2	−6	−12	±25°	5	+4	+4	+6	+12
−24°	3	+9	−1	−3	−15	±70°	4	−2	+3	+8	+14

10. Determining the coefficient of galactic condensation per magnitude per degree from the first and last rows only, and in each case taking the simple mean of the four determinations, we have the result—

Coefficient of galactic condensation.

Unobscured region −·00065. Obscured region −·00142.

The first result falls between those for Algiers and Perth, the second is slightly larger than either Algiers or Perth (see *Monthly Notices*, lxxv. p. 608). They do not lend support to the unusually high value in the case of the Cape Zone −41°. The mean value —·00104, however, lends some support to the suggestion that there is an increase in the amount of galactic condensation as we proceed southwards.

Nizamiah Observatory, Hyderabad:
1917 *January* 17.

Errata in the Double-Star Measures of the "Monthly Notices," Volumes LXVI. to LXXV. By Eric Doolittle.

In the course of copying all the double-star measures published in the *Monthly Notices* for the past ten years, the following errors have been detected. It is hoped that this list is an exhaustive one: perfectly obvious misprints, such as can cause no inconvenience, are, however, for brevity, not referred to.

Volume LXVI.

Royal Observatory, Greenwich.

Page 24. Ho 57, BC. Apparently this may be some other pair.

„ 26. Hu 482. Apparently the wrong star. The measures describe β 1115, and may belong to this.

„ „ Hu 483. Apparently the wrong star; (= See 277?).

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Page 27. Ho 74. A large error in the angle. (For 30°·7 write 130°·7 ?)
„ 28. Σ 2349. The distance is about 6″ too small; (= 7″·28 ?).
„ „ Σ 2352. The measured distance is about 12″ too small.
„ 29. Ho 89. Misprint in distance. Probably should be 6″·43.
„ 503. For Ho 328 read Ho 238.
„ 507. Σ 2170. Misprint in angle; (= 70° ?).
„ 508. Σ 2341. This measure apparently belongs to Σ 2344.
„ „ Σ 2367. The measured distance is evidently in error.
„ „ Hu 676. The wrong star (?).
„ 509. Σ 2703. The measures belong to some other pair.
„ „ Σ 2726. Either misprints in angle and distance, or the wrong pair.
„ „ Σ 3113. The R.A. is 19 hrs. too great and the Decl. 16° too great.
„ 508. + 14° (3718). The Decl. is 1° in error.
„ 509. Σ 2802. The R.A. is 10 min. in error.
„ 510. For β 89 read β 989.

Espin, pages 141–144.

Holmes (17 hrs. 52·1 min.) is Σ 2246.
Hh 583. The companion is not new; measured by β in 1879.
For Sh 348 read Sh 358.

Volume LXVII.

Espin.

Page 190. H 5450. This is G.C. **58**, 52 sec. and 6′ away from H's place. The latter star was not found by β.
„ „ H 631. The R.A. is 1 hr. too great. These measures apparently belong to the neighbouring H 1065 = G.C. **558**.
„ 191. H 524. H and Scheiner reverse the quadrant.
„ „ H 587. The R.A. is 2 hrs. too small.
„ 191. H 263. The R.A. is 2 hrs. too small.
„ 192. H 1834. The star C was noticed by β in 1905, but not fully measured by him.

Volume LXVIII.

Royal Observatory, Greenwich.

Page 43. Ho 542. The first magnitude is 8·8, not 6·8.
„ 44. Lewis (15 hrs. 24 min.); this is Ku 50 = G.C. **7281**.

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Page 527. Espin 278. This is identical with Σ 3114.
„ 530. Ho 442. The position and magnitudes are incorrect.
„ 532. The unidentified star at 20 hrs. 16 min. is apparently Ho 125.
„ 533. Ho 179. The R.A. is 4 min. too small.

Espin, page 203.

H 495. The R.A. is 2 hrs. too small.
H 1651. Error of 100° in angle (?).
H 1657. This pair is +47° (3462); it was measured by β in 1911. The pair measured by Espin is new.
H 1705. According to the description on page 204, the Decl. is here 12′ too great.

Volume LXIX.

Espin.

Page 219. H 2667. Identified by Espin with +48° (3136), but this is an error, for this star is in a different place. Espin's position nearly agrees with +49° (2236), and the pair may be this star.
„ „ H 1274. Identified by Espin with +42° (2604). If this is correct, the place given is 2 min. in error. No DM star in the position given.
„ „ H 1478 (1). This pair is H 1472. Measured by β and Doolittle.
„ „ H 1478 (2). From Espin's position this pair seems to be 25 sec. following and 5′ N of H 1478. The latter pair was measured by me on three nights in 1907.
„ 222. β—. For G.C. **994** read G.C. **944**.

Royal Observatory, Greenwich.

Page 594. β 928. The R.A. is 20 min. too small.
„ 596. For AG 167 read AG 197.
„ 597. Ku 43. The position given is that of Ku 53, but the measure does not describe either pair. I have no record of any pair in Lewis's position answering to his description.
„ „ OΣ 310. This measure apparently belongs to the neighbouring pair, Ho 405.
„ „ Weisse 31. This measure apparently belongs to some other pair.
" 600. Aitken 368. The measures belong to this pair, but the position given is that of OΣ 368.

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Volume LXX.

Espin, page 239.

Kustner 23 should be Krueger 23.
For Hussey 394 read 794.

Royal Observatory, Greenwich.

Page 636. Σ 3072. There is an error in the distance, or this is the wrong pair.
„ 646. AG 219. An error in distance, or some other pair.

Volume LXXI.

Phillips, page 216.

Σ 401. The R.A. is 1 hr. too great.
Σ 427. The R.A. is 10 min. too small.

Espin, page 218.

H 2428. The R.A. is 1 hr. too small.
+48° (1900). The Decl. is 40′ too small.
+51° (2487). If this identification is correct the R.A. is 1 hr. too small and the Decl. is 20′ too large. No DM star in Espin's place.

Royal Observatory, Greenwich.

Page 734. AG 122. The measures evidently belong to Aitken 122. The position stated and the magnitudes are incorrect.
„ 735. Σ 1114. Angle 200° too large, probably from a misprint.
„ 737. Perrotin. There is evidently an error of about 100° in the angle.
„ 739. OΣ 276. Evidently an error in the printed distance of AC.
„ 740. Σ 1912. The distance is in error.
„ „ Ku 50. The distance is in error.
„ 741. Hu 748. An error in the distance (?).
„ 743. Σ 2165. The angle about 200° in error.
„ „ β 130. The R.A. is in error.
„ 745. OΣ 362. The measures belong to some other star.
„ „ Lewis (19 hrs. 4 min.). No position given; meaning of angle printed is uncertain.
„ 749. Ho 480. Entered twice, the second time with an erroneous R.A.
„ „ Σ 2900. Apparently the measures belong to Σ 2990. The R.A. and Decl. are incorrect.

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Volume LXXII.

Jonckheere, page 157.

J. 43. Both angle and distance disagree with the values published in *J.A.*, i. p. 151.

Espin, page 192.

+49° (3131). Evidently either this measure or the one published in *M.N.*, lxix. p. 221, is in error.
+49° (3216). The Decl. is 9′ too large.

Volume LXXIII.

Royal Observatory, Greenwich.

Page 95. β 1315 and Hu 1044. These pairs are identical.
,, 101. Aitken 224. The measures belong to some other pair (β 339?).
,, 102. H 570 should be Hussey 570.
,, ,, 1324. This is β 1324.
,, ,, 1672. This is Σ 1672.
,, 103. AG 192. An error in distance, or some other pair.
,, 105. Hu 661. Evidently an error in the angle.
,, 106. Lewis 14. Evidently a large error in this distance, or else in that of 1900 (*M.N.*, lxxi. p. 486).
,, 113. AG 267. There is an error in the distance.

Phillips.

Page 159. Σ 87. The R.A. is 10 min. in error.
,, 161. H 1317. These measures belong to a faint pair first noticed by Miller in 1904. The H pair is 31 sec. following and 20′ S. (See *Pub. Flower Observatory*, iv.)

Volume LXXIV.

Royal Observatory, Greenwich.

Page 139. Σ 930 should be Σ 950.
,, 141. β 858 should be β 585. The magnitudes are in error.
,, ,, Aitken. This is A 2131; it forms the principal star of Ho 357.
,, 142. Σ 3121. The angle is evidently in error.
,, ,, Σ 1372. The second measure belongs to some other star; possibly to Ho 369, which is 13 min. following.
,, 143. Σ 1634. There is evidently a large error in the distance.

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Page 144. Σ 1700. Evidently an error in distance.
„ 145. Σ 1834. Error in angle (= 91°·1 ?).
„ 152. OΣ 382. A large error in angle, or some other pair.

Espin.

Page 244. +46° (1476) should be +45° (1476).
„ „ Aitken 1774 should be A 1744.
„ 246. Σ 2658. *a* was also measured by Espin in 1910.
„ „ H 1635. The angle differs 100° and the distance 4″·2 from the measure of Espin in 1909. Misprints, or some other star.
„ „ +49° (4189). The description in exact agreement with that of +49° (4188), which was measured by Espin in 1909, but the latter pair is 11′ N. The two pairs identical (?).

Phillips and Acocks, pages 443–444.

Σ 2264. For 359°·0 read 259°·0 (?).
Σ 2279. The printed distance incomplete.
Σ 3008. Misprint in angle; probably should be 228°·3 or 238°·3.

Franks.

Page 531. σ 434. For 10° 50′ read −10° 50′.
„ 534. H 5524. This is either a more distant companion, not noticed before, or else some other pair.

Volume LXXV.

Franks, page 101.

Σ II, 9. The distance is incompletely printed.

Voûte.

Page 432. Σ 517. The measures belong to some other pair.
„ 433. I 15·72, Howe 37. This is apparently not I 15·72, nor does it answer the description of Howe 37. It is just possible that the pair is Σ 3095, with slight errors in the angle and distance.

The Flower Observatory,
University of Pennsylvania:
1916 *November* 1.

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Preliminary Values of the Variation of Latitude at Greenwich for 1916.

(*Communicated by the Astronomer Royal.*)

The results given in this paper have been derived from measurements of 216 plates taken with the Cookson Floating Zenith Telescope.

The paper is a continuation of the preliminary discussion by Mr. Jones which appeared in the *Monthly Notices* for 1915 May, and the data for star places and proper motions are those found and used in that paper.

The aberration has been assumed to be 20″·47. No corrections have been applied for stellar parallax.

The minimum number of plates obtained in a section (tenth part) of the year was 8, and the maximum number 29.

The probable error of a result from one plate is taken as ±″·056 (*M.N.*, vol. lxxv. p. 544), and from this the result in the last column of the table is derived.

Variation of Latitude at Greenwich in 1916.

Section.	Epoch.	Variation.	Prob. Error.
		″	″
44	1916·03	− ·201	± ·013
45	·16	− ·347	·020
46	·26	− ·234	·011
47	·37	− ·202	·011
48	·42	− ·052	·014
49	·50	+ ·032	·014
50	·58	+ ·165	·010
51	·69	+ ·218	·014
52	·77	+ ·279	·011
53	·92	+ ·110	·014
54	1917·08	− ·179	·015

A table of corrections for latitude variation on selected dates applicable to observed N.P.D.'s is appended.

These corrections were read from a smooth curve drawn through the above results plotted as points on squared paper.

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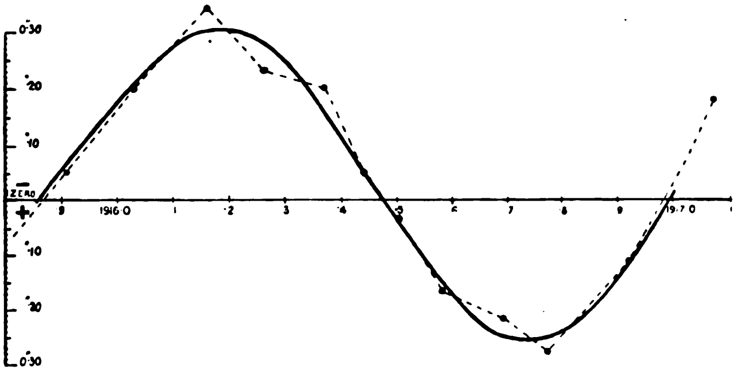

Latitude Variation at Greenwich during 1916.

Table of Corrections applicable to observed N.P.D. for Variation of Latitude.
(Longitude of Greenwich.)

Date.	Corrn.	Date.	Corrn.
	"		"
1916 Jan. 1	+0·18	1916 July 1	−0·03
11	·21	11	·07
21	·23	21	·11
Feb. 1	·26	Aug. 1	·15
11	·28	11	·18
21	·30	21	·21
Mar. 1	·30	Sept. 1	·23
11	·31	11	·24
21	·30	21	·25
Apr. 1	·29	Oct. 1	·25
11	·27	11	·24
21	·24	21	·23
May 1	·21	Nov. 1	·21
11	·18	11	·18
21	·14	21	·15
June 1	·09	Dec. 1	·12
11	·05	11	·08
21	+0·01	21	−0·04
July 1	−0·03	31	0·00

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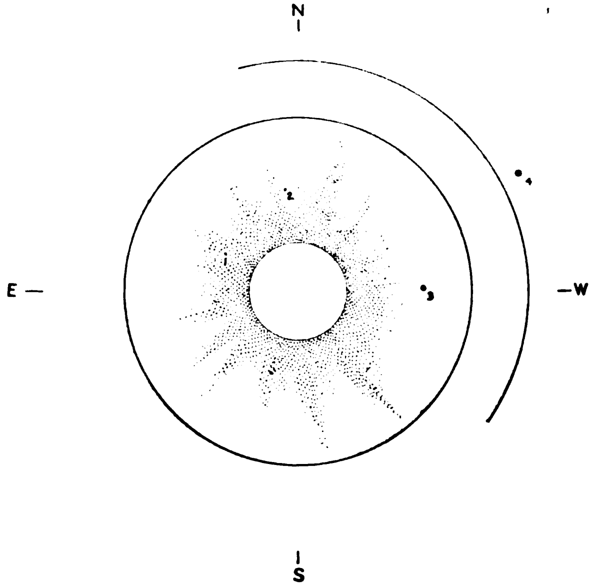
On the Opportunity afforded by the Eclipse of 1919 *May* 29 *of verifying Einstein's Theory of Gravitation.* By Sir F. W. Dyson, M.A., LL.D., F.R.S.

According to Einstein's theory, a ray of light passing near the Sun will be deflected by an amount $1''\cdot75\frac{r_0}{r}$ where r_0 is the radius of the Sun and r the perpendicular distance of the ray from the Sun's centre. An examination was made of four photographs taken by Mr. Davidson with the astrographic equatorial at the eclipse of 1905. The brightest stars near the Sun at the time were:—

No.	Name.	Photog. Mag.	Dist. from Sun's Centre.	Displacement.
		m	′	″
1	Mayer 470	7·2	29	0·96
2	Lal. 20558	8·4	35	0·79
3	49 Leonis	5·7	43	0·64
4	ρ Leonis	3·9	86	0·32

Their positions with a rough sketch of the corona are shown on the accompanying diagram. The two stars ρ Leonis and 49



Leonis were easily shown. It was thought that Mayer 470 was faintly visible. Accordingly, a key-plate was taken with the astrographic equatorial, and was placed film to film with the eclipse photographs in turn (these had been reversed owing to the employment of a cœlostat mirror at the eclipse), and the plates were carefully measured differentially and reduced. It was disappointing to find that the results were not sufficiently accordant

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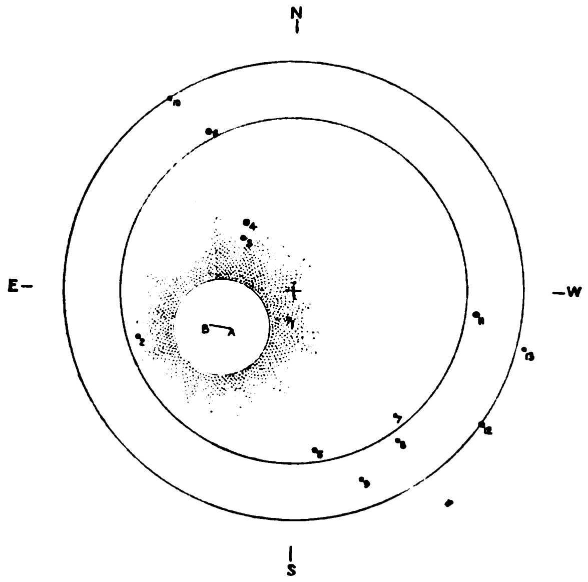
to serve as a verification, or the contrary, of Einstein's theory. It is probable that if the plates had been treated differently the star Mayer 470 would have been shown, but they were developed so as to give pictures of the corona. They show, however, that astrographic object-glasses are very suitable for this important observation, and the purpose of this note is to draw attention to the unique opportunities afforded by the eclipse of 1919 May 29.

The field of stars is shown in the accompanying diagram. There



are an unusual number of bright stars, and with weather conditions as good as those at Sfax in 1905—which were by no means perfect—no less than thirteen stars might be obtained. These are—

No.	Name.	Photog. Mag.	Dist. from Sun's Centre.	Displacement.
		m	′	″
1	B.D. 21° 641	7·0	23	1·20
2	Piazzi IV. 82	5·8	29	0·95
3	κ^2 Tauri	5·5	32	0·86
4	κ^1 Tauri	4·5	37	0·75
5	Piazzi IV. 61	6·0	55	0·50
6	υ Tauri	4·5	69	0·40
7	B.D. 20° 741	7·0	69	0·40
8	B.D. 20° 740	7·0	71	0·39
9	Piazzi IV. 53	7·0	71	0·39
10	72 Tauri	5·5	82	0·33
11	56 Tauri	5·5	88	0·31
12	53 Tauri	5·5	96	0·29
13	51 Tauri	6·0	105	0·26

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This should serve for an ample verification, or the contrary, of Einstein's theory. Unfortunately, the track of the eclipse is across the Atlantic and near the Equator. Mr. Hinks has kindly undertaken to obtain for the Society information of the stations which may be occupied. I have brought the matter forward so that arrangements for observing at as many stations as possible may be made at the earliest possible moment.

Royal Observatory, Greenwich:
March 2.

Two Applications of Jacobi's Integral.

By Harold Jeffreys, M.A., M.Sc.

1. *Note on the Moulton-Gyldèn Theory of the Counterglow.*

Suppose that a planet of mass μ is moving in a circular orbit about the sun; if a very small body is free to move about the two together, then it is known that the only first integral of its equations of motion is the Jacobi integral, namely, in Moulton's notation *

$$\nu \equiv x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - x'^2 - y'^2 - z'^2 = \text{const.}$$

Further, if fixed axes OX, OY, OZ be taken, coinciding with the moving axes when $t = 0$, we have

$$x^2 + y^2 = X^2 + Y^2; \; x'^2 + y'^2 + z'^2 = (X' + Y)^2 + (Y' - X)^2 + Z'^2.$$

Thus the integral can be expressed in terms of the co-ordinates and velocities referred to fixed axes without introducing the time explicitly. It follows at once by Jeans's theorem † that a system of such particles can be in a steady state if, and only if, the number of particles whose co-ordinates and velocity-components lie in the element $dXdYdZdX'dY'dZ'$ is $\chi(\nu)dXdYdZdX'dY'dZ'$, where χ is any function. It is easily proved that $\partial(XYZX'Y'Z')/\partial(xyzx'y'z') = 1$, and hence at any point the numerical density ρ is $\iiint\chi(\nu)dx'dy'dz'$.

Now, $x^2 + y^2 + \dfrac{2(1-\mu)}{r_1} + \dfrac{2\mu}{r_2} = 2U$ in Moulton's notation; the surfaces obtained by giving different values to U are the "surfaces of zero velocity." Put then $x' = \xi U^{\frac{1}{2}}$; $y' = \eta U^{\frac{1}{2}}$; $z' = \zeta U^{\frac{1}{2}}$.

Then $\rho = \iiint\chi\{U(2 - \xi^2 - \eta^2 - \zeta^2)\}d\xi d\eta d\zeta$, the limits being $-\infty$ to $+\infty$ in all cases. Hence ρ is a function of U only.

According to the theory of Gyldèn and Moulton, ‡ the counterglow is caused by light reflected from particles moving in the neighbourhood of the straight-line position of Lagrange's third particle. For these U must lie between two slightly different values, both

* *Celestial Mechanics*, 1914, p. 281.
† *Monthly Notices*, vol. lxxvi., 1915, p. 78.
‡ *Bulletin Astronomique*, t. 1; *Astronomical Journal*, No. 483.

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being near that corresponding to the surface of zero velocity that has a double point.

The luminosity of the sky as seen from the earth is proportional to $\int \rho dr_2/r_1^2$, and as r_1 does not vary widely, it is nearly proportional to the depth (along a line through the earth) between two neighbouring surfaces of the system. It can be shown that this depth is greatest in the direction through the double point, so that the part of the sky just opposite to the sun is the brightest. An independent proof of the discovery of Gyldèn and Moulton is thus obtained.

2. *Orbital Motion in a Resisting Medium.*

It can be seen at once from the above that the mean value of each of x', y', and z' among the particles at any point is zero; in other words, however much the velocities may vary among themselves, the condition of steady motion requires the *average* velocity at any point to be the same as if the whole system was rotating like a rigid body.

Now, let x, y, z be the co-ordinates of a third body, large compared with the particles considered, but small compared with S and J. Then its equations of motion are

$$x'' - 2y' = \frac{\partial U}{\partial x} - \kappa x',$$

$$y'' + 2x' = \frac{\partial U}{\partial y} - \kappa y',$$

$$z'' \quad = \frac{\partial U}{\partial z} - \kappa z',$$

where κ is a small quantity, depending only on U.

Multiply by x', y', z' respectively, and add. Then

$$\frac{d}{dt}(x'^2 + y'^2 + z'^2 - 2U) = -2\kappa(x'^2 + y'^2 + z'^2).$$

If the quantity C be *defined* now as equal to $2U - x'^2 - y'^2 - z'^2$, so that in the case of motion without a resisting medium it is the Jacobi constant, then it follows that resistance causes C to increase steadily, as has been stated by See. Thus the space that the third body is confined to will steadily diminish till it is ultimately forced to move either as an interior planet or a satellite of J. Lagrange's particles, corresponding to a minimum point of U, will be secularly unstable. As the general motion of the medium is direct, retrograde satellites will experience more resistance than direct ones, and therefore approach their primaries more rapidly. If then we can grant that the attraction of the other planets produces no important effect on the secular changes in the orbits, the fact that the known retrograde satellites are all on the outside of their systems renders it very unlikely that they have been captured by the action of a medium in a state of steady motion.

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The Viscosity of the Earth. (Third Paper.)
By Harold Jeffreys, M.A., M.Sc.

In a former paper with a similar title to the present * I showed that if the materials of the earth be assumed to obey the "elastico-viscous" law used by Sir G. H. Darwin, and the degree of viscosity were chosen to make the lunar secular acceleration due to tidal friction agree with the observed value, then the Eulerian nutation would die down in a few days. The observed fourteen-monthly variation of latitude could not then be a free vibration. This appeared improbable, and several alternative explanations were offered; one was that the elastico-viscous law was perhaps widely different from the truth. This now appears to be the most plausible of them. The object of the present paper is to indicate the consequences of a new type of solid viscosity, which is here termed "firmo-viscosity," and which enables the formerly apparently conflicting facts to be reconciled.

The Viscosity of Solids.

The existence of some kind of dissipation of energy in the interior of a solid body when a stress is applied is generally recognised, but hitherto little work seems to have been done on the precise laws that govern this dissipation. The law known as that of "elastico-viscosity" was used by Sir G. H. Darwin in his work on tidal friction; the equations of motion of a solid obeying it were first given by Mr. J. G. Butcher, but the original idea appears to have been due to Maxwell.† If a perfectly elastic body be suddenly strained, and inertia can be neglected, then the strain immediately attains the value given by the equation

$$nS = F \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where S is the strain and F the stress, and n is a constant proportional to the coefficient of rigidity. No further change takes place till the stress is removed, when the strain at once sinks to zero. If the body be elastico-viscous, however, the stress-strain equation is most conveniently expressed in the form

$$nS = F + \frac{1}{t_1}\int F dt \quad . \quad . \quad . \quad . \quad (2)$$

where t_1 is a constant. In this case a sudden stress gives an immediate strain F/n as before, but the solid then proceeds to flow at a uniform rate F/nt_1 till the stress is removed; the strain then does not sink to zero, but to FT/nt_1, where T is the time for which the stress acted. A permanent set is therefore possible if this law holds.

An alternative law was suggested to me by Sir Joseph Larmor

* *Monthly Notices*, vol. lxxv., 1915, p. 648.
† *Phil. Mag.*, vol. xxxv., 1868, p. 134.

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some time after the publication of my former paper. According to this, the stress-strain equation is

$$n\left(S + t_2\frac{dS}{dt}\right) = F \quad . \quad . \quad . \quad . \quad (3)$$

There is a frictional term proportional to the rate of straining, which appears decidedly probable. When a constant stress is applied the strain approaches asymptotically the value F/n, and when it is removed the strain returns asymptotically to zero. Permanent set is therefore absent; this kind of viscosity does not imply weakness, but an additional stiffness. For this reason it will henceforth be referred to as "firmo-viscosity." The two types of internal resistance may be combined in the single formula

$$n\left(S + t_2\frac{dS}{dt}\right) = F + \frac{1}{t_1}\int F dt \quad . \quad . \quad . \quad (4)$$

The strain tends to approach asymptotically to the value $\frac{1}{n}\left(F + \frac{1}{t_1}\int F dt\right)$, which on account of permanent set increases steadily. When the stress is removed the strain sinks asymptotically to the constant value FT/nt_1. The observations of Trouton and Andrews on substances like pitch, wax, and glass* in a plastic condition appear to agree very well with this law.

If in this we put $t_1 = \infty$, the viscosity follows Larmor's law, while if $t_2 = 0$ it follows Maxwell's. If t_1 is not infinite, it is evident that the body cannot remain in equilibrium unless there are no deforming forces acting, *i.e.* unless it is in a hydrostatic state. If, on the other hand, $t_1 = \infty$, permanent equilibrium is possible when constant deforming forces act. This evidently corresponds with the condition of materials used in building, such as girders and struts.

It appears that it will be impossible to frame a plausible law of viscosity that will permit an unlimited flow when a stress is indefinitely prolonged, without permanent set occurring; the converse is also true. For, first suppose a permanent set to remain after a stress has been applied and removed. Then F and all its derivatives are zero; the only difference between the actual body and one that has never been strained is that in the actual case $\int F dt$ is not zero. Hence permanent set implies the presence of this integral in the stress-strain relation. But this integral increases steadily with the time when the force is uniform, and the strain must therefore also increase indefinitely. Conversely, if the strain increase indefinitely when a uniform stress is applied, the strain must depend on $\int F dt$, and hence a permanent set will remain when the stress is removed. Indefinite flow under constant stress and permanent set are thus completely associated.

* *Phil. Mag.*, ser. vi., vol. vii., 1904, p. 347.

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If a problem has been solved for the case of perfect elasticity ($t_1 = \infty$, $t_2 = 0$), the rigidity being n, the solution of the general case can be obtained from this by replacing n by the operator

$$c = n\left(1 + t_2\frac{d}{dt}\right)\Big/\left(1 + \frac{1}{t_1\frac{d}{dt}}\right).$$

In the previous paper t_2 was assumed to be zero. It was found that the lunar secular acceleration made $t_1 = 1$ day nearly, so that for movements with a fourteen-monthly period $t_1 d/dt$ and therefore c were exceedingly small. The earth would therefore behave like a perfect fluid ($n = 0$), and a Eulerian nutation would be impossible. With any value of t_1 of the order of a day, and any value of t_2 less than a year, this would still hold. We may safely say, then, that the accepted theories of tidal friction and the variation of latitude cannot be reconciled if Maxwell viscosity is important. Hence t_1 must be practically infinite; in other words, there is no permanent set in the earth for small stresses.

A confirmation of this may be found in the imperfection of isostasy. Uncompensated surface inequalities exist which must cause a considerable shearing stress even at a depth of thousands of kilometres; yet they are supported, which could not happen if the interior of the earth showed any permanent set for small strains.

Next, we may consider the case of firmo-viscosity. Now

$$c = n(1 + t\,d/dt).$$

The suffix may be dropped without ambiguity.

For a homogeneous earth, let E denote the amplitude of the actual semidiurnal bodily tide divided by that of the equilibrium tide, and ϵ its lag. Then in the notation of the previous paper

$$Ee^{-\iota\epsilon} = \left\{\frac{19n}{2g\rho a}(1 + \iota p t) + 1\right\}^{-1}.$$

E is about 0·70, and the secular acceleration of the moon requires ϵ to be about $1°\ 5'$. We find then

$$\tan^{-1} \cdot 7pt = 1°\ 5', \text{ whence } t = 6^{m}\ 11^{s}.$$

If $1/\tau_0$ be the ellipticity of the Maclaurin ellipsoid rotating in 24^{h}, then the pitch of the Eulerian nutation is given, in the notation of the complex variable, by the equation

$$\frac{86400\tau_0 p}{2\pi} = \frac{7(1 + \iota pt)}{7(1 + \iota pt) + 3},$$

the unit of time being 1 second.

Putting $\tau_0 = 232$, we find that one root of this makes

$$p = (1{\cdot}89 \times 10^{-2} + 4{\cdot}6 \times 10^{-7}\iota)/1 \text{ day}.$$

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The motion of the pole is then a circular one of period 331 days, with an amplitude proportional to $e^{-4 \cdot 6 \times 10^{-7}t}$. The amplitude thus dies down very slowly, the life of the vibration being of the order of 10,000 years. The Eulerian nutation can therefore persist on this hypothesis.

The other root makes $p = \frac{260\iota}{1 \text{ second}} + \frac{2\pi}{232 \text{ days}}$.

The amplitude of this vibration thus declines to zero in a few minutes. Its physical meaning is, that if the earth were originally rotating about its axis of symmetry, and the axis of rotation were suddenly displaced without being given any transverse motion, it would rapidly return to rest along the axis of symmetry on account of viscosity. The neglect of inertia in calculating the elastic strain produced is probably not justified for this motion, but the order of magnitude of the importance of viscosity is unlikely to be wrong, and in any case such a movement is obviously of little astronomical interest.

The theories of tidal friction as an explanation of the secular acceleration of the moon's mean motion and of the Eulerian nutation as an explanation of the fourteen-monthly variation of latitude can therefore be reconciled if the viscous parts of the earth are supposed to obey Larmor's law instead of Maxwell's.

The former Viscosity of the Earth.

If we accept the hypothesis that the earth was formerly gaseous, it follows at once that at some date it was liquid. If, on the other hand, the Planetesimal Hypothesis were true, it is somewhat uncertain whether the earth could have been liquid or not. The velocities with which the planetesimals struck the earth must have been of the same order of magnitude as planetary velocities, and therefore great enough to cause fusion if the heat were retained; if the accretion were very slow, however, heat might be radiated away sufficiently fast to prevent fusion. Even on the hypothesis designed to render unnecessary the hypothesis of a molten earth, it is then very doubtful whether the earth could always have been solid. It will here be assumed that it was once liquid.

If the first section of this paper be considered to have established that the Maxwell coefficient of viscosity is infinite, the physical arguments that I have elsewhere based * on its continuous increase from a very small amount to a very large one, through the value that would make tidal evolution proceed at its maximum rate, will still hold in their entirety. On the hypothesis of a formerly molten earth, then, Darwin's theory of tidal friction as an important factor in the past history of the earth and moon seems necessarily true. When the changes were taking place most rapidly elastico-viscosity must have been much more important

* *Phil. Mag.*, vol. xxxii., 1916, p. 590; or *Monthly Notices*, vol. lxxvi., 1915, p. 85.

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than firmo-viscosity. A certain amount of information may yet be obtained from the theory of firmo-viscosity alone.

The secular retardation of the earth's rotation was found by Darwin * to be 1900^s E sin 2ϵ "in a century," the semidiurnal tide alone being considered. But the moon's secular acceleration in a century is $413''$ E sin 2ϵ,† and is observed to be $4''\cdot5$. It follows that if v be the earth's angular velocity,

$$\frac{dv}{dt} = -621'' \div (1 \text{ century})^2.$$

Now the estimates of the age of rocks from the ratio of uranium to lead in them have led to the conclusion ‡ that since the oldest known rocks became solid not less than $1\cdot6 \times 10^9$ years have elapsed. The change in v in this time is found to be 75·9 degrees per day. This estimate is, however, only a minimum, for during part of the time elastico-viscosity would probably be important, and the rate of change would consequently be more rapid. Further, the moon's tide-raising power is supposed to have been constant, whereas it must have decreased on account of the increasing separation of the two bodies. For both reasons we conclude that at the commencement of the period considered the earth's angular velocity was greater than 360 + 75·9, or 435·9 degrees per day. Hence the period of rotation was less than 0·827 days. The ellipticity of the earth's surface corresponding to such a period would be about $\frac{1}{202}$. Yet at the present time the strata of equal density within the earth have, to a considerable degree of accuracy, the hydrostatic ellipticity throughout. In accordance with previous results,§ then, the earth must since have adjusted itself to the hydrostatic form in consequence of some weakness in its structure. This adjustment would cause compression at the equator and tension at the poles; if it took place suddenly, a series of mountain chains with a north and south trend would have been produced round the equator. It seems more probable that the adjustment occurred in a series of small stages, as no relics of the large mountains required by sudden adjustment can now be traced. A series of small adjustments at long intervals would probably be more easily hidden by denudation and crust-movement than a single very large one. Continuous adjustment is not impossible, as it would only require a very slow permanent set for small strains; t_1 might be many thousands of years without making the process too slow, and the hypothesis of the first section that it is infinite would remain sufficiently accurate. The whole change of ellipticity here considered would yet be insufficient to cause set or fracture in the materials of the earth under the conditions prevailing at the surface. Small adjustments, therefore, imply that the materials

* *Sci. Papers*, vol. ii. p. 68. † *Ibid.*, p. 73.
‡ Holmes, *Geol. Mag.*, decade vi., vol. ii., 1915, p. 108.
§ *Memoirs R.A.S.*, vol. lx., 1915, p. 197.

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of the interior of the earth are much weaker than those of the crust; Barrell's theory * of a layer of solid but weak matter below the crust agrees with this, and it would not be surprising if the weakness extended to the centre of the earth, especially as the centre of the earth is the point that has cooled least since solidification, and hence may be expected to be the place of easiest fusion.

Application to the Theory of Earthquake Waves.

In a transverse earthquake wave it is well known that the displacement at any point can be derived † from the complex function $\frac{1}{r}\exp\iota\kappa(Vt-r)$, where r is the distance from the focus, V the velocity of transmission, and $2\pi/\kappa$ the wave-length. We have then

$$V^2=\frac{c}{\rho}=\frac{n}{\rho}(1+\iota p\mathfrak{t})$$

in the present notation, ρ being the density of the rock.

Hence, if $p\mathfrak{t}$ is small,

$$V=V_0(1+\tfrac{1}{2}\iota p\mathfrak{t}),\text{ where }\left(\frac{n}{\rho}\right)^{\frac{1}{2}}=V_0.$$

Then the displacement is proportional to

$$\frac{1}{r}\exp\iota\kappa\left\{\left(1+\frac{\iota p\mathfrak{t}}{2}\right)V_0t-r\right\}$$

$$=\frac{1}{r}\exp\left(-V_0\frac{p\kappa\mathfrak{t}t}{2}\right)\exp\iota\kappa(V_0t-r).$$

A damping factor is thus introduced, and becomes important after a time $2/p\kappa\mathfrak{t}V_0$. But $p=\kappa V_0$, and therefore the time needed is $2/p^2\mathfrak{t}$. In this time the wave has travelled a distance $2V_0/p^2\mathfrak{t}$, and thus the slowest vibrations get furthest from their source. The period of earthquake waves is usually of the order of 20 seconds.‡ V_0 is about 4×10^5 cms. per second at the surface, and probably not much more than 6×10^5 cms. per second at the centre of the earth. If then $\mathfrak{t}$ were as small as one second, transverse earthquake waves could not penetrate to more than about 100 kilometres from their source. The actual value of $\mathfrak{t}$, already obtained as a statistical weighted average for the whole earth, is 371 seconds, so that the existence of these waves is conclusive evidence against the uniform distribution of so high a degree of viscosity. The viscosity may, however, be concentrated into a small bulk, possibly very near the centre of the earth, without preventing the transmission of earthquake waves.

* *Journal of Geology*, 1914–15.
† Love, *Theory of Elasticity*, 1906, p. 282.
‡ G. W. Walker, *Modern Seismology*, p. 51.

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Need for Further Inquiry.

The evidence here obtained from the fourteen-monthly variation of latitude and from earthquake waves appears to leave little room for imperfection of elasticity within the earth except in a small region, and bodily tidal friction can occur only inside this small region, if at all. At present the sole evidence of imperfection of elasticity is that afforded by the secular accelerations of the sun and moon, and the apparent trustworthiness of this is somewhat diminished by the fact that these are not in the theoretical ratio.* The unduly large secular acceleration of the sun then requires explanation if the datum given by the moon is accepted. It is possible that when the theory of earthquake waves has been considerably improved the behaviour of these may give further information about the viscosity near the centre of the earth. Several attempts have been made to measure the lag of the bodily tides directly by means of the horizontal pendulum;† but although this would give valuable information if it could be accomplished, success seems very unlikely. If the ocean were absent the problem might be soluble, but the tides in it lag to a tremendous extent quite independently of viscosity, the lag being in some places positive and in others negative, and due entirely to the irregular forms of the bed and the coast.‡ The attraction of these tides and their pressure on the ocean bottom deform the solid earth to an extent comparable with the bodily tide and inseparable from it,§ and must transmit to the solid earth their lags, thus causing retardations in some places and accelerations in others so much greater than the true lag of the bodily tide that to separate the latter from them will be practically impossible. As the secular acceleration cannot be produced except by a dissipative force, and these oceanic lags are not caused by viscosity, it follows that the sum of their effects (direct and indirect) on the moon must necessarily be zero. When only a finite number of observing stations is available, however, they are sufficient to mask the true effects of viscosity completely. Oceanic viscosity itself would produce in an ocean of uniform depth‖ a lag of $0^{s}\cdot001$, which would be quite imperceptible, and the order of magnitude of the dissipation of energy in the sea, and hence its contribution to secular accelerations, is not likely to be changed by the irregular form of the bed.

More information on the present secular accelerations is particularly desirable on account of its bearing on the past history of the earth. It has already been shown that the value of the

* J. K. Fotheringham, *Monthly Notices*, vol. lxxv., 1915, p. 396; *Monthly Notices*, vol. lxxvi., 1915, p. 85.

† Michelson, *Astrophysical Journal*, vol. xxxix., 1914, p. 122.

‡ H. Lamb and L. M. Swain, *Phil. Mag.*, June 1915; Goldsbrough, *Proc. Lond. Math. Soc.*, vol. xv., 1916, p. 64.

§ K. Terazawa, *Phil. Trans.*, vol. ccxvii.*a*, 1916, pp. 35–50.

‖ *Monthly Notices*, vol. lxxv., 1915, p. 658.

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secular acceleration of the moon generally accepted implies frequent adjustments of the earth to the hydrostatic form during geological time. This would not be necessary if tidal friction did not at present exist; the moon might have attained its present orbit before the earliest known sedimentary rocks were formed.

Summary.

The existence of the Eulerian nutation and the transmission of transverse earthquake waves together imply that nearly the whole of the earth may be regarded as perfectly elastic except for forces with periods of a century or more. If bodily tidal friction be regarded as the cause of the secular acceleration of the moon's mean motion, it must be confined to a comparatively small firmo-viscous bulk, probably near the centre of the earth. Elastico-viscosity can nowhere be important at present, although it must have been a very important factor in controlling the past history of the system. The time it takes to cause permanent set for small strains must be many years. On the basis of the observed secular acceleration of the moon, it has been shown to be most likely that the earth adjusted itself to the hydrostatic state during geological time by a series of small steps.

Occultation of B.D. + 21° 1714, *by Saturn's Ring,* 1917 *Feb.* 9.

By Naval-Instructor M. A. Ainslie, R.N., B.A.

1917 Feb. 9, at about $8^h\ 45^m$ G.M.T., I noticed a star of about the 7th magnitude north of, and preceding, the N. pole of Saturn. It was of a golden-yellow colour.

Circumstances prevented further observation until after 10 p.m., when the planet was again observed, but in poor seeing.

It was not until about 10.15 that the star was again seen, when it appeared as a very conspicuous cream-white spot, very small, and apparently projected on the extreme edge of ring B; it passed into the division and travelled along it, remaining very conspicuous and, as nearly as I could judge, as bright as when clear of the planet. It does not appear to have passed behind any portion of ring B, as its brightness was so little affected; the appearance of its being projected on the ring was probably an effect of irradiation, the B ring being at its outer edge very bright.

At about 10.30 it was seen on the outer edge of the division, and (owing to bad seeing) disappeared for a few minutes; at about $10^h\ 35^m$, however, it was very easily seen, though greatly reduced in brightness, through ring A.

It remained visible to the end of the observation, and traversed

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the ring with little or no variation in brightness, though at $11^{h}\ 8^{m}$ it increased very rapidly, not quite instantaneously, to perhaps double intensity, at which it remained for from 10 to 15 seconds, fading again as rapidly as it had brightened. A second brightening was observed at $11^{h}\ 8^{m}$, this time for not more than about 5 seconds.

The star finally appeared from behind ring A at $11^{h}\ 10^{m}$, and was seen separated from the ring by a distinct black interval at $11^{h}\ 15^{m}$.

During the passage of the ring over the star, and when seen through the Cassini division, the yellow colour of the star was much less pronounced than when clear of the planet before and after the observation; it was more in the nature of a creamy white. The image throughout was very "stellar," especially when seen through the Cassini division; there may have been a very slight enlargement of the image when seen through the ring. I should estimate that the star lost perhaps three-quarters of its light when seen through the ring, but any estimate of the brightness of the star was difficult on account of the glare of the planet. During the brief "brightenings" in ring A, the star did not attain to anything like its normal brilliancy.

The seeing was poor at first, but improved considerably; for a few minutes at about $10^{h}\ 20^{m}$, and again from about $10^{h}\ 35^{m}$ to $11^{h}\ 15^{m}$, the seeing was good, and the image of the star small and round.

A 9-inch silvered reflector was used in the observation, powers 180 and 270.

The alteration in brightness of the star on the two occasions described above, behind ring A, was very rapid, but not instantaneous; I should estimate that it occupied about half a second, and that the final brightening when clearing the ring took about the same time. It was certainly nothing like the disappearance and reappearance of a star in the case of an ordinary lunar occultation.

The times given above for the two brightenings in ring A, and for the final reappearance are, I believe, correct to the nearest minute.

The apparent position of the star for 1917 Feb. $9^{d}\ 10^{h}$ G.M.T. was:—

$$7^{h}\ 49^{m}\ 23^{s}.163,\ +21^{\circ}\ 19'\ 15''.43.$$

The phenomenon described above was also observed by Mr. J. Knight, of Rye, who has kindly furnished me with some details. He used a 5-inch refractor, power 180.

He states that he never lost sight of the star behind the ball, but that it was projected on the edge thereof; probably, like the projection witnessed by myself on the edge of ring A, an effect of irradiation. He describes the star as seeming to "eat its way" into the ring "in a varying manner," with occasional appearances and disappearances, and as having the exact appearance of a

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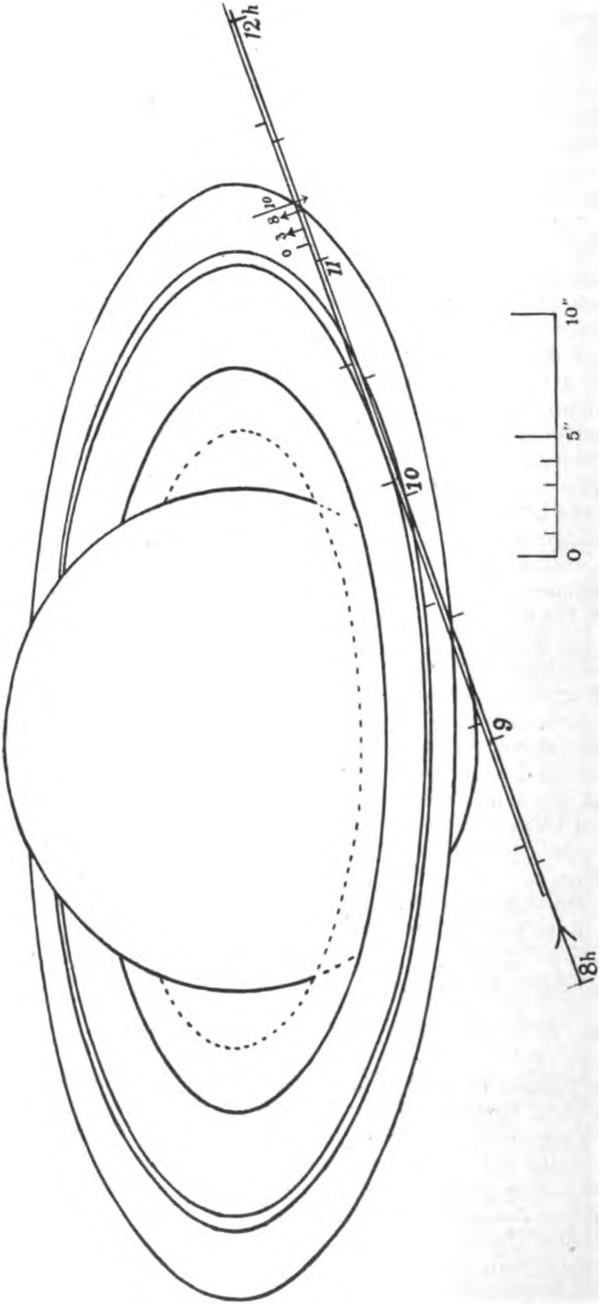

Occultation of B.D. +21° 1714, by Saturn's Ring, 1917 Feb. 9.

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satellite of Jupiter in transit; also as seeming to "float in the ring." He also mentions that when seen through the Cassini division its image appeared elongated. I cannot help thinking that these appearances must be, to a large extent, due to the bad seeing which he unfortunately experienced; certainly, to my eyes, the star never looked in the least like a satellite in transit.

He further states that he saw the star pass along the Cassini division "in an exceptionally distinct manner"; but adds, "when it left that division I could not be sure that I saw it at all until it emerged."

He further concurs with me as to the pale appearance of the star when in the Cassini division, and states that, although very distinct in the division, it was by no means as bright as when clear of the planet.

The attached diagram has been drawn to scale from the data given in the *N.A.* for the rings and ball; the lower of the two parallel straight lines represents the path of the star relative to the planet, from positions of the planet and star kindly furnished to me by Dr. Crommelin, the computed relative positions at 8^h, 10^h, and 12^h being as there shown.

The upper of the two straight lines shows the relative path which best represents my observations, although it would seem that this relative path would involve, if correct, the disappearance of the star behind the ball, which does not seem to have taken place. The polar semidiameter of the ball has been corrected for the tilt of the axis, and the position of the planet for parallax.

It seems fairly clear, from comparison of Mr. Knight's observations with my own, that—

(1) the star was visible through ring A;

(2) that while behind ring A it was seen, though not with anything like its full brilliancy, through Encke's division, and through another exterior to the same;

(3) that it did not actually pass behind the ball, although it must have made a very close approach thereto; and

(4) that, as far as can be judged from the present observation, the measures of the rings, on which the data given in the *N.A.* are based, would appear to be somewhat too small.

I have, up to date, heard of no other observations of this phenomenon.

Blackheath:
1917 *March* 6.

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The Surface Currents of Jupiter during the Apparition of 1916–17. By Scriven Bolton. (Plate 3.)

The main factors concerning the observations of 1916–17 may be expressed here—

Jupiter in opposition	1916 October 23.
Period covered by the observations .	1916 September 6 to 1917 February 12 inclusive.
Instrument used	26-inch reflector.
Powers employed on same . . .	300, 380, 500.

The scope of the work executed during the above-named period is as under:—

Total number of C.M. transits of markings recorded .	4115
Total number of C.M. transits utilised in the final reduction	3896
Number of hours observed	209
Average number of C.M. transit estimates obtained per hour	19·7
Total number of sketches of the surface accumulated .	338

With the aid of the aperture above referred to, diligent attention was paid to the high temperate and polar regions. Since the faintest spots were readily visible and could be studied without much effort, and because of the larger image usually obtainable without detriment to definition, the transit estimates of the fainter spots proved in my own case more trustworthy than usual.

The following table has been prepared from a reduction of all the observations selected:—

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Table of Mean Rotation Periods of Markings in Different Latitudes.

Chart Reference.	Markings recorded.	Approximate Latitude.	Average Drift in 30 Days.	Number of Selected Objects.	Average Number of Observations.	C.M. Transits utilised.	Date of First Observation.	Date of Last Observation.	Mean Rotation Period.
		°	°				1916.	1917.	h m s
A	Dark spots, N. edge S. Polar Regions.	−50	+ 5·34	5	5·00	25	Oct. 12	Feb. 6	9 55 33·3
B	White spots, S.S. Temperate Zone .	−43	+24·99	19	8·53	162	,, 6	,, 9	9 55 6·4
C	Dark spots, S.S. Temperate Band .	−40	+23·61	22	9·59	211	Sept. 6	,, 6	9 55 8·3
D	White spots, S. Temperate Zone. .	−32	+15·70	23	11·74	270	,, 10	,, 7	9 55 19·1
E	Dark spots and streaks, S Temperate Band	−28	+15·64	19	13·26	252	,, 6	,, 6	9 55 19·2
F	White spots, S. Tropical Zone . .	−21	+10·25	12	10·17	122	,, 10	,, 6	9 55 26·6
	p end, Great S. Tropical Disturbance	−18	+ 9·50	1	18·00	18	,, 13	,, 2	9 55 27·6
	f end, Great S. Tropical Disturbance .	−18	+ 9·75	1	17·00	17	Oct. 3	,, 5	9 55 27·3
F, G	White and dark spots within Great S. Tropical Disturbance	−18 to −21	+ 9·12	14	8·14	114	Sept. 23	,, 4	9 55 28·1
G	Dark crests, S. edge S. Equatorial Belt	−18	+10·50	17	9·47	161	,, 10	,, 6	9 55 26·3
P, S, F, S,	Red spot Bay, *p* and *f* shoulders and middle	−18	+ 3·00	3	17·00	51	,, 15	,, 6	9 55 36·5
H	White spots, central rift, S. Equatorial Belt	−13	+ 4·60	5	5·80	29	,, 28	,, 5	9 55 34·5
I	Dark spots, N. edge S. Equatorial Belt	− 9	+ 6·76	24	11·17	268	,, 6	,, 6	9 50 20·9
J	Dark streaks in Equatorial Band .	± 0	+ 6·77	6	8·00	48	Oct. 9	,, 5	9 50 20·9
K	White spots, S. edge N. Equatorial Belt	+ 8	+ 9·50	15	13·40	201	Sept. 6	,, 9	9 50 17·2
L	Dark crests, S. edge N. Equatorial Belt	+ 9	+ 9·88	21	12·90	271	,, 6	,, 12	9 50 17·7
L	Dark streaks, S. edge N. Equatorial Belt	+ 7	+10·10	4	12·25	49	Oct. 12	,, 12	9 50 16·4
M, N	White and dark spots, N. edge N. Equatorial Belt	+15 to	+ 6·54	22	14·55	320	Sept. 6	,, 6	9 55 31·7
		+18	+19·90	8	12·38	99	,, 23	,, 12	9 55 13·4
O	Dark spots, N. Temperate Band . .	+23	+ 6·78	28	10·50	294	,, 6	,, 9	9 55 31·4
P	White spots, N. Temperate Zone .	+29	+16·85	27	9·11	246	,, 10	,, 6	9 55 17·6
Q	Dark spots and streaks in N.N. Temperate Band	+36	− 7·03	22	8·14	179	,, 20	,, 6	9 55 50·3
R	White spots, N N. Temperate Zone .	+45	+15·91	28	8·50	238	,, 28	,, 9	9 55 18·9
S	Dark spots and streaks, edge N. Polar Region	+49	+ 7·90	25	10·04	251	,, 6	,, 6	9 55 29·8
	Totals	..	..	371	..	3896	..	..	..
	Average	..	..	15·5	11·0	162·3	Sept. 26	Feb. 7	..
I, J, K, L	All light and dark spots in Equatorial Current	− 9 to +9	+ 8·60	70	11·54	837	Sept. 6	Feb. 12	9 50 18·6
M, N	All light and dark spots along N. edge N. Equatorial Belt	+15 to +18	+13·22	30	13·47	419	,, 6	,, 12	9 55 22·6
	P and *f* ends S. Tropical Disturbance .	−18	+ 9·63	2	17·50	35	,, 13	,, 5	9 55 27·5

Notes on certain deduced Rotation Periods.—In the foregoing table the rotation periods representing the N. Temperate Zone, the S. Polar Band, and the N.N. Temperate Zone differ somewhat from

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those usually assigned to the same latitudes. In reducing these observations the results of other observers in the past have not been overlooked.*

The quickened rate of velocity of the Equatorial current proved altogether abnormal, and appeared (to some extent) to be commensurate to the disturbed condition of this part of the globe. The period worked out at 7·7 seconds shorter than that for the last apparition (1915 September–1916 February).†

The white and dark spots and crests along the S. edge of the S. Equatorial Belt, including those within the S. Tropical Disturbance, moved in reasonable agreement with the motion of this disturbance, whereas during the previous apparition they appeared to drift in unison with the Red Spot.

As a result of the progressive diminution in velocity of the S. Tropical Disturbance, and the accelerated motion of the Red Spot, the rotation periods of these two objects differed by only 9 seconds.

The white and dark spots fringing the N. edge of the N. Equatorial Belt (many of which possessed abnormal size and intensity) exhibited two separate rates of motion, the line of demarcation occurring at about $\lambda 204^\circ \pm$. Preceding this longitude the surface rotated in about $9^h\ 55^m\ 31{\cdot}7^s$ (22 spots), while following it to about $\lambda 295^\circ \pm$ an accelerated velocity was evidenced by a period of $9^h\ 55^m\ 13{\cdot}4^s$ (8 spots). Farther to the right the spots exhibited a gradual diminution in speed, until at and beyond $\lambda 312^\circ$ the velocity corresponded with the average value of the zone (excluding the exceptional region). A not dissimilar phenomenon was witnessed in this latitude by Molesworth in 1899.

Aspect of the Planet.—The surface features on the whole exhibited somewhat greater activity than for many years past. White, almost phosphorescent, spots abutted on both edges of the intensified N. Equatorial Belt (the darkest stripe across the disc), and formed a series of large bays therein. The N. Temperate Belt was much intensified, and appeared semi-duplex. The S. Equatorial Belt presented unusual activity, as did also the S.S. Temperate Belt, which was widely duplicate. The Red Spot was faintly visible until 1916 December, its disappearance (which was anticipated) being occasioned by the approach of the S. Tropical Disturbance, the preceding end of which caught up to the spot probably about 1917 February 6. During the last fourteen years the spot has practically disappeared during each conjunction with the disturbance named, and the hollow in which it lies has every time grown apparently whiter, a probable contrast effect.

Respecting the colour of the planet, I can fully confirm the late Dr. Percival Lowell and Mon. E. M. Antoniadi with regard to the cherry-red hue of the belts and poles.

* Seeming discrepancies have not infrequently arisen through an alternate manifestation of spots along both edges of a zone or belt participating in two currents.

† *Monthly Notices*, vol. lxxvi., No. 7.

Bramley, Yorkshire:
1917 *Feb.* 22.

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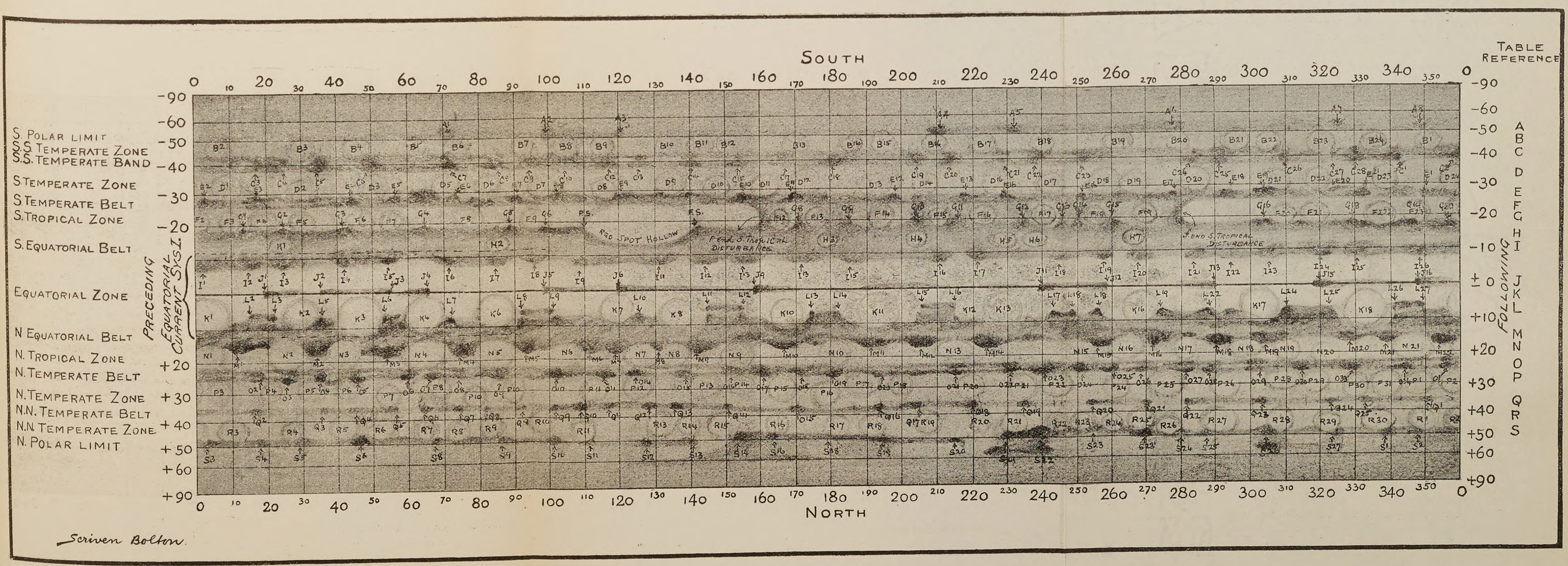

Jupiter during the apparition of 1916–17. Key to the markings. Longitudes reduced to date 1916 December 10.

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Occultations observed at Eltham in 1915.
By Maurice E. J. Gheury.

The following occultations were observed at Eltham in 1915. Calls made by the war and a protracted illness prevented their reduction from being carried out, and as there seems to be no likelihood of this being done in the near future, the observations alone are given here. The notation and method of the two previous papers (see *Monthly Notices*, vol. lxxiv. p. 540, and vol. lxxv. p. 710) have been followed throughout. The times have been corrected for the latest clock errors, kindly supplied by Mr. T. Lewis.

No.	Date.		G.M.T.			Estimated Probable Error.	Star.	Mag.	Dis. or Reap.	Posi-tion.†
	1915.		h	m	s	s				
54	Feb.	2	11	24	2·6	±0·6	75 Leonis	5·4	R	D
55		2	12	32	19·3	±0·7	76 Leonis	6·0	R	D
56*		21	6	35	27·1	±0·6	B.D. + 24° 543	8·7	D	D
57		21	7	4	45·9	±0·7	18 Tauri	5·6	D	D
58*		21	7	14	27·7	±0·8	B.D. + 24° 548	8·8	D	D
59*		21	8	20	11·6	±0·7	B.D. + 24° 563	8·4	D	D
60*		21	8	54	50·8	±0·7	B.D. + 24° 571	7·1	D	D
61*		21	10	46	5·9	±0·7	B.D. + 24° 583	8·5	D	D
62	Mar.	25	14	5	2·0	±0·8	μ_2 Cancri	5·4	D	B
63*	Apr.	18	9	54	38·6	±0·8	B.D. + 28° 732	8·0	D	B
64*	June	19	10	20	36·1	±0·8	B.D. + 0° 2782	6·4	D	B
65	July	3	13	48	14·8	±0·9	51 Piscium	5·7	R	D
66	Sept.	19	11	5	33·2	±0·8	Rumker 8872	6·6	D	A
67		21	6	59	57·1	±0·8	67 Aquarii	6·3	D	D
68	Oct.	26	11	34	18·3	±0·7	W.Z.C. 341	6·6	R	D
69		29	11	24	10·3	±0·9	B.A.C. 2544	6·3	R	D
70*	Dec.	11	5	41	25·0	±1·0	B.D. − 10° 5804	8·5	D	B
71*		11	6	49	27·2	±1·1	B.D. − 9° 5896	8·6	D	B

Notes.—Nos. 54, 57, 60, 62, 64, 66, 70: instantaneous.
Nos. 58, 63: difficult.
Nos. 59, 61: haze.
No. 67: haze and bright moon; difficult.
No. 71: just at limit of visibility; the observation may be bad.

* Particulars of these occultations are not given in the *Nautical Almanac.*

† See *Monthly Notices*, vol. lxxiv. p. 541. The places of observation are close to one another: mean astronomical latitude, 51° 27′ 13″·7 N.; mean geocentric latitude, 51° 15′ 55″·1 N.; mean longitude, $0^h\ 0^m\ 15^s{\cdot}16$ E.; mean altitude, 250 feet above sea-level.

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The positions of the stars not given in the *Nautical Almanac* are given below for 1915·0:—

No.	Mean R.A. h m s	Mean N.P.D. ° ′ ″	Catalogue.
56	3 39 15·92	65 23 36·0	*Astr. Gesel.*, 1875.
58	3 40 16·10	65 20 35·6	,,
59	3 41 58·81	65 26 36·1	,,
60	3 43 1·75	65 16 21·7	,,
61	3 46 19·56	65 5 34·5	,,
63	4 56 37·74	61 49 58·5	,,
64	11 18 56·86	89 24 4·3	,,
70	21 54 33·98	100 10 8·9	*Astr. Gesel.*, 1900.
71	21 57 36·71	99 41 30·2	Mean of Cambridge (*u.s.*) 1912, and Vienna, Ottakring, 1904.

Richborough:
1917 *January*.

Differential Star Corrections. By W. E. Cooke. M.A.

This is still one more attempt to shorten the work of the meridian observer, and has special application to zone observations.

Consider a supposititious case, reduced to its simplest elements.

Let P be a known star, and Q an unknown, and let P and Q be observed with the transit circle and all instrumental errors applied. In order to obtain the R.A. (or N.P.D.) of Q on January 1, we may apply the "star-corrections" to the observed values, subtract the P value thus obtained from the known position of P on January 1, and add (or subtract) this difference to the Q value and thus obtain the Q position on January 1.

Suppose the star corrections are $+3^s{\cdot}50$ and $+3^s{\cdot}00$. It will give exactly the same result if we apply these corrections, or if we apply $+0^s{\cdot}50$ and $0^s{\cdot}00$. It is the *difference* between the corrections that matters.

I now propose to obtain values for such differences, rather than to determine the complete "star-corrections."

In order to focus our ideas, let us consider the R.A. side only. Using the old notation (prior to 1916) of the *Nautical Almanac*, the correction is

$$Aa + Bb + C_1c + D_1d + C_2c + D_2d,$$

where $C_1 = t - {\cdot}025 \sin 2L,$

$C_2 = -{\cdot}342 \sin \Omega + {\cdot}004 \sin 2\Omega,$

$D_1 = -0''{\cdot}551 \cos 2L,$

$D_2 = -9''{\cdot}21 \cos \Omega + 0''{\cdot}09 \cos 2\Omega.$

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Still further focussing our ideas, let us consider one term only, *e.g.* Aa,

$$Aa = A \cdot \tfrac{1}{15} \cos \alpha \sec \delta,$$

and its differential effect upon the two stars P and Q. First let us imagine the two stars P and Q to have the same declination, and to differ by 1^{h} in R.A. The differential correction due to Aa would be

$$K \text{ (say)} = A \cdot \tfrac{1}{15} \sec \delta d(\cos \alpha),$$

where $d(\cos \alpha)$ means the change in $\cos \alpha$ for the 1^{h} difference in α.

Next let us imagine a third star R of the same R.A. as Q, but differing 1° in N.P.D. The differential correction due to Aa between Q and R would be

$$L \text{ (say)} = A \cdot \tfrac{1}{15} \cos \alpha d(\sec \delta),$$

where $d(\sec \delta)$ means the difference in $\sec \delta$ for the 1° difference in N.P.D.

The differential correction between P and R would be

$$K + L = A \cdot \tfrac{1}{15} \sec \delta d(\cos \alpha) + A \cdot \tfrac{1}{15} \cos \alpha d(\sec \delta).$$

This is a general expression, as far as the term Aa is concerned, for the differential correction between any two stars differing 1^{h} in R.A. and 1° in δ.

Similarly, for any two stars differing by m^{h} and n°, if $d(\cos \alpha)$ and $d(\sec \delta)$ represent suitable average values for 1^{h} and 1°, the differential correction becomes

$$K + L = m \cdot A \cdot \tfrac{1}{15} \sec \delta d(\cos \alpha) + n \cdot A \cdot \tfrac{1}{15} \cos \alpha d(\sec \delta).$$

So far we seem to be only complicating matters, and the final formulæ, if written in full, look rather formidable: but they eventually straighten out and lend themselves to a very simple tabulation.

Applying the above process throughout, we obtain the following scheme:—

$a_1 = \frac{1}{15} \sec \delta d(\cos \alpha)$.	$a_1' = + \sin \delta d(\sin \alpha)$.
$b_1 = \frac{1}{15} \sec \delta d(\sin \alpha)$.	$b_1' = - \sin \delta d(\cos \alpha)$.
$c_1 = 1 \cdot 336 \tan \delta d(\sin \alpha)$.	$c_1' = - 20 \cdot 045 d(\cos \alpha)$.
$d_1 = \frac{1}{15} \tan \delta d(\cos \alpha)$.	$d_1' = + d(\sin \alpha)$.
$a_2 = \frac{1}{15} \cos \alpha d(\sec \delta)$.	$a_2' = - \tan \omega d(\cos \delta) + \sin \alpha d(\sin \delta)$.
$b_2 = \frac{1}{15} \sin \alpha d(\sec \delta)$.	
$c_2 = 1 \cdot 336 \sin \alpha d(\tan \delta)$.	$b' = - \cos \alpha d(\sin \delta)$.
$d_2 = \frac{1}{15} \cos \alpha d(\tan \delta)$.	

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$K_\alpha = Aa_1 + Bb_1 + C_1c_1 + D_1d_1$. $K_\delta = Aa_1' + Bb_1' + C_1c_1' + D_1d_1'$.

$L_\alpha = Aa_2 + Bb_2 + C_1c_2 + D_1d_2$. $L_\delta = Aa_2' + Bb_2'$.

$M_\alpha = C_2c_1 + D_2d_1$. $M_\delta = C_2c_1' + D_2d_1'$.

$N_\alpha = C_2c_2 + D_2d_2$. $N_\delta = 0$.

$d(\sin \alpha)$ = change in $\sin \alpha$ for 1^h in α.

$d(\cos \alpha)$ = ,, $\cos \alpha$,,

$d(\tan \delta)$ = ,, $\tan \delta$ for $1°$ in N.P.D.

$d(\sec \delta)$ = ,, $\sec \delta$,, ,,

Further, let $\alpha_0\delta_0$ be some arbitrarily selected position near the middle of the group to be reduced for any given evening, $\alpha\delta$ the R.A. and Dec. for any star.

Then

$$d\alpha = (\alpha_0 - \alpha)(K_\alpha + M_\alpha) + (\delta_0 - \delta)(L_\alpha + N_\alpha).$$

$$d\delta = (\alpha_0 - \alpha)(K_\delta + M_\delta) + (\delta_0 - \delta)L_\delta.$$

The quantities A, B, C_1, D_1, have already been published in the *Monthly Notices of the R.A.S.*, vol. lxvi. p. 544. They have been called A°, B°, C°, D°, and are given for every tenth day, with t as argument.

C_2D_2 are given on p. 545, where they are called x and y: and are tabulated for every 5° of ☊.

The object of the present communication is to recommend the tabulation of

$$K_\alpha L_\alpha M_\alpha N_\alpha \qquad K_\delta L_\delta M_\delta$$

by any astronomer who is making any form of zone observations.

By suitably arranging the work, the tabulation for the zone of, say, 15° is not a very lengthy undertaking, and will be found to lead to a considerable simplification of the daily routine.

The practical arrangement of the tables and method of use can best be understood by an extract from those computed for the Sydney Zones, 50°–65° south declination.

Let us suppose that on 1906 July 22 we were observing stars between 2^h and 4^h in R.A. and 50°–60° south declination.

Denoting by t the interval in days since the beginning of the year, and ☊ being the mean longitude of the Moon's ascending node, we have

$$t = 202, \quad ☊ = 132°.$$

We shall select $3^h : 55°$: as our standard position, and from the tables extract the necessary quantities for 2^h, 3^h, and 4^h. The following is a portion of the table for 2^h:—

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t.	K_a (per hour). 50.	55.	60.	65.	L_a (per degree). 50–55.	55–60.	60–65.
180	− ·64	− ·72	− ·84	− 1·01	− ·037	− ·050	− ·069
	− 5	− 6	− 6	− 8	+ 7	+ 9	+ 13
	− 3	− 4	− 5	− 5	+ 7	+ 10	+ 14
200	− ·72	− ·82	− ·95	− 1·14	− ·023	− ·031	− ·042
	− 2	− 2	− 3	− 3	+ 7	+ 10	+ 15
	0	0	0	− 1	+ 8	+ 11	+ 15
220	− ·74	− ·84	− ·98	− 1·18	− ·008	− ·010	− ·012
	+ 2	+ 2	+ 2	+ 2	+ 7	+ 10	+ 14
	+ 3	+ 3	+ 3	+ 4	+ 7	+ 10	+ 13
240	− ·69	− ·79	− ·93	− 1·12	+ ·006	+ ·010	+ ·015

☊.	M_a.				N_a.		
120°	+ 16	+ 19	+ 23	+ 28	− ·003	− ·004	− ·005
	0	0	+ 1	+ 1	− 5	− 6	− 8
			0	0	− 4	− 6	− 8
140	+ 16	+ 19	+ 24	+ 29	− ·012	− ·016	− ·021
	0	0	− 1	− 1	− 4	− 5	− 7
	− 1	− 1	− 1	− 1	− 4	− 5	− 7
160	+ 15	+ 18	+ 22	+ 27	− ·020	− ·026	− ·035

From this and corresponding tables for 3^h and 4^h we obtain the following:—

		55°.	50–55.	55–60.
2^h	K_a L_a	− ·82	− ·022	− ·029
	M_a N_a	+ ·19	− ·009	− ·011
	Sum	− ·63	− ·031	− ·040
3^h		− ·72	− ·043	− ·055
		+ ·20	− ·003	− ·002
	Sum	− ·52	− ·046	− ·057
4^h		not	− ·060	− ·079
		required	+ ·004	+ ·005
	Sum		− ·056	− ·074

The sums form the skeleton of the following extended table. It should be noticed that, under the heading 55°, the first sum − ·63 represents the relative correction between 2^h and 3^h, and so is the correction to be applied to 2^h to reduce it to our adopted standard 3^h. The correction to 3^h is of course zero. The quantity − ·52 is the relative correction between 3^h and 4^h, and so + ·52 is to be applied to 4^h to reduce it to 3^h.

Also note that in order to avoid troubling about signs with the argument δ, the signs under the heading 55–60 have been reversed, so that − ·040 (to reduce 55° to 60°) becomes + ·040 (to reduce 60° to 55°, our adopted standard).

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	Arg. R.A. only.	Arg. R.A. and Dec. 50–55.	55–60.
h m	s	s	s
2 00	−0·63	−0·031	+0·040
10	− ·52	− ·034	+ ·043
20	− ·41	− ·037	+ ·046
30	− ·30	− ·040	+ ·049
40	− ·20	− ·042	+ ·052
50	− ·10	− ·044	+ ·055
3 00	0·00	− ·046	+ ·057
10	+ ·09	− ·048	+ ·060
20	+ ·18	− ·050	+ ·063
30	+ ·27	− ·052	+ ·066
40	+ ·36	− ·054	+ ·069
50	+ ·44	− ·055	+ ·072
4 00	+ ·52	− ·056	+ ·074

This table is now ready for use. To each star we apply two quantities, one from the second column, with argument R.A., and the other from the third or fourth column, with arguments R.A. and Dec. In the latter case the tabular quantity is for 1° and must be multiplied by the difference between the star's Dec. and 55°. The following may be taken as an example—

$$\text{R.A. } 2^h\ 35^m{\cdot}2,\ \delta\ 52^\circ{\cdot}3.$$

The first quantity is $-0^s{\cdot}25$, direct from table.
The second quantity is $-{\cdot}041 \times 2{\cdot}7 = -0^s\ 11$
$(2{\cdot}7$ being $55^\circ \backsim 52^\circ{\cdot}3)$.

The total correction therefore is $-0^s{\cdot}36$.

It is advisable to apply these corrections to all the standard stars, then subtract the observed positions thus corrected from mean positions on January 1. Each star should theoretically give the same difference, and the variations are the accidental errors of observation. The mean of all the differences then becomes a constant quantity to be added to all stars for that evening, and the successive steps may be summarised thus:—

Mean of wires.
Instrumental errors.
Tabular correction, arg. R.A.
" " " R.A. and δ.
Constant quantity.
Sum = mean R.A. Jan. 1.

This takes a long time to explain, but only a short time to compute. Even the original tables, though the formulæ look so ponderous,

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are very soon formed. It must of course be understood that the standard stars are within the zone covered by the tables.

The declination correction is similar but more simple, because M_δ contains no declination argument, and $N_\delta = 0$.

In conclusion, it may perhaps be pointed out that other corrections can easily be embodied in the above daily table. For example, the instrumental corrections can be combined with L + N as a function of the declination, and the clock rate with K + M.

It is also worth while, if there are many stars in the one evening, to prepare a rough preliminary table for reducing the standard stars, so as to obtain the "constant quantity" for the evening, and then embody this in the more complete table. Thus only two quantities, taken from the table, will reduce each star from "Mean of wires" to "Mean R.A. Jan. 1."

Sydney Observatory:
1916 *December* 6.

Wireless Time Signals: Some Suggested Improvements.
By W. E. and F. B. Cooke.

Anticipating a redetermination of Australian longitudes by means of trans-Pacific radio-signals, some experimental work has been recently undertaken at the Sydney Observatory, with a view of eliminating certain sources of error.

Prior to the war great and rapid progress was made with the new methods, culminating in the determination of the difference of longitude between Paris and Washington by means of radio-signals across the Atlantic. In the course of this work every possible precaution to ensure great accuracy was taken. The results were undoubtedly good, but one gathers from the remarks of M. Baillaud (Director of the Paris Observatory) and others that there are still some outstanding difficulties which will have to be met and overcome.

With those of a purely astronomical character we are not here concerned. The object of this present paper is to indicate a method whereby two clocks or chronometers, separated by some 5000 miles or so, may be compared with an error not exceeding one-hundredth of a second, and with entire elimination of personal equation.

In order that our remarks may be followed, it will be advisable to indicate briefly the general features of the whole operation, and the special difficulties to be attacked.

Three, or perhaps four, stations are concerned. Let us call the two astronomical stations (or those which contain the clocks to be compared) A and B. A third station C (the transmitting or

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signalling station) contains some piece of apparatus which radiates a series of "dots" regularly spaced at intervals of *nearly*, but not quite, one second apart. This station should preferably be about midway between A and B, but if this is impracticable, we shall probably have two stations C1 and C2 close to A and B respectively, and each acting alternately as a transmitting station. For definiteness we shall suppose a single station C. For example, if A and B were the Sydney and Lick Observatories, C might be at Honolulu; or with A and B at Sydney and Honolulu, C might be at Nauru (near the equator).

All we expect from C is a series of regularly spaced "dots" at nearly a second interval. The exact times of the "dots" are quite immaterial. It is the object of the observers at A and B to receive this series of arbitrary signals and compare it with the beats of their respective clocks, and this is accomplished by the well-known method of coincidences. Our problem now resolves itself into

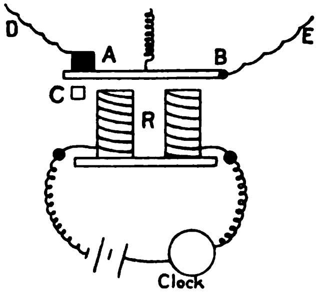

FIG. 1.

this: to determine the exact seconds at which the C signals coincide with the beats of our A (or B) clock. The principal difficulty seems to be this—that a very short sharp "dot" cannot be radiated and received over long distances, and that if the "dot" is of an appreciable length different observers take different parts of it to form their coincidences with their clock-beats. The personal element also arises from the fact that the sound of the radio-signal generally differs greatly from the sound of the clock-beat, however it is introduced. We propose to avoid both these difficulties and eliminate the personal element by means of the following simple mechanical device.

The clock circuit, as usual, passes from the clock contacts through a battery to the relay R, and for all practical purposes R is the clock. The armature of the relay is pivoted at B, and normally rests against the stop at A. When the seconds-impulse comes from the clock the contact A is broken, and the armature rests against the stop C.

Let us now interpose the contact A in the path of the incoming signals. That is, let D A B E be in series with the aerial. *That is all.*

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Let us follow the sequence, supposing that the period of the radio-dots is $0^{\mathrm{s}}{\cdot}99$, thus giving a coincidence, with a gain of one second upon the local clock, in every hundred seconds.

With relay open, as shown in the above diagram, the radio-dots come through uninterruptedly, but with relay closed the radio-circuit is broken and we hear nothing. For brevity let us call the clock circuit *c* and the radio-circuit *r*, and note that it is essential that the duration of the periodic closing of *c* shall be greater than the duration of the *r* signal. *E.g.*, if the signal occupies $0^{\mathrm{s}}{\cdot}1$, then the *c* must close every second for $0^{\mathrm{s}}{\cdot}15$ or any period longer than this. There need be practically no limitation to the length of the *r* signal, provided that the *c* impulse is a trifle longer.

There will thus occur occasionally a condition when *r* tries to come through but finds *c* closed, and consequently no path available, and we shall then cease to hear the *r* signals for a time. Taking the above figures, this overlap will be $0^{\mathrm{s}}{\cdot}05$, and with *r* gaining about $0^{\mathrm{s}}{\cdot}01$ per second, the *r* signals will find their path blocked for five consecutive seconds, and we shall have silence for that period. In addition, the few *r* signals immediately preceding this silence will be cut short. They will commence to pass, but

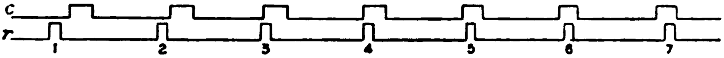

Fig. 2.

almost immediately their path will be blocked by the closing of *c*, and we shall notice a gradual diminution until they finally cease. The coincidence may be taken either as the last audible signal or a second later. Theoretically, it never occurs at any exact second, and we must adopt either the one immediately preceding or following. Practically it makes no difference which we adopt, provided we act systematically; and then even the third place of decimals will, in the long run, be just about correct. To those accustomed to chronographs, the following diagram may help to make the matter clear. The whole sequence is purposely exaggerated.

These two irregular lines show the records as they would appear if one could transfer them to a chronograph.

At 1 and 2 the signals come through without interruption. At 3 the signal starts, but is broken when *c* closes. At 4 the same thing happens, but the making and breaking of *r* is almost simultaneous, and we hear a very short sharp dot. At 5, 6, and 7 *r* cannot get through, and we hear nothing. Real coincidence occurs between 4 and 5, and we may make a rule to take either, provided we adopt the same rule at both A and B. It may perhaps be as well to point out that the shortening of the *r* signals in this way is quite different from the (unsuccessful) attempt to *send* short sharp dots by shortening the duration of key-closing at the transmitter. In our case the signal may normally be of any

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duration, and we do not interfere with it until it actually begins to sing. *Then* our clock automatically cuts it out. In practice we have not yet had an opportunity of testing the method over any great distance. We have, however, taken advantage of some signals sent by the Riefler clock at Adelaide (about 700 miles). The determination of coincidence was found to be a very simple matter and quite free from strain. It is far more easy to concentrate the attention upon one series of signal dots and ascertain when they cease than to determine the moment of coincidence of two series, and requires no special training. We occasionally obtained the assistance of friends who had no experience of time observations, and their determinations of the exact moments of coincidence always agreed with our own.

Incidentally a few side issues may be worth glancing at.

(1) It is obvious that *one* transmitting station, where possible midway between A and B, is preferable to two. We may then feel pretty certain that the arbitrary signals reach A and B practically simultaneously.

(2) In Australia the spark and crystal system still holds sway, and we understand that it is also in use in England and France. Through the courtesy of Mr. Ormiston, an American operator, we have been introduced to the arc system, and have been so fortunate as to obtain one of the latest forms of oscillating Audion receivers. In our opinion this combination is greatly superior, for several reasons. In the spark system no current passes until the key is closed, and then some little time must elapse before radiations actually commence. In the arc system the current is flowing through the aerial continuously, and the action of pressing the key simply shorts a few turns of wire in the inductance and thus alters the wave-length. It is quite probable, though we have not yet had the opportunity of testing the question, that very short sharp dots can be sent by this method if required.

Then again the tuning of the arc can be made far sharper than the spark, and the Audion is very much more sensitive than the crystal; *e.g.* with our little aerial at the Observatory we have no difficulty in hearing Tuckerton (New Jersey) at a distance of about 10,000 miles. It would be a simple matter to choose some wave-length sufficiently distinct to be free from interference, and to tune sharply, so that we should hear nothing except the time signals and the atmospherics.

Finally, the receiving observer makes his own "tune." That is, he can control the oscillation of his Audion so as to bring the signals to any musical pitch he desires, and by raising this sufficiently he can cause such a difference between the sound of the dots and the lower growl of the atmospherics as to reduce very considerably the annoyance from the latter.

(3) The main principle can easily be used for the comparison of local clocks or chronometers, in particular for comparing mean and sidereal times. In this case the mean time clock takes the

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place of *c*, and the sidereal of *r*; and the latter, instead of being in series with the aerial, is placed in series with a battery and high-pitch buzzer. With this arrangement not only can coincidences be obtained with the greatest ease and certainty, but the slightest want of beat (irregularity of seconds) can be immediately and unmistakably detected.

(4) Alternatively, the radio-signals can be made to *lose*, instead of gain, about one second in 100, and this appears to us to be rather better. In this case coincidence occurs at the *first* perceptible tick after the silence, instead of the last heard. It seemed to us to be easier to describe the gaining arrangement, but our own personal feeling is in favour of the losing ticks.

(5) It is an essential point that the duration of the closing of *c* should be slightly longer than the length of the *r* ticks: and

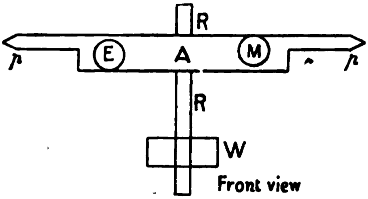

FIG. 3.

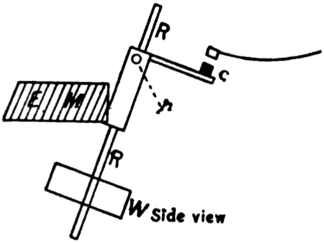

FIG. 4.

to ensure the practicability of always obtaining this adjustment, it is advisable to have some form of slow-acting relay. There are several on the market, but amongst those which we have seen nothing appeared to be quite suitable, so the following form was adopted.

In the above—

> EM are the electromagnets.
> A the armature, pivoted at *pp*.
> R a light rod passing through the armature and carrying a weight W.
> *c* the contact between the armature and a light spring.

Normally the clock circuit is closed, and broken momentarily every second. Therefore, as a rule, the armature rests against the magnets; but when the current is broken it acts as a pendulum, swinging away and forming contact at *c* for a period that is determined by the position of the weight W. This arrangement has been in use for another purpose at the Sydney Observatory for several months, and works very satisfactorily.

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Comet Observations made at the Melbourne Observatory, 1914 and 1915.

Micrometer Observations.

Instrument: 8-inch equatorial, square bar micrometer. **Observer:** J. M. Baldwin.

Comet 1914 *e* (*Campbell-Westland*).

	Melbourne Mean Time.	α − α*.	δ − δ*.	Comp.	α app.	log pΔ.	δ app.	log p′Δ.	Star Reduction to Apparent Place. dα.	dδ.	Star.
1914	h m s	m s	′ ″		h m s		° ′ ″		s	″	
Sept. 21	8 32 49	−4 37·22	− 6 1·7	8	2 35 27·01	9·9314n	−57 5 46·5	0·4825n	+3·90	+34·0	1
,, 22	9 6 36	+0 17·43	+ 2 55·9	10	1 58 10·79	9·8937n	− 55 35 56·7	0·1133n	+4·48	+32·0	2
,, 23	9 8 6	−8 7·78	+11 7·9	7	1 24 16·18	9·8398n	−53 26 47·4	9·7939n	+4·72	+30·3	3
,, 25	8 37 4	+1 47·01	−12 12·5	6	0 28 13·05	9·7485n	−47 42 27·4	9·7877n	+5·02	+25·1	4
,, 30	8 28 27	+3 0·49	− 7 53·2	8	23 5 4·11	9·4438n	−31 12 17·0	0·1745n	+4·60	+18·3	5
Oct. 3	9 9 18	+4 24·91	+ 3 27·1	10	22 38 53·60	8·9860n	−23 1 17·9	0·3559n	+4·32	+17·0	6
,, 6	7 42 40	−3 39·68	− 2 58·0	10	22 22 12·87	9·3221n	−16 50 0·7	0·5213n	+4·11	+17·4	7
,, 7	8 36 16	+1 44·91	− 1 21·7	10	22 17 41·51	8·9260n	−15 0 17·9	0·5329n	+4·03	+16·9	8
,, 10	8 54 36	+4 7·26	− 2 36·1	2	22 7 23·07	6·2345	−10 32 12·9	0·6026n	+3·86	+16·8	9
,, 12	7 46 27	+1 20·64	− 4 41·3	10	22 2 24·25	9·0500n	− 8 11 12·8	0·6386n	+3·78	+17·2	10
,, 15	8 9 30	+0 25·87	− 1 1·7	10	21 55 35·87	7·4430n	− 5 11 35·9	0·6737n	+3·68	+17·5	11
,, 18	8 12 47	+4 28·63	+ 3 34·5	10	21 52 29·73	7·9664	− 2 47 51·0	0·7009n	+3·55	+17·5	12
,, 20	8 2 36	+1 50·33	+ 6 17·9	10	21 50 28·99	7·9348	− 1 26 10·3	0·7153n	+3·51	+17·9	13
,, 21	8 1 4	−1 19·89	+ 2 56·2	10	21 49 38·79	8·1813	− 0 50 20·7	0·7214n	+3·49	+18·3	14

Comet 1913 *f* (*Delavan*).

	Melbourne Mean Time.	α − α*.	δ − δ*.	Comp.	α app.	log pΔ.	δ app.	log p′Δ.	dα.	dδ.	Star.
1915	h m s	m s	′ ″		h m s		° ′ ″		s	″	
Feb. 12	15 39 26	+1 48·02	− 1 46·8	10	17 36 21	9·6730n	−24 27 6	0·5812n	+0·40	− 7·7	15
,, 23	15 8 55	−0 40·21	− 2 32·5	10	17 45 9·12	9·6825n	−27 50 58·7	0·5380n	+0·73	− 6·7	16
Mar. 9	15 46 46	+1 40·98	+ 7 36·3	10	17 52 24·79	9·5896n	−32 20 9·6	0·2802n	+1·23	− 5·1	17
Apr. 12	11 31 32	−0 4·69	+ 3 52·7	10	17 42 40·91	9·7907n	−44 2 39·2	0·3896n	+2·80	− 2·2	18
,, 16	11 48 50	−3 16·44	+ 1 37·0	10	17 38 5·03	9·7728n	−45 26 10·1	0·1953n	+3·01	− 2·1	19
,, 20	11 47 31	−2 18·97	+ 5 21·1	10	17 32 38·26	9·7627n	−46 47 16·9	0·0172n	+3·25	− 2·4	20

Comet 1915 *a* (*Mellish*).

	Melbourne Mean Time.	α − α*.	δ − δ*.	Comp.	α app.	log pΔ.	δ app.	log p′Δ.	dα.	dδ.	Star.
1915	h m s	m s	′ ″		h m s		° ′ ″		s	″	
Apr. 12	13 54 54	+1 52·33	+ 6 7·5	10	18 19 2·47	9·5262n	− 3 48 59·8	0·7021n	+1·88	−13·4	21
,, 20	13 16 43	+1 58·98	+ 1 39·7	10	18 29 25·85	9·5567n	− 6 9 3·9	0·6869n	+2·09	−11·6	22
,, 30	15 50 37	+2 33·86	− 5 28·4	6	18 43 45·49	8·6701n	−10 40 31·8	0·6011n	+2·39	− 8·6	23
May 17	11 29 35	+0 35·12	− 6 11·7	10	19 17 8·43	9·6632n	−26 25 43·8	0·5297n	+3·07	− 0·2.	24
,, 31	10 30 4	+0 28·88	−11 4·1	4	20 33 1·53	9·9110n	−55 36 5·7	0·3642n	+4·47	+14·6	25
June 1	11 2 20	+4 35·16	− 1 59·3	10	20 45 13·59	9·9335n	−58 14 22·6	0·1680n	+4·68	+16·0	26
,, 2	10 50 12	+0 14·55	− 1 54·5	10	20 59 2·97	9·9738n	−60 46 1·3	0·2570n	+4·78	+18·2	27

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Mean Places of Comparison Stars.

Star.	α_{1914}	δ_{1914}	Authorities.
	h m s	° ′ ″	
1	2 40 0·33	−57 0 18·8	Gou. 2916, GZ 2^h, 1073, Cp_{80} 1120.
2	1 57 48·88	−55 39 24·6	Gou. 2010, GZ 1^h, 1505.
3	1 32 19·24	−53 38 25·6	Gou. 1556, GZ 1^h, 807, Cp_{80} 636.
4	0 26 21·02	−47 30 40·0	Cp_{00} 145.
5	23 1 59·02	−31 4 42·1	Gou. 31382, GZ 22^h, 1757, Perth 1963.
6	22 34 24·37	−23 5 2·0	Gou. 30863, GZ 22^h, 988.
7	22 25 48·44	−16 47 20·1	RC_{90} 6036, AG Wash. 8394.
8	22 15 52·57	−14 59 13·1	AG Wash. 8345.
9	22 3 11·95	−10 29 53·6	AG Camb., M. 7818.
10	22 0 59·83	− 8 6 48·7	$W_{(1)}$ 21^h 1334, $SF_{(3)}$ 93, 1110, $Me_{(5)}$ (1914).
11	21 55 6·32	− 5 10 51·7	AG Strass. 7681.
12	21 47 57·55	− 2 51 43·0	AG Strass. 7640.
13	21 48 35·15	− 1 32 46·1	Abbadia (1900).
14	21 50 55·19	− 0 53 35·2	Abbadia (1900).
	α_{1915}	δ_{1915}	
15	17 34 33	−24 25·2	C.P.D. − 24° 5927.
16	17 45 48·60	−27 48 19·5	GZ 17^h, 2928.
17	17 50 42·58	−32 27 40·8	Gou. 24327, Cp_{80} 9763.
18	17 42 42·80	−44 6 29·7	Cp_{00} 6417.
19	17 41 18·46	−45 27 45·0	Cp_{00} 6410.
20	17 34 53·98	−46 52 35·6	Cp_{00} 6371.
21	18 17 8·26	− 3 54 53·9	AG Strass. 6141.
22	18 27 24·78	− 6 10 32·0	$Val._{1885}$. Vol. 5. Karlsruhe.
23	18 41 9·24	−10 34 54·8	W 18^h 949 (1); Münch. 17333 (3); AG Camb. M. 6433 (5).
24	19 16 30·24	−26 19 31·9	Gou. 26496; Cp_{80} 10489; Cp_{00} 2665.
25	20 32 28·18	−55 25 16·2	Gou. 28232 (3); GZ 20^h 974 (2).
26	20 40 33·75	−58 12 39·3	Gou. 28428 (3); GZ 20^h 1224 (2).
27	20 58 43·64	−60 44 25·0	Me_1 1066 (3); Gou. 28835 (3); GZ 20^h 1767 (1).

Comet 1914 *e* (*Campbell-Westland*).

Sept. 21. Concentrated towards centre, distribution of light nearly symmetrical round centre, and gradually falling away. Diameter 12′. Naked-eye magnitude 3½.

Sept. 22. More condensed than on Sept. 21, but nucleus still rather diffuse, surrounded fairly symmetrically by extensive coma, perhaps more pronounced to the north. Diameter of coma about 12′. Magnitude (naked eye) 3½.

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Sept. 23. First half of observations somewhat troubled by cloud. Latter half good. Mag. 3½.
Sept. 30. Bright moonlight.
Oct. 3. Comet very difficult in bright moonlight.
Oct. 6. Comet nucleus diffuse, and difficult to observe. Mag. 6½. Definition very bad.
Oct. 7. Comet diffuse and difficult. Definition poor.
Oct. 15. Comet faint and difficult.
Oct. 18. Comet faint and very difficult.
Oct. 20. Comet faint, diffuse, and very difficult.
Oct. 21. Comet very faint, diffuse, and very difficult.
Nov. 6. Very faint and diffuse, no pronounced concentration. Quite impossible to observe.

Comet 1913*f* (*Delavan*).

Feb. 12. Well-defined nucleus, mag. 8.
Feb. 23. Good nucleus, mag. about 8. Extended coma, with trail traceable 40′ in finder, fairly bright for 20′. Width through nucleus at right angles to tail 2′.
Mar. 9. Moonlit field. Well-defined nucleus surrounded by slight coma. Mag. of nucleus 9·5 as seen in telescope.
Apr. 12. Well-defined nucleus, mag. 10. Surrounded by extensive coma elongated to N*p*. Diameter of coma 2′ or 3′.
Apr. 16. Nucleus 10 mag., faint coma.
Apr. 20. Nucleus 10 mag. Coma. At first tail suspected for 20′, but not seen afterwards. Comet observations difficult from faintness of object and further difficulties from two faint stars slightly preceding the comet. Fainter than April 16, when mag. 9 would probably be better.

Comet 1915*a* (*Mellish*).

Apr. 12. Diffuse nucleus surrounded by coma more extended to N*p*. Maximum diameter 5′. Mag. 8.
Apr. 20. Total light 8 mag. Concentrated in middle; no stellar nucleus; extensive coma; diameter 6′; elongated towards preceding.
Apr. 30. Comet faint and diffuse against bright background, moon nearly full. Comet fairly difficult throughout, and sky too bright after sixth set. Foggy.
May 17. Fairly sharp nucleus, 8½ mag., surrounded by extensive coma, with apparently a fainter concentration 30″ preceding main nucleus. Central coma bright, rendering observations somewhat difficult.
May 31. Observations difficult throughout, owing to bright moonlight and haze.

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June 1. Good nucleus, mag. 8½; easy to observe, but definition and steadiness poor. Extensive coma. Bright moonlight.

June 2. Faint bright patch in tail. Nucleus 8·5; spot extremely faint.

Photographic Observations.

Instrument: astrograph. Observer: J. M. Baldwin. Measurer: C. Peel.

Comet 1913*f* (*Delavan*).

	Melbourne Mean Time.	Astrographic Positions.		Annual Aberration.		Parallax Factors.		Catalogue.
		α_{1915}	δ_{1915}	$d\alpha$	$d\delta$	$\log p\Delta$	$\log p'\Delta$	
1915	h m s	h m s	° ′ ″	s	″			
May 6	9 10 30	17 2 33·83	−51 30 21·4	+1·85	+ 3·7	9·8569$_n$	0·2815$_n$	Cp. Ast$_{00}$.
,, 17	7 53 48	16 35 36·30	−53 47 15·2	+2·21	+ 0·4	9·8828$_n$	0·2683$_n$	Gou.
June 1	6 47 23	15 56 38·33	−55 17 25·7	+2·32	− 5·1	9·8800$_n$	9·9846$_n$	Gou.
July 1	9 10 49	15 0 38·05	−54 14 13·9	+1·41	−12·5	9·1791	0·3816	Gou.
,, 13	10 57 56	14 50 26·28	−53 17 45·8	+0·92	−13·2	9·7878	9·3277	Gou.

Comet 1915*a* (*Mellish*).

1915	h m s	h m s	° ′ ″	s	″			
July 3	6 30 13	5 42 52·54	−48 5 8·7	−1·98	+ 4·6	9·8084	0·7433$_n$	Cp. Ast$_{00}$.

The astrographic positions are referred to the equator and equinox of 1915·0, and are corrected for the part of the aberration due to the earth's motion.

Melbourne Observatory:
1917 *Jan.* 22.

The following reports of the Sydney and Melbourne Observatories were received too late for insertion in the Annual Report of the Council:—

Sydney Observatory.
(*Director, Mr. W. E. Cooke, Government Astronomer.*)

After four years' strenuous exertions to obtain the removal of the Observatory to a more favourable locality, the Government Astronomer was definitely informed in August that on account of the war the reorganisation must once more be postponed. Under these circumstances the Astronomer has definitely ceased his efforts in that direction, and arrangements are being made for him to reside in the old Observatory quarters and do the best he can under the old conditions.

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Considerable attention has been given to the astrograph. It has been structurally modified and completely readjusted, and is now very nearly ready for a fresh start.

The main energies of the staff have been directed to the reduction of the accumulated meridian observations. The assistance of two young ladies has been obtained, and steady progress has been made.

During the year a Board of Visitors was appointed by the Government, and several meetings have been held. Most of the business consisted of a discussion as to what use, if any, was to be made of the Melbourne measures of the Sydney Catalogue plates. Eventually it was decided to refer the matter to the Astronomer Royal, and a letter has just (1917 January) been received from him advising the publication of the measures already made. The Board has not yet had time to consider Sir Frank Dyson's letter, but the reduction work has been temporarily laid aside, and all available computing energy has been transferred to the preparation of the photographic measures for the press. They are fairly well spread over the regions having plate centres at 52°–59°, and comprise 652 regions out of a possible 960.

An attempt is being made to utilise the chart plates. Reproduction as carried out elsewhere is considered impracticable. There is a valuable collection of these plates, having their centres at 53°, 55°, 57°, and 59° declination. They cover nearly completely, but without overlap, the whole zone from 52° to 60°. Their epochs range from 1893 to 1910, and there has been much duplication. Some of the regions have been taken quite a number of times. The earliest received single exposures from 30 minutes up to 2 hours, and were taken on Cadett and Marion plates, and the latest series have triple exposures of 30 minutes and were taken on the standard Ilford plates. The earlier ones seem to have been focussed for the centre of the plate, and the images are therefore bad in the corners. The triple exposure charts are much better. Altogether there is a mass of material too valuable to lose, so two schemes for utilising it have been considered, and one is already in operation. Dealing first with the latter, pairs of plates covering the same region, and having epochs at least ten years apart, are being selected and sent to Mr. Innes, Director of the Union Observatory of South Africa. There they are being examined with the blink microscope, and a catalogue will eventually be formed of all measurable proper motions. From the first preliminary survey it appears probable that the entire zone 52°–60°, with the exception of 115 regions, is available for this treatment; and our first care, as soon as a fresh start is made, will be to retake those missing regions.

The second project is to select the best photograph of each region and reproduce it in some form or other, so as to make a photographic atlas of the entire zone; but that is at present only in the experimental stage.

It will be seen that the main scientific work of the Observatory

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at present necessarily consists of the digestion and most practical utilisation of the great mass of accumulated observations, both photographic and meridian, taken during the last quarter of a century. It will be unwise to recommence a new programme of observational work, except such as is required to complete the existing gaps, until the whole of the material has been definitely handled.

The University, practical, and popular activities of the Observatory have been continued as usual.

Melbourne Observatory.

(Dr. J. M. Baldwin, Acting Government Astronomer.)

Transit Circle (8-*inch*).—The observations for the year include:—

	Number of Observations.	
	R.A.	Z.D.
Clock stars . . .	421	242
Azimuth stars . . .	136	106
List stars . . .	534	537
Miscellaneous . . .	74	75
Total . .	1165	960

Collimation has been obtained 70 times; level, 130; nadir, 118; and runs 21 times.

The exceptionally cloudy weather of the latter half of the year has interfered very seriously with the amount of observing, on occasions ten to fifteen days elapsing without an observation being possible.

Astrograph.—The following plates have been secured:—

Catalogue	4
Triple chart	18
Oxford areas	1
Pole	15
Adjustment	2
Rejected	2
	42

The work on the mounting for the 6-inch doublet has been continued during the year, and the instrument is now ready for hoisting into position. It will be placed in the dome above the roof of the main building formerly occupied by the $4\frac{1}{2}$-inch tele-

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scope, which now serves as guiding telescope for the 6-inch. The width of the shutter opening has been doubled, and for the new shutter the mode of balancing described by Dr. Maw (*M.N.*, lxxiv. 2, 1913) has been adopted, and has proved entirely satisfactory.

8-inch Equatorial.—Observations, as opportunity offered, have been made of the variable 191033, but no other work has been done with this telescope during the year.

The instrument has, as usual, been set apart for visitors on three nights in each lunation.

Reduction of Meridian Observations.—The apparent places of all stars are completed to date, and the reduction from apparent to mean place for 1915 is in progress.

The introduction to the General Catalogue 1890 has just been received, and will be printed as soon as possible.

The manuscript of the General Catalogue 1900 has been prepared to $19^h\ 53^m$ and is being proceeded with.

Reduction of Astrographic Plates. — The reduction of the measured co-ordinates to millimetres, and the application of the réseau corrections, have been completed to $1^h\ 36^m\ -74^\circ$.

The measured co-ordinates and diameters have been copied for the printer to $14^h\ 50^m\ -67^\circ$.

The intercomparison of overlapping plates has been completed to $2^h\ 6^m\ -66^\circ$ with $2^h\ 10^m\ -67^\circ$.

The identification of C.P.D. stars has been extended to $14^h\ 50^m\ -67^\circ$.

Twenty-three pairs of astrographic plates, with a minimum time interval of fifteen years, were forwarded to Mr. Innes at Johannesburg for examination with the blink microscope. The results of the examination will be published by Mr. Innes.

Annular Eclipse of the Sun, 1916 *July* 30.—This was observed at Glen Thompson, a station on the central line of eclipse, by Mr. Merfield. The longitude of his station was determined by telegraphic exchange of time-signals with the Observatory on four nights, the latitude and time being determined with an altazimuth. The first contact was observed through cloud, the first internal contact under favourable conditions, and the second internal contact through cloud. The last contact could not be observed. The reductions of these observations are almost complete.

Magnetic Observations and Reductions.—The absolute value of the earth's magnetic field has been determined fortnightly throughout the year, and the scale value of the Adie recording instruments has also been determined at regular intervals. Owing to the lack of assistance, no curves have been measured during the year, but the classification of the curves has been commenced from 1916 January 1, and the results have been forwarded quarterly to De Bilt.

During the year sanction was obtained for the erection of a new magnetic observatory. The building will be of wood and insulated. The general design is similar to that of the U.S. Coast

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and Geodetic Survey magnetic observatories, the total thickness of insulation being 18 inches.

It has been decided to erect the building near Toolangi, about 35 miles N.E. of Melbourne, and over 8 miles from the nearest railway line.

An intercomparison of the Melbourne absolute instruments with those of the Carnegie Institution of Washington, used by Mr. Parkinson, was obtained on May 31.

Seismograph.—Records have been taken as usual, and have been read off regularly since January 1, and the results forwarded to Shide Observatory.

The time service has continued as in 1915, and so have the registration of tides at Williamstown, the rating of chronometers, and other miscellaneous services.

The verification of the secondary standard weights and measures and their adjustment has proceeded at intervals during the year as time allowed, and is not yet completed. With the depleted staff it is found impossible to give continuous attention to this or any other long piece of work.

Personal.—None of the vacancies mentioned in last year's report have been filled, and the staff has been further reduced by the resignation of the last temporary assistant from the astrographic bureau and by the enlistment of Mr. Cornell (astrographic observer), so that now there are only two computers engaged on the astrographic reductions.

Errata in Prof. de Sitter's Papers on "Einstein's Theory of Gravitation."

First Paper:

Vol. lxxvi. p. 714, equation (28), first term should be $r\gamma_2'$.

,, p. 714, footnote, middle formula B, insert ν'' before the + sign.

,, p. 715, equation (33) B, should read—

$$\alpha = \beta = \frac{2\lambda^2}{r} m(r) + 2\lambda^2[\mathrm{N} - n(r)].$$

,, p. 719, line 8 (formula), *for* $-1''\!\cdot\!75$ *read* $+1''\!\cdot\!75$.

Second Paper:

Vol. lxxvii. p. 162, formula (82), first line. In denominator of second term, *for* r_ν^2 *read* r_ν^3; and on right-hand side, *for* $4\lambda^2$ *read* $4\lambda^4$.

Erratum in Annual Report.

Vol. lxxvii. p. 394. In list of persons who have contributed to the Library, etc., *insert* E. B. Knobel, Esq.

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Vol. LXXVII. No. 5.

THE NUMBER OF STARS OF DIFFERENT MAGNITUDES IN THE HYDERABAD ASTROGRAPHIC CATALOGUE, ZONE −17°.

BY

R. J. POCOCK, B.A., B.Sc.

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The Number of Stars of Different Magnitudes in the Hyderabad Astrographic Catalogue, Zone – 17°. By R. J. Pocock, B.A., B.Sc.

1. The first volume of the *Hyderabad Astrographic Catalogue*, containing the diameters and measured co-ordinates of over 63,000 stars on plates with centres approximately in Decl. – 17°, is now completed. The numbers of stars recorded under different diameters have been counted, and the results are discussed in the present note. As the printing of the volume is not yet complete, it may be best to begin with a brief description of the Hyderabad practice in determining diameters.

2. As each star is measured an estimate of the diameter is made and entered on the measure sheets. Since each star is measured twice in different positions of the plate; two estimates of the diameter are made, and these are added together and the sum printed in the catalogue under the heading d (thus following the Greenwich plan). In order to convert these numbers into magnitudes a formula of the form $m = a - b\sqrt{d}$ has been adopted, since other observatories have found such a formula suitable. The numbers of stars of each tenth of a magnitude among the reference stars used for this zone, taken from the *Washington A.G. Catalogue*, were counted, and by means of these counts the Washington magnitudes were converted to a scale comparable with that of Chapman and Melotte.* These modified magnitudes (m) were then compared with the diameters (d) separately for each plate, and hence the constants a, b determined. Several different groups of plates gave values of b all close to 1·09, which was therefore adopted for the whole zone, and the values of a determined separately for each plate. The resulting formulæ, however, very often result in giving much too faint a magnitude for the fainter stars on the plate. This is due to the fact that most of the measurers used in reality two distinct scales, one for the bright stars and another for the faint stars, estimating all the stars as over 20 or under 10 say, with very few between, so that a formula determined

* See *Astronomical Journal*, vol. xxix., No. 688.

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from the bright stars alone necessarily makes the faint stars too faint, and *vice versa*. We shall have evidence of this fact later. Another peculiarity is that some of the measurers estimate a very large proportion of their stars as 4 or 5 (printed as 8 or 9); on the other hand, other measurers go to the other extreme, and never use these low numbers at all. Since there were five different measurers working throughout, and six towards the end of the work, and the work of the different measurers is well mixed up in each part of the zone, this can have little effect on the final counts.

3. Since one of the objects of this paper is to look for discontinuities in the ratio of faint to bright stars, it is well to consider first what artificial discontinuities may exist, owing to seasonal changes, etc. The zone here considered is particularly favourable for such a consideration; the whole work occupied just two years (1914 Dec.–1916 Dec.), and only two different batches of plates were used. In the earlier part of the work (1914 Dec.–1915 May) the plates used belonged to an old batch that had been in India for some considerable time, and had lost much of their sensitiveness. The exposures given were at first fifteen minutes, and towards the end of this period were increased to eighteen and in a few cases to twenty minutes. All the plates from three hours to ten hours belong to this group, as also a few plates outside these limits of R.A. Since this region was taken within a comparatively short period of time and with similar plates, there is little likelihood of the artificial introduction of any discontinuity in the series. With a few exceptions (mostly in $11^h - 13^h$) all the plates from 10^h to 3^h were taken between 1916 March and 1916 December with a different batch of plates. A few of these newer plates were given an exposure of fifteen minutes, but as they were found to be much more rapid than the older ones, the exposure was soon diminished to twelve minutes. The great majority of these plates were exposed during the period 1916 March 1–1916 May 7, while of the plates exposed later in the year, most were exposed in October–November. Hence here again there is very little chance of any discontinuity. It is true that in the period December–March there is a very great rise in temperature, but this occurs slowly and should cause no sudden break. There is, on the other hand, a sudden change about the middle of June, when the monsoon sets in, but in June, July, or August it is very rarely possible to secure plates, and there are very few in this volume taken in these months.

4. Turning to the numbers themselves, and denoting by N_x the number of stars whose recorded diameters (*i.e.* the sum of two estimations) are not less than x, I have tabulated in Table I. the totals for each hour of R.A. of N_{50}, N_{30}, N_{20}, N_{12}, N_8, the latter being equal to the total number of stars measured. Since the hours of R.A. contain alternately eight and seven plates, the numbers have first been multiplied by a factor to make them equivalent to ten plates in each hour of R.A.

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TABLE I.

Cumulative Totals for Ten Plates in each Hour of R.A., Zone − 17°.

R.A. h	N_{50}	N_{30}	N_{20}	N_{12}	N_{8}
0	93	553	909	1413	1940
1	60	379	656	1075	1620
2	53	366	726	1313	1777
3	59	308	729	1272	1665
4	94	353	832	1500	2310
5	248	1121	1841	2694	3270
6	236	1016	2059	3383	4620
7	164	1108	3100	6401	10720
8	123	724	1554	3067	5220
9	79	378	871	1610	2750
10	51	259	554	1116	1966
11	51	252	581	1182	2341
12	69	316	613	1249	1974
13	100	434	799	1520	2799
14	107	637	1180	2044	3990
15	106	514	1151	2220	4236
16	119	494	821	1490	2917
17	182	739	1356	2720	5910
18	110	611	1214	2490	6167
19	91	330	1046	2189	5282
20	89	587	1206	2273	4034
21	108	708	1146	1864	2620
22	87	429	820	1376	2203
23	64	446	789	1261	1861
C. and M. Mag.	9·14	10·77	11·55	12·25	12·94
Calcd.	9·16	10·62	11·51	12·40	12·95
Diff.	− ·02	+ ·15	+ ·04	− ·15	− ·01

5. The most noticeable feature about the table is the sudden very large increase in the star density between 6^h and 7^h, which, moreover, the table shows is entirely due to the fainter stars. At the foot of each column is given the corresponding magnitude on the C. and M. scale, determined from the table in *Monthly Notices*, vol. lxxv. p. 145. Below these are given the magnitudes calculated from the formula $m = 15·44 - 0·88 \sqrt{d}$. Now the value of b, viz. 0·88 in this formula, is sensibly different from that mentioned above, viz. 1·09, adopted for the whole zone. It will be seen that the formula just given suits very well both for faint and bright stars, whereas if we use the value 1·09 and only take account of the bright stars we obtain the formula $16·92 - 1·09 \sqrt{d}$, which will

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make the faint stars 13·84 instead of 12·95, nearly a whole magnitude too faint; this is an instance of the fact mentioned above, that the bright stars are estimated too bright relatively to the faint stars. The formulæ given in the printed volume must therefore be considered as only suitable for the bright stars; at about mag. 11·0 they will make the stars 0·5 mag. too faint, and below 12·0, 0·9 mag. too faint.

6. Next, taking the logarithms of the numbers in Table I. and treating them in the same way as Professor Turner has done in several papers (see *Monthly Notices*, lxxv. p. 63), that is, subtracting the mean of each column from each number in the column and then subtracting one column from the remainder, we form Table II. Since the column N_{30} has been taken as standard and subtracted from the remainder, this column is omitted, and the signs of the column headed N_{50} are changed so as to be comparable with the other columns.

TABLE II.

Values of log N′ to log N′$_{30}$ for Selected Magnitudes in Units of 0·01

R.A. h	N_{50}	N_{20}	N_{12}	N_{8}	Gal. Lat.
0	− 6	− 9	− 16	− 25	− 77
1	− 9	− 7	− 11	− 16	− 75
2	− 13	− 1	− 1	11	− 64
3	0	+ 7	+ 4	− 6	− 51
4	+ 14	+ 6	+ 6	+ 2	− 38
5	+ 6	− 9	19	− 33	− 24
6	+ 8	0	− 4	− 14	− 11
7	− 11	+ 14	+ 20	+ 19	+ 1
8	− 6	+ 2	+ 6	+ 6	+ 14
9	+ 3	+ 6	+ 6	+ 7	+ 25
10	+ 1	+ 2	+ 7	+ 9	+ 35
11	+ 2	+ 6	+ 10	+ 17	+ 42
12	+ 5	− 2	+ 3	0	+ 45
13	+ 8	− 4	− 2	+ 2	+ 44
14	− 6	− 4	− 6	0	+ 39
15	+ 3	+ 4	+ 7	+ 12	+ 31
16	+ 10	− 8	− 9	− 2	+ 20
17	+ 10	− 4	0	+ 11	+ 8
18	− 3	− 1	+ 4	+ 21	− 4
19	+ 16	+ 20	+ 26	+ 41	− 17
20	− 11	0	+ 2	+ 4	− 30
21	− 10	− 10	− 15	− 23	− 43
22	+ 2	− 2	− 6	− 8	− 57
23	− 13	− 6	− 12	− 17	− 69

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Looking at the column headed N_8, it is at once evident why the lines have been drawn separating the region 7^h to 20^h from the rest. We have here a discontinuity similar to those found by Professor Turner in other zones. Following Professor Turner, I shall speak of the region 7^h to 20^h as unobscured, and of the region 21^h to 6^h as obscured, simply for convenience, without intending to imply that the difference is necessarily due to any actual obscuring matter in space. It may be noted that the obscured region from 21^h to 6^h fits very well into the diagram given by Professor Turner in *Monthly Notices*, lxxv. p. 480.

7. Now, taking the numbers given in Table I. and multiplying by such factors as to reduce the total number of stars in each hour of R.A. to about 2000, we can determine the amount of the obscuration without giving undue weight to the galactic regions. Without giving the details, we arrive at the results shown in Table III., where I., II. are the obscured and unobscured regions respectively. Table III. is similar in form to Tables V., VII., XIII., XIV. in *Monthly Notices*, lxxv. p. 470. In this case log N_{50} has been subtracted from the other logarithms.

Table III.

Amount of Obscuration, Zone − 17°. *Logarithms in Units of* 0·01.

Mag.	9·14.	10·77.	11·55.	12·25.	12·94.
I.	867	3788	3878	5645	5774
II.	742	2924	4020	7288	13318
log I.	0	64	65	82	83
log II.	0	60	73	99	125
log I.-II.	0	+4	−8	−17	−42

8. It will be seen that the final figure comes out −42 in exact agreement with the number found in other zones; and, comparing the amount of obscuration with the two adjacent zones as given in *Monthly Notices*, lxxv. p. 480, we get—

Mag.	10·0.	11·5.	13·0.
Algiers −1°	−10	−18	−42
Hyderabad −17°	0	−7	−42
Perth −32°	0	0	−42

So that the Hyderabad results fall naturally between the other two. In the Hyderabad zones the obscuration begins about 11·0.

9. Now, arranging the numbers of Table II. according to galactic latitude, we obtain the results shown in Table IV.

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TABLE IV.

Amount of Galactic Condensation.

Obscured Region.						Unobscured Region.					
Mean Gal. Lat.	No. in Group.	9·14.	11·55.	12·25.	12·94.	Mean Gal. Lat.	No. in Group.	9·14.	11·55.	12·25.	12·34.
−71°	4	−10	−6	−10	−17	+41°	5	+2	0	+2	+6
−50°	3	−3	−2	−6	−12	±25°	5	+4	+4	+6	+12
−24°	3	+9	−1	−3	−15	±70°	4	−2	+3	+8	+14

10. Determining the coefficient of galactic condensation per magnitude per degree from the first and last rows only, and in each case taking the simple mean of the four determinations, we have the result—

Coefficient of galactic condensation.

Unobscured region −·00065. Obscured region −·00142.

The first result falls between those for Algiers and Perth, the second is slightly larger than either Algiers or Perth (see *Monthly Notices*, lxxv. p. 608). They do not lend support to the unusually high value in the case of the Cape Zone −41°. The mean value —·00104, however, lends some support to the suggestion that there is an increase in the amount of galactic condensation as we proceed southwards.

Nizamiah Observatory, Hyderabad:
1917 *January* 17.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LXXVII. April 13, 1917. No. 6

Major P. A. MacMahon, D.Sc., Sc.D., LL.D., F.R.S., President, in the Chair.

James Bootherstone, 38 St. Annes Road, Chorlton-cum-Hardy, Manchester,

was balloted for and duly elected a Fellow of the Society.

The following were proposed by the Council as Associates of the Society:—

Professor C. G. Abbot, Director of the Astrophysical Observatory of the Smithsonian Institution, Washington, D.C., U.S.A.;
Professor Giovanni Celoria, Director, Reale Osservatorio di Brera, Milan, Italy;
M. Pierre Henri Puiseux, Observatoire, Paris; and
Professor Charles Edward St. John, F.R.A.S., Solar Observatory, Mount Wilson, California, U.S.A.

Fifty-one presents were announced as having been received since the last meeting, including, amongst others:—

David Brunt, The Combination of Observations, presented by the author.

Rome, Specola Vaticana, Aggiunte al Catalogo dell' Atlas Stellarum Variabilium di J. G. Hagen, presented by the Observatory.

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A Determination of the Constant of Aberration. By S. S. Hough, M.A., F.R.S., H.M. Astronomer at the Cape. (Plates 4, 5.)

During the year 1911 the new reversible transit-circle of this Observatory was devoted exclusively to the observation of a series of close circumpolar stars. A discussion of the observations with a view to the determination of the most accurate positions of the stars is contained in *Cape Annals*, vol. xii. part 3, and it is there indicated how the observations may be utilised to derive a determination of the constant of aberration with no inconsiderable weight. As this determination did not form part of the programme originally intended when the observations were made, no special provision was made to secure observations under the most favourable conditions for yielding large aberration factors. The object of the present note is primarily to correct inaccuracies in the provisional procedure adopted in the *Cape Annals*, and to derive the value of the aberration constant by a more rigorous discussion of the material; and further, to consider how the programme of observations might best be modified with a view to repetition, so as to throw as much weight as possible into the determination of this quantity.

For the purposes of the former discussion the observations were divided into twenty-six groups, each contained within a limited period not exceeding about a fortnight in duration, during which the azimuth of the meridian marks and the instrumental latitude were regarded as sensibly constant. This same grouping has been adhered to in the present work.

The first step was to evaluate for each star in each group mean values of the aberration factors as involved in the computation of the "star correction."

Denoting by Δk the correction to the tabular value of the aberration constant (20″·47) as used in the *Nautical Almanac*, the additional corrections to be applied to the observations to reduce from apparent to mean place, consequent on this change in the value of the constant of aberration (k), are given by the formulæ

$$\Delta\alpha \cos\delta = -h \sin(\mathrm{H}+\alpha)\Delta k/k,$$
$$\Delta\delta \qquad = -h \sin\delta \cos(\mathrm{H}+\alpha)\Delta k/k.$$

In estimating the coefficients, $\sin\delta$ was throughout taken as -1, and the values of H, h/k for the separate groups were derived from the following rough table (Table I.).

A rough mean value of α for each separate star was adopted.

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TABLE I.

	H.		h/k.		H.		h/k
	h	m			h	m	
Jan. 16–26	22	0	·98	July 4–19	10	48	·99
Jan. 28–Feb. 8	21	0	·96	July 20–Aug. 8	9	44	·97
Feb. 16–23	20	0	·94	Aug. 12–26	8	24	·95
Feb. 24–Mar. 3	19	20	·92	Aug. 28–Sept. 2	7	40	·93
Mar. 13–20	18	16	·92	Sept. 4–11	7	0	·92
Mar. 21–30	17	40	·92	Sept. 12–19	6	40	·92
Apr. 1–8	17	0	·93	Sept. 28–Oct. 7	5	20	·92
Apr. 10–24	16	0	·94	Oct. 9–20	4	30	·92
Apr. 28–May 12	15	0	·96	Oct. 25–28	3	40	·94
May 14–27	14	0	·98	Oct. 30–Nov. 6	3	10	·95
May 30–June 9	13	0	·99	Nov. 7–12	2	40	·96
June 10–21	12	20	1·00	Nov. 13–19	2	12	·97
June 25–July 3	11	30	1·00	Nov. 20–29	1	44	·98

Retaining Δk as an algebraic quantity, associated with numerical coefficients derived from this table, means of these coefficients in the different group combinations were taken out, following exactly the same procedure as that used in the original discussion, in which the polar corrections in R.A. and latitude corrections in declination, and subsequently the star places, were derived. For instance, the expressions for polar deviations in R.A. are given in the following table.

TABLE II.

Polar Deviation in Right Ascension, including Terms due to Aberration Correction.

	Δp.			Δp.	
	s			s	
Jan. 16–26	−0·030	−1·44 Δk	July 4–19	+0·014	·00 Δk
Jan. 28–Feb. 8	−0·014	−1·27 Δk	July 20–Aug. 8	+0·046	·00 Δk
Feb. 16–23	−0·022	−1·16 Δk	Aug. 12–26	+0·029	+0·24 Δk
Feb. 24–Mar. 3	+0·012	−1·05 Δk	Aug. 28–Sept. 2	+0·019	+0·30 Δk
Mar. 13–20	+0·013	−0·85 Δk	Sept. 4–11	+0·018	−0·46 Δk
Mar. 22–30	−0·061	−0·72 Δk	Sept. 12–19	+0·021	−0·55 Δk
Apr. 1–8	+0·025	−0·60 Δk	Sept. 28–Oct. 7	+0·011	−0·88 Δk
Apr. 12–24	−0·033	−0·46 Δk	Oct. 9–20	+0·030	−0·83 Δk
Apr. 28–May 12	+0·010	0·00 Δk	Oct. 25–29	+0·015	−0·87 Δk
May 14–27	−0·013	0·00 Δk	Oct. 30–Nov. 6	+0·032	−0·88 Δk
May 30–June 9	+0·040	0·00 Δk	Nov. 7–12	−0·001	−0·93 Δk
June 10–21	+0·005	0·00 Δk	Nov. 13–19	+0·032	−1·28 Δk
June 25–July 3	+0·031	0·00 Δk	Nov. 19–29	+0·007	−1·18 Δk

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The constant parts in these expressions correspond with those of the original discussion, the terms involving Δk being now added. The coefficients of Δk during the period April 28–August 8 are throughout zero, as the determinations of azimuth therein involved are based solely on observations of the same stars at adjacent upper and lower culminations.

At these epochs the effects of an error in the adopted tabular aberration constant will be thrown entirely into the derived mean right ascensions of the stars. In the case of the stars observed at double transit it will be always associated with a small coefficient, as the observations are made near the times of sunrise and sunset, when the value of $H + a$ is near zero or twelve hours. This, however, will not be the case for other stars observed in the same watches whose right ascensions sometimes differ by as much as four hours from the former, giving a factor of Δk, in the expression for $\Delta a \cos \delta$, amounting in the extreme to about $\pm 0{\cdot}87$.

The coefficients of Δk appropriate to the other epochs thus involve the mean differences between the factors for each star at the epoch at which the value of Δp is sought and that at which the provisional determinations of right ascension were made. In the former discussion the second part of the factor was treated (erroneously) as zero, both in the right ascensions and in the analogous treatment of the declinations.

The revised expressions for the latitude corrections, including the aberration terms, are as follows :—

TABLE III.

Latitude Corrections, including Aberration Terms.

	$\Delta\phi$.			$\Delta\phi$.	
Jan. 16–26	$+0''{\cdot}25$	$-1{\cdot}25\ \Delta k$	July 4–19	$-0''{\cdot}16$	$0{\cdot}00\ \Delta k$
Jan. 28–Feb. 8	$-0{\cdot}47$	$-1{\cdot}27\ \Delta k$	July 20–Aug. 8	$-0{\cdot}04$	$0{\cdot}00\ \Delta k$
Feb. 16–23	$-0{\cdot}13$	$-0{\cdot}43\ \Delta k$	Aug. 12–26	$-0{\cdot}06$	$-0{\cdot}21\ \Delta k$
Feb. 24–Mar. 3	$-0{\cdot}29$	$-0{\cdot}23\ \Delta k$	Aug. 28–Sept. 2	$-0{\cdot}18$	$-0{\cdot}39\ \Delta k$
Mar. 13–20	$-0{\cdot}33$	$+0{\cdot}42\ \Delta k$	Sept. 4–11	$+0{\cdot}04$	$-0{\cdot}97\ \Delta k$
Mar. 22–30	$-0{\cdot}48$	$+0{\cdot}40\ \Delta k$	Sept. 12–19	$-0{\cdot}05$	$-1{\cdot}25\ \Delta k$
Apr. 1–8	$-0{\cdot}66$	$+0{\cdot}15\ \Delta k$	Sept. 28–Oct. 7	$+0{\cdot}12$	$-1{\cdot}24\ \Delta k$
Apr. 12–24	$-0{\cdot}56$	$+0{\cdot}20\ \Delta k$	Oct. 9–20	$+0{\cdot}05$	$-1{\cdot}40\ \Delta k$
Apr. 28–May 12	$-0{\cdot}43$	$0{\cdot}00\ \Delta k$	Oct. 25–28	$+0{\cdot}07$	$-1{\cdot}44\ \Delta k$
May 14–27	$-0{\cdot}28$	$0{\cdot}00\ \Delta k$	Oct. 30–Nov. 6	$-0{\cdot}01$	$-1{\cdot}47\ \Delta k$
May 30–June 9	$-0{\cdot}43$	$0{\cdot}00\ \Delta k$	Nov. 7–12	$+0{\cdot}29$	$-1{\cdot}48\ \Delta k$
June 10–21	$-0{\cdot}49$	$0{\cdot}00\ \Delta k$	Nov. 13–19	$-0{\cdot}07$	$-1{\cdot}02\ \Delta k$
June 25–July 3	$-0{\cdot}34$	$0{\cdot}00\ \Delta k$	Nov. 20–29	$+0{\cdot}22$	$-1{\cdot}10\ \Delta k$

In like manner the appropriate factors of Δk were calculated for correcting the derived right ascensions and declinations of the stars as contained in Table XXV. of the former paper. The following expressions for $\Delta a \cos \delta$, $\Delta\delta$ were thus derived.

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TABLE IV.

Aberration Corrections for Separate Stars.

	Δα cos δ.	Δδ.
Lacaille 9745	+0·1 Δ*k*,	−1·0 Δ*k*
Lacaille 23	+0·1 Δ*k*,	−1·0 Δ*k*
o Octantis	+0·1 Δ*k*,	−0·9 Δ*k*
β Hydri	+0·2 Δ*k*,	−0·9 Δ*k*
Lacaille 228	+0·2 Δ*k*,	−0·9 Δ*k*
Lacaille 248	+0·1 Δ*k*,	−0·9 Δ*k*
Gilliss P.Z. 939	+0·1 Δ*k*,	−1·0 Δ*k*
Gilliss P.Z. 1219	+0·1 Δ*k*,	−1·0 Δ*k*
Lacaille 634	+0·3 Δ*k*,	−1·0 Δ*k*
Lacaille 760	+0·2 Δ*k*,	−0·9 Δ*k*
Lacaille 764	+0·2 Δ*k*,	−1·0 Δ*k*
Lacaille 1884	+0·3 Δ*k*,	−0·9 Δ*k*
Lacaille 1029	+0·3 Δ*k*,	−0·9 Δ*k*
Lacaille 1076	+0·2 Δ*k*,	−0·9 Δ*k*
Lacaille 1203	+0·4 Δ*k*,	−0·9 Δ*k*
Lacaille 1848	+0·3 Δ*k*,	−0·9 Δ*k*
Lacaille 1414	+0·5 Δ*k*,	−0·9 Δ*k*
Lacaille 1839	+0·6 Δ*k*,	−0·7 Δ*k*
Lacaille 1707	+0·5 Δ*k*,	−0·7 Δ*k*
Cape 1880, 2449	+0·7 Δ*k*,	−0·7 Δ*k*
Lacaille 2296	+0·5 Δ*k*,	−0·5 Δ*k*
Cape 1880, 2901	+0·5 Δ*k*,	−0·4 Δ*k*
Lacaille 2512	+0·5 Δ*k*,	−0·4 Δ*k*
Lacaille 3274	+0·5 Δ*k*,	+0·4 Δ*k*
Gilliss P.Z. 5752	+0·2 Δ*k*,	0·0 Δ*k*
A Octantis	+0·3 Δ*k*,	+0·4 Δ*k*
Brisbane 2007	+0·2 Δ*k*,	+0·6 Δ*k*
θ Chamæleontis	+0·2 Δ*k*,	+0·8 Δ*k*
Lacaille 3759	+0·2 Δ*k*,	+0·8 Δ*k*
Gilliss P.Z. 6020	+0·2 Δ*k*,	+0·4 Δ*k*
ζ Octantis	0·0 Δ*k*,	+0·9 Δ*k*
Lacaille 4342	−0·1 Δ*k*,	+0·9 Δ*k*
C.G.A. 14444	−0·1 Δ*k*,	+9·9 Δ*k*
C.G.A. 14481	−0·1 Δ*k*,	+0·9 Δ*k*
Lacaille 4510	−0·1 Δ*k*,	+0·9 Δ*k*
δ² Chamæleontis	−0·1 Δ*k*,	+0 8 Δ*k*
Lacaille 4578	−0·1 Δ*k*,	+0·9 Δ*k*
η Octantis	−0·1 Δ*k*,	+0·9 Δ*k*
Lacaille 4708	−0·1 Δ*k*,	+0·8 Δ*k*
Lacaille 4731	−0·1 Δ*k*,	+0·9 Δ*k*
Gilliss P.Z. 7980	−0·2 Δ*k*,	+0·9 Δ*k*
C.G.A. 15761	0·0 Δ*k*,	+0·9 Δ*k*
Lacaille 4865	-0·1 Δ*k*,	+0·9 Δ*k*
Lacaille 4991	−0·1 Δ*k*,	+0·9 Δ*k*
Lacaille 5096	−0·1 Δ*k*,	+1·0 Δ*k*
Lacaille 5107	−0·1 Δ*k*,	+0·9 Δ*k*
Lacaille 5235	−0·2 Δ*k*,	+0·8 Δ*k*
ι Octantis	−0·2 Δ*k*,	+0·9 Δ*k*
Lacaille 5325	−0·2 Δ*k*,	+0·9 Δ*k*
Lacaille 5452	−0·2 Δ*k*,	+1·0 Δ*k*
Lacaille 5444	−0·2 Δ*k*,	+1·0 Δ*k*
κ Octantis	−0·2 Δ*k*,	+0·9 Δ*k*
C.G.A. 18500	−0·2 Δ*k*,	+1·0 Δ*k*
δ Octantis	−0·2 Δ*k*,	+0·9 Δ*k*
Brisbane 4614	−0·3 Δ*k*,	+0·9 Δ*k*
Gilliss P.Z. 10018	−0·3 Δ*k*,	+0·8 Δ*k*
Lacaille 5882	−0·4 Δ*k*,	+0·9 Δ*k*
ε Octantis	−0·3 Δ*k*,	+1·0 Δ*k*
ρ Octantis	−0·3 Δ*k*,	+0·9 Δ*k*
Lacaille 6441	−0·5 Δ*k*,	+0·7 Δ*k*
Lacaille 6545	−0·5 Δ*k*,	+0·8 Δ*k*
Gilliss P.Z. 11448	−0·5 Δ*k*,	+0·8 Δ*k*
C.G.A. 23027	−0·6 Δ*k*,	+0·7 Δ*k*
Lacaille 7078	−0·6 Δ*k*,	+0·6 Δ*k*
χ Octantis	−0·5 Δ*k*,	+0·4 Δ*k*
Brisbane 6229	−0·5 Δ*k*,	+0·1 Δ*k*
Lacaille 7442	−0·4 Δ*k*,	+0·1 Δ*k*
σ Octantis	−0·4 Δ*k*,	−0·4 Δ*k*
Lacaille 7751	−0·4 Δ*k*,	−0·1 Δ*k*
Gilliss P.Z. 13504	−0·2 Δ*k*,	−0·2 Δ*k*
Lacaille 8094	−0·3 Δ*k*,	−0·4 Δ*k*
Lacaille 8257	−0·2 Δ*k*,	−0·6 Δ*k*
C.G.A. 28663	0·0 Δ*k*,	−0·8 Δ*k*
C.G.A. 28714	0·0 Δ*k*,	−0·8 Δ*k*
Lacaille 8551	0·0 Δ*k*,	−0·8 Δ*k*
Lacaille 8751	0·0 Δ*k*,	−0·9 Δ*k*
Lacaille 8720	0·0 Δ*k*,	−0·8 Δ*k*
B Octantis	0·0 Δ*k*,	−0·9 Δ*k*
Lacaille 8738	0·0 Δ*k*,	−0·9 Δ*k*
υ (C.) Octantis	+0·1 Δ*k*,	−0·9 Δ*k*
β Octantis	+0·2 Δ*h*,	−0·9 Δ*k*
τ Octantis	+0·1 Δ*k*,	−0·9 Δ*k*
Lacaille 9464	+0·1 Δ*k*,	−0·9 Δ*k*
Lacaille 9563	+0·1 Δ*k*,	−0·9 Δ*k*
Lacaille 9596	+0·1 Δ*k*,	−0·9 Δ*k*

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In this table the order of arrangement for the different stars is that of R.A. It will be seen that with the actual arrangement of the observations the effect of an error in the adopted aberration constant would be to produce a pronounced periodic error in the resulting declinations, and a similar periodic error, but of a less pronounced character and in complementary phase, in the right ascensions.

In the discussion given later the aim has been to re-arrange the observations through the year in such a manner as to throw these periodic errors into higher relief, and at the same time provide means for strengthening their determination.

Still retaining Δk in algebraic form, residuals were now derived by comparing the separate observations (combined in groups, but uncorrected for latitude and azimuth) with the final results derived from the combination of all groups. For example, taking the period Sept. 4–11, the right ascension observations yielded the following residuals.

TABLE V.

Residuals (O–C). September 4–11.

Above-pole Observations.	s		No.	Below-pole Observations.	s		No.
χ Octantis	+0·023	+0·2 Δk	4	Lacaille 2296	−0·049	−0·3 Δk	4
Brisbane 6229	+0·020	+0·2 Δk	4	Cape 1880, 2901	−0·004	−0·3 Δk	3
Lacaille 7442	+0·017	0·0 Δk	5	Lacaille 2512	−0·029	−0·2 Δk	3
Lacaille 7751	0·000	−0·1 Δk	3	Lacaille 3274	−0·005	+0·1 Δk	2
σ Octantis	+0·012	−0·2 Δk	3	Gilliss P.Z. 5752	−0·046	+0·4 Δk	1
Gilliss P.Z. 13504	+0·046	−0·4 Δk	1	A Octantis	−0·031	+0·3 Δk	3
Lacaille 8094	+0·002	−0·3 Δk	3	Brisbane 2007	−0·040	+0·4 Δk	3
Lacaille 8257	+0·006	−0·5 Δk	2	θ Chamæleontis	−0·055	+0·5 Δk	2
C.G.A. 28663	+0·024	−0·7 Δk	2	Lacaille 3759	−0·013	+0·6 Δk	2
C.G.A. 28714	−0·006	−0·9 Δk	2	ζ Octantis	−0·040	+0·8 Δk	1
Lacaille 8551	+0·004	−0·9 Δk	1	Lacaille 4342	−0·014	+1·0 Δk	1
Lacaille 8751	−0·001	−0·9 Δk	1	C.G.A. 14481	−0·016	+1·1 Δk	1
Lacaille 8720	−0·037	−0·9 Δk	1	Lacaille 4510	−0·005	+1·1 Δk	1
B Octantis	+0·015	−0·9 Δk	1	Lacaille 4578	−0·027	+1·0 Δk	1
Lacaille 8738	+0·007	−0·9 Δk	1	η Octantis	−0·019	+1·1 Δk	1
υ (C.) Octantis	−0·018	−1·1 Δk	1	Lacaille 4708	−0·075	+1·1 Δk	1
τ Octantis	+0·008	−1·1 Δk	1	Lacaille 4731	−0·028	+1·1 Δk	1
Lacaille 9464	+0·008	−1·0 Δk	1	C.G.A. 15761	−0·042	+1·0 Δk	1
Lacaille 9563	−0·002	−1·1 Δk	1	Lacaille 4865	−0·038	+1·0 Δk	1
Lacaille 9596	−0·006	−1·1 Δk	1	Lacaille 4991	−0·014	+1·0 Δk	1
	+0·010	−0·38 Δk,	39		−0·030	+0·40 Δk,	34

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The means, taken with weight proportional to the number of observations of each star, give a revised determination of the polar correction, Δp, as originally applied during the interval Sept. 4–11. From a combination of the above- and below-pole results we derive

$$\Delta p = +0^{s}{\cdot}019 - 0{\cdot}39\Delta k$$

as compared with the value $+0^{s}{\cdot}018 - 0{\cdot}46\Delta k$ contained in Table II.

Subtracting this value from those derived from the separate stars, the following equations were derived which were regarded as equations of condition for the determination of Δk. In this application they are regarded as expressing the condition that, apart from accidental variation, the true value of Δp is sensibly constant throughout the period Sept. 4–11.

TABLE VI.

Equations of Condition for Period September 4–11.

	s			s	
χ Octantis	+0·004	+0·6 Δk=0	Lacaille 2296	−0·030	−0·7 Δk=0
Brisbane 6229	+0·001	+0·6 Δk=0	Cape 1880, 2901	+0·015	−0·7 Δk=0
Lacaille 7442	−0·002	+0·4 Δk=0	Lacaille 2512	−0·010	−0·6 Δk=0
Lacaille 7751	−0·019	+0·3 Δk=0	Lacaille 3274	+0·014	−0·3 Δk=0
σ Octantis	−0·007	+0·2 Δk=0	Gilliss P.Z. 5752	−0·027	0·0 Δk=0
Gilliss P.Z. 13504	+0·027	0·0 Δk=0	A Octantis	−0·012	−0·1 Δk=0
Lacaille 8094	−0·017	+0·1 Δk=0	Brisbane 2007	−0·021	0·0 Δk=0
Lacaille 8257	−0·013	−0·1 Δk=0	θ Chamæleontis	−0·036	+0·1 Δk=0
C.G.A. 28663	+0·005	−0·3 Δk=0	Lacaille 3759	+0·006	+0·2 Δk=0
C.G.A. 28714	−0·025	−0·5 Δk=0	ζ Octantis	−0·021	+0·4 Δk=0
Lacaille 8551	−0·015	−0·5 Δk=0	Lacaille 4342	+0·005	+0·6 Δk=0
Lacaille 8751	−0·020	−0·5 Δk=0	C.G.A. 14481	+0·003	+0·7 Δk=0
Lacaille 8720	−0·056	−0·5 Δk=0	Lacaille 4510	+0·014	+0·7 Δk=0
B Octantis	−0·004	−0·5 Δk=0	Lacaille 4578	−0·008	+0·6 Δk=0
Lacaille 8738	−0·012	−0·5 Δk=0	η Octantis	0·000	+0·7 Δk=0
υ (C.) Octantis	−0·037	−0·7 Δk=0	Lacaille 4708	−0·056	+0·7 Δk=0
τ Octantis	−0·011	−0·7 Δk=0	Lacaille 4731	−0·009	+0·7 Δk=0
Lacaille 9464	−0·011	−0·6 Δk=0	C.G.A. 15761	−0·023	+0·6 Δk=0
Lacaille 9563	−0·021	−0·7 Δk=0	Lacaille 4865	−0·019	+0·6 Δk=0
Lacaille 9596	−0·025	−0·7 Δk=0	Lacaille 4991	+0·005	+0·6 Δk=0

Forming normal equations from these equations of condition, with weights proportional to the number of observations of each star, we find

from above-pole observations $8{\cdot}44\Delta k + 0^{s}{\cdot}145 = 0$

from below-pole observations $9{\cdot}23\Delta k - 0^{s}{\cdot}016 = 0$

from all observations $17{\cdot}67\Delta k + 0^{s}{\cdot}129 = 0.$

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Similar normals have been derived from each group. In the following table the results from the separate groups are given in the form of normal equations which may be combined by addition.

TABLE VII.

Normal Equations from Separate Groups of Right Ascension.

	s	
Jan. 16–26	−0·038	+ 1·5 Δk=0
Jan. 28–Feb. 8	−0·003	+ 1·3 Δk=0
Feb. 16–23	+0·029	+ 0·5 Δk=0
Feb. 24–Mar. 3	+0·031	+ 0·3 Δk=0
Mar. 13–20	−0·019	+ 0·3 Δk=0
Mar. 21–30	+0·041	+ 0·6 Δk=0
Apr. 1–8	+0·176	+ 7·3 Δk=0
Apr. 12–24	+0·118	+ 5·7 Δk=0
Apr. 28–May 12	−0·204	+12·5 Δk=0
May 14–27	+0·056	+ 7·0 Δk=0
May 30–June 9	+0·093	+ 5·5 Δk=0
June 10–21	+0·149	+12·7 Δk=0
June 25–July 3	−0·156	+11·0 Δk=0
July 4–19	−0·221	+11·1 Δk=0
July 20–Aug. 8	−0·056	+ 6·7 Δk=0
Aug. 12–26	+0·019	+13·4 Δk=0
Aug. 28–Sept. 2	−0·012	+ 9·0 Δk=0
Sept. 4–11	+0·129	+17·7 Δk=0
Sept. 12–19	+0·040	+16·7 Δk=0
Sept. 28–Oct. 7	+0·317	+25·1 Δk=0
Oct. 9–20	−0·318	+17·0 Δk=0
Oct. 25–28	+0·216	+13·9 Δk=0
Oct. 30–Nov. 6	−0·140	+12·2 Δk=0
Nov. 7–12	+0·090	+ 4·5 Δk=0
Nov. 13–19	−0·035	+ 5·5 Δk=0
Nov. 20–29	−0·121	+ 8·9 Δk=0
	+0s·181	+227·9 Δk=0

From the normals as thus combined we therefore derive

$$\Delta k = -0^{s}{\cdot}0008 \pm 0^{s}{\cdot}0020$$
$$= -0''{\cdot}012 \pm 0''{\cdot}030.$$

The probable error here quoted is derived from the final factor of Δk, as indicating its weight referred to a unit weight derived from the discordances between the indications of the separate epochs.

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The analysis also yields the following revised determinations of the instrumental polar corrections for the different epochs.

TABLE VIII.

Instrumental Polar Corrections in R.A.

Epoch.	Δp.		Inst.
	s		
Jan. 16–26	−0·030	− 1·28 Δk	W.
Jan. 28–Feb. 8	−0·015	− 1·14 Δk	E.
Feb. 16–23	−0·026	− 1·05 Δk	W.
Feb. 24–Mar. 3	+0·008	−0·95 Δk	E.
Mar. 13–20	+0·014	−0·79 Δk	E.
Mar. 21–30	−0·057	−0·69 Δk	W.
Apr. 1–8	+0·028	−0·52 Δk	E.
Apr. 12–24	−0·029	−0·37 Δk	W.
Apr. 28–May 12	+0·013	−0·27 Δk	E.
May 14–27	−0·015	−0·08 Δk	W.
May 30–June 9	+0·036	−0·12 Δk	E.
June 10–21	+0·005	−0·12 Δk	W.
June 25–July 3	+0·030	−0·09 Δk	E.
July 4–19	+0·017	−0·04 Δk	W.
July 20–Aug. 8	+0·043	−0·04 Δk	E.
Aug. 12–26	+0·030	0·00 Δk	W.
Aug. 28–Sept. 2	+0·013	−0·03 Δk	E.
Sept. 4–11	+0·019	−0·39 Δk	W.
Sept. 12–19	+0·019	−0·53 Δk	E.
Sept. 28–Oct. 7	+0·013	−0·74 Δk	W.
Oct. 9–20	+0·029	−0·83 Δk	E.
Oct. 25–28	+0·015	−0·86 Δk	W.
Oct. 30–Nov. 6	+0·033	−0·87 Δk	E.
Nov. 7–12	0·000	−0·91 Δk	W.
Nov. 13–19	+0·032	− 1·19 Δk	E.
Nov. 20–29	+0·007	− 1·18 Δk	W.

Assuming that these are primarily due to changes in the azimuth of the meridian marks produced by the motion of the earth's axis of rotation within the earth itself, we may equate them to the values of the latter derived from the investigations of the international latitude stations. Combining pairs of adjacent determinations taken in opposite clamps, in order more efficiently to eliminate any outstanding errors in the adopted collimation correction, and introducing a constant term ΔA which may arise from an erroneous value in the adopted mean azimuth of the marks, we thus derive the following twenty-four equations of condition.

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Table IX.

Equations of Condition from Comparison with International Latitude Results (R.A.).

Date.		O.	C.
		s	s
1911·07	ΔA − 1·21 Δk	−0·023	= −0·015
·11	ΔA − 1·10 Δk	−0·021	= −0·016
·15	ΔA − 1·00 Δk	−0·009	= −0·013
·21	ΔA − 0·74 Δk	−0·022	− −0·010
·24	ΔA − 0·61 Δk	−0·015	= −0·006
·28	ΔA − 0·45 Δk	−0·001	= −0·002
·32	ΔA − 0·32 Δk	−0·008	= +0·002
·36	ΔA − 0·18 Δk	−0·001	= +0·007
·40	ΔA − 0·10 Δk	+0·011	= +0·012
·44	ΔA − 0·12 Δk	+0·021	= +0·015
·47	ΔA − 0·10 Δk	+0·018	= +0·018
·50	ΔA − 0·07 Δk	+0·024	= +0·021
·55	ΔA − 0·04 Δk	+0·030	= +0·023
·61	ΔA − 0·02 Δk	+0·037	= +0·025
·65	ΔA − 0·02 Δk	+0·022	= +0·024
·67	ΔA − 0·21 Δk	+0·016	= +0·024
·69	ΔA 0·46 Δk	+0·019	= +0·024
·72	ΔA − 0·64 Δk	+0·016	= +0·023
·77	ΔA − 0·79 Δk	+0·021	= +0·020
·81	ΔA − 0·85 Δk	+0·022	= +0·017
·83	ΔA − 0·87 Δk	+0·024	= +0·015
·85	ΔA − 0·89 Δk	+0·017	= +0·014
·86	ΔA − 1·05 Δk	+0·016	= +0·013
1911·88	ΔA − 1·19 Δk	+0·020	= +0·011

The absolute terms (O) on the left are those derived from the present investigation. Those on the right (C) depend on Albrecht's determination of the polar motion. The latter are derived from smoothed curves, and it is difficult either to assign any suitable variation of weight among themselves or to compare their absolute accuracy with that of the quantities O. Hence it seemed best to combine the observations with equal weight. This yielded the following normals:

$$24\Delta A - 13{\cdot}03\Delta k = -0^{s}{\cdot}012$$

$$-13{\cdot}03\Delta A + 10{\cdot}97\Delta k = -0^{s}{\cdot}009$$

whence $\Delta k = -0^{s}{\cdot}0008$, weight 3·9

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The probable error corresponding to unit weight, as dependent on the residuals in the least square solution, is

$$\pm 0^s{\cdot}0047.$$

Thus finally

$$\Delta k = -0^s{\cdot}0008 \pm 0^s{\cdot}0024$$
$$= -0''{\cdot}012 \pm 0''{\cdot}036.$$

The declination observations have been treated in an exactly analogous manner, the latitude corrections playing a role equivalent to that of the instrumental polar corrections in right ascension. The following are the normal equations resulting from the separate groups.

TABLE X.

Normal Equations from Separate Groups of Declinations.

Jan. 16–26	$27{\cdot}4\ \Delta k$	$+\ 2''{\cdot}02 = 0$
Jan. 28–Feb. 8	$43{\cdot}4\ \Delta k$	$+\ 9{\cdot}15 = 0$
Feb. 16–23	$48{\cdot}5\ \Delta k$	$+12{\cdot}90 = 0$
Feb. 24–Mar. 3	$28{\cdot}4\ \Delta k$	$+\ 6{\cdot}14 = 0$
Mar. 13–20	$11{\cdot}2\ \Delta k$	$+\ 2{\cdot}86 = 0$
Mar. 21–30	$15{\cdot}8\ \Delta k$	$-\ 1{\cdot}44 = 0$
Apr. 1–8	$11{\cdot}9\ \Delta k$	$+\ 8{\cdot}48 = 0$
Apr. 12–24	$9{\cdot}4\ \Delta k$	$-\ 0{\cdot}78 = 0$
Apr. 28–May 12	$12{\cdot}6\ \Delta k$	$+\ 3{\cdot}86 = 0$
May 14–27	$3{\cdot}0\ \Delta k$	$+\ 0{\cdot}45 = 0$
May 30–June 9	$1{\cdot}3\ \Delta k$	$+\ 1{\cdot}42 = 0$
June 10–21	$4{\cdot}5\ \Delta k$	$-\ 1{\cdot}50 = 0$
June 25–July 3	$2{\cdot}6\ \Delta k$	$+\ 1{\cdot}02 = 0$
July 4–19	$1{\cdot}9\ \Delta k$	$-\ 0{\cdot}88 = 0$
July 20–Aug. 8	$1{\cdot}4\ \Delta k$	$-\ 0{\cdot}73 = 0$
Aug. 12–26	$11{\cdot}2\ \Delta k$	$-\ 3{\cdot}07 = 0$
Aug. 28–Sept. 2	$10{\cdot}9\ \Delta k$	$+\ 0{\cdot}65 = 0$
Sept. 4–11	$6{\cdot}8\ \Delta k$	$-\ 0{\cdot}20 = 0$
Sept. 12–19	$5{\cdot}4\ \Delta k$	$-\ 2{\cdot}39 = 0$
Sept. 28–Oct. 7	$11{\cdot}3\ \Delta k$	$-\ 1{\cdot}67 = 0$
Oct. 9–20	$9{\cdot}4\ \Delta k$	$+\ 1{\cdot}90 = 0$
Oct. 25–28	$12{\cdot}3\ \Delta k$	$+\ 9{\cdot}28 = 0$
Oct. 30–Nov. 6	$9{\cdot}1\ \Delta k$	$-\ 0{\cdot}86 = 0$
Nov. 7–12	$4{\cdot}4\ \Delta k$	$+\ 2{\cdot}00 = 0$
Nov. 13–19	$33{\cdot}6\ \Delta k$	$+\ 4{\cdot}71 = 0$
Nov. 20–29	$34{\cdot}9\ \Delta k$	$-\ 3{\cdot}13 = 0$
	$372{\cdot}6\ \Delta k$	$+50{\cdot}19 = 0$

with the solution

$$\Delta k = -0''{\cdot}135 \pm 0''{\cdot}033.$$

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The revised corrections to the adopted instrumental latitude for the different epochs are as follows :—

TABLE XI.

Corrections to Instrumental Latitude.

Epoch.	$\Delta\phi$.		Inst.
Jan. 16–26	+0.27″	−1.06 Δk	W.
Jan. 28–Feb. 8	−0.40	−0.96 Δk	E.
Feb. 16–23	−0.01	−0.37 Δk	W.
Feb. 24–Mar. 3	−0.27	−0.45 Δk	E.
Mar. 13–20	−0.40	+0.05 Δk	E.
Mar. 21–30	−0.50	+0.14 Δk	W.
Apr. 1–8	−0.80	0.00 Δk	E.
Apr. 12–24	−0.57	+0.01 Δk	W.
Apr. 28–May 12	−0.48	+0.05 Δk	E.
May 14–27	−0.11	−0.01 Δk	W.
May 30–June 9	−0.45	−0.02 Δk	E.
June 10–21	−0.53	+0.04 Δk	W.
June 25–July 3	−0.31	+0.03 Δk	E.
July 4–19	−0.12	+0.02 Δk	W.
July 20–Aug. 8	0.00	−0.03 Δk	E.
Aug. 12–26	−0.03	−0.34 Δk	W.
Aug. 28–Sept. 2	−0.17	−0.62 Δk	E.
Sept. 4–11	+0.06	−0.92 Δk	W.
Sept. 12–19	−0.09	−1.10 Δk	E.
Sept. 28–Oct. 7	+0.14	−1.25 Δk	W.
Oct. 9–20	+0.05	−1.35 Δk	E.
Oct. 25–28	+0.05	−1.38 Δk	W.
Oct. 30–Nov. 6	−0.03	−1.43 Δk	E.
Nov. 7–12	+0.27	−1.36 Δk	W.
Nov. 13–19	−0.05	−0.96 Δk	E
Nov. 20–29	+0.26	−1.16 Δk	W.

A constant difference between results obtained with clamp E and clamp W may result from the fact that different parts of the divided circle are brought into use in the neighbourhood of the pole in the two positions. This difference may be eliminated by combining into means the results of adjacent groups in opposite clamps. Retaining also a constant $\Delta\phi_0$ to represent a constant error in the adopted mean latitude, we form the equations of condition.

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TABLE XII.

Equations of Condition from Comparison with International Latitude Results (Dec.).

Date.		O.	C.
1911·07	$\Delta\phi_0 - 1·01\ \Delta k$	−0·06	= −0″·12
·11	$\Delta\phi_0 - 0·67\ \Delta k$	−0·21	= −0·15
·15	$\Delta\phi_0 - 0·41\ \Delta k$	−0·14	= −0·19
·21	$\Delta\phi_0 + 0·10\ \Delta k$	−0·45	= −0·26
·24	$\Delta\phi_0 + 0·07\ \Delta k$	−0·65	= −0·28
·28	$\Delta\phi_0 + 0·01\ \Delta k$	−0·68	= −0·32
·32	$\Delta\phi_0 + 0·03\ \Delta k$	−0·53	= −0·35
·36	$\Delta\phi_0 + 0·02\ \Delta k$	−0·30	= −0·36
·40	$\Delta\phi_0 - 0·02\ \Delta k$	−0·28	= −0·38
·44	$\Delta\phi_0 + 0·01\ \Delta k$	−0·49	= −0·38
·47	$\Delta\phi_0 + 0·04\ \Delta k$	−0·42	= −0·38
·50	$\Delta\phi_0 + 0·03\ \Delta k$	−0·22	= −0·37
·55	$\Delta\phi_0 - 0·01\ \Delta k$	−0·06	= −0·31
·61	$\Delta\phi_0 - 0·19\ \Delta k$	−0·02	= −0·23
·65	$\Delta\phi_0 - 0·48\ \Delta k$	−0·10	= −0·18
·67	$\Delta\phi_0 - 0·77\ \Delta k$	−0·05	= −0·16
·69	$\Delta\phi_0 - 1·01\ \Delta k$	−0·02	= −0·13
·72	$\Delta\phi_0 - 1·18\ \Delta k$	+0·03	= −0·09
·77	$\Delta\phi_0 - 1·30\ \Delta k$	+0·10	= −0·03
·81	$\Delta\phi_0 - 1·37\ \Delta k$	+0·05	= +0·02
·83	$\Delta\phi_0 - 1·41\ \Delta k$	+0·01	= +0·04
·85	$\Delta\phi_0 - 1·40\ \Delta k$	+0·12	= +0·06
·86	$\Delta\phi_0 - 1·16\ \Delta k$	+0·11	= +0·07
1911·88	$\Delta\phi_0 - 1·06\ \Delta k$	+0·11	= +0·09

Here the quantities O are derived from the series of observations under discussion, while the quantities C depend on the investigations of the international latitude stations. Combining the equations with equal weights we obtain the normals

$$24·00\Delta\phi_0 - 13·14\Delta k = -0''·23$$
$$-13·14\Delta\phi_0 + 14·89\Delta k = +0''·45$$

whence $\Delta k = +0''·041$, weight 7·74.

The probable error corresponding to unit weight derived from the residuals is $\pm 0''·11$, and that of the derived value of Δk is therefore $\pm 0''·034$.

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We thus have the following four virtually independent determinations of the aberration constant:—

(1) from observations of right ascension,

$$k = 20''\cdot46 \pm 0''\cdot030;$$

(2) from observations of azimuth, compared with Albrecht's determination of the polar motion,

$$k = 20''\cdot46 \pm 0''\cdot036;$$

(3) from declination observations,

$$k = 20''\cdot33 \pm 0''\cdot033;$$

(4) from latitude determinations in combination with Albrecht's determination of the polar motion,

$$k = 20''\cdot51 \pm 0''\cdot034.$$

The simple mean of these four results gives

$$k = 20''\cdot44 \pm 0''\cdot017,$$

where the probable error is estimated from the internal agreement of the constituent material of the four separate results. The discordances between the results themselves point to a somewhat higher value of this probable error, chiefly on account of the outstanding determination (3). The actual result will not be materially changed by any reasonable alteration of the combining weights. Supposing, however, that each result is liable to its own peculiar systematic errors, we may derive the probable error of the combination from the agreement of the four separate determinations as

$$\pm 0''\cdot026.$$

The final value of k ($20''\cdot44$) is somewhat smaller than that derived from the preliminary discussion of the same material contained in the *Cape Annals.*

The erroneous computation of the coefficients of Δk, referred to above, will tend to modify the determinations (1) and (4) of that discussion in favour of the two determinations (2) and (4) of the present one. It appears also that in the determinations (2) and (3) of the former paper, the quantities which figure as corrections to the tabular aberration constant were interpreted with erroneous sign. Hence these determinations should be replaced by

$$k = 20''\cdot36, \; k = 20''\cdot465.$$

These two determinations, with additional material included, are now replaced by the determination (3) of the present paper.

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Suggested re-arrangement of Observations for the Determination of the Aberration Constant.

The aberration factors in right ascension involve the expression $\sin(H+\alpha)$ associated with a negative sign.

Hence at any epoch they will be greatest for those stars for which $H+\alpha=18$ hours, and least for those stars for which $H+\alpha=6$ hours. But we have approximately, on neglecting the obliquity of the ecliptic and the eccentricity of the Earth's orbit,

$$H = 18 \text{ hours.} - \odot,$$

where $\odot$ denotes the Sun's longitude. Hence the maximum aberration factors will occur with stars for which

$$\alpha = \odot,$$

and the minimum with stars for which

$$\alpha = \odot - 12 \text{ hours.}$$

It follows that the most favourable times for observing transits for the determination of the aberration constant are near midday and midnight.

A star for which α approximates to $\odot$ will reach its upper culmination near midday and its lower culmination near midnight; and if both culminations could be observed, the effects of an error in the aberration constant would be entirely eliminated from the resulting determination of azimuth, but thrown with full effect into the determination of R.A. We should have, in fact, from the mean of the two observations,

$$\text{true R.A.} = \text{observed R.A.} + \Delta k \sec \delta \quad . \quad . \quad (a)$$

After an interval of six months, when the value of $\odot$ has increased by 12 hours, the same star will again present itself for observation at midday and midnight, but in reversed culminations, *i.e.* the upper culmination would occur at midnight and the lower culmination at midday. In this case, both at upper and lower culminations, the aberration correction will again have its maximum value, but with the reversed sign. For the second epoch we have

$$\text{true R.A.} = \text{observed R.A.} - \Delta k \sec \delta \quad . \quad . \quad (b)$$

Denoting the observed R.A.'s at the two epochs by α_1, α_2, the combinations of the two equations (a) (b) gives

$$2\Delta k = (\alpha_2 - \alpha_1) \cos \delta.$$

If the same process be repeated with a number of stars, the weight of the final determination would depend on the number of adjacent days and nights on which each star can be observed sufficiently near to midnight and midday, say within one hour or one hour and a half, and also on the number of stars used. By

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making observations at three consecutive epochs of maximum aberration effect instead of two, we could provide for the elimination of any errors of proper motion, while, as changes due to parallax have the same period as those due to aberration, but with complementary phase, the parallax factors throughout will approximate to zero.

The process is, however, inadmissible, as it would require that at least half the observations should be made by daylight, and in the southern hemisphere there are no stars in the immediate vicinity of the pole sufficiently bright to permit of daylight observing. In this latitude consecutive upper and lower transits can be observed during about four months in the year, but then only near the times of sunrise and sunset, when the aberration factor approximates to zero. Recourse must therefore be had to less direct methods. In the light of the analysis of the previous observations, and having regard to the conditions indicated which yield the largest aberration factors, the following process is suggested.

Consider the stars involved divided into four groups centred about R.A.'s 0^h, 6^h, 12^h, 18^h, but which in part overlap.

Group I. consists of stars which at one epoch can be observed at lower transit in the evening and at upper transit in the morning. These stars are included roughly between R.A. $20\frac{1}{2}$ hours and R.A. $3\frac{1}{2}$ hours.

Group II. consists of stars having right ascensions between the approximate limits $1\frac{1}{2}$ hours and $10\frac{1}{2}$ hours, which can be observed at lower culmination at or near midnight at a time of year when azimuths can be derived from consecutive transits, *i.e.* between the middle of April and the middle of August.

Group III. contains stars which can be observed at the upper transit in the evening and at lower transit in the morning at one and the same epoch. The limiting right ascensions are about $8\frac{1}{2}$ hours and $15\frac{1}{2}$ hours.

Group IV. contains stars whose upper culmination occurs near midnight within the period during which the consecutive transits can be observed. The limiting R.A.'s are about $13\frac{1}{2}$ hours and $22\frac{1}{2}$ hours.

If now the observations of any one night during the winter months be divided into three short watches (distributed round six hours before midnight, midnight, and six hours after midnight, when the same series of stars can be observed in the first and third of these watches), from the *mean* of the evening and morning observations of R.A. we may derive the true R.A. of stars of Groups I. and III. independent of azimuth error, provided only that the azimuth of the meridian marks remains sensibly constant through the night. On the other hand, the *difference* between the upper and lower culminations for each star observed at both culminations will yield a determination of azimuth, or rather of instrumental polar error, in the form

$$\Delta p = \tfrac{1}{2}(a_{\text{above}} - a_{\text{below}}) \sec \delta.$$

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This determination is quite independent of the tabular right ascensions, and also of any aberration effects except those due to diurnal aberration, which may be allowed for in the usual way.

During the midnight watch stars which reach their lower culmination will belong to Group II., and those which reach their upper culmination belong to Group IV. In reducing these observations the instrumental correction Δp must be supplied from the evening and morning observations, and the further reductions from apparent to mean place require corrections $+\Delta k \sec \delta$ for stars of Group II. and $-\Delta k \sec \delta$ for stars of Group IV.

In other words, the mean right ascensions from the midnight observations, derived with the tabular value of the aberration constant, may be regarded as determinations of

$$\alpha_{\text{II}} - \Delta k \sec \delta, \quad \alpha_{\text{IV}} + \Delta k \sec \delta,$$

where α_{II} or α_{IV} denotes a typical *mean* R.A. of a star belonging to Group II. or Group IV.

Thus, from observations during the winter months alone, we may derive values of

$$\alpha_{\text{I}}, \quad \alpha_{\text{II}} - \Delta k \sec \delta, \quad \alpha_{\text{III}}, \quad \alpha_{\text{IV}} + \Delta k \sec \delta$$

for each star involved in Groups I., II., III., IV. respectively.

In so far as consecutive groups overlap, the differences between the determinations for the stars common to two groups will in all cases yield a determination of Δk with a factor $\pm$ 1. This factor is relatively small, and the restricted ranges of R.A. common to consecutive groups will limit the number of suitable stars available for such direct use. It is necessary, therefore, to consider how best the determinations may be strengthened by observations taken at epochs of the year when the stars available for observation can be seen at a single culmination only.

At these times the star places, as derived by the preceding processes dependent on the winter observations, may be regarded as tabular places to be used in conjunction with the additional observations for the determination of the instrumental polar correction at the instant of each of these additional observations. This polar correction is computed by the formula

$$\Delta p = (\alpha_o - \alpha_c) \cos \delta \quad . \quad . \quad . \quad . \quad \text{(A)}$$

for stars at upper culmination, or

$$\Delta p = (\alpha_c - \alpha_o) \cos \delta \quad . \quad . \quad . \quad . \quad \text{(B)}$$

for stars at lower culmination; where α_o denotes an observed R.A. and α_c the corresponding tabular R.A., both being considered as computed with the tabular value of the aberration constant.

Now both α_o, α_c as here involved will be liable to error in consequence of an erroneous value of the aberration constant used in reduction. (The comparison is supposed to be made through the reduced *mean* places.) This error will vary from star to star, and consequently an erroneous aberration constant will exhibit

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itself as an apparent change in the polar error in the course of a night.

If the tabular places a_c were accurately known, the correction to Δp as derived by the formulæ (A) or (B) (*i.e.* either from upper or lower culminations) will go through approximately the same sequence of changes each day, ranging from $-\Delta k$ at midday to $+\Delta k$ at midnight.

By comparing determinations at the extreme range available we shall secure the most favourable factors for determination of Δk—*i.e.* it is desirable that observations should be made at midnight on the one hand, and as near to dusk or dawn on the other as the intervention of daylight will permit.

The requisite conditions would be attained if all the stars concerned, whether observable at midnight, or at dusk or dawn, were included in Groups I. and III., since we have seen that the tabular mean places for such stars can be derived freed from errors in aberration.

At midsummer, however, the stars which present themselves for observation at midnight belong to Groups II. and IV., Group II. being available for above-pole determinations and Group IV. for below-pole determinations. For these stars the tabular right ascensions, as derived from observations associated with double transits in the winter months, require corrections $+\Delta k \sec \delta$ for upper culminations and $-\Delta k \sec \delta$ for lower culminations. Hence the value of Δp derived from midnight observations will require an additional correction Δk, making a total correction $+2\Delta k$.

The dusk and dawn observations will depend on stars belonging to Groups I. and III. The error in Δp resulting from an erroneous aberration constant will therefore be solely that introduced through the medium of the observed quantity a_o. The factor associated with Δk for these observations will be small, but (considered as a correction to Δp) of the same sign as that involved in midnight observations. In the most unfavourable circumstances (at exact midsummer when the nights are shortest) the least value of this factor for this latitude will be about 0·5. We thus have

true polar correction = polar correction from midnight observations + $2\Delta k$

true polar correction = polar correction from dusk or dawn observations + $0·5\Delta k$,

whence, if the true polar correction may be considered invariable

$1·5\Delta k$ = (polar correction from dusk or dawn observations) − (polar correction from midnight observations).

As the night lengthens, on either side of midsummer, the range of factor will increase and more favourable determinations of Δk will be possible so long as the stars presenting themselves at dusk, midnight, and dawn belong to the same groups as above.

Still more favourable will be the circumstances when stars

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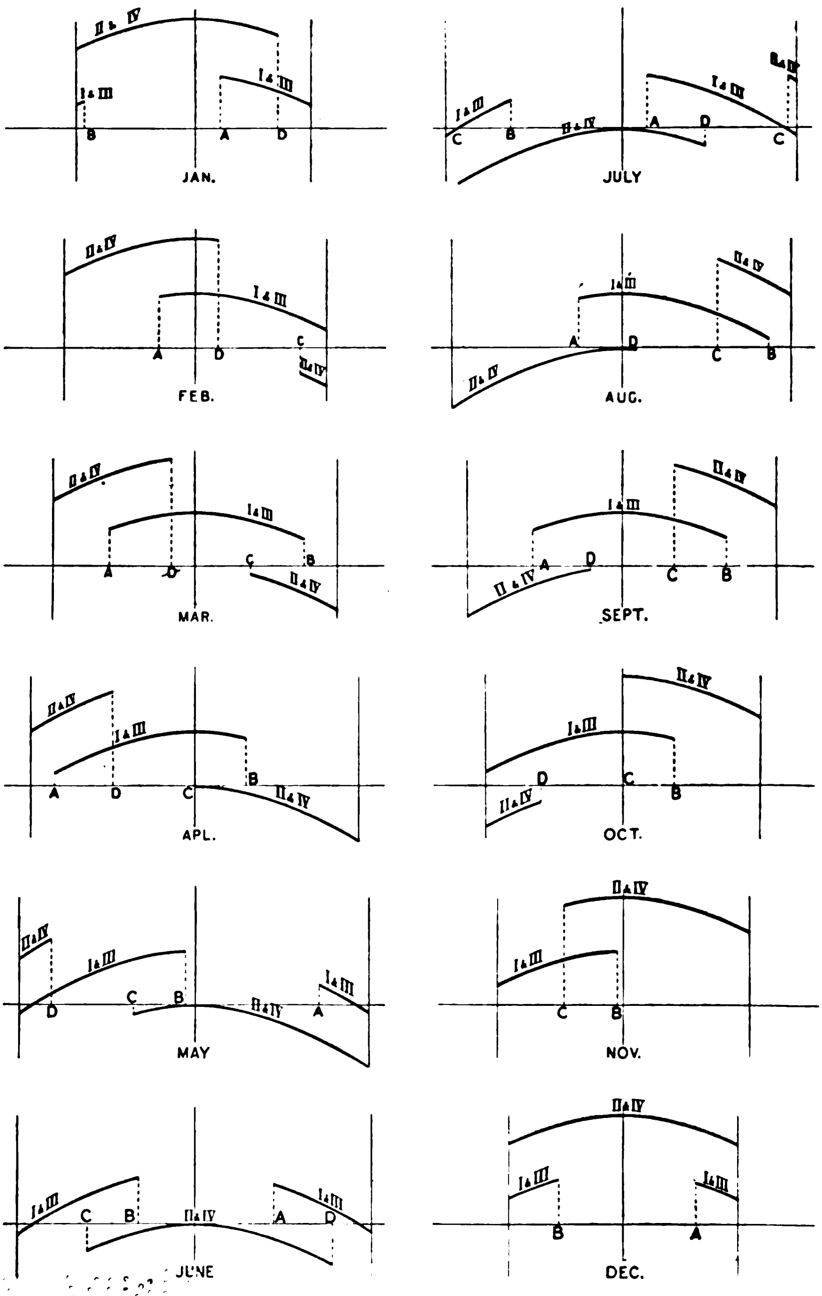

Aberration Factors in Azimuth.

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belonging to Groups II. and IV. are available for either evening or morning determinations with the same groups, but at opposite culminations, at or near midnight. These circumstances will occur at intermediate times between midsummer and midwinter. For the dusk or dawn observations of stars of Groups II. and IV. the factor associated with Δk, in Δp, must be diminished by unity on account of the error introduced through the medium of the adopted tabular place. The factor introduced through the medium of the observation will be small and generally positive, say about 0·2. Hence the total factor for the dusk or dawn observation will be about $-0·8$ as compared with $+2·0$ for the midnight observation, giving a total range in Δp for a night's observing of $2·8\Delta k$.

The most favourable circumstances for each month in the year are indicated graphically in the annexed diagram, Plate 4, the different figures in which illustrate the circumstances as computed for the latitude of the Cape at about the middle of each month.

The abscissæ represent times of the night, the central ordinate corresponding in all cases with midnight, and the extreme ordinates with the times when the Sun is 6° below the horizon.

The curves, which all form parts of curves of sines, represent the aberration factors in Δp in the sense in which they must be applied as corrections to the observed values, derived through the medium of tabular places dependent on the winter observations, and reduced with the tabular value of the aberration constant. The discontinuities in the curve result from the method in which the tabular places are derived, whether from dusk or dawn observations (Groups I. and III.) or from midnight observations (Groups II. and IV.).

Groups I. and III. are regarded as commencing with the star Lacaille 8257, which transits above pole at $20^h\ 23^m$ and below pole at $8^h\ 23^m$, and ending with Lacaille 1414, which transits above pole at $3^h\ 40^m$ and below pole at $15^h\ 40^m$. These times are indicated on the figures by the ordinates marked A and B.

Groups II. and IV. start with Gilliss P.Z. 1219 at $1^h\ 30^m$ or $13^h\ 30^m$, and finish with Lacaille 9102 at $10^h\ 30^m$ or $22^h\ 30^m$. The times of transit of these stars are indicated by the ordinates marked C and D.

The most favourable times of observing at any epoch are those which approximate to the greatest and least ordinates in the corresponding figure. Where one of these extreme ordinates intersects two branches, the particular branch which yields the greatest or least factor is clearly seen, and the tabular place to be used is that of the star derived as a member of the corresponding group.

Similar figures are also given, Plate 5, for the declination observations, in which latitude determinations play an analogous role to the quantity Δp involved in the right ascensions. Here,

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however, the factor introduced through the medium of the observation itself decreases continuously from dusk to dawn, without passing through a maximum or minimum value, while at midsummer the additional factors introduced through the medium of the tabular position as derived in winter combine with this in a cumulative direction. The tabular declinations of Stars of Groups II. and IV. will be free from aberration error, while those of stars in Groups I. and III. will be in error by an amount $\pm\Delta k$. The total variation in apparent latitude between dusk and dawn at midsummer amounts to about $3{\cdot}6\Delta k$. This factor will, however, rapidly decrease as the winter is approached. On the whole, an increase in latitude would be more favourable for R.A. determinations and less favourable for declination determinations.

The essential difference between the proposed method and the classical methods employed at Pulkovo consists in the use of a continuous sequence of stars following one another at short intervals throughout the twenty-four hours of right ascension, instead of restricting the observations to a limited star list.

In the investigation dealt with in the first part of this paper, a considerable number of observations were involved which added little or no weight to the ultimate derivation of the aberration constant, though they were effective in strengthening the determination of the star position. Further, owing to the absence of midnight observations of right ascension in the winter and relatively short duration of the watches in the summer, the extreme factors attained probably averaged only about half those attainable.

There should be little difficulty, provided the observing strength is concentrated on those observations most effective for the aberration problem, in arriving at a determination carrying about three or four times the weight of the former as the outcome of a single year's observations.

By extending the programme over a longer interval, complete provision could be made for the elimination of proper motion and precession effects.

The parallax factors in right ascension are zero at midnight and have their greatest values, but with opposite signs, near dusk and dawn. If, therefore, the observations used for comparison with the midnight observations of right ascension be symmetrically distributed between dusk and dawn, the tendency will be to eliminate any small residual effects due to parallax. The most effective observations of declination are always made near dusk and dawn when the parallax factors are insignificant.

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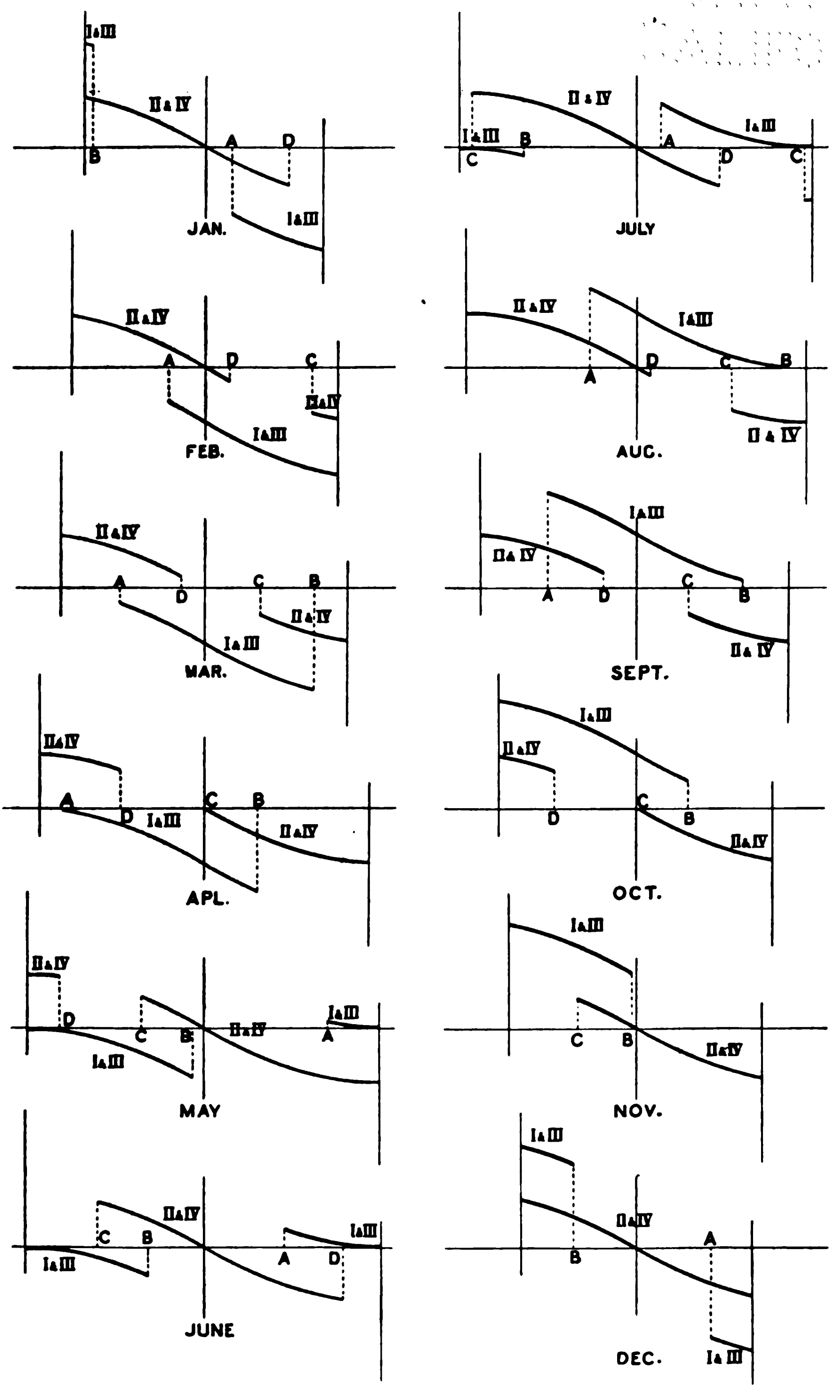

Aberration Factors in Latitude.

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The Motion of the Perihelion of Mercury deduced from the classical Theory of Relativity. By L. Silberstein, Ph.D., Lecturer in Natural Philosophy at the University of Rome.

It is well known that as early as 1845 Le Verrier found that the motion of the perihelion of Mercury, as derived from observations of transits, was greater by 38″ per century than it should be from the perturbation due to all the other planets of our system. A recent discussion of the subsequent investigations has shown the excess of motion to be about 5″ greater, viz., per century,

$$\delta\varpi = +42''{\cdot}9.$$

Equally well known are the attempts of Newcomb and of Seeliger to account for this excess of motion of Mercury's perihelion. A discussion of Seeliger's results (which, broadly speaking, were very satisfactory) and a modification of his treatment have been given by H. Jeffreys (*M.N.*, vol. lxxvii. p. 112). On the other hand, a great sensation has been recently produced among astronomers by the surprising circumstance that Einstein's newest "*generalised* theory of relativity" has yielded for the said excess just its full value, *i.e.*, in round figures, 43″. In fact, Einstein gives in his recent paper,* for the angle ϵ through which the elliptic orbit of a planet is turned, in the direction of motion, per period T, the formula

$$\epsilon = 24\pi^3 \frac{a^2}{T^2c^2(1-e^2)}. \qquad \text{(G)}$$

where a, e stand for the major semi-axis and the eccentricity of the orbit, and c is the velocity of light in empty space. Substituting $a = 0{\cdot}3871 \,.\, 1{\cdot}49 \,.\, 10^6$ km., $e = 0{\cdot}206$, $T \doteq 87{\cdot}97$ days, $c = 3 \,.\, 10^5$ km./sec., the reader will find, for $\delta\varpi$ per century, $43''{\cdot}1$, which is the desired angle. The reason why I have denoted the above formula by (G) is, with all respect due to Einstein, that identically the same formula was given eighteen years earlier by Gerber,† whose investigation, entirely independent of any relativity ("old" or "new"), seems to have passed unobserved, most likely owing to its badly supported fundamental assumptions. To enter upon these latter would not answer the purposes of the present paper. It may, however, be interesting to notice that Gerber replaces Newton's potential $\frac{M}{r}$ by $\frac{M}{r\left(1 - \frac{1}{c}\frac{dr}{dt}\right)^2}$, where c is the "velocity of propagation of the gravitation potential";

* *Annalen der Physik*, vol. xlix., 1916, pp. 769–822. See also Professor de Sitter's papers in *M.N.*, vol. lxxvi., 1916, p. 699, and vol. lxxvii. p. 155.

† P. Gerber, *Zeitschr. math. Phys.*, xliii., 1898, pp. 93–104. A short account of Gerber's theory is given in *Enc. d. math. Wiss.*, vol. V. 1, pp. 49–51; a still shorter, and very unfair, account is given by Herr E. Gehrcke in *Annalen der Physik*, li., 1916, pp. 122–124.

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rejecting the third, and the higher, powers of dr/cdt, Gerber obtains for any (isolated) planet in its motion round the central body the above formula. It is historically interesting that Gerber does not identify c with the velocity of light, but determines its value from the observed excess of the secular motion of the perihelion of Mercury, and finds $c = 305500$ km./sec., *i.e.* "surprisingly near the light velocity." Thus, whatever his theory, the formula (G), accounting for *the full excess* of Mercury's perihelion motion, will appropriately be called *Gerber's formula.*

Now, to repeat it, Gerber has deduced his formula from an untenable theory, or at least from one which has not been based upon well-established general principles. Einstein, eighteen years later, but undoubtedly without knowing Gerber's formula, has rediscovered it by deducing it from his "generalised" theory of relativity, which, in its turn, is again very far from being well established. In fact, notwithstanding its broadness and mathematical elegance, it certainly offers many serious difficulties in its very foundations, while none of its predictions of new phenomena, as the deflection of a ray by the sun, have thus far been verified. And even the fact that Einstein's new theory gives Gerber's formula, and therefore the *full* excess of $43''$ for Mercury, does not seem to be decisive in its favour. As far as I can understand from Jeffreys's investigation,* it would rather alleviate the astronomer's difficulties if the Sun by itself gave only a *part* of these 43 seconds.

Under such circumstances it has seemed worth while to investigate, in as general a way as possible, how much of that excess will be accounted for by the ordinary or, as Einstein now calls it, the "old" theory of relativity, retaining the *constancy* of the velocity of light, and, of course, its independence of any gravitational field. It will be remembered that this "old" theory is not only unobjectionable in its foundations and considerably simpler than the "new" theory of relativity, but that it has also the physical advantage of accounting for a good number of important facts: it harmonises with the null-effects of Michelson-Morley's and of Trouton-Noble's experiments, it stands the test of experience in the domain of the best observations on β-particles, and it represents Fresnel's dragging coefficient in the most direct way, viz. as a simple corollary of kinematics.

So much to justify a renewal of the investigation of planetary motion on the basis of the "old" theory of relativity, as laid down in Einstein's famous paper of 1905, and developed formally by Minkowski two years later. The planetary motion has been treated by Minkowski himself in an appendix to his memoir (*Grundgleichungen, etc.*, of 1907 Dec. 21), in which, however, he limits himself to a short sketch and concludes by saying that the proposed relativistic modification would hardly show any observable deviations from the results of ordinary Newtonian mechanics. In

* *Loc. cit.*, see especially p. 113, and the final paragraph of the paper, p. 118.

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fact, his law of gravitation would correspond in our notation (*cf. infra*) to $N = M_0 m_0/\gamma r^2$, and this gives, for a small planet (m_0/M_0 negligible),

$$\epsilon = 0 \quad . \quad . \quad . \quad . \quad . \quad (M)$$

In more recent times the problem of planetary motion has been treated relativistically by Dr. Wacker (*Dissertation*, Tübingen, 1909), whose assumptions amount to adopting the law of force $N = M_0 m_0/r^2$, leading to

$$\epsilon = \tfrac{1}{6}(G), \text{ i.e. } \delta\varpi = 7''\cdot 2 \text{ for } ☿ \quad . \quad . \quad . \quad (W)$$

in words, to $\frac{1}{6}$ of Gerber's value quoted at the outset.* This is at the same time just $\frac{1}{6}$ of the full observed excess for Mercury. Now, there is nothing particular about these two special laws. We shall see that the problem can be treated more broadly, and so as to account for the full observed excess. In our opinion the retouching of Newton's gravitation law must be guided by experience (observation) and not by guesswork or predilection for particular integers. At the same time we shall use the opportunity of simplifying the mathematical treatment of the corresponding equation of motion, which is usually, and without any intrinsic reasons, being made formidably complicated.

Let **r** be the vector (x, y, z) drawn from the Sun to the planet, a system attached to the former being our reference system S. Let t be the S-time, and $\mathbf{v} = d\mathbf{r}/dt$ the (ordinary) vector velocity of the planet with respect to S. Write

$$\gamma = (1 - \beta^2)^{-\frac{1}{2}}, \ \beta = v/c \quad . \quad . \quad . \quad (1)$$

where c is the velocity of light, assumed to be constant and invariant, of course. Let m_0 be the *rest-mass* of the planet, a constant by assumption, and

$$m = m_0 \gamma \quad . \quad . \quad . \quad . \quad (2)$$

Finally, let **N**, an ordinary three-dimensional vector (like **r** or **v**) be the "Newtonian force" acting upon the planet. Then the equation of motion of the planet will be †

$$\frac{d}{dt}(m\mathbf{v}) \equiv \frac{d}{dt}\left(m\frac{d\mathbf{r}}{dt}\right) = \mathbf{N} \quad . \quad . \quad . \quad (3)$$

This can be read: The rate of increase of momentum, $m\mathbf{v}$, is equal to the force **N**. The only departure from Newtonian mechanics is that the inertia-coefficient m is not a constant but a function of velocity, precisely as the familiar transversal mass of an electron.

* In revising the MS., I learn from Professor Eddington that de Sitter obtains, in *M.N.*, vol. lxxi., 1911, on Minkowski's and Lorentz's law of force, for Mercury, $\delta\varpi = 0$ and $\delta\varpi = 7''\cdot 15$ respectively, which are evidently Minkowski's and Wacker's results.

† The reader unacquainted with the "old" theory may consult the author's *Theory of Relativity*, Macmillan, 1914, pp. 192 *et seq.*, where m is identical with the present m_0.

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If the reader does not like even the "old" relativity, but is progressive enough to admit the inertia of energy, he may write $m = m_0 + \frac{1}{2} m_0 \frac{v^2}{c^2}$, or $m = m_0(1 + \frac{1}{2}\beta^2)$, which to all purposes is the same thing as $m_0\gamma$. In fact, we shall ultimately retain but the second power of β, rejecting all higher powers of this exceedingly small fraction. Thus (3) as an equation of motion has nothing unfamiliar about it.

Next, without yet specifying the dependence of the force upon distance, let **N** be a central force, say,

$$\mathbf{N} = -\mathrm{N}\mathbf{u}, \quad . \quad . \quad . \quad . \quad (4)$$

where **u** is a radial unit vector drawn away from the Sun, and N the absolute value of the force. With this symbol we shall have also $\mathbf{r} = r\mathbf{u}$. Under these circumstances the vector product **VrN** will vanish, and therefore by (3)

$$0 = \mathrm{V}\mathbf{r}\frac{d}{dt}(m\mathbf{v}) = \frac{d}{dt}(m\mathrm{V}\mathbf{r}\mathbf{v}).$$

Thus

$$m\mathrm{V}\mathbf{r}\mathbf{v} = \mathbf{k}$$

will be a constant vector both in size and in direction, relative to the system of reference S, of course. The planet moves in a plane normal to **k**, as it would according to Newtonian mechanics. The only difference is that m is slightly greater than its constant rest-value m_0. In other words, the *moment of momentum*, **k**, is a constant vector, as with Newton, but the inertial coefficient m is slightly variable.* Writing k for the absolute value of **k**, we have, in terms of the usual polar co-ordinates, r, θ,

$$mr^2\frac{d\theta}{dt} = k \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Using this equation in (3), we have, by (4),

$$-\mathrm{N}\mathbf{u} = \frac{k}{mr^2}\frac{d}{d\theta}\left(m\frac{d\mathbf{r}}{dt}\right) = \frac{k^2}{mr^2}\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{d\mathbf{r}}{d\theta}\right).$$

Now, remembering that $\mathbf{r} = r\mathbf{u}$, the reader will easily verify the identity

$$\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{d\mathbf{r}}{d\theta}\right) = -\left(\frac{d^2\rho}{d\theta^2} + \rho\right)\mathbf{u},$$

where $\rho = \frac{1}{r}$. Thus the equation of motion gives the scalar differential equation for the orbit,

$$\frac{d^2\rho}{d\theta^2} + \rho = \frac{mr^2}{k^2}\mathrm{N}, \ \rho = \frac{1}{r} \quad . \quad . \quad . \quad (6)$$

where N is the absolute value of the central attractive force.

* These somewhat lengthy remarks are intended for those readers who are not at all familiar with relativity.

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Thus far we have only assumed that **N** is a central force, N**u**, without specialising its absolute value or intensity N. Now, according to Newton's gravitation law we should have, in appropriate units, $N = M_0 m_0 / r^2$, where M_0 is the constant and, as we shall assume, relativistically invariant *rest-mass** of the Sun. Instead of this let us take

$$N = \frac{M_0 m_0}{r^2} f(\gamma) \quad . \quad . \quad . \quad . \quad (7')$$

where f is an *arbitrary function* of $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$, to be determined from experience, *i.e.* from astronomical observations. To begin with, we know only that, apart from second-order effects, Kepler's three laws, and therefore Newton's law of force, have to reappear. Thus we shall require $f(1) = 1$, and since the higher powers of β^2 are ultimately rejected, we can without any loss to generality write $f(\gamma) = \gamma^{n-1}$, that is,

$$N = \frac{M_0 m_0}{r^2} \gamma^{n-1} \quad . \quad . \quad . \quad . \quad (7)$$

where n is *any real number.* Its value (or the form of f) will be determined from the observed excesses of the motion of perihelia, which are second-order effects in the precise, technical sense of the word. Meanwhile we shall leave n undetermined.

As to relativistic requirements, the vector $\gamma \mathbf{N} = -\gamma N \mathbf{u}$ as given by (7′) or (7) is the space-part of a genuine four-vector or physical quaternion, as it should be. Thus the equation of motion (3), with (7), satisfies the Principle of Relativity. (For proof of the above statement see Note at the end of the paper.)

Introducing the law (7) into (6), we have, as the equation to the planetary orbit,

$$\frac{d^2\rho}{d\theta^2} + \rho = \frac{M_0 m_0^2}{k^2} \gamma^n \doteq \frac{M_0 m_0^2}{k^2}\left(1 + \frac{n}{2}\beta^2\right) \quad . \quad . \quad (8)$$

Next, multiplying the original equation of motion by **v**, scalarly, we have the equation of energy

$$c^2 m_0 \frac{d\gamma}{dt} = (\mathbf{N}\mathbf{v}) = M_0 m_0 \gamma^{n-1} \frac{d\rho}{dt},$$

whence, for any $n \neq 2$,† and denoting by E the integration constant, which is the total energy,

$$\gamma^{2-n} = (2 - n)\left(\frac{M_0 \rho}{c^2} + \frac{E}{m_0 c^2}\right),$$

* It will be noted that rest-masses may, but need not necessarily, be relativistic invariants. See, for instance, pp. 247 *et seq.* of my *Theory of Relativity.*

† And if $n = 2$, then $\log \gamma = \frac{M_0 \rho}{c^2} + \frac{E}{m_0 c^2}$, and, up to β^4, $\frac{1}{2}\beta^2 = \frac{M_0 \rho}{c^2} + \frac{E}{m_0 c^2}$, as in (9), apart from the meaning of the constant term.

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or, up to β^4, and including the constant additive term $\frac{1}{n-2}$ in the energy constant,

$$\tfrac{1}{2}\beta^2 = \frac{M_0\rho}{c^2} + \frac{E}{m_0c^2} \quad . \quad . \quad . \quad . \quad (9)$$

Introducing this value into (8) and writing

$$g^2 = 1 - n\left(\frac{M_0m_0}{kc}\right)^2, \qquad C = \frac{M_0m_0^2}{g^2k^2}\left(1 + \frac{nE}{m_0c^2}\right) \quad . \quad (10a)$$

(and zero instead of the last term in the case $n = 2$), we have the differential equation of the orbit

$$\frac{d^2\rho}{d\theta^2} + g^2\rho = Cg^2 \quad . \quad . \quad . \quad . \quad (10)$$

where $\rho = 1/r$. Thus if θ is reckoned from the perihelion

$$\frac{1}{r} = \left(\frac{1}{r_0} - C\right)\cos g\theta + C,$$

whence also, for $\theta = \frac{\pi}{2}$, $1/C = r_{\pi/2} = a(1 - e^2)$, while $r_0 = a(1 - e)$.

Finally, therefore, the orbit becomes

$$\frac{1}{r} = \frac{1 + e\cos g\theta}{a(1 - e^2)} \quad . \quad . \quad . \quad . \quad (11)$$

i.e., if $e < 1$, *an ellipse with moving perihelion.* At the same time we have, by (10*a*), and since $C = 1 : r_{\pi/2}$,

$$\frac{1}{a(1 - e^2)} = \left(1 + \frac{nE}{m_0c^2}\right)\frac{M_0m_0^2}{g^2k^2} \quad . \quad . \quad . \quad (11a)$$

The angle ϵ through which the perihelion moves (or the orbit is turned round) in the direction of motion, *per period* of revolution, is, by the first of (10*a*),

$$\epsilon = 2\pi\left(\frac{1}{g} - 1\right) = 2\pi\left[1 - n\left(\frac{M_0m_0}{kc}\right)^2\right]^{-\frac{1}{2}} - 2\pi\,;$$

that is, with the rejection of the fourth power of the small fraction M_0m_0/kc,

$$\epsilon = n\pi\left(\frac{M_0m_0}{kc}\right)^2.$$

Here we can write, with more than sufficient accuracy, by (5), $k = mr^2\dot\theta \doteq m_0r^2\dot\theta = 2\pi abm_0/T$; and, by (11*a*), $M_0 = 4\pi^2a^3/T^2$, where T is the period of revolution and $b = a\sqrt{1 - e^2}$, the minor semi-axis.

Ultimately, therefore, the motion of the perihelion, per period, becomes

$$\epsilon = 4n\pi^3\frac{a^2}{T^2c^2(1 - e^2)}.\text{*} \quad . \quad . \quad . \quad (12)$$

* The two cases treated by Minkowski and by Dr. Wacker, as mentioned before, correspond to $n = 0$ and $n = 1$ respectively. Gerber's formula (G) follows for $n = 6$.

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Thus the law of force (7),

$$N = M_0 \frac{m_0}{r^2} \gamma^{n-1} = M_0 \frac{m}{r^2} \gamma^{n-2},$$

or its four-dimensional equivalent, satisfying the Principle of Relativity, gives the motion of perihelion (12), where n is *any* real number. The *numerical value* of n, it would seem, *is to be decided upon the basis of astronomical observations.* If it is true that Mercury's excess of 43″ per century has to be thrown *entirely* upon the Sun, then make

$$n = +6;$$

and if only in part, then make it less than six. There is, however, a strong presumption that the whole excess is to be attributed to the Sun's gravitation. If so, then n, determined from Mercury's $\delta\varpi$, is just (or very nearly) *six*, and gives for Venus the value $\delta\varpi = 8''\cdot 6$. The required law of force would then be

$$N = M_0 \frac{m_0}{r^2} \gamma^5 = M_0 \frac{m}{r^2} \gamma^4 \quad . \quad . \quad . \quad (7a)$$

where $m = m_0\gamma \doteq m_0\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$ is, according to the well-established proportionality of heavy and inert mass, the *heavy* mass of the planet. Why n is just six, or the exponent of γ in the latter formula just four, I am sure I do not know. But as little do we know "why" the exponent of r is "just" or exceedingly nearly equal -2.

Such a naturalistic method of improving Newton's law of gravitation seems a great deal safer than those based on fantastic constructions or rash generalisations. Since the capital part of that great law, the value of the exponent of r, has (through Kepler's empirical laws) been ultimately derived from and solely supported by experience, viz. by the observations on the shape of the orbits, it seems but natural that the exponent of a retouching factor, such as γ, should be determined from refined experience, viz. from the observed true excess of the secular rotation of the axes of those orbits. At any rate, the "old" Principle of Relativity has turned out to be broad enough to give us a (very simple and) sufficiently general form of the retouching factor of Newton's law, as required for the said secular excesses.

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1917 *March.*

Note to page 507.—In order to see that the law of force (7′), and therefore also (7), satisfies the Principle of Relativity, let us return to the general equation of motion (3). If τ be the proper time of the planet, then $d/dt = d/\gamma d\tau$, so that (3) becomes

$$\gamma \mathbf{N} = m_0 \frac{d}{d\tau}(\gamma \mathbf{v}) = m_0 \frac{d^2\mathbf{r}}{d\tau^2}.$$

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Now, m_0 is, by assumption, a relativistic invariant, and $d^2\mathbf{r}/d\tau^2$ is the space-part of a relativistic four-vector or (as I call it) of a physical quaternion, viz. $d^2q/d\tau^2$, where $q = \mathbf{r} + ict$; and τ, as is well known, is an invariant. Thus the relativistic requirement is that $\gamma\mathbf{N}$ (not $\mathbf{N}$ itself) *should be the space-part of a four-vector* (or the vector of a physical quaternion) X, viz.

$$\gamma\left[\mathbf{N} + \frac{i}{c}(\mathbf{N}\mathbf{v})\right] = X = \frac{d^2q}{d\tau^2} \quad . \quad . \quad . \quad (13)$$

Now, let $Y = dq/d\tau$ be the four-velocity of the planet, and similarly $Y_s = dq_s/d\tau_s$ that of the Sun, and write $R = q - q_s = \mathbf{r} - \mathbf{r}_s + ic(t - t_s)$, so that Y, Y_s, R are all four-vectors. With these symbols, and denoting by ( ) the "scalar product" of two four-vectors, write

$$X = \frac{F(YY_s)}{(RY_s)^3}\{(Y_sY)R - (YR)Y_s\} \quad . \quad . \quad (14)$$

where F is any function of (YY_s). Then, the bracketed expression being a four-vector, and its scalar coefficient even an invariant, X will be a genuine four-vector as required. In fact, the above X is but a large sub-case of the four-vectors constructed by Poincaré (*Rend. del Circolo Mat. di Palermo*, xxi., fasc. i., 1906). In comparing with his memoir notice that Poincaré's C is our $-\frac{1}{c^2}(YY_s)$. Now, the space-part of (14) is precisely of the form of the law of force (7′). To see this, remember that our reference system S has been the rest-system of the attracting centre (the Sun), and this will be a legitimate system provided that m_0/M_0 is small.* This means that $\mathbf{r}_s = 0$, $\mathbf{v}_s = 0$, and therefore $Y_s = ic\gamma_s = ic$, and $c(t - t_s) = -r$, *i.e.* $R = \mathbf{r} - ir$. Under these circumstances we have

$$(YY_s) = -c^2\gamma, \quad (RY_s) = cr,$$

and therefore for $\gamma\mathbf{N}$, or the vector of the quaternion (14),

$$\gamma\mathbf{N} = F(-c^2\gamma) \cdot \frac{\mathbf{r}}{c^3r^3} = \frac{F(-c^2\gamma)}{c^3}\frac{\mathbf{u}}{r^2},$$

or N equal an arbitrary function of γ divided by r^2, as in formula (7′), of which the adopted law of force (7) is but a particular case.

* In the case of Mercury this is at least as small as 1 : 5,600,000, and, according to Newcomb, even as 10^{-7}. It is also sufficiently small for Venus, etc. For a double star, of course, (14) itself would have to be used, without the simplification which has been possible in our case.

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The Third Line Spectrum of Oxygen. By A. Fowler, F.R.S., and J. Brooksbank, B.Sc., D.I.C. (Plate 6.)

One of the most interesting problems of experimental astrophysics is that of the reproduction of lines which occur in the spectra of stars of early type, and in nebulæ. Assuming the identity of celestial and terrestrial matter, the method of experiment would seem to be clearly indicated by the important results which have previously been obtained in this connection. The investigations of silicon, carbon, nitrogen, and sulphur, carried out by Lockyer and his assistants,* have established that in the case of these elements the spectrum can be successively changed by increasing the energy of excitation, and that the lines which appear at the different stages in the laboratory experiments appear with greatest intensity at successively higher stages of the stellar sequence. Several lines of the early B stars were identified by Lockyer in this way with lines produced by relatively strong discharges in the case of the elements above named. Subsequent work on helium † showed that a modified spectrum of this gas, also produced by very strong discharges, accounted for the line 4686 and the lines of the Pickering series, which occur not only in the earliest B stars, but also in Wolf-Rayet stars and gaseous nebulæ. The more recent work of Merton ‡ similarly revealed a modification of the line spectrum of carbon beyond that observed by Lockyer, and showed that these lines were probably identical with some of the most characteristic lines in the spectra of Wolf-Rayet stars.

It has seemed to us important to extend such observations to as many substances as possible, with a view to the identification of additional stellar lines, and to enlarge the range of spectrum observed for the purpose of investigating the constitution of the spectra produced at the various stages. One of the substances we have studied is oxygen, and although but few of the lines developed under the most intense discharges fall within the range of spectra which can be observed in celestial bodies, it may be useful to present a brief account of the results obtained, in so far as they may be of use in connection with stellar spectra.

Two distinct line spectra of oxygen were already well known, under the names "compound line" and "elementary line" spectra,

* *Proc. Roy. Soc.*, vol. lxxiv. p. 296 (1904); vol. lxxxii. p. 532 (1909), and other papers.

† A. Fowler, *M.N.*, vol. lxxiii. p. 62 (1912); *Phil. Trans.*, A, vol. ccxiv. p. 254 (1914).

‡ *Proc. Roy. Soc.*, A, vol. xci. p. 498 (1915).

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assigned by Schuster in 1879.* These spectra are produced respectively by uncondensed and comparatively weak condensed discharges, and the names given by Schuster were based upon the supposition that complex and simplified molecules were involved in the two cases. In connection with their discovery that the lines of the compound spectrum could be arranged in well-defined series, Runge and Paschen † proposed to call this the "series spectrum" of oxygen, but the name does not seem particularly appropriate, as it is by no means certain that the elementary line spectrum will not also be found to include series lines.

The lines of oxygen which were shown to be present in the solar spectrum by Runge and Paschen ‡ are the components of a narrow triplet in the extreme red (near λ 7770) which forms part of the compound line spectrum. It was the elementary line spectrum, requiring greater energy for its production, which was discovered in some of the B stars by McClean in 1897,§ and subsequently confirmed by Gill and by Lockyer.

By analogy with metallic spectra the compound line and the elementary line spectra would appear to correspond with arc and spark spectra respectivelv, so that the elementary line spectrum might properly be called the spark or enhanced line spectrum of oxygen. Such a designation would be in entire agreement with the occurrence of the elementary line spectrum in early type stars.

The probability of a further modification of the line spectrum of oxygen was first suggested by the early observations of Schuster and Roscoe,|| who found that two lines in the green and greenish yellow "only appeared at very high temperatures,"—that is, when a condensed discharge was passed through the gas in a short and narrow capillary tube. The wave-length of one of these lines was given as 5591, which may be put at 5592 on the scale of Rowland; the position of the second line was not stated, and we have not identified it with certainty. The line at λ 5592 is apparently identical with one of those assigned to carbon by Merton, but our experiments agree with those of Schuster and Roscoe in showing it to be due to oxygen. It appears with undiminished strength as the oxygen is purified, and persists when all known lines of carbon, including those discovered by Merton in association with λ 5592, have disappeared. It is probable that in Merton's experiment the line was derived from the oxygen of carbon dioxide.

In 1906 additional lines of the type of 5592 were observed by Lunt, though he was apparently unacquainted with the above-mentioned observation by Schuster and Roscoe. These lines—in

* *Phil. Trans.*, vol. clxx. p. 41 (1879).
† *Astrophys. Journ.*, vol. viii. p. 70 (1898).
‡ *Ibid.*, vol. iv. p. 317 (1896).
§ *Proc. Roy. Soc.*, vol. lxi. p. 215; vol. lxii. p. 418.
|| *Mem. Manchester Lit. and Philos. Soc.*, 3rd series, vol. vii. p. 82 (1880).

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the near ultra-violet—were found by Lunt to be brought out by reducing the pressure of the gas, or by widening the air-gap in the secondary circuit when the pressure of the gas was such as would not otherwise have given rise to them. The brightest of them were at wave-lengths 3754·9, 3757·4, 3760·0, and 3961·8.* Lunt concluded that these lines were probably due to oxygen, but, in view of their peculiar behaviour, the possibility of their being due to an impurity was not overlooked.

Our observations are in entire agreement with those of Lunt, and we have found no reason to suppose that any of the lines in question are due to substances other than oxygen. These lines, in fact, only appear with any considerable strength under the action of very strong discharges, and with greater facility when the pressure of the gas is low; they increase in relative importance as the intensity of the discharge becomes greater. It would seem that they form part of a third line spectrum, which may be distinguished, in general terms, as the "super-spark" spectrum of oxygen. On the plan adopted by Lockyer in the case of the four groups of lines of silicon, we propose to designate the compound line, the elementary line, and the third line spectra as O. I., O. II., and O. III., corresponding respectively to the arc, spark, and super-spark stages. As in the case of other elements, the third spectrum might be expected to occur without the second under favourable conditions, such as may possibly be found in some of the stars.

Our observations of the oxygen spectra have extended over the range λ 7000 to λ 2000, and they show that although lines of O. III. are very numerous in the ultra-violet, there are no conspicuous members of this class between Schuster and Roscoe's λ 5592 and Lunt's 3961·8. The chief lines of O. III. between λ 7000 and λ 3700 are given in Table I., which also includes the brighter lines of O. II. in the near ultra-violet in order to indicate the difference of behaviour. The first two columns give the wave-lengths, on Rowland's scale, of lines belonging to the two groups, while the third and fourth show the variations in the intensities of the lines in passing from moderate to very strong discharges. Lunt's wave-lengths and intensities for corresponding lines are also given for comparison; this column includes all the lines of O. III. given by Lunt, but only the brighter lines of O. II. which are tabulated by him.

It will be observed that Lunt's spectrum is intermediate between the two given under O. II. and O. III. Any intermediate stage can, in fact, readily be obtained by suitable adjustment of the gas pressure, diameter of capillary tube, and intensity of discharge (see Plate 6).

* *Annals of the Cape Observatory*, vol. x., part ii., p. 26B (1906).

TABLE I.

The Chief Lines of Oxygen III. in the Region λ 7000 – λ 3700, and of Oxygen II. in the Region λ 4000 – λ 3700.

Oxygen II.	Oxygen III.	Intensity. Moderate Discharge.	Intensity. Strong Discharge.	Lunt. λ.	Lunt. Intensity.
	5592·57	...	6	Out of range	
Many lines					
3982·87	...	5	5	3982·93	6 *
3973·41	...	10	10	3973·51	9 *
	3961·75	...	6	3961·82	3
3954·52	...	7	7	3954·64	7 *
3945·18	...	5	5	3945·26	6 *
3919·48	...	5	5	3919·51	6 *
3912·10	...	8	8	3912·19	7 *
3882·34	...	6	6	3882·37	6 *
	3791·47	...	5	3791·46	1
	3774·23	...	5	3774·22	1
	3760·09	...	8	3760·04	6
	3757·40	...	4	3757·42	3
	3754·86	...	6	3754·88	5
3749·71	...	10	10	3749·63	7 *
3740·10	...	5	5	3739·95	0
3727·55	...	9	9	3727·49	6 *
	3715·25	...	4	3715·23	0
3712·80	...	8	8	3712·87	5 *
	3707·41	...	3		
	3703·50	...	2	3703·53	1
	3702·95	...	2		
	3698·86	...	1–2	3698·87	0

Occurrence of O. III. Lines in Stellar Spectra.

In view of the relative behaviour of the different groups of lines in the laboratory, it might be expected that lines of O. III. would appear in the spectra of B stars of somewhat earlier type than those which exhibit the lines of O. II. The stellar observations are less consistent and complete than might be desired for the purpose of this inquiry, chiefly because the strongest lines of O. III. lie outside the region to which most attention has been given in the observations of stellar spectra. Photographic records in the green and yellow regions of stellar spectra are unfortunately

* Observed by Lunt in stellar spectra taken at the Cape Observatory, chiefly in β Crucis and ε Canis majoris, which are both of type B1.

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very few, and published wave-lengths for the ultra-violet beyond K appear to have been based chiefly on plates obtained with small dispersion. Nevertheless, the evidence for the occurrence of O. III. lines in stars is fairly consistent, as may be gathered from the data given in Tables II. and III. Table II. includes all the lines of O. III. of intensity 5 or greater in the region λ 7000 to λ 3700, and for purposes of comparison Table III. shows the behaviour of some of the chief lines of O. II. in the region λ 4000 to λ 3900.

Details of the stellar spectra are taken from the South Kensington * and Harvard † publications. Lockyer and Baxandall's typical stars are γ Orionis (B2) and ε Orionis (B0). Those of Harvard are σ Sagittarii (B3), γ Orionis (B2), β Centauri (B1), and ε Orionis (B0) for all the lines except 5592, in which case the stars are λ Scorpii (B2), β Crucis (B1), and ε Orionis (B0). In the Harvard tables "0" indicates that a satisfactory negative failed to show the line, while ". ." means that the line was possibly obscured by some other line, or may have been missed through imperfect focus or other causes.

For details of the Wolf-Rayet star B.D. + 30° 3639 we are indebted to Dr. W. H. Wright, of the Lick Observatory, who has very kindly placed an unpublished table of wave-lengths at our disposal.

TABLE II.

Lines of Oxygen III. in Stellar Spectra.

Oxygen III.		Stars, Lockyer and Baxandall.			Stars, Harvard.					B.D. + 30° 3639. Wright.
λ.	Intensity.	λ.	B2.	B0.	λ.	B3.	B2.	B1.	B0.	λ.
5592·57	6	Out of range			5588	0	0	0	2	5593
3961·75	6	3961·8	1–2	2–3	3962	0	0	0	2	3962·2
3791·47	5	Out of range			3790·9	0	. .	. .	2	3793·6±
3774·23	5	,,								
3760·09	8	,,			3760·1	0	. .	5	5	} 3760±
3754·86	6	,,								

TABLE III.

Lines of Oxygen II. in Stellar Spectra.

Oxygen II.		Stars, Lockyer and Baxandall.			Stars, Harvard.					B.D. + 30° 3639. Wright.
λ.	Intensity.	λ.	B2.	B0.	λ.	B3.	B2.	B1.	B0.	
3982·87	5	3982·9	1–2	1	3982·8	0	1	2	2	Not seen
3973·41	10	3973·4	2	1–2						,,
3954·52	7	3954·6	1–2	2	3954·8	1	2	2	2	,,
3945·18	5	3945·3	1–2	1–2	3945·0	1	2	2	2	,,
3912·10	8	3912·2	2		3912·2	1	3	3	2	,,

The line 5592 is well known in the spectra of Wolf-Rayet stars, and Wright's wave-length is in sufficient agreement with that determined in the laboratory to make the identification with O. III. very probable. The Harvard wave-length 5588 is rather low, but

* *Publications of the Solar Physics Committee*, 1914.

† *Annals H.C.O.*, vol. xxviii., part ii. pp. 235, 237 (1901).

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the behaviour of the line in the stars corresponds precisely with that of other O. III. lines, and it may reasonably be supposed to be identical with 5592.

The presence of λ 3961·75 in stars appears to be well established by agreement in wave-length, though there is a notable discordance between the two sets of stellar data as regards the occurrence of this line in γ Orionis (B2); possibly the Harvard dispersion was insufficient to separate it from the adjacent helium line λ 3964·9.

The line λ 3791·47 corresponds closely with the stellar line 3790·9 and with the Wolf-Rayet line λ 3793·6 ±. The strongest of the O. III. lines, λ 3760·09, is apparently represented in the Wolf-Rayet star, and also by a comparatively strong line in the B1 and B0 stars; the occurrence of this line in the B1 star, and its high intensity both in B1 and B0, however, suggest that a line of some other element may possibly also be involved. Dr. Wright also points out that a line about this position occurs in the planetary nebula N.G.C. 7027, where no line in the position of λ 5592 has yet been photographed.

Two of the brightest O. III. lines, 3774·23 and 3754·86, do not appear in the Harvard lists, but it is possible that they may have escaped record on account of their proximity to the hydrogen lines 3770·8 and 3750·2; the Harvard plates have, in fact, similarly failed to reveal the strong O. II. line 3973·4, given by Lockyer and Baxandall, and by Lunt, apparently because it is too close to the hydrogen line 3970·2.

It will be seen that there is substantial evidence that the brighter lines of O. III. are present in some of the stars, and a comparison with Table III. shows that the circumstances of their appearance are distinctly different from those of the lines of the O. II. group. The tables indicate that while the latter first appear faintly in stars of type B3, and cease to be visible in the Wolf-Rayet star, the O. III. lines certainly do not appear earlier than type B2, and have their greatest development in type B0, besides persisting to the Wolf-Rayet type. This difference in the behaviour of the two sets of lines in the stars is in strict accordance with the variations of the oxygen spectrum observed in the laboratory.

The stellar lines which are thus probably to be identified with the third line spectrum of oxygen are few in number, but it is interesting to have found in oxygen another example of the occurrence in stars of the earliest types of lines which only appear in experimental spectra when the strongest discharges are employed. It is of special interest to note that oxygen is now probably to be added to hydrogen, helium, and carbon as representing the few known elements which exhibit their spectra in stars of the Wolf-Rayet class. The importance of such identifications does not end with the recognition of the chemical elements which are involved; they may be of even greater significance as tending to prove that in the early-type stars, and probably also in nebulæ, we are presented with phenomena which result either from the effects of powerful electrical actions or of very elevated temperatures.

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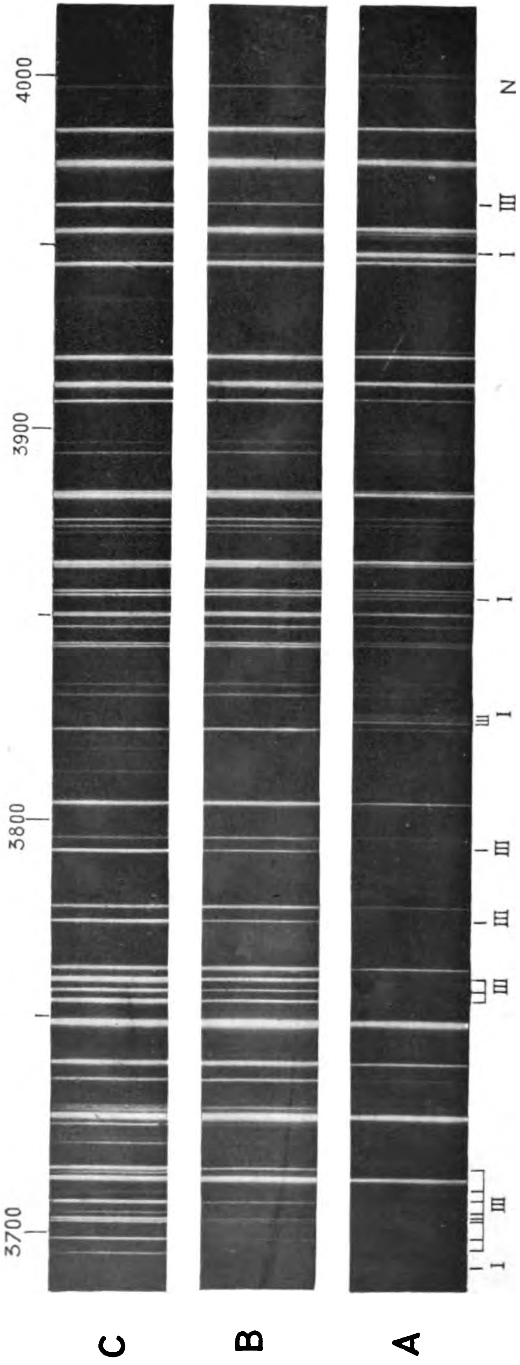
LINE SPECTRUM OF OXYGEN.



C. Strong condensed discharge.
B. Moderate condensed discharge.
A. Feeble condensed discharge.

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Description of Plate.

The photographs reproduced have been enlarged from negatives obtained with a quartz spectrograph of the Littrow type, giving a linear dispersion of 10 A. per mm. in the neighbourhood of λ 3800. They represent the spectra produced in a capillary tube containing sensibly pure oxygen, by discharges from a 12-inch coil with varying capacity in the secondary circuit. In passing from A to B, and from B to C, the capacity was increased, and the pressure of the gas reduced.

A shows lines of O. I. and O. II., while those of O. III. are absent.

B shows the disappearance of lines of O. I., and the appearance of lines of O. III.

C shows the increased intensity of lines of O. III. as compared with those of O. II. when very strong discharges are passed through the gas at low pressures.

The chief lines may be identified by the scale of wave-lengths, and reference to Table I. The line at λ 3995 is due to a trace of nitrogen. In comparing intensities in the different spectra, it should be noted that spectrum A is rather underexposed in comparison with B and C.

Missing B.D. Stars. By J. G. Hagen, S.J.

The list of 81 B.D. stars within the zone +40° to +53° Dec., which were found "missing or faint" by the Rev. T. E. Espin (*M.N.*, Nov. 1916), forms a valuable addition to those published by Chandler, Pickering, and others. In view of their great number it seems rather remarkable that the writer met with only 28 cases of the kind in an area of 590 square degrees, which is that covered by the "Atlas of Var. Stars" (if we except Series V.). They may be found in the "Aggiunte al Catalogo dell' A.S.V.," *Spec. Vat.*, xi. (1916), 234, table xii*b*. Only one of them occurs also in Rev. T. E. Espin's list, No. 3 = +47° 192. The star is not really missing, for it is identical with +47° 193. The duplicate number had its origin in an erroneous Dec., and was suppressed by Schönfeld himself in *A.N.*, cxvi. p. 272.

This is a good illustration of "error in the original observation," one of the three sources to which Sir William Herschel reduces the missing stars (*Phil. Trans.*, lxxiii. [1783], 250–254).

"Variation," if periodical, may cause the missing of a B.D. star at intervals, but if progressive, it will cause the star to be missing for ever. This latter case, however, is extremely rare. As an example may be adduced B.D. + 16° 354, now Nova Arietis 1854. It is on the charts of Argelander and of Chacornac, near T Arietis, but was missed by Winnecke and Schönfeld and has never been seen again (*A.N.*, 4523 and 4579).

"Proper motion" may exceptionally shift a star as much as 5′ or more in half a century, and thus make it apparently disappear. As a rule, however, the shift will remain far within the errors of the B.D. In fact, not a single B.D. star was missed on account of P.M. on all the 290 charts of the Atlas (not counting Series V.).

What the writer wishes especially to mention from his own experience are two other kinds of errors that have caused spurious

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stars in the B.D. One of them may be called *bad focus*, through which faint stars close together are observed as one bright star. In the Atlas no less than 70 B.D. stars are resolved into components. A striking example is $+39°$ 4866, $9^{m}\cdot5$. It consists of three stars, the brightest of which (to the north) hardly reaches mag. 11·5. Variation is excluded, since none of them showed a change on the Harvard plates between 1891 and 1906, and the B.D. position is assured by two "finder zones" 1856 Aug. 30, Sept. 26. The star $9^{m}\cdot5$ was entered on the working chart S Lacertæ (III. 8068) at $+2^{m}\ 46^{s}\ +17'\cdot1$, but afterwards cancelled as not belonging there.

A rather curious error, which has apparently never been pointed out before, consists in the *combination of the R.A. of one star with the Dec. of another.* The spurious star then appears at the right angle of a triangle of which the real stars form the vertices of the acute angles. Six cases of this kind were found during the work on the Atlas: $-21°$ 1062, $+7°$ 1288, $+19°$ 4513, $+32°$ 3524, $+33°$ 469, $+57°$ 558, a rather small number, when we remember that the two co-ordinates were written down by different persons, one at the telescope and the other, beneath the floor, in front of the clock.

Vatican Observatory:
1917 *March* 6.

Observations made during the Partial Eclipse of the Sun on 1917 *January* 22 *at the temporary University Observatory, Rostow-on-Don.* By Professor S. D. Tscherny.

Phenomenon.	Rostow-on-Don. Mean Time. h m s	Remarks.
First contact	20 52 25·2	Good
Occultation of the following border of sun-spot 1 by the Moon	21 11 12·7	Good
First contact of the Moon and the large umbra of sun-spot 2	21 33 10·7	
Second contact of the Moon and the large umbra of sun-spot 2	21 33 31·7	
Occultation of the following border of sun-spot 3 by the Moon	21 35 37·7	
Occultation of the following border of sun-spot 4 by the Moon	21 37 21·7	
First contact of the Moon and the umbra of sun-spot 5	21 39 18·7	
Second contact of the Moon and the umbra of sun-spot 5	21 39 54·7	
First contact of the Moon and the large umbra of sun-spot 6	21 48 5·7	
Second contact of the Moon and the large umbra of sun-spot 6	21 48 39·7	
Last contact	23 8 45·7	Satisfactory. Through Str.-Cum.

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Instruments used:—

(1) 30-mm. Bahrenfeld altazimuth, power 31. (First contact.)
(2) 71-mm. Vion refractor, power 71. (All other observations.)

The magnitude of the eclipse in Rostow-on-Don was 0·64.

Position of Observatory: latitude, 47° 13′ 9″·4 N.; longitude, $2^h\ 38^m\ 49^s{\cdot}48$ E. The comparison of the observed times with those calculated * for our Observatory gives—

	Observ.-Calcul.
First contact . .	$+3^s{\cdot}6$
Last contact . .	$-8^s{\cdot}9$

The following meteorological observations were made by B. Yankowich:—

Summary of Barometer and Thermometer Readings.

Local M.T.	Aneroid Barometer.	Thermometer.
h m		°
20 42	759·2	− 10·0 C.
21 32	757·8	− 9·5 (max.)
22 0	757·6	− 9·8 (min.)
22 55	757·5	− 9·2
23 9	757·5	− 8·8

The amount of cloud varied from 2 to 5.

University Observatory,
Rostow-on-Don:
1917 *January* 25.

Note on the possible Attraction between Photographic Images.†

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In 1910 a copy of a photograph ‡ taken at Oxford was shown to the Society, in which the image of a bright star had apparently caused a considerable distortion in a neighbouring réseau line (see *The Observatory*, vol. xxxiii. p. 467, 1910 Dec.). Such a disturbance is certainly not systematic, for we have scrutinised all the cases of stars brighter than 4·5 mag. (R.H.P.)

* "Elements of the Eclipse of the Sun," *Connaissance des Temps pour l'an 1917*, p. 594.

† Since this paper was put in type, attention has been drawn to the interesting note by W. W. Campbell in *L.O.B.*, No. 284, "On Systematic Errors in the Measured Interval between Close Lines."

‡ Plate 2712, R.A. $5^h\ 56^m$, Dec. +25°. The bright star is 139 Tauri, mag. 4·90 R.H.P.

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on our Oxford plates without finding another instance. It may have been due to some accident; but the cause is still obscure, as in the case cited many years ago by Professor Pritchard of a deformation amounting to 1″·2 (*M.N.*, xlvi. p. 442). Still what may happen once or twice grossly may happen in minor degree at other times; and it seems desirable to watch the behaviour of closely contiguous images with some care.

2. What recalled to mind the instances cited was the following circumstance. The war having made it impossible to obtain réseaux from the former makers, the Director of the National Physical Laboratory very kindly undertook to rule one, and sent us a successful result some months ago, which has ultimately been despatched to India, where the need of it was greatest. But, to test its accuracy at Oxford, it was twice copied on to the same plate, turning it round through 90° between the exposures. Measurement of the distances apart of the two copies in various parts of the plate, while they do not give an absolute determination of error, are quite sufficient to indicate whether the errors are small: for quite different parts of the réseau are brought into juxtaposition, and moreover the x lines are compared with the y lines. But the difference of orientation was not quite 90°, so that one set of lines crossed the other at a small angle. The differences should decrease uniformly if the lines are straight: but it was found that in the neighbourhood of the crossing point they did not do so. The observed separation gradually became less than the theoretical, on both sides of the crossing point. There is no need to give details of so simple an operation: the general result of a good many measures was as follows:—

Separation in réseau intervals .	0·015	020	·030	·040	·050	·060	·070
Mean error of separation in units of 0·0001 . .	+11	+13	+1	+7	+5	+2	0

Beyond a separation of 0·070 (or one-third of a millimetre, or 20″ in *Astrographic Catalogue* measure) the error was sensibly zero. From there it gradually increased to ·0012 (·006 of a millimetre, or 0″·36 for the *Astrographic Catalogue* measures) at the point where the images were practically in contact, within which measures were not made. After noticing the effect on the plate made to test the réseau, some other plates were made with longer exposures in order to see whether denser and thicker images gave more "attraction": but no sensible increase of the phenomenon was found, much to our surprise.

3. It seemed desirable to examine the measures of close double stars given in the *Oxford Astrographic Catalogue*, and Mr. Bellamy has done this, finding much the same order of error for distances less than 7″, though nothing sensible beyond this. His results are given in a separate note. It does not follow, of course, that there is any actual displacement of the photographic film. If we visualise two photographic images as similar to two probability curves, when the maxima are close together, the portion between

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is the sum of the two ordinates: and it is easily seen that the maxima will tend to approach. But whatever the origin of the error, it should be noted and allowed for in measuring close images. Probably in most measures for stellar parallax the images are too far apart to be affected. But in measures of spectrum lines, is it not possible that we have here a source of error, for closely contiguous lines, which has not been entirely eliminated? Tests could be made by photographing a straight line (from a réseau say) crossing a spectrum at a slight inclination to the spectrum lines. It might show distortions of the kind above noted.

Systematic Errors in Photographic Distances of Double Stars.

By F. A. Bellamy, M.A.

The recent examination of a new réseau, by making two copies on the same plate, called attention to the fact that the crossing of two réseau lines at a slight inclination showed an interference of one image with the other; and it seemed probable that photographic measures of double stars might afford some information upon the point.

Some years ago I undertook the computation of the distances and position-angles of all pairs of stars—to a definite limit in distance—which are given in the seven volumes of the Oxford Astrographic Catalogue, zones $+31°$ to $+25°$. I specially selected those, in the first instance, which are given in Burnham's "General Catalogue of Double Stars," then recently published.

At that time two of the Oxford zones were not printed, and the investigation has remained in MS. awaiting time for its completion of the stars in zones $+25°$ and $+27°$. But when the question of the effect of one photographic image on another was raised, it was a simple matter to make use of some of this material to test this question.

Zone $+26°$ was used in the first instance, but there being too few stars below the distance of $9''$, all the pairs in zones $+28°$ to $+31°$ were collected into groups and the results are tabulated in Table I., where the mean groups for Burnham's Δ are given in columns 1 and 2, $O-\beta$ in columns 3 and 4 are the differences of Oxford *minus* Burnham for distance and position-angle, r_1+r_2 is the half sum of the Oxford diameters (d) of the stars expressed in seconds of arc in column 5, and the last column gives the number of pairs used: altogether 436 pairs are included.

Table I.

Means of the Groups for O − β.

Group.	Mean. Δ.	O − β. Δ.	O − β. ∠.	Oxford. r_1+r_2.	No. Pairs.
″ ″	″	″	°	″	
3·0–3·9	3·7	−0·76	0·0	8·1	3
4·0–4·9	4·5	−0·46	+0·3	5·1	9
5·0–5·9	5·5	−0·38	+0·2	6·6	9
6·0–6·9	6·5	−0·14	−0·5	7·2	29
7·0–7·9	7·6	−0·15	0·0	6·6	20
8·0–8·9	8·5	−0·06	−1·2	7·5	19
9·0–14·9	11·9	+0·01	−0·3	8·7	102
15–20	17·7	−0·01	+0·2	7·8	66
20–25	22·2	−0·02	0·0	8·1	48
25–35	29·6	−0·07	+0·3	10·2	51
35–50	41·2	−0·05	+0·1	10·2	22
50+	75·7	+0·01	−0·2	14·1	58

The accordance in position-angle is satisfactory, but in distance there seems to be some systematic error in the first three groups. We may therefore give the first 21 pairs in detail as in Table II. The columns are similar to those in Table I., with the addition of the Oxford and Burnham numbers and epochs, and the measured diameters of the star disks in seconds of arc for each star—instead of the sum of their radii.

Table II.

Oxford.	β.G.C.	β. Δ.	O − β.	β. ∠.	O − β.	Epochs. Oxf.	Epochs. β.	d.	d.
°		″	″	°				″	″
29 68671–0	12453	3·42	−0·45	276·1	−5·1	1895	1902	8·4	7·8
31 64607–8	10949	3·78	−0·57	22·7	−1·8	1902	1902	9·3	7·8
30 15717–6	3399	3·97	−1·27	353·3	+6·8	1903	1902	8·4	6·6
31 60562–3	10388	4·18	−0·01	156·0	+2·6	1902	1902	5·7	5·7
28 42628–9	7656	4·27	−0·76	106·5	+3·1	1901	1902	3·6	3·0
29 38806–7	7656	4·27	−0·58	106·5	−2·4	1906	1902	3·3	3·0
30 17928–9	3897	4·39	−0·49	162·2	−4·2	1903	1886	6·3	6·0
30 18515–6	3897	4·39	−0·07	162·2	+4·7	1903	1886	6·9	6·6
26 70777–6	10865	4·41	−0·63	215·2	+9·3	1897	1901	8·1	2·7
26 39801–0	7764	4·55	−0·13	136·4	+4·8	1899	1901	6·6	6·0
26 1212–0	289	4·84	−0·64	266·1	+4·5	1907	1897	4·2	3·6
26 1578–7	289	4·84	−0·97	266·1	+3·4	1895	1897	7·8	3·6
31 46619–20	9280	5·00	+0·34	68·2	+5·3	1902	1903	6·9	5·4
31 680–1	191	5·03	−0·08	167·5	−1·8	1902	1903	7·5	6·6
28 33047–8	5866	5·21	−1·01	87·4	−1·7	1901	1904	9·6	9·0
29 20785–6	3871	5·31	+0·03	125·8	+2·3	1906	1904	4·2	3·6
31 27401–0	5950	5·41	−0·27	241·7	+7·5	1903	1898	9·6	9·0
28 43968–9	7915	5·58	−0·33	17·6	+6·1	1901	1903	8·1	6·3
28 73437–6	12606	5·80	−0·10	356·9	−6·4	1894	1905	6·3	3·6
29 69412–1	12606	5·80	−1·01	356·9	0·0	1895	1905	7·8	5·4
30 17807–8	3871	5·82	−0·48	125·5	+3·1	1903	1894	6·0	4·5

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It may be remarked that four pairs are repeated, being in two zones or on overlapping plates. Attention is also drawn to the fact that there are a number of cases where the epochs differ by more than ten years; that the sum of the radii is, for 15 of the 21 pairs, greater than the Oxford measured distance—indicating that there is no complete separation of the two images on the photographs; that the measures used from Burnham's Catalogue are often on one night by one observer; and that in some stars the visual and photographic magnitudes of the pairs differ relatively, with the result that the primary star used is not the same and the angles differ by 180°: this is, however, unimportant.

Besides the pairs given in Table II. there are 13 pairs—one pair (12 and 13) being in two zones—of an exceptional character; these are shown in Table III., in a similar form to Tables I. and II.

TABLE III.

	Oxford.	β.G.C.	Δ. Oxf.	Δ. β.	∠. Oxf.	∠. β.	Epochs. Oxf.	Epochs. β.	d. (in arc).	d. (in arc).
	°		″	″	°	°			″	″
1	+29 19190–1	3706	1·08	6±	16·0	5±	1907	1820	6·6	5·7
2	+29 63747–8	11353	1·89	3·47	161·8	174·6	1894	1904	2·4	2·1
3	+31 23554–5	5125	2·34	3·58	130·3	144·1	1904	1901	6·6	5·4
4	+30 1508–7	523	2·46	3±	194·1	25±	1902	1820	3·9	3·0
5	+26 10136–4	2064	2·85	8±	297·0	304·7	1893	1831	7·2	3·6
6	+30 66842–1	11122	4·32	6±	262·8	88·0	1895	1828	3·0	3·0
7	+29 14209–8	2905	4·44	5·79	207·7	194·5	1899	1891	14·4	6·6
8	+28 9010–1	2337	4·47	3±	137·3	143·4	1899	1831	7·5	4·2
9	+29 29311–0	5377	4·47	4·81	311·7	331·4	1906	1896	14·7	13·5
10	+26 52969–70	8829	4·71	4·30	54·9	67·6	1897	1901	7·8	4·2
11	+29 15993–4	3147	5·22	1?	156·9	160±	1896	1820	4·8	3·0
12	+28 47920–1	8570	5·22	3±	12·4	10·8	1901	1828	6·6	6·0
13	+29 45002–3	8570	5·61	3±	15·2	10·8	1900	1828	6·0	4·8

The pairs numbered 2, 3, 7, 9, and 10 show large differences for O – β; no sensible error has been found at Oxford. May these differences be partly due to actual motion? No. 2 (β.G.C., 11353) is somewhat peculiar, and the following words may be quoted from Burnham's Catalogue, p. 962: "There are large errors in some of the distances

1893·28	356°·4	1″·84	2*n*	Ho.
1904·74	175·5	2·37	3*n*	Doo.
1904·87	354·6	3·47	1*n*	Lewis."

The rest of the pairs in this table have Herschel's estimated distances, which are usually several seconds too small; but nothing later is given by Burnham in the catalogue or in the

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volume of notes. It seems of little use to reprint so much approximate information, unless connected with some modern accurate measures.

With one or two exceptions the measurement of double stars has been greatly neglected in this country for the last twenty-five or thirty years by possessors of small equatorials of 4 to 8 inches aperture. There must be a number of such instruments available and suitable for double star work that would fall within the power of the smaller equatorials—stars of about 3″ to 30″ distance for example, which are too often avoided or neglected by the more powerful instruments, and perhaps rightly so. A perusal of Burnham's Catalogue would show that there is an abundance of double star work to be done, if only the measurement of those stars with approximate distances or angles be included in the working list.

It seems clear from the results in Table I. that there are systematic errors in *photographic* measures of close pairs: so that, however useful photography may be for wider pairs, it cannot at present replace visual work for close pairs.

The measures from which the Oxford distances and angles have been determined may not be quite as good as could be made by re-measurements and by taking the means in x and y to the fourth decimal (unit 0″·03), but there is danger of prejudice in the bisections, now it is known what is required. The images of the second, and sometimes the third, exposure could also be measured for such an investigation as is briefly given in this paper; the printed measures have, however, only been used now. The formula adopted for these computed angles and distances are (12) and (13) given by Professor H. C. Plummer in his paper (*M.N.*, lxix. p. 102).

To show the distribution of the values of the mean distance residuals (Oxford *minus* Burnham), Tables IV. and V. are given; they correspond to the results shown in Table I., and give the number of pairs with differences of +1″·25 to +0″·75, etc., according to the distances in the heading of each column. Table IV. is with the Oxford distance as the argument, and Table V. is the same for the Burnham distances. There is an apparent change in the visual and photographic distances for the closer pairs.

TABLE IV.

Oxford Distance.

Value of o − β.	3″.	4″.	5″.	6″.	7″.	8″.	9″–15″.	15″–20″.	20″–25″.	25″–35″.	35″–50″.	50″+.
+1″·25 to +0″·75	0	0	0	0	2	1	10	0	7	3	1	6
+0″·75 to +0″·25	0	0	0	0	6	8	27	9	8	11	3	10
+0″·25 to −0″·25	0	4	3	7	5	7	33	29	20	22	10	28
−0″·25 to −0″·75	5	1	7	9	6	3	27	22	8	16	3	11
−0″·75 to −1″·25	3	1	4	6	2	3	5	3	2	3	2	4
Sum	8	6	14	22	21	22	102	63	45	55	19	59

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TABLE V.

Burnham Distance.

Value of o − β.	3″.	4″.	5″.	6″.	7″.	8″.	9″–15″.	15″–20″.	20″–25″.	25″–35″.	35″–50″.	50″+
+1″·25 to +0″·75	0	0	0	3	0	0	10	3	4	3	1	6
+0″·75 to +0″·25	0	0	0	3	7	6	26	9	8	10	3	10
+0″·25 to −0″·25	0	3	2	10	5	6	33	28	21	22	10	28
−0″·25 to −0″·75	2	4	6	10	4	4	27	21	10	13	6	11
−0″·75 to −1″·25	1	2	2	4	4	3	8	2	3	3	2	4
Sum	3	9	10	30	20	19	104	63	46	51	22	59

1917 *April* 13.

Erratum in Annual Report.

Vol. lxxvii. p. 388, line 6, *for* earlier *read* later.

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LXXVII. May 11, 1917. No. 7

Major P. A. MacMahon, D.Sc., Sc.D., LL.D., F.R.S., President, in the Chair.

Arthur Brinley Mayne, B.A. (Oxon.), Charterhouse School, Godalming, Surrey; and 3 Bryn-y-mor Crescent, Swansea, South Wales;

Wilfred C. Parkinson, Department of Terrestrial Magnetism of the Carnegie Institution of Washington, The Observatory, Perth, Western Australia;

Thomas James Richmond, B.Sc. (Lond.), Hydrographic Department, Admiralty; and "Eastdene," St. James' Road, Sutton, Surrey; and

Rohani Thirunal Raja M. Rama Varma Raja, Prince of Mavelikara Palace, Assistant Director and Astronomer, H.H. the Maharaja's Observatory, Trivandrum, Travancore, S. India,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Philip Francis Everitt, B.Sc., Optical Instrument Designer, St. Heliers, Oxgate Gardens, Cricklewood, N.W. 2 (proposed by H. H. Turner);

E. H. S. Flood, Canadian Trade Commissioner, Barbados, West Indies (proposed by Capt. L. A. Demers); and

Lieut. Hugh Duncan Grant, R.N.V.R., L.A., F.R.Met.Soc., N1 Section, Naval Air Department, Air Board Office, Admiralty (proposed by Hugh Walsham).

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Forty-six presents were announced as having been received since the last meeting, and the thanks of the Society were returned to the respective donors.

On the Origin of Ptolemy's Catalogue of Stars.
By J. L. E. Dreyer, Ph.D.

For more than a hundred years it has been the prevailing opinion among astronomers that Ptolemy had borrowed all his star-places from the catalogue of Hipparchus. This was supposed to be proved by the fact that Ptolemy's longitudes are on an average fully a degree too small, while he assumed the constant of precession to be 36″ instead of 50″. The epoch of Ptolemy's catalogue is A.D. 137, that of Hipparchus 129 B.C.; the interval is therefore 265 years, and $265 \times 14'' = 1^\circ 2'$, *q.e.d.*

But the question is not, after all, so easily decided. It is interesting to see that the late C. H. F. Peters, to whose labours, jointly with those of Mr. Knobel, we owe the recently published invaluable edition of the catalogue, three years before his death pointed out that the constant error of the longitudes might very well have arisen in some other way.* Deferring for the present the consideration of this question, we shall first consider some information concerning the catalogue of Hipparchus which has come to light since Peters' death, and which strongly militates against the prevailing view.

That Hipparchus was a careful observer who must have produced a number of star-places as accurate as the imperfect instruments then in use would allow, is shown by his list of 44 time-stars, which follow each other at intervals of exactly one hour. Schjellerup has shown that these stars had been so well chosen that they gave the time every hour within a minute.† We also know that Hipparchus possessed a better knowledge of the true value of the constant of precession than Ptolemy did; for though he first announced that the equinoxes moved at least 36″ per annum, he afterwards stated that he had found the longitude of Spica to have increased 2° in the 154 years elapsed since the time of Timocharis, which gives 46″·8.‡ It is therefore much to be regretted that his star-catalogue has not been preserved. If Ptolemy had merely added a certain constant quantity to the longitudes of Hipparchus to bring them up to his own epoch, it would be reasonable for us to assume that the 1025 stars in Ptolemy's catalogue must all have occurred in the catalogue of

* Short report of a speech made by Peters at the Kiel meeting of the Astronomische Gesellschaft in 1887, *Vierteljahrsschrift*, xxii. p. 269.

† "Sur le chronomètre céleste d'Hipparque," in the journal *Copernicus*, vol. i. p. 25 (Dublin, 1881). This valuable paper appears to be quite unknown to Dr. Manitius, editor of *Hipparchi in Arati et Eudoxi Phænomena Commentarii* (Lipsiæ, 1894).

‡ *Syntaxis*, vii. 2, ed. Heiberg, ii. p. 13.

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Hipparchus. It is therefore of importance for us to know how many stars the older catalogue contained.

The only information on this point which we possessed until lately was given by an anonymous commentator to Aratus.* According to him, Hipparchus said that there were 1080 stars inside the constellations; that is, not counting the "informes" or stars not actually inside the figures of the constellations. Subtracting the informes, Ptolemy has only 918 stars inside the constellations. But, as remarked by Boll in the paper presently to be mentioned, it is incredible that Ptolemy should have omitted 162 or nearly one-sixth of the whole number. Either the passage is corrupt, or it is simply an estimate of the number of more or less conspicuous stars, like that given by Pliny when he says that some people have counted 1600 stars.†

But in 1892 there was first brought to light a hitherto unknown Greek list of the constellations, attributed to Hipparchus; and since then several copies of it have been found, the best one being part of Cod. Parisinus 2506, an astrological MS. of the fourteenth century. The title is simply *ἐκ τῶν ὑπάρχων ἀστέρων*. It has been discussed by F. Boll in a paper published in 1901,‡ in which the whole of the short text is given; it simply counts up the constellations and adds to each a number, being the number of stars in it, as follows :—

Ursa major	24	Triangulum	3	Canis major	21
Ursa minor	7	Perseus	19	Canis minor	3
Draco	15	Auriga	8	Cancer	16
Bootes	19	Hydra	27	Leo	19
Corona borealis	9	Crater	10	Virgo	19
Hercules	24	Corvus	7	Libra	4
Ophiuchus	17	Argo	13	Scorpio	15
Lyra	10	Centaurus	26	Sagittarius	16
Cygnus	14	Lupus	13	Capricornus	26
Aquila	4	Ara	4	Aquarius	18
Sagitta	4	Corona australis	...	Pisces	41
Delphinus	9	Piscis australis	12	Aries	17
Pegasus	18	Cetus	14	Taurus	18
Cepheus	19	Eridanus	...	Gemini	19
Cassiopea	14	Orion	18		
Andromeda	20	Lepus	...		

* *Petavii Uranologion*, p. 262, second ed. in *Petavii Opus de doctrina temporum*, t. iii. (Antwerp, 1703), p. 145. Maas, *Commentariorum in Aratum reliquiæ*, Berlin, 1898, p. 128.

† In this passage (*Hist. Nat.*, ii., § 110) it is also said that there are 72 constellations. Ptolemy has only 48. If we add to these Coma and Antinous (which he omitted) and his 22 groups of informes, we get 72. Can that perhaps be the explanation ?

‡ F. Boll, "Die Sternkataloge des Hipparch und des Ptolemaios," in the *Bibliotheca mathematica*, 3rd series, vol. ii. pp. 185–195.

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In all 46 constellations. Of Ptolemy's only Equuleus and Serpens are missing, the former possibly because it was not used by Hipparchus, the latter certainly only owing to a slip on the part of the original compiler. In 43 cases the number of stars is given. In 30 cases the numbers are exactly the same as in the Katasterismoi of Pseudo-Eratosthenes, now known to be a compilation made in the third (or the end of the second) century A.D. Of the other 12 cases (neglecting Argo, the number belonging to which must be very erroneous), only one, Cancer, has a smaller number than the Katasterismoi. In the 11 remaining constellations (Bootes, Hercules, Lyra, Centaurus, Lupus, Cetus, Orion, Canis major, Sagittarius, Capricornus, Aquarius) Hipparchus has from one to five stars more than Pseudo-Eratosthenes.

According to Boll, everything seems to confirm the idea that this list really gives the number of stars observed by Hipparchus. In his earlier work, the Commentary to Aratus, Hipparchus gives more stars to some constellations than this list does. But it should be remembered that there is no finality about lists of that kind. In 11 constellations Ptolemy has fewer stars than the Katasterismoi and the new list, viz. Corona, Cepheus, Cassiopea, Hydra, Crater, Canis major, Canis minor, Aries, Gemini, Cancer, Pisces. But this is probably solely caused by drawing the constellations in a different manner.* To draw the figures of the constellations so as to make them resemble human figures or other objects, it was necessary to put some stars among the ἀμόρφωτοι (informes), as already done by Aratus; and this was probably also done by Hipparchus, so that he in his great star-catalogue gave some constellations narrower limits than he had done in his earlier work, the Commentary to Aratus.

If we omit Argo, we have then 42 constellation figures containing 640 stars. In the same (Aquarius without the water and Ophiuchus without the serpent) Ptolemy has 772 stars.† In the remaining seven figures (Serpens, Aqua, Equuleus, Eridanus, Lepus, Argo, Corona australis) Ptolemy has 146 stars. Assuming in these the same proportion between the two catalogues, it would appear that Hipparchus must have had 121 stars, so that his catalogue must have contained $640 + 121 = 761$ stars. But this estimate does not include the informes, though there are some important stars among them, *e.g.* Arcturus. It is possible that Hipparchus (or the compiler of the list) included these in the nearest constellation, as Ptolemy did. If not, we must add some informes to the 761, and as Ptolemy has in all 108, Hipparchus may have had about 90. The star-catalogue of Hipparchus therefore contained at least 851 stars, and at most 20 or possibly 30 more. For it is most improbable that, in the seven constellations for which we have no

* Compare *Syntaxis*, vii. 4, at end (Heib., ii. p. 37).

† 774, but two of these are duplicates, Herculis 29 = Bootis 9 and Tauri 21 = Aurigæ 11.

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numbers, the proportion should be very different from that of the 42 constellations.

Assuming, then, that it has been proved by Boll in his paper (with which I have only lately become acquainted) that the catalogue of Hipparchus did not contain more than about 850 stars, I shall now try to show that *if* Ptolemy made use of these 850 star-places we are probably able to pick out most of the remaining 175 stars, which he must either have observed himself or taken from some other source.

In Ptolemy's catalogue the longitudes and latitudes are given to fractions of a degree, the fractions being generally multiples of $\frac{1}{6}$, as if the instrument used had been divided to 10′. But in a certain number of cases the fractions $\frac{1}{4}$ and $\frac{3}{4}$ of a degree occur, as if a different instrument had been used.* 142 stars distributed fairly well through the constellations have latitudes of which the fractions are either $\frac{1}{4}$ or $\frac{3}{4}$, while only three stars, all in Virgo, have longitudes distinguished in this way.† On reading Boll's paper it struck me at once that, as about 175 stars cannot in any case have been taken from the catalogue of Hipparchus, it was not unlikely that they were represented by the 145 stars plus some others the minutes of which are 30 or 60, and may therefore have been observed either with an instrument divided to $\frac{1}{6}°$ or with one divided to $\frac{1}{4}°$. The following table gives for each of the 42 constellations the number of stars in Ptolemy's catalogue, the number of stars of $\frac{1}{4}°$ or $\frac{3}{4}°$, the difference between the two sets, and the number of stars observed by Hipparchus.

Constellation.	Stars in Ptolemy's Cat.	Stars of $\frac{1}{4}$ and $\frac{3}{4}$.	Diff. 2-3.	Hipparchus.
Ursa minor	7	0	7	7
Ursa major	27	3	24	24
Draco	31	5	26	15
Cepheus	11	3	8	19
Bootes	21	2	19	19
Corona borealis	8	1	7	9
Hercules	29	5	24	24
Lyra	10	1	9	10
Cygnus	17	1	16	14
Cassiopea	13	2	11	14
Perseus	26	6	20	19
Auriga	13	1	12	8
Ophiuchus	24	6	18	17
Sagitta	5	0	5	4
Aquila	9	0	9	4

* Peters in his speech in 1887 pointed out that two instruments must have been used.

† One of the three stars is among the informes Virginis.

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Constellation.	Stars in Ptolemy's Cat.	Stars of ¼ and ¾.	Diff. 2–3.	Hipparchus.
Delphinus	10	2	8	9
Pegasus	20	1	19	18
Andromeda	23	0	23	20
Triangulum	4	0	4	3
Aries	13	1	12	17
Taurus	33	6	27	18
Gemini	18	2	16	19
Cancer	9	1	8	16
Leo	27	4	23	19
Virgo	26	3	23	19
Libra	8	2	6	4
Scorpius	21	1	20	15
Sagittarius	31	2	29	16
Capricornus	28	4	24	26
Aquarius	22	5	17	18
Pisces	34	7	27	41
Cetus	22	1	21	14
Orion	38	8	30	18
Canis major	18	4	14	21
Canis minor	2	0	2	3
Hydra	25	4	21	27
Crater	7	0	7	10
Corvus	7	1	6	7
Centaurus	37	6	31	26
Lupus	19	1	18	13
Ara	7	1	6	4
Piscis australis	12	3	9	12
Total	772	106	666	640

Some of the figures in the third column are of course too small, because, as already mentioned, we are unable to disentangle some of the additional stars from those observed by Hipparchus, $\frac{1}{2}$° being both $= \frac{2}{4}$° and $\frac{3}{6}$° and $1° = \frac{4}{4}$° and $\frac{6}{6}$°.* Otherwise the agreement between the last two columns would probably be still better. Two-thirds of the additional 145 stars are of magnitudes 4 to 6 (fully $\frac{1}{3}$ being of the fourth magnitude), and there is only one star of the second magnitude, Pollux. It is needless to say that a star like Pollux could not have been passed over by Hipparchus, but

* Of the 106 additional stars, 63 have latitudes with the fraction ¼ and 41 with ¾, while there are two stars of which the longitude has ¼°.

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it may have been used by his successor as a connecting link between the earlier catalogue and the stars observed to fill gaps here and there. It is certainly strange that, with the exception of the three stars in Virgo, it is only latitudes which show signs of having been observed with a different instrument. The differences of longitude and of latitude of the new stars from a small number of standard stars must have been observed with the instrument described by Ptolemy, called astrolabon,* and the latitude circle must have been graduated to $\frac{1}{4}°$ only.

The next question is: By whom were the additional stars observed, by Ptolemy or by some astronomer living in the interval between him and Hipparchus?

Of astronomical activity during that period of more than 250 years we know extremely little. Ptolemy only quotes a very few observations made by Menelaus and Agrippa during the last decade of the first century; but some details about the planetary orbits given by Pliny (especially the positions of the apsides of the excentrics) show that serious work in astronomy must have been done during that long interval. Some idea of this kind, combined with the error of Ptolemy's longitudes, must have suggested to Al Sufi the following remarkable statement †:—"As to the positions of the stars with regard to the signs of the Zodiac, I have found that Ptolemy built on the observations of Menelaus made in the year 845 of Nabonassar,‡ and that he has given in his book the positions of the fixed stars for the first year of Antoninus, that is, the year 886 of Nabonassar. The interval between the observation of Menelaus and the epoch of Ptolemy is, then, 41 years. Ptolemy says that Menelaus found the distance of Spica from the beginning of Cancer to be 86° 15′, and the distance of the most northerly of the three in the brow of the Scorpion from the autumnal equinox 35° 55′; whence it follows that Spica ought then to be in 26° 15′ of Virgo, and the most northerly of three in the brow of the Scorpion § in 5° 55′ of Scorpius. Ptolemy believed the motion to be of 1° in 100 years. Adding then to each of the two stars 25′, which results from the motion in the interval of time between him and Menelaus, that is, 41 years, he fixed Spica in 26° 40′ of Virgo and the north star in the brow of the Scorpion in 6° 20′ of Scorpius; afterwards he added this same quantity to all the stars, and it is thus that he has prepared the tables which are in his book *Al-madjisti*."

There are several flaws in this very positive assertion. First, the epoch of Ptolemy is not 886 of Nabonassar, but 885,‖ so that the interval between him and Menelaus was 40 years, and the amount of precession (at the rate of 36″ per annum) 24′ 0″, not

* By Tycho Brahe called Armillæ Zodiacales.
† *Description des étoiles fixes*, ed. by Schjellerup, p. 42.
‡ Began in July, A.D. 97.
§ β Scorpii.
‖ This slip had already been made by Al Battani, *Opus astron.*, cap. 51, ed. Nallino, p. 124.

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25′, a mistake Ptolemy is not very likely to have made. Secondly, Ptolemy does not say that Menelaus found the longitudes of Spica and β Scorpii to be 170° 15′ and 215° 55′. He merely quotes two conjunctions of the moon with these stars observed by Menelaus in January A.D. 98, and from these he himself deduces from his own lunar tables the positions of the moon and consequently of the two stars for that year. It is, however, quite true that these positions differ from those in Ptolemy's catalogue by 25′. Ptolemy uses them for determining the amount of precession by comparison with the observations of Timocharis in the year 454 Nabonassar. Otherwise he never mentions Menelaus. What is then Al Sufi's authority for the assertion that *all* Ptolemy's star-places are due to Menelaus, 25′ having been added to the longitudes? There is no sign anywhere in Al Sufi's book that he had access to any Greek astronomical observations, except those recorded in the *Syntaxis*. He accepted Al Battani's estimate of annual precession, 1° in 66 years or 54″·5 per annum; he knew that Al Battani, the greatest Arabian astronomer up to his own time, had (like Ptolemy) made use of some observations made by Menelaus to fix that constant, and he must have jumped to the conclusion that as Ptolemy's erroneous precession was due to the use he had made of observations by Menelaus and Agrippa 40 years before his own time, the whole of the star-catalogue must be due to one of them. Of Agrippa the Arabs knew nothing, while they were well aquainted with the mathematical writings of Menelaus, particularly with his valuable work in three books on spherical geometry. Al Sufi therefore made up his mind that Menelaus was the real author of the star-catalogue.

An attempt to prove that the Arabs must have possessed some information about observations made by Menelaus which is not accessible to us was made in 1901 by the late Dr. Björnbo of Copenhagen.* He founded this on a passage in chapter li. of Al Battani's work on astronomy, in which are given, not only the above-mentioned longitude of β Scorpii for the year A.D. 98, but also longitudes of Regulus and Sirius for the same year, supposed by Björnbo to be taken from some book by Menelaus, and not mentioned by Ptolemy. Björnbo gives the following table:—

	Menelaus.	Ptolemy.	Al Battani.
	° ′	° ′	° ′
Sirius	77 0	77 40	88 50
Regulus	122 10	122 30	134 0
Spica	176 15	176 40	[187 50] †
β Scorpii	215 55	216 20	227 45

As regards the two last stars, the difference between Menelaus and Ptolemy is 25′, but the two first stars differ respectively 40′

* "Hat Menelaos aus Alexandria einen Fixsternkatalog verfasst?" *Bibliotheca mathematica*, 3rd Series, ii. pp. 196–212.

† Not given in Björnbo's table; taken from Al Battani's star-catalogue.

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and 20′. The conclusion is therefore drawn that Al Battani must have known observations by Menelaus which have not come down to us, and that their number was probably considerable. Still, Björnbo acknowledges that the statement of Al Sufi was only guesswork, and that Al Battani cannot have had before him a catalogue by Menelaus comparable in extent to that of Ptolemy, since he, who placed so much reliance on Menelaus, would then have based his own catalogue on the older catalogue in preference to that of Ptolemy.

But these conclusions are founded on the old and imperfect translation of Al Battani's book by Plato of Tivoli, and they are disproved by the new translation by Nallino, which includes the star-catalogue of Al Battani, not translated before.* The following is a literal translation of the passage in question: † "Una ex observationibus stellarum qua Ptolemæus usus est, fuit observatio quam Menelaus narravit, ex anno 845 dicti regis Nabonassaris; et narravit stellam borealem ex iis quæ sunt inter duos oculos Scorpii fuisse eo anno, cum eam sphæra armillari metitus esset per lunam, in 5° 55′ Scorpii. Et necesse erat, juxta id quod descripsit in libro suo, ut esset Cor Leonis eo anno memorato in $2\frac{1}{6}$° Leonis ac esset Sirius in 17° Geminorum." The subject of "descripsit in libro suo" is Ptolemæus, not Menelaus; and the meaning is, that Al Battani, from the data in Ptolemy's book, estimated that α Leonis in A.D. 98 must have been in 2° 10′ Leonis and Sirius in 17° 0′ Geminorum. He then, from a comparison with his own observations, finds that the longitudes from A.D. 98 to A.D. 880 had increased by 11° 50′. He therefore adds 11° 10′ to Ptolemy's longitudes to form those of his own catalogue of 533 stars. Why he should in one case have subtracted 40′ from Ptolemy's longitude, and in the other case 20′, to form his estimate of the places of the two stars for A.D. 98, is not clear. But it is by no means the only time that there is some confusion or even error in Al Battani's quotations from Ptolemy.

It must therefore be acknowledged that we have no reason to believe that Ptolemy borrowed any star-places from Menelaus, or that the Arabs possessed any knowledge about observations made by him, beyond what we read in the *Syntaxis*. As Menelaus is the only man after Hipparchus of whom it has ever been asserted that he supplied Ptolemy with star-places, we must now inquire whether Ptolemy may not himself have observed *at least* the stars not observed by Hipparchus and possibly others.

That Ptolemy was an observer of the fixed stars is most distinctly stated by himself. In the chapters on precession ‡ he tells us first that the interval between the observations of Hipparchus and the first year of Antoninus, "in which we also have observed most

* *Pubbl. del Reale Osservatorio di Brera in Milano*, No. 40: *Al-Battani sive Albatenii Opus astronomicum*, ed. C. A. Nallino. (3 vols.)

† *Loc. cit.*, ii. p. 270, among notes, compare t. i. p. 124. In the latter place, line 24, there must be a misprint, for 2° 40′ *read* 2° 10′.

‡ *Syntaxis*, vii. 2 and 3, Heib. ii. p. 12 *sqq.*

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of the places of the fixed stars," is 265 years. Next, that "when we by observation determined for each star its latitude, we found almost exactly the same values as had been found at the time of Hipparchus, or at least with only small differences which might be due to errors of observation. But when we by observation determined the distances from the equator along the declination circle, we found values differing greatly from those of Hipparchus, while the latter differed very much from those handed down from the astronomers of the school of Timocharis." This is illustrated by giving for eighteen stars the declinations observed by Timocharis, Hipparchus, and Ptolemy. In the introduction to the star-catalogue* the method of determining the longitude and latitude of the stars is set forth. Ptolemy says: "Making use again of the same instrument, whose astrolabe circles turn round the pole of the ecliptic, we have observed as many stars down to the sixth magnitude as it was possible to point to." And he goes on describing in detail how the various circles of the instrument were set, so as to give him the difference of longitude and of latitude of a star already determined and a star under observation. It would be impossible to affirm more distinctly that he has made a large number of observations, though Ptolemy does not say that he has observed every single star in the catalogue. Unless we are prepared to accuse that distinguished mathematician and astronomer of deliberate fraud, it is impossible to maintain that he copied the latitudes of the stars from Hipparchus and merely added 2° 40′ to his longitudes. And now that the discovery of the list of Hipparchian stars renders it at least extremely probable that the catalogue of Hipparchus only contained about 850 stars, even the believers in the unacknowledged use of that catalogue must at any rate credit Ptolemy with the 175 additional stars.

But how are we then to account for the constant error of fully a degree, by which amount Ptolemy's longitudes are too small, as if the epoch of the catalogue was about A.D. 60 and not A.D. 137? Is it possible that Ptolemy's equinox for the year 137 can be affected by so large an error?

We know in a general way how Ptolemy determined the absolute longitude of a standard star. He measured the difference of longitude of sun and moon just before sunset, and as soon as the stars began to appear, the difference of longitude between the moon and a star; and, allowing for the moon's motion during the interval, this gave the difference of longitude between the star and the sun, and consequently the longitude of the star. This was of course only done for a very small number of stars, with which the rest of the stars in the catalogue were then connected. But it is easy to see that considerable systematic errors were introduced in this way. In the only example given by Ptolemy, the observation of Regulus on February 23, A.D. 139, he says expressly that the sun was just setting. Therefore the distance between the sun and the moon must have been diminished by nearly the full amount of

* *Syntaxis*, vii. 4, p. 34 *sqq.*

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refraction, about half a degree, and all standard stars observed in this manner and all the catalogue-stars connected with them would be affected by the same error. The interval of time between the two parts of the operation was no doubt made as short as possible in order to eliminate errors arising from imperfect knowledge of the moon's motion in the interval. But Ptolemy had taken over the whole of the solar theory of Hipparchus without alteration; and this was a serious mistake, since the value of the excentricity was too great, which produced an error in the equation of the centre reaching a maximum of 22′.* Again, the instrumental errors must have been considerable, since the observations did not reveal the fact that the adopted latitude of the Museum of Alexandria, 30° 58′, was 13′ or 14′ too small. Is it any wonder, then, that Ptolemy's equinox was about a degree in error?

But it should also be noticed (which nobody seems to have done hitherto) that observations of the kind just described were not the only ones on which Ptolemy depended when he fixed his equinox. To determine the constant of precession he combines observations made by Timocharis and Hipparchus with later observations in the following manner:—

1. The observation of Regulus made by himself, described above, combined with the place of the star given by Hipparchus.

2. A conjunction of the moon with the Pleiades, observed January 29, 283 B.C., by Timocharis, combined with another observed November 29, A.D. 92, by Agrippa in Bithynia.†

3. An occultation of Spica observed March 9, 294 B.C., by Timocharis, and a conjunction observed by him November 8, 283 B.C., combined with an occultation observed by Menelaus at Rome January 10, A.D. 98.

4. A conjunction of the moon with β Scorpii, observed by Timocharis December 20, 295 B.C., combined with another observed by Menelaus January 13, A.D. 98.

The first observation we have already considered. A worse selection of material for the object in view than the seven conjunctions of the moon with stars it would be difficult to conceive. From these observations Ptolemy deduces the positions of η Tauri, Spica, and β Scorpii for the various years, by calculating the place of the moon by means of his own lunar theory. It must be remembered that the latter was very imperfect. He did not know the lunar variation, of which the maximum effect is 39′, though his "prosneusis" would account for part of this, leaving, however, in the octants errors which might rise to 24′. Neither did he know the annual equation, of which the maximum is 11′. It is, therefore, not surprising that the star-places derived from calculated places of the moon are very poor, and that the resulting constant

* P. Tannery, *Recherches sur l'histoire de l'astronomie ancienne*, p. 171.

† For particulars about these conjunctions see Schjellerup's paper in *Copernicus*, vol. i. (1881), p. 223, and Dr. Fotheringham's paper, *Monthly Notices*, vol. lxxv. (1915), p. 377.

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of precession was very erroneous. The following table gives the longitudes thus found compared with the true ones:—

	−293, 294.	−282.	+92.	+98.
	° ′	° ′	° ′	° ′
η Tauri, Ptol.	...	29 30	33 15	...
True	...	28 17	33 29	...
Error	...	−1 13	+14	...
Spica, Ptol.	172 20	172 30	...	176 15
True	172 0	172 10	...	177 25
Error	−20	−20	...	+1 10
β Scorpii, Ptol.	212 0	...	...	215 55
True	211 19	...	...	216 44
Error	−41	...	...	+49

By a curious piece of ill luck the longitudes for the time of Timocharis were all too great and those for the end of the first century too small, which produced the faulty precession of 1° in 100 years. And the longitude of Regulus deduced from the observation described above was 122° 30′, and was therefore 1° 33′ too small, the true longitude for A.D. 139 being 124° 3′. We have no information whatever as to how many standard stars were compared with the sun, nor how often this was done. And, judging from the important part played by conjunctions of stars with the moon in determining the amount of precession, it does not seem improbable that stars may also sometimes have been measured from the moon, and the place of the moon have been taken from Ptolemy's tables, without observing the moon's elongation from the sun. But even if this was never done, the mean of the errors in the adopted longitudes of the four stars mentioned, if we refer them to the year A.D. 137 with Ptolemy's precession, is +63′. This corresponds to the amount of precession in 75½ years, making the real epoch of Ptolemy's catalogue A.D. 62. From 349 zodiacal stars Peters found the epoch to be A.D. 58. Is it then necessary to believe that Ptolemy borrowed his star-places from Hipparchus?

To sum up, the following conclusions may reasonably be drawn:—

1. That the star-catalogue of Hipparchus did not contain more than about 850 stars, so that Ptolemy cannot have borrowed the whole of his catalogue from Hipparchus.

2. That if he did borrow the relative positions of these 850 stars (of which there is no proof), 175 stars, probably identical with the stars observed with an instrument, of which the latitude circle was only divided to ¼°, must at any rate have been observed by Ptolemy.

3. That the assertion of one Arabian astronomer who lived eight hundred years later, that Ptolemy borrowed his catalogue from Menelaus, must be rejected as being totally unfounded.

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4. That the methods adopted by Ptolemy for determining the longitudes of standard stars of necessity introduced large systematic errors in the results.

5. That the only four longitudes, of which he gives the values thus determined, are on an average 63′ too small, which agrees perfectly with the average error of the stars in his catalogue.

6. That this is therefore an error in the equinox determined by him ; and

7. That there is therefore no reason to disbelieve his positive statement, that he had made extensive observations of fixed stars.

14 *Staverton Road, Oxford:*
1917 *May.*

Thermal Diffusion and the Stars. By S. Chapman, M.A., D.Sc.

It has recently been discovered theoretically,* and verified experimentally,† that a temperature gradient in a mixture of two gases will in general produce diffusion, or a tendency to diffusion. A steady state is possible only when this tendency has become neutralised by its own action in producing a concentration gradient, or variation in the relative proportions of the two gases. The direction of the effect is such that the heavier molecules tend towards the cooler regions, except that if the masses are very nearly equal, and the diameters are very unequal, the larger molecules will pass towards the place of lower temperature.‡ If the masses and diameters (or laws of interaction at collision) are the same for the two gases, thermal diffusion does not occur at all.

It is to be expected that such a phenomenon may have interesting consequences in cosmical physics, especially in the stars, where large temperature gradients are known to exist. The letters which I have received from various friends, containing suggestions or inquiries on this point, have led me to examine the more obvious possibilities. Though the result does not favour the idea that thermal diffusion has very much play in stellar physics, I have thought that it may be of interest to publish a brief note on the matter, in order to make clear why this conclusion seems probable.

The feature which adds special interest to thermal diffusion is that, since the temperature of a star diminishes from centre to surface, the tendency is for the heavier molecules, on this account, to be raised to the higher levels. It is a fact of observation, of a

* Chapman, *Phil. Trans.*, A, ccxviii. p. 1 (received May 19, 1916, but not yet published), and *Proc. Roy. Soc.*, A, xciii. p. 1 (1916). The same phenomenon has also been discovered by rather different mathematical analysis, almost simultaneously, by D. Enskog (*Inaugural Dissertation*, Upsala, 1917).

† Chapman and Dootson, *Phil. Mag.*, March 1917.

‡ *Phil. Mag.*, 1917 (as yet unpublished).

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rather remarkable character, that heavy elements *are* observed near the surface of the sun. It is perhaps tempting to connect the two phenomena in a theory of the latter. In the accompanying note on "Convection and Diffusion within Giant Stars," I have discussed the case as regards the interior of a giant star. It appears that the thermal effect is far outweighed by the contrary influence of "pressure diffusion," which tends to produce stratification into layers of increasingly heavy elements towards the centre of the star. The effect of thermal diffusion is in amount only a small percentage of that of pressure diffusion. The reason, as there indicated, is that the concentration gradients which the two actions tend to set up are proportional to the gradients of log p and log T; the constant of proportionality, moreover, being two or three times as great in the former as in the latter case. Within a giant star $p \propto T^n$, where n is probably 4, so that the gradient of log p is four times as great as that of log T.

In the case of a stellar atmosphere, such as that of the sun, the conditions will in many respects differ widely from those prevailing within a giant star. For pressure diffusion to be overcome by thermal diffusion, however (so that heavy molecules may be raised), the exponent n in $p \propto T^n$ would have to be a fraction (say $\frac{1}{3}$), and if Boyle's law is approximately valid, the density ρ would then have to increase outwards according to some such law as $\rho \propto T^{-\frac{2}{3}}$. It is very difficult to suppose that this is the case in fact.

Moreover, even if thermal diffusion were predominant over pressure diffusion, the convective motion as observed is so considerable that it is hardly to be expected that the state of diffusive equilibrium could establish itself. Convection has a powerful mixing effect, and it seems most likely that the presence of elements of widely different weights in the solar atmosphere must be attributed largely to this cause.

Convection and Diffusion within Giant Stars.

By S. Chapman, M.A., D.Sc.

§ 1. Professor Eddington's recent paper* "On the Radiative Equilibrium of the Stars" has given us an altogether new insight into the conditions probably existing within a star, especially within a "giant" star, of low density. He has shown that in the latter case the temperature and pressure gradients towards the centre must be much less than was formerly supposed, the condensing influence of gravity being largely counteracted by the pressure of the outflowing radiation. Definite numerical estimates of various quantities relating to such stars are deducible from his theory, and have proved very successful wherever comparison with

* Eddington, *M.N.R.A.S.*, lxxvii. p. 16, 1916 November.

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observational data was possible. The theory may well serve, therefore, as a working hypothesis on which to base the consideration of further problems concerning giant stars. In the present paper this plan is adopted in the attempt to determine the relative importance of convection * and diffusion within a star; this question has, of course, an important bearing on the degree of uniformity of composition in the star.

§ 2. The illustrative model of a giant star which will be considered here is that described as an example in Professor Eddington's paper. The following are its principal data :—

Mass . . .	$2{\cdot}87 \,.\, 10^{33}$ grm. (about $1{\cdot}5 \times$ sun)
Radius . . .	$7 \,.\, 10^{11}$ cm. (about $10 \times$ sun)
Mean density . .	$0\,.\,002$ grm. cm.$^{-2}$
Effective temperature	$6500°$

These correspond to a giant star of spectral type F–G.

The theory of radiative equilibrium indicates that the laws of variation of density ρ, pressure p, and temperature T within the star resemble those occurring in the adiabatic equilibrium of a mass of gas for which the ratio of specific heats (γ) is $\frac{4}{3}$. Therefore

$$p \propto T^4.$$

(If Boyle's law is assumed to hold, also $\rho \propto T^3$.)

The law of variation of T along the radius of a sphere of gas in adiabatic equilibrium has been worked out by Emden (*Gaskugeln*, p. 80) for various values of γ. The following table of values of T, ρ, and p has been deduced from Emden's table 7 (corresponding to $\gamma = \frac{4}{3}$), in combination with Professor Eddington's calculated values of the central pressure (p_0) and temperature (T_0), viz.

$$p_0 = 1{\cdot}19 \,.\, 10^{12} \text{ dynes (or } 1{\cdot}18 \,.\, 10^6 \text{ atmospheres).}$$
$$T_0 = 7{\cdot}12 \,.\, 10^6 \text{ degrees.}$$

Fraction of Radius.	Temperature.	Density in grm.-cm.$^{-2}$ (Water = 1.)	Pressure in Atmospheres.
0·0	7,120,000°	0·109	1,180,000
0·1	6,580,000	0·086	860,000
0·2	5,340,000	0·046	370,000
0·3	4,000,000	0·0194	118,000
0·4	2,870,000	0·0072	31,000
0·5	2,040,000	0·0026	8,000
0·6	1,360,000	0·00076	1,600
0·7	870,000	0·00020	260
0·8	500,000	0·000038	29
0·9	230,000	0·0000037	1·3
0·98	34,000	$1{\cdot}1 \,.\, 10^{-7}$	$6 \,.\, 10^{-4}$
1·0	0	0	0

* By convection is meant any motion caused by a pressure gradient, whether or not the latter is due to non-uniform temperature.

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For comparison with the densities in the above table, we may note that the density of air at 0° C. and 1 atmosphere pressure is 0·0013.

The details of the above table must naturally not be pressed too far, but they probably give a sound general indication of the conditions existing within the star. One important assumption implied by the table is that the substance of the star is homogeneous, a condition which is the subject of examination in this paper. It will be sufficient for the present, however, to adopt the above numbers as a basis for discussion.

§ 3. The principal calculation to be made will deal with the amount of stratification, or separation of the different constituents of the star into concentric layers, which is to be expected if diffusion is supposed to act undisturbed by any mixing influences. If the results do not agree with observation, we may then conclude that the latter influences are present, and may perhaps infer roughly to what depths they extend. The principal agent tending to restore homogeneity of composition is, of course, convective motion. If the star is sufficiently highly agitated by currents and eddies, the composition may be rendered almost uniform throughout. But we do not know what eddy-producing forces exist within a star, and cannot obtain any direct estimate of their amount. In the case of the sun (which is, however, a dwarf star) we have visible evidence of considerable convection near the surface, but we do not know to what depths this extends. It is a notable fact that in the sun some of the heaviest elements appear in the high levels, *e.g.* lead (atomic weight 207) in the photosphere, and ytterbium (172) even in the chromosphere. The case seems to be similar in some of the giant stars, though the facts regarding a star are less easy to obtain than are those relating to the sun. At the suggestion of the Astronomer Royal, I examined the data for Arcturus, which is of magnitude 0·2 and parallax 0″·066 (Yale), and must therefore be a giant star *: its spectral type is K_0. Adams † has shown that the spectrum of this star resembles that of a sunspot, and he has identified at least 17 elements in it, the one of highest atomic weight being lanthanum (139). As the identification of an element in a star spectrum is more difficult than in the solar spectrum, it seems not unlikely that still heavier atoms will be identified in this and other giant stars. The presence of heavy elements in the outer layers, alongside those of much smaller atomic weight, is a remarkable fact, and one which requires explanation.

§ 4. We will first consider the state of affairs which would be brought about by diffusion alone, assuming that there is no

* Its light emission must be about 300 times that of the sun, so that if the surface luminosity is similar in the two cases, Arcturus must be of large radius and presumably of low density.

† Adams, *Ap. J.*, xxiv. p. 69.

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relative motion in the star. The causes which are effective in producing differences of composition are—

(*a*) external forces acting unequally per unit mass on the different constituents;
(*b*) a gradient of pressure;
(*c*) a gradient of temperature.*

The direction in which (*a*) will act is obvious; (*b*) will cause the heavy elements to tend towards regions of higher pressure; while (*c*) will cause the heavier molecules to move towards the cooler regions, except that if two sets of molecules are nearly alike in mass, and of widely different diameters, the larger ones will tend towards the cooler regions.† The latter case, however, hardly concerns us in the present instance.

In the star under consideration, the external forces acting on the molecules are those of gravity and radiation pressure. If the atoms are split up into positive and negative particles at the centre of the star, owing to the high temperature prevailing there, electrical forces would also come into play which might be considerable in certain circumstances. These will not be considered here, however.‡ As regards the two forces just mentioned, gravity acts proportionately to the mass of each molecule, and we may assume, for the time being, that so also does radiation pressure. They will consequently not produce diffusion.

In the mathematical treatment of the other two agents, (*b*) and (*c*), it is convenient to idealise the problem by supposing that only two gases are present, of molecular weights m_1 and m_2 (say $m_1 > m_2$), and relative proportions, by volume, λ_1 and λ_2 (so that $\lambda_1 + \lambda_2 = 1$). The equation of steady state, as regards (*b*) and (*c*), may be written

$$\frac{\partial \lambda_1}{\partial x} = -\frac{\partial \lambda_2}{\partial x} = k_p \frac{\partial \log p}{\partial x} - k_{\mathrm{T}} \frac{\partial \log \mathrm{T}}{\partial x},$$

where k_p and k_{T} are certain diffusion factors which are defined in the memoir already cited. The gradient of relative concentration (λ_1 or λ_2) is therefore compounded of two terms proportional respectively (over ranges in which k_p and k_{T} are nearly constant) to the gradients of log p and log T. In the case of the star in question,

$$p \propto \mathrm{T}^4, \quad \log p = \text{constant} + 4 \log \mathrm{T};$$

* The last agent, which I have termed thermal diffusion, appears to have remained unknown until published, as a theoretical discovery, in *Phil. Trans.*, A, ccxvii. p. 1 (received 1916 May 19). An abstract is given in *Proc. Roy. Soc.*, A, xcviii. p. 1. Enskog (*Inaugural Dissertation*, Upsala, 1917) has also discovered the same phenomenon, almost simultaneously. Some experiments confirmatory of the theory are given in *Phil. Mag.*, 1917 March (Chapman and Dootson).

† *Phil. Mag.*, July 1917.

‡ They would, of course, be such as to oppose any general separation of the opposite charges.

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so that, other things being equal, thermal diffusion will not be so powerful an agent as pressure diffusion. Moreover, except when the molecular masses are nearly equal, k_p seems to be twice or thrice as great as k_T, so that the concentration gradient due to the pressure effect will be ten or twelve times as large as that due to the temperature gradient. The two effects are in opposition in the giant star: the rise of temperature towards the centre neutralises, to the extent of perhaps about 10 per cent., the central condensing influence which the rise of pressure exerts on the heavier gas. This conclusion, it should be noted, does not depend on the actual magnitudes of the pressure and temperature gradients, but only on the relation $p \propto T^4$. If the star were in adiabatic equilibrium, and $\gamma > \frac{4}{3}$, p will vary according to a lower power of T: thus for a monatomic gas, $\gamma = \frac{5}{3}$, and the corresponding relation between p and T is $p \propto T^{\frac{5}{2}}$. In such a case thermal diffusion would be more effective than in the above instance: perhaps something of this kind may occur in a dwarf star, where radiation pressure is relatively unimportant compared with gravitation.

The constant k_p is given by

$$k_p = \lambda_1 \lambda_2 \frac{m_1 - m_2}{\lambda_1 m_1 + \lambda_2 m_2},$$

and k_T also contains the factor $\lambda_1 \lambda_2$, though the remainder of the expression cannot be written down in simple terms. We shall incorporate the effects of thermal and pressure diffusion in one equation, for simplicity, taking $k_p = \frac{5}{2} k_T$ and $p \propto T^4$. Then we have

$$\frac{\partial \lambda_1}{\partial x} = -\frac{\partial \lambda_2}{\partial x} = \frac{18}{5} k_p \frac{\partial \log T}{\partial x}$$

as our equation of composition.

§ 5. This will now be applied to the problem of a star composed of two elements, one of which is four times as heavy as the other. The lighter one, for instance, may be thought of as iron (56) and the other as lead (207) or radium (226). In this case $k_p = \frac{7}{2} \frac{3\lambda_1 \lambda_2}{1 + 3\lambda_1}$. Writing the equation of composition in the form

$$\frac{\partial \log \lambda_1}{\partial x} = \frac{7}{2} \frac{3\lambda_2}{1 + 3\lambda_1} \frac{\partial \log T}{\partial x}$$

we see that

$$\lambda_1 \propto T^{\frac{7}{2} \frac{3\lambda_2}{1 + 3\lambda_1}}$$

over a range throughout which the index is nearly constant. If the proportion of the heavy element is large, it changes only very slowly. If, on the other hand, it is very small, $\lambda_1 \propto T^{10 \cdot 5}$. Suppose, for instance, that at the centre of the star there are 9 parts by volume of the heavy element to 1 of the light element. This proportion will at first vary very gradually: a rough calcula-

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tion indicates that the proportions at other fractions of the total radius will be as follows, if we neglect the change in the pressure and temperature gradients caused by the varying composition along the radius :—

r (Fraction of Total Radius).	T.	ρ.	λ_1.	λ_2 (to 3 Decimal Places).	Fraction of Mass within Radius r.		
					(a)	(b)	(c)
·000	7,120,000	0·109	0·900	0·100	0·000	0·000	0·000
·109	6,500,000	0·083	0·877	0·123	0·060	0·076	0·078
·217	5,120,000	0·040	0·790	0·210	0·312	0·380	0·397
·290	4,150,000	0·022	0·675	0·325	0·519	0·599	0·635
·362	3,290,000	0·0107	0·519	0·481	0·695	0·749	0·811
·435	2,560,000	0·0050	0·316	0·684	0·820	0·825	0·912
·507	1,970,000	0·0023	0·120	0·880	0·903	0·850	0·960
·580	1,490,000	0·0010	0·018	0·982	0·951	0·854	0·982
·652	1,110,000	0·00041	0·0010	0·999	0·979	0·855	0·993
·725	790,000	0·00015	·000034	1·000	0·994	0·855	0·999
·870	310,000	0·00001	$2·1 . 10^{-9}$	1·000	0·999	0·855	1·000

According to this table, the heavy gas will practically not extend further from the centre of the star than three-quarters of the total radius. This result, however, certainly much overstates the outward extension of the heavy gas. In the last three columns of the table are given the proportions of the total mass, within radius *r*, (*a*) in the case of a homogeneous star, the only case in which the above values of T and ρ are strictly correct (this column has been calculated from the table by Emden already mentioned), and (*c*) in the present case. Column (*b*) gives the proportion of the mass of the heavy gas within radius *r* to the total mass of the star. The greater concentration of mass towards the centre will, however, make the pressure gradient greater than that which corresponds to a homogeneous star, and this will increase the central condensation of the heavy gas: thus, if we consider a star composed of 85 per cent. of the heavy gas to 15 per cent. of the lighter gas (reckoning by mass), there will be more than 90 per cent. of the heavy gas at the centre (by volume), while at a central distance of six- or seven-tenths of the radius there will be only a very small fraction of 1 per cent. of the heavy constituent.

Since it does not seem probable that in an actual star there would be so large a proportion (by mass) of the heavy constituents as 85 per cent., I have also calculated a similar table, as follows, starting from the hypothesis that at the centre of the star the proportion of heavy gas, by volume, is $\frac{2}{3}$:—

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r.	T.	ρ.	λ_1.	λ_2.	Fraction of Mass within r. (a)	(b)	(c)
·000	7,120,000	0·109	0·667	0·333	0·000	0·000	0·000
·036	7,050,000	0·106	0·660	0·340	0·003	0·005	0·006
·072	6,830,000	0·096	0·628	0·372	0·019	0·037	0·042
·109	6,500,000	0·083	0·555	0·445	0·060	0·108	0·125
·145	6,080,000	0·068	0·427	0·573	0·125	0·205	0·247
·181	5,610,000	0·054	0·228	0·772	0·212	0·290	0·376
·217	5,120,000	0·040	0·0465	0·954	0·312	0·331	0·481
·254	4,630,000	0·030	0·0011	0·999	0·417	0·339	0·565
·290	4,150,000	0·022	$3·7 . 10^{-5}$	1·000	0·519	0·339	0·640
·362	3,290,000	0·0107	$1·2 . 10^{-8}$	1·000	0·695	0·339	0·772
·435	2,560,000	0·0050	$2·4 . 10^{-13}$	1·000	0·820	0·339	0·867
·652	1,110,000	0·00041	...	1·000	0·979	0·339	0·984
·870	310,000	0·00001	...	1·000	0·999	0·339	0·999

As the table shows, in this case the proportion, by mass, of the heavy constituent is approximately one-third, and this is almost entirely concentrated within the central region of one-quarter the total radius, while all traces of it have vanished at the half radius. As before, these figures underestimate the centralisation of the heavy gas, owing to the more rapid pressure gradient which would result from the heterogeneity of composition.

The proportion, by mass, of the heavy gas is still large in the last example, viz. one-third, and the difference of concentration between the two cases indicates how greatly the concentration increases as the proportion diminishes. If, for instance, only one-tenth of the mass of the star were due to the heavy substance, this would settle down completely within a radius of one-fifth, or less.

In an actual star there would be, not merely two elements, but a large number of different ones with molecular weights extending over a wide range. It is difficult to discuss this case in detail, since we know so little about the relative amounts of the different substances. The result of the graded nature of the mixture will be a less sharp separation between the layers of different composition. As a rough approximation we may take the elements in groups covering a range of 50 or 80 in atomic weight. The mass ratio between two groups will be less than in the above examples, and to illustrate the effect of this we will consider a further example in which there are two gases (only) with a mass ratio of 2 in place of 4. We will also suppose that there is one part in four, by volume, of the heavy gas at the centre of the star. This is found to correspond to a proportion, by mass, in the whole star, of 0·258 of the heavy gas. The proportions by volume at different distances are as follows:—

Distance r	0·000	0·072	0·145	0·217	0·290	0·362	0·435	0·507
λ_1	0·250	0·225	0·126	0·024	0·0009	$6 . 10^{-5}$	$1·6 . 10^{-6}$	$2 . 10^{-8}$

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Thus the heavy element practically does not appear beyond the half radius. In the former case, however, with a larger amount of the heavy gas, the centralisation was greater.

As regards the light gases like hydrogen, these should, of course, be on the surface in rather sharply defined layers. If we consider the two elements iron and hydrogen, m_1/m_2 is about 50, and when λ_2 is nearly 1 the value of λ_1 will be proportional to T^{170} approximately. In the model star considered, the temperature gradient near the surface is comparable with the maximum gradient (one-sixth, in fact), so that the iron layer should thin out very rapidly at its upper surface. The same is true, to a slightly less extent, for the lighter elements such as magnesium (24).

§ 6. The above calculations are only roughly illustrative of the conditions which would exist in a star if diffusion could act undisturbed by mixing influences, but they indicate a degree of stratification according to density which seems to be incompatible with the observed presence of heavy elements at the surface of the stars. It is therefore necessary to consider what explanation can be given for their appearance in the upper levels.

One possibility is that radiation pressure does not act proportionately to the mass of the atoms, but is more effective per unit mass on the heavy than on the light elements. The facts on this point are not known; should experimental data become available, it would be an easy matter to take account, in this calculation, of any observed departure from the assumed law, but it has not seemed worth while to consider hypothetical cases of this kind.

The most likely explanation would seem to lie in the presence of convective motion, which, to be effective in raising heavy elements from their natural (diffusion) levels towards the surface, must extend inwards to a depth of one-half or one-third the radius of the star. The degree of convection necessary to counteract the diffusive tendency to stratification is hardly a matter which can be the subject of calculation at present. The case can be illustrated, however, by consideration of what takes place in the earth's atmosphere. There the pressure gradient (p diminishing upwards) should produce by diffusion a change of 1 per cent. in the proportions of oxygen and nitrogen in every 5 kilometres: the temperature gradient would neutralise this tendency to the extent of a few per cent. of its whole amount. But careful tests have revealed no such change of composition up to a height of 15 kilometres, a circumstance which is natural in view of the continual agitation of the troposphere by convection currents. In the stratosphere or isothermal layer, however, it is generally supposed that convection is absent, so that the various constituents of the atmosphere settle out more or less definitely into layers according to their molecular weights.

The absence of convection in the upper air is referable partly to the diminution of disturbing forces and also to the fact that the upper air is much more viscous, in proportion to its mass, than that of the troposphere. The viscosity μ of a gas is independent

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of the pressure p or density ρ (the temperature being kept constant) down to very low pressures. Hence the kinematic viscosity μ/ρ, on which depends the damping of differential motions such as occur in eddying masses of gas, increases inversely as the density. At the earth's surface it is 1·3 C.G.S. units; at a height of 100 kilometres it is about one million times as great.

The independence of μ on p fails both for very low and very high pressures; in the case of carbon dioxide* at pressures of 60 atmospheres, or more, μ seems to vary as p^n, where n is 2 or even more. Also μ varies with temperature, as T^m, where m seems always to lie between $\frac{1}{2}$ and 1. In the case of a star, if these laws hold good under the conditions found there, $\mu \propto p^2T^{\frac{1}{2}}$ at least, *e.g.* as $T^{9/2}$. Hence $\mu/\rho \propto T^{3/2}$, and the kinematic viscosity should increase to a high value towards the centre of the star. The disturbing forces tending to produce eddies are probably greater near the surface than far within the star, so that we may expect that convective mixing will be least important in the innermost regions. Again, the time taken to bring about the steady state by diffusion is proportional to $1/D$, where D is the coefficient of diffusion. Under ordinary conditions D is a small multiple of the kinematic viscosity μ/ρ, and varies approximately in the same manner. Hence if μ/ρ increases towards the centre of the star, not only will any tendency to convective mixing be hindered, but the process of diffusion will be accelerated. We may therefore expect that a certain amount of settling-out of the heaviest substances takes place near the centre, but that convection extends to a very considerable depth within the star, sufficiently to raise many heavy elements to the surface layers.

[*Note added* 1917 *May* 30.] § 7. The above calculations would to a large extent lose their point should it prove that the substance of the interior of the star is not in the physical condition which is there contemplated. It may be the case, for instance (as Mr. Jeans suggested in his recent Bakerian lecture, 1917 May 17), that at the high temperatures in the interior all molecules become split up into their component electrons, so that differences of molecular weight or constitution cease to be. The ordinary chemical elements would then occur only in an upper layer, of unknown thickness, where the temperature is sufficiently low to permit of their continued existence. If so, the above calculations would not begin to apply until this layer was reached; the conclusions reached would probably be modified so that a smaller demand would be made on such mixing agencies as convection, to account for the presence of heavy elements in the surface layers.

* *Cf.* Jeans' *Dynamical Theory of Gases*, 2nd ed., p. 297.

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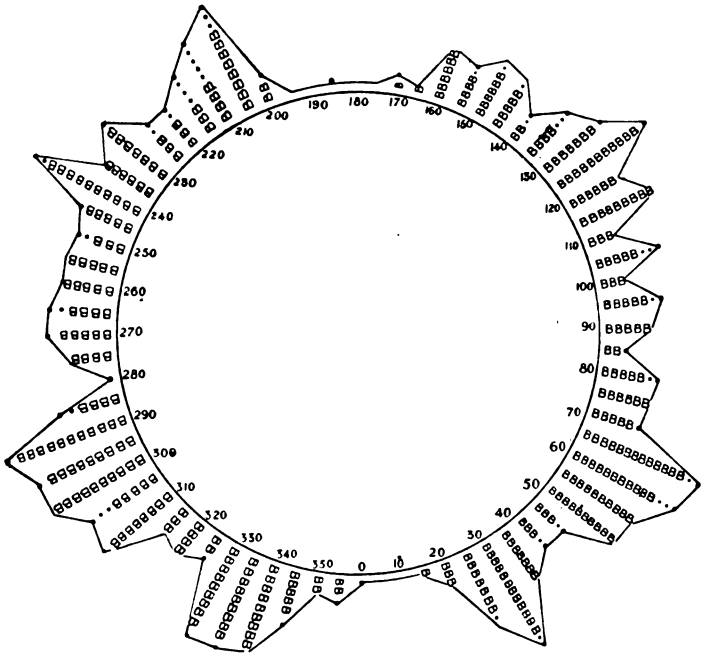
Solar Prominences, 1916. By G. J. Newbegin.

The weather conditions of the months January and December were such that in the course of the year no more than sixty-three observations were recorded.

For enumeration on the plan of the International Solar Union,



N.

Distribution of Prominences, 1916.

in which the lower limit of height is taken as 30″, 343 prominences are available, which, when plotted into diagrammatic form, affect 425 position-angles.

The diagram indicates a growth in activity all round, the polar regions only showing a want of occupation, though they are much more filled up than in 1915.

A general trend of activity towards the northern hemisphere is noticeable.

The dark absorption has been observed on thirty occasions, on 5 faint and 25 bright prominences.

The general brightness has increased, and faint prominences are proportionally fewer.

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Observations of (Ch. 7085) RT Cygni in 1912–17.

By A. N. Brown, M.A.

I began to observe this Long-Period Variable, the Harvard designation of which is 194048, in 1908, the observations, with light curves, down to the beginning of 1912, having been printed in *Monthly Notices*, vols. lxix.–lxxii. The 212 observations which follow continue the series, and, as I have not previously shown the dates of maxima and minima in tabular form, they are now given from 1908 to the beginning of this year. It has not been thought necessary to reproduce the curves drawn from the later observations.

Maxima.			Minima.		
Date.	Julian Date.	Mag.	Date.	Julian Date.	Mag.
			1908 Apr. 2	8034	11·1
1908 June 29	8122	7·3	1908 Oct. 11	8226	11·3
1909 Jan. 2±	8309	7·7	1909 Apr. 24	8421	11·1
1909 July 20	8508	7·2	1909 Oct. 31	8611	11·0
1910 Jan. 20	8692	6·6	1910 May 6	8798	10·7
1910 July 17	8870	7·1	1910 Nov. 24	9000	11·5
1911 Feb. 17	9085	8·1	1911 May 22	9179	11·2
(1911 Aug. 21)	(9270)	...	1911 Nov. 21	9362	11·2
1912 Feb. 4±	9437	7·8	1912 June 5	9559	11·2
1912 Sept. 2	9648	7·8	1912 Dec. 14	9751	10·9
1913 Feb. 23	9822	6·5	1913 June 28	9947	10·9
1913 Sept. 18	0029	7·8	1913 Dec. 25	0127	10·9
1914 Mar. 16	0208	7·0	1914 July 9	0323	11·3
1914 Oct. 8	0414	7·5	(1915 Jan. 21)	(0519)	...
(1915 Apr. 10)	(0598)	...	1915 July 20	0699	11·1
1915 Oct. 16	0787	7·4	1916 Feb. 9±	0903	11·3
1916 May 6	0990	7·0	1916 Aug. 17	1093	10·9
1916 Nov. 4	1172	7·3	1917 Feb. 16	1276	11·2

The symbol ± above implies that the date is somewhat uncertain owing to paucity of observations, while the dates given in brackets are conjectural, the observations being interrupted at these periods. But in the case of none of the phases indicated above, even where the curve is most pointed, is it strictly accurate to speak of the maximum or minimum as having occurred definitely on any particular day, since this star, as indeed is the case with all Long Periods with which I am familiar, always remains

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at maximum and minimum light during several days. On assigning the above dates, therefore, I have endeavoured to give the day when the middle point of each phase had been reached. Neither at maximum nor minimum has the curve any fixed shape. Sometimes it is more or less rounded, sometimes comparatively flat. But the rise is invariably more rapid than the fall, on the average by 27 days. The period as deduced from these observations is 192·4 days. The Harvard period is 190·5 days.

By far the most remarkable of the maxima is that which is assumed to have occurred on 1915 Oct. 16. If the observations between Sept. 19 and Dec. 12 in this year are plotted, it will be seen that there was a rise to a maximum of 7·4 mag. on Oct. 16; a fall to 7·7 mag. on Nov. 2; and again a rise, this time to 7·3 mag., on Nov. 17. After this the light faded quickly, the estimate for Nov. 26 being 8·0 mag. Here then seems to be a clear case of a double maximum. M. Félix de Roy, a member of the Variable Star Section of the B.A.A., has kindly allowed me to compare his observations made at this time with mine. On the whole the two series agree well, and M. de Roy's—he considers the date of maximum to have been Oct. 18—certainly show indications of a fall on about Nov. 4 which continued for several days and was followed by an abrupt rise. But unfortunately, just at the period when I think the second maximum occurred, his observations are somewhat scanty in number. As the date in October corresponds much more closely with the mean period of the Variable, I have entered it in the table given above.

In 1911 I noted (*M.N.*, vol. lxxii. p. 417) a case of a very faint maximum (8·1 mag.) being preceded by an unusually slow fall to minimum. In subsequent years so far there has been no minimum as faint as 8·1 mag. The three faintest are assumed to have occurred on 1912 Feb. 4 ± ; 1912 Sept. 2; and 1913 Sept. 18, all of 7·8 mag. The maximum immediately before the first of these was not observed, but the falls to the minimum which preceded each of the other two took 122 ± and 125 days respectively, the average time being 109·6 days. It almost seems as if the energy of the Variable is to some extent dissipated during these slower falls to minimum, so that it is incapable of rising to its normal maximum brilliance.

The arrangement of the different columns showing the observations is the same as that adopted in my observations of V Cassiopeiæ which were published last year in *M.N.*, lxxvi. pp. 606–612, and is there fully explained; but in Hagen's Atlas (series iv.) Harvard (H.P.) magnitudes are given for only some of the comparison stars for RT Cygni. I have adopted these values for each comparison star which I have used when they are there given, and in all other cases have employed Hagen's own magnitudes.

The telescopes used were 5-inch and 3-inch refractors with powers varying from 25 to 200.

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The Observations.

Date.		Julian Date.	Comparisons.	Ded. Mag.
1912				
Jan.	26	9428	5−1·5, 7+1	7·9
	27	9429	5−1·5, 7+0·5	8·0
	29	9431	5−1, 7+1	7·9
Feb.	2	9435	5+1	7·7
	4	9437	4−2·5, 6+0·5; 5+1·5	7·8
Apr.	3	9496	14−2, 22+6	8·9
	29	9522	41+1	10·3
May	24	9547	=44	11·0
June	2	9556	44−1·5	11·2
	8	9562	44−2	11·2
	10	9564	44−2	11·2
	20	9574	44+1	10·9
	22	9576	43−3, 44+1	10·9
	28	9582	41−1·5, 43+0·5	10·6
July	4	9588	41+2	10·2
	11	9595	22−7, 41+2	10·2
	14	9598	25−1	9·8
	16	9600	23−1·5, 25+1	9·6
	25	9609	14−3, 18+1; 15−3·5, 20+2; 16−4, 19+2	9·1
	30	9614	15−1, 20+3·5; 16−1·5, 19+4·5	8·9
Aug.	1	9616	15+1; 16−1	8·8
	29	9644	4−1·5, 6+1	7·8
	30	9645	4−1·5, 6+1	7·8
Sept.	2	9648	4−1·5, 6+1	7·8
	17	9663	5−1·5, 7+0·5	8·0
	20	9666	5−1·5, 7+0·5	8·0
	23	9669	5−2, 7+0·5	8·0
Oct.	2	9678	7−3, 10+1	8·3
	3	9679	6−3·5, 10+1	8·3
	9	9685	10−2, 14+1	8·6
	10	9686	13+1	8·4
	14	9690	14−1, 19+5; 15+1	8·8
	17	9693	14−1·5, 19+5	8·8
Nov.	3	9710	22−1; 23−0·5, 25+1	9·6
	29	9736	41−1·5, 43+0·5	10·6
Dec.	10	9747	43−3, 44+1	10·9
	12	9749	43−3, 44+1	10·9
	16	9753	43−2·5, 44+1	10·9
	18	9755	43−3, 44+1	10·9
	29	9766	41−1·5, 43+1	10·5
1913				
Jan.	1	9769	41+1·5	10·3
	5	9773	30−1·5, 41+3	10·1
	12	9780	23−1·5, 25+1	9·6
	26	9794	6−5, 12+0·5; 10−1, 13+0·5; 14+2	8·5
Feb.	2	9801	7−3, 10+1	8·3
	11	9810	3−7, 4+2	7·4
	16	9815	2−1, 3+0·5	6·6
	18	9817	2−1, 3+0·5	6·6
	20	9819	2−1, 3+0·5	6·6
	22	9821	2−0·5, 3+1·5	6·5
Mar.	2	9829	2−1, 3+0·5	6·6
	11	9838	3−6, 4+4	7·3
	30	9857	5−1, 7+1	7·9
May	1	9889	14−2, 19+4; 15−1, 20+4	8·9
	6	9894	14−4·5, 19+1·5; 15−3, 20+1·5	9·1
	10	9898	18−1, 19+1	9·2
	13	9901	15−4, 19+1	9·2
	16	9904	19−1·5, 22+1	9·4
	24	9912	30−2·5, 41+1	10·3
	30	9918	41−0·5, 43+1	10·5
June	2	9921	41−1, 43+1	10·5
	5	9924	41−1, 43+0·5	10·5
	7	9926	41−1·5, 43+0·5	10·6
	23	9942	43−2·5, 44+1	10·9
	27	9946	43−3, 44+1	10·9

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The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1913			
July 3	9952	43 − 2·5, 44 + 1	10·9
11	9960	41 − 1·5, 43 + 0·5	10·6
13	9962	41 − 1·5, 43 + 0·5	10·6
20	9969	25 − 2·5, 30 + 1	9·9
24	9973	= 22	9·5
30	9979	15 − 1·5, 20 + 3·5; 16 − 2, 19 + 4	8·9
Aug. 28	0008	3 − 5, 4 + 5	7·2
Sept. 6	0017	3 − 8, 4 + 2	7·5
7	0018	3 − 7, 4 + 2	7·4
15	0026	4 − 0·5, 6 + 2	7·7
24	0035	3 − 6·5, 4 + 3	7·3
28	0039	3 − 7, 4 + 2	7·4
Oct. 6	0047	4 − 1, 6 + 2	7·7
Nov. 5	0077	14 − 4·5, 19 + 2	9·1
19	0091	19 + 1	9·2
22	0094	30 − 2, 41 + 2	10·2
28	0100	41 + 1	10·3
Dec. 14	0116	43 − 3, 44 + 1	10·9
18	0120	43 − 3, 44 + 1	10·9
28	0130	43 − 3, 44 + 1	10·9
30	0132	43 − 3, 44 + 1	10·9
1914			
Jan. 3	0136	43 − 2·5, 44 + 1	10·9
5	0138	43 − 2·5, 44 + 1	10·9
23	0156	= 30	10·0
26	0159	25 − 2, 30 + 1	9·9
Feb. 15	0179	7 − 5, 14 + 1·5	8·5
22	0186	3 − 7, 4 + 2	7·4
28	0192	3 − 5, 4 + 5	7·2
Mar. 21	0213	3 − 3, 4 + 6·5	7·0
27	0219	3 − 3, 4 + 6·5	7·0
31	0223	3 − 7, 4 + 2	7·4
Apr. 25	0248	5 − 0·5, 7 + 1·5	7·9
27	0250	7 − 1, 10 + 3	8·1
May 15	0268	15 − 4, 20 + 1; 16 − 5, 19 + 1	9·2
17	0270	15 − 4, 20 + 1; 16 − 5, 19 + 1	9·2
1914			
May 21	0274	= 22; = 23	9·5
24	0277	25 − 1	9·8
26	0279	= 25	9·7
June 4	0288	41 − 0·5, 43 + 1·5	10·5
14	0298	43 − 2·5, 44 + 1	10·9
21	0305	= 44	11·0
23	0307	= 44	11·0
28	0312	44 − 2	11·2
July 4	0318	44 − 2	11·2
9	0323	44 − 2·5	11·3
17	0331	44 − 2	11·2
29	0343	44 − 1	11·1
Aug. 14	0359	41 − 0·5, 43 + 1·5	10·5
26	0371	14 − 4·5, 19 + 2	9·1
29	0374	14 − 4·5, 19 + 2; 15 − 3, 20 + 1·5	9·1
Sept. 5	0381	7 − 3, 10 + 1	8·3
8	0384	5 − 1·5, 7 + 0·5	8·0
11	0387	5 − 1, 7 + 1	7·9
19	0395	4 − 2, 6 + 1; = 5	7·8
24	0400	4 − 1, 6 + 2	7·7
29	0405	= 4	7·6
Oct. 5	0411	3 − 8, 4 + 2	7·5
10	0416	3 − 8, 4 + 2	7·5
18	0424	4 − 1, 6 + 1	7·8
Nov. 1	0438	7 − 3, 10 + 1	8·3
16	0453	14 + 1	8·6
18	0455	10 − 2, 14 + 0·5	8·6
Dec. 1	0468	16 − 3·5, 19 + 2·5	9·1
10	0477	23 − 1·5, 25 + 1	9·6
16	0483	25 − 1·5, 30 + 1·5	9·9
1915			
Jan. 5	0503	43 − 2, 44 + 1·5	10·8
11	0509	43 − 3, 44 + 1	10·9
17	0515	= 44	11·0
Apr. 18	0606	3 − 4, 4 + 6·5	7·0
23	0611	3 − 2·5, 4 + 7·5	6·9
29	0617	3 − 6·5, 4 + 3	7·3
May 7	0625	3 − 6·5, 4 + 3	7·3

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The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1915			
May 23	0641	10−1, 14+2	8·5
30	0648	14−1·5, 19+5	8·8
June 6	0655	14−4·5, 19+1·5	9·1
15	0664	=19	9·3
9	0688	43−1·5, 44+3	10·7
15	0694	44−1	11·1
July 26	0705	43−3, 44+1	10·9
28	0707	43−2·5, 44+1	10·9
Sept. 11	0752	14−5·5, 19+0·5	9·2
19	0760	14−1; =16	8·7
25	0766	7−1, 10+2·5	8·1
29	0770	5−0·5, 7+1·5	7·9
Oct. 4	0775	=4	7·6
14	0785	3−7, 4+2	7·4
18	0789	3−7, 4+2	7·4
Nov. 2	0804	4−0·5, 6+2	7·7
5	0807	=4	7·6
9	0811	3−8, 4+2	7·5
14	0816	3−6·5, 4+3	7·3
17	0819	3−6, 4+4	7·3
26	0828	=7	8·0
Dec. 8	0840	14−1, 19+5·5	8·8
12	0844	14−5, 19+1·5	9·2
1916			
Jan. 5	0868	43−2·5, 44+1	10·9
8	0871	43−3, 44+1	10·9
13	0876	43−3, 44+0·5	10·9
22	0885	44−1	11·1
Feb. 6	0900	44−3	11·3
Apr. 2	0956	10−1·5, 14+1·5	8·5
9	0963	5−1, 7+0·5	7·9
14	0968	4−1, 5+0·5	7·7
May 3	0987	3−5, 4+5	7·2
15	0999	3−3, 4+6·5	7·0
19	1003	3−7, 4+2	7·4
28	1012	4−1, 5+0·5	7·7
June 6	1021	=5	7·8
21	1036	14−4·5, 19+1·5	9·1
30	1045	=25	9·7
1916			
July 8	1053	43−1, 44+2·5	10·7
22	1067	=43	10·6
25	1070	43−3, 44+1	10·9
Aug. 8	1084	43−3, 44+1	10·9
19	1095	=44	11·0
27	1103	43−2·5, 44+1	10·9
Sept. 13	1120	22+1	9·4
21	1128	15−4, 20+1; 16−4·5, 19+1·5	9·2
27	1134	15−2·5, 20+2·5	9·1
Oct. 15	1152	3−7, 4+2	7·4
19	1156	3−8, 4+2	7·5
21	1158	3−7, 4+2	7·4
24	1161	3−7, 4+2	7·4
28	1165	3−6, 4+4	7·3
31	1168	3−6·5, 4+3	7·3
Nov. 2	1170	3−7, 4+2·5	7·4
8	1176	3−6·5, 4+3	7·3
12	1180	3−6·5, 4+3	7·3
16	1184	3−5, 4+5	7·2
20	1188	3−7, 4+2	7·4
24	1192	3−3, 4+6·5	7·0
26	1194	3−5, 4+5	7·2
Dec. 3	1201	4+1	7·5
25	1223	14−3, 19+3	9·0
27	1225	14−4·5, 19+1·5	9·1
30	1228	14−5, 19+1	9·2
1917			
Jan. 4	1233	=19; 22+1	9·4
10	1239	25−2, 30+1	9·9
27	1256	Glimpsed. ∠41	∠10·4
Feb. 7	1267	44−1	11·1
13	1273	44−2	11·2
26	1286	=44	11·0
Mar. 7	1295	43−3, 44+1	10·9
16	1304	41+1	10·3
22	1310	=41	10·4
27	1315	30−2·5, 41+1	10·3
Apr. 15	1334	13−1, 14+0·5	8·6

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The Comparison Stars Used.

Hagen's Number.	Magnitude. H.P.	Magnitude. Hagen.	Hagen's Number.	Magnitude. H.P.	Magnitude. Hagen.
2	6·50		16	8·71	
3	6·67		18		9·1
4	7·64		19		9·3
5		7·8	20		9·3
6		7·9	22		9·5
7		8·0	23		9·5
10	8·39		25		9·7
12		8·5	30		10·0
13	8·54		41		10·4
14	8·67		43		10·6
15		8·8	44		11·0

Ludgrove, New Barnet, Herts:
1917 *May* 2.

Baxendell's Observations of Variable Stars. Ninth Instalment. No. 23, S Vulpeculæ, and No. 24, S Aquilæ; with notes on No. 13, T Herculis, and No. 18, R Serpentis. Edited by Professor H. H. Turner and Miss Mary A. Blagg. (Plate 7.)

No. 23. *S* (2) *Vulpeculæ* (7106 *Chandler*).
(No. 1773 Greenwich 12-year Cat. 1840.)

$19^h\ 42^m\ 27^s\ +26^\circ\ 55'\cdot7$ (1855·0)
$19\ \ 44\ \ 18\ \ +27\ \ 2\ \cdot3$ (1900·0)

The Comparison Stars.—For a long time no chart of the comparison stars could be found, and application was made to Mr. Joseph Baxendell, "junior," for help in clearing up the matter. He suggested that the name "Greenwich 1773," which was in use near the time of discovery, might be on the chart; and with this clue a chart was found among some papers which had not been identified. It is labelled simply "26° 57′ North 1773," particulars which are amply sufficient to identify it when once a clue is furnished, but which, in the absence of any R.A.; of any interpretation of the number 1773; of any scale of magnitude for the stars charted, or of length for their positions; and, finally, of any indication which side is *p* or *f*, and whether north is up or down,—was, as it stood, a serious puzzle, only solved by Mr. J. Baxendell's further kind help.

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The difficulties will be better understood by a glance at this very interesting chart, which is reproduced on Plate 7. It is natural to look first for the identification of the stars with the smaller numbers attached, seeing that on almost any principle of numbering the brighter stars would be numbered first. But the stars numbered 4 and 5, for instance, are actually so faint that they are not marked on Father Hagen's chart (in the A.S.V.) at all. What increased the trouble was that (although star 4 was never used) star 5 was nominally used once by Baxendell, and from this single observation a magnitude 8·6 can be fairly inferred. A star so bright would certainly have been on the A.S.V. chart. It seems clear that the "5" of 1866 Nov. 3 must be a simple mistake for "8."

When, however, a few stars had been identified by Mr. Baxendell "junior," it was seen at once that the chart was very accurately made, and must represent many hours of careful work at the telescope. Most of the stars are included in the *Oxford Astrographic Catalogue* (though they are divided between two plates with centres $19^h\ 40^m\ +27°$ and $19^h\ 48^m\ +27°$), and it was easy to compare the places there given with Baxendell's positions as measured from his chart. For this comparison measures of the chart were made with a millimetre scale to a tenth of a millimetre, by naked eye, without magnification. The scale of the chart is apparently 1600″ to 3 inches, so that 1 mm. = 21″, and 0·1 mm. = 2″·1. It is possible that the chart is worth measuring under a microscope, but the figures of Table I. suggest that it would be very difficult to judge whether any observed displacements are due to errors in the chart or to movements in the stars.

Baxendell's No. 7 is just outside the réseau on this plate, but is measurable with quite sufficient accuracy, and the Oxford measures have been added in brackets.

Baxendell's No. 22 is so faint that it was not noticed in the measuring, and can only be seen with difficulty. Measures have however, been made, and added.

Attention is attracted by the large residuals for Baxendell's Nos. 18 and 37. Is it possible that these represent real displacements since 1861? Fortunately we have them on two plates of the *Oxford Astrographic Catalogue*,

No. 1112, centre $19^h\ 48^m\ +27°$: exposed 1897 Sept. 3,
and No. 2389, centre 19 44 +26 : exposed 1904 Oct. 3,

so that there is an interval of seven years (one-fifth of the interval from Baxendell's epoch 1861 to that of the first plate) during which any proper motion so large as that indicated would easily declare itself. But comparison of these plates shows no sensible proper motion at all, so that the displacements shown in Table I. must be due to errors in Baxendell's chart. It affords an additional check on the Oxford results, if such be needed, that a comparison was made in the first instance with the Franklin-

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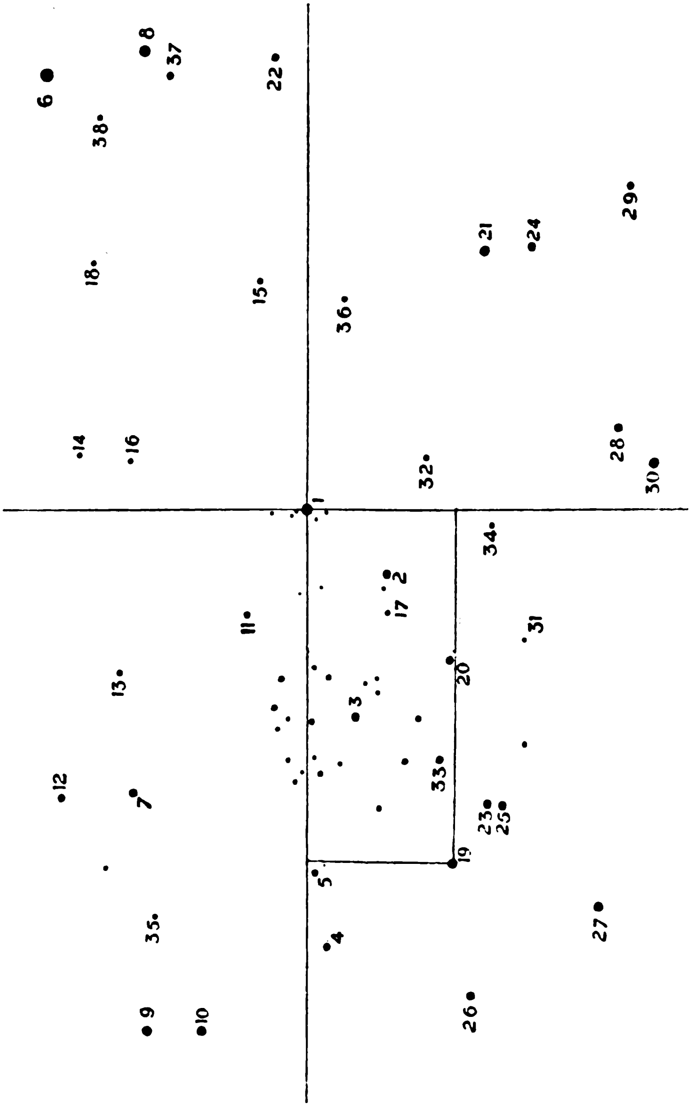

26° 57′ North 1773.

Baxendell's Chart of S Vulpeculæ.

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Adams chart of the region, and these displacements were first noticed then.

TABLE I.

Comparison of Baxendell's Chart of 1861 *with Oxford Plate No.* 1112, *centre* $19^h\ 48^m + 27°$, *exposed* 1897 *Sept.* 3.

Oxford.				Bax. Chart.			Bax. – Oxford.	
No.	*d.*	*x.*	*y.*	No.	X.	Y.	Δ*x.*	Δ*y.*
					mm.	mm.	″	″
...	20	(1̄·877)	(11·811)	7	−37·7	−22·9	−15	−21
53317	13	1·000	11·670	13	−21·8	−24·9	−12	− 9
53451	20	1·926	14·137	2	− 8·8	+10·8	− 6	+15
53409*	17	2·543	13·407	1	0·0	0·0	− 3	0
53320	11	2·941	11·701	16	+ 6·2	−23·7	+ 6	+ 6
53540	24	3·000	16·670	30	+ 6·7	+46·1	+ 6	+ 6
53321	9	3·026	11·304	14	+ 6·8	−30·2	− 3	−15
53453	12	3·050	14·528	32	+ 7·0	+15·8	− 3	+ 3
53541	23	3·354	16·306	28	+11·4	+41·3	+ 3	+12
53412	10	4·498	13·692	36	+27·8	+ 4·9	+ 6	+18
53368	10	4·648	12·903	15	+30·1	− 6·3	+ 6	+15
53323	8	4·947	11·154	18	+32·1	−28·2	−42	+69
53500*	23	4·971	15·067	21	+34·2	+23·6	0	+ 3
53501	18	4·992	15·524	24	+35·0	+29·8	+15	+ 3
53545	17	5·599	16·429	29	+43·5	+43·0	+15	+ 6
53324	12	6·198	11·420	38	+51·3	−27·5	− 9	+ 3
53370	12	6·562	12·434	37	+57·2	−18·3	+ 9	−75
...	5	(6·588)	(13·142)	22	+59·6	− 4·5	+51	−21
53293*	40	6·610	10·896	6	+57·0	−34·5	−12	+ 9
53327*	27	6·835	11·867	8	+60·4	−21·9	− 6	−15

It seems probable that Baxendell's measures were made with a position micrometer, and that there is an error of 60″ in the *distance* of star 18. The residuals −42″ and +69″ would then become 0 and +27″, which do not differ excessively from neighbouring ones. The error for No. 37 corresponds to about 3° in position angle.

The following stars of Baxendell's chart have been identified on the other Oxford plate :—

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TABLE II.

Stars on Oxford Plate No. 2583, *centre* $19^h\ 40^m + 27°$,
exposed 1907 *Sept.* 20.

Oxford.				Bax. Chart.			Bax. – Oxford.	
No.	*d.*	*x.*	*y.*	No.	X.	Y.	Δ*x.*	Δ*y.*
					mm.	mm.	"	"
...	5	20·654	13·288 }	4	−57·8	+ 1·5	+11	−11
...	5	20·745	13·338 }					
52088	13	21·427	13·171	5	−48·5	0·0	−10	+ 2
52090	14	22·873	13·619	3	−27·0	+ 6·0	+10	+ 1
52057	14	23·836	12·676	11	−14·0	− 8·5	+ 1	−21
52092	14	23·840	13·949	17	−13·5	+10·0	+ 5	−10
52093	23	24·193	13·928	2	− 8·5	+10·2	+ 5	0
52096*	22	24·819	13·217	1	0·0	0·0	+ 2	+ 1

Baxendell's No. 4 was not measured on the Oxford plate, but on search being made a pair of very faint stars was found and measured.

For these comparisons the chart measures in mm. were first multiplied by ·07 to convert them into units approximately comparable with Oxford measures; next the constants

for plate $19^h\ 48^m$, $x = 2·670$, $y = 13·380$
and for plate $19^h\ 40^m$, $x = 24·820$, $y = 13·220$

were added; and then approximate linear formulæ were found and applied to the differences.

This discussion is a digression, because Baxendell used only very few of the stars plotted. But it will serve to illustrate the general nature of his work on the charts of his variables.

TABLE III.

List of Comparison Stars used.

Bax. No.	A.S.V. No.	A.S.V. Grades.	Deduced Mag.	H.P.	Adopted.
8	10	26, 22	7·64	7·91	85
9	29	61, 71	8·98	...	90
10	45	82, 94	9·58	...	96
2	52	92, 105	9·87	9·70	99

The deduced magnitudes in the fourth column were derived by plotting curves for the two sets of grades against the 17 H.P. stars given in the A.S.V., then converting the grades into magnitudes and taking the mean. Two of the stars are, however, given directly in the H.P. as in the fifth column, and it will be seen Baxendell's 8 is sensibly fainter in H.P. than as given by the A.S.V. grades. Such indications as are afforded by Baxendell's

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observations would make it fainter still, and accordingly the magnitude 8·5 has been adopted. For his star 2, on the other hand, Baxendell agrees more nearly with the A.S.V. But the material for comparison is scanty, as will be seen from Table IV.

Table IV.

Baxendell's Differences between his Comparison Stars.

Years.	90–85.	96–90.	99–96.	99–90.	99–85.	96–85.
1861–4	$5\cdot6_{15}$	$6\cdot2_{18}$	$3\cdot6_{20}$	$9\cdot0_{24}$	$14\cdot0_{17}$	$11\cdot5_{13}$
1865–6	$6\cdot0_{2}$	$5\cdot8_{20}$	$4\cdot9_{8}$	$11\cdot9_{7}$	$18\cdot3_{2}$	$12\cdot3_{2}$
1867–8	? $5\cdot0_{1}$	$6\cdot6_{11}$	...	$10\cdot0_{1}$	? $15\cdot0_{1}$	...
1878–80	$4\cdot3_{2}$	$8\cdot3_{4}$	...	...	...	...
1884–7	$5\cdot0_{1}$	$8\cdot7_{25}$	...	...	...	$13\cdot0_{1}$
Mean	5·5	7·0	4·0	9·7	14·5	11·7
No. of Obs.	21	78	28	32	20	16
Reduced Mean	4·5	6·1	3·4	8·0	12·0	9·7

The interval between the stars 9 and 10 (96–90) increases almost steadily.

The telescopes used were A 68 in the first series (to end of 1868) and E 80 in the later series, except that R (13) 81 was used 1864 Oct. 7 and 1868 July 27; R (7) 39, 1863 Aug. 15; and *a* (22) 39, 1863 Sept. 19. Also, on the first four dates of the whole series, the telescope and power are not given.

Table V.

Baxendell's Observations of S (2) Vulpeculæ.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.	Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1861	240+			1862	240+		
Nov. 27	1107	99 − 9, 90 − 1	90	Oct. 29	1443	99 − 6, 85 + 3, 90 − 1	90
1862				Nov. 24	1469	85 + 7, 99 − 1, 90 + 3, 96 − 1	94
Apr. 29	1260	85 + 4, 99 − 6, 90 + 6, 96 − 2	91	1863			
June 28	1320	85 + 6, 99 − 5, 90 + 2, 96 − 4	92	Aug. 15	1733	85 + 5, 99 − 3	93
				Sept. 5	1754	99 − 7, 90 − 2	90
July 28	1350	85 + 4, 99 − 6	91	7	1756	85 + 3, 90 − 2	88
Sept. 20	1404	85 + 6, 99 − 4, 90 + 3, 96 − 2	:93	19	1768	*90 + 3, *96 − 1, 85 + 7, 99 − 6	? 93

* Baxendell writes, Sept. 20, "Qu. as to last night's comparisons with 9 and 10?" and on Sept. 19 he writes, "Does 2 change?"

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TABLE V.—*Baxendell's Observations of S (2) Vulpeculæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1863	240+		
Sept. 20	1769	85+4, 90−3, 99−6	90
Oct. 9	1788	85+7, 99−4, 90+2, 96−3	93
22	1801	85+12, 99−4, 90+3, 96−2	95
30	1809	85+11, 99−3, 90+3, 96−2	95
Nov. 19	1829	85+5, 99−9, 90+1, 96−4	*91
Dec. 17	1857	90+4, 96−1, †85+8, 99−4	94
27	1867	85+8, 99−6, 90+3, 96−2	93
1864			
May 13	2005	85+8, 96+0, 99−5	94
July 4	2057	90+1, 96−3, 99−6	92
14	2067	90+3, 96−3, 99−6, 85+9	93
29	2082	90+3, 96−3, 99−6	93
Aug. 21	2105	90−1, 99−8	90
Sept. 9	2124	90+1, 96−3, 99−6	92
Oct. 7	2152	99−6	‡93
9	2154	90+4, 96−2, 99−6	94
17	2162	90+3, 96−3, 99−7	93
24	2169	90−1, 96−6, 99−7	90
Oct. 31	2176	90−1, 99−9	90
Nov. 23	2199	90+0	90
Dec. 6	2212	96+0, 99−4	§96
21	2227	96−3	‖93
1865			
Apr. 15	2342	90+4, 96+0, 85+8, 99−5	94
29	2356	90+5, 96−1, 85+10, 99−6	94
May 15	2372	90+0, 96−4	91
20	2377	90+1, 96−4	92
June 9	2397	90+0	¶90
13	2401	90+3, 96−3, 99−7	93
17	2405	90+3, 96−2	94
July 1	2419	90+4, 96−1	94
14	2432	90+1, 96−3	92
Aug. 3	2452	90+1, 96−3	92
25	2474	90+3, 96−2	94
Sept. 7	2487	96−1, 99−3	96
28	2508	90+1, 96−4	92
Nov. 1	2542	90+3, 96−3	93
1866			
Apr. 23	2715	99−9, 90+2, 96−4	91
May 17	2739	96+0, 99−3	96
24	2746	90+4, 96−2, 99−5	94
June 25	2778	90+0, 99−10	90
July 11	2794	90+2, 96−3	92
22	2805	90+0	90
Aug. 27	2841	90−1	89

* "☽ and v. hazy."

† Baxendell writes, "Does 8 vary?"

‡ This figure is followed by a double note of admiration by Baxendell in his list of deduced magnitudes.

§ "☽ and hazy."

‖ "v. low and clouds troublesome."

¶ This is followed by a double note of admiration by Baxendell in his list of deduced magnitudes.

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TABLE V.—*Baxendell's Observations of S (2) Vulpeculæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1866	240+		
Sept. 17	2862	90−1, 96−5	90
Oct. 6	2881	90+3, 96−3	93
16	2891	96+0	96
Nov. 3	2909	90−1, 96−6, 99−9 [? *85+5]	90
9	2915	90−1	89
19	2925	90−1	89
25	2931	90−0	90
30	2936	90+0	90
Dec. 16	2952	90+3, 96−2	94
30	2966	90+3, 96−3	93
1867			
Jan. 11	2978	90−1 [? †85+3], 99−9	89 (or 90)
June 5	3123	90+0	90
Sept. 5	3215	90+2, 96−3	92
10	3220	90+3, 96−3	93
16	3226	90+3, 96−3	93
24	3234	90+3, 96−3	93
Oct. 23	3263	90−1	89
Nov. 1	3272	90−1	89
19	3290	90+3, 96−3	93
Dec. 12	3313	90+1	91
1868			
Apr. 11	3434	96+0	96
23	3446	90+3, 96−3	93
May 11	3464	90+0	90
20	3473	90+1	91
31	3484	96−1	95
June 7	3491	96−2	94
16	3500	96−2	94
19	3503	90+4, 96−2	94
28	3512	90+3, 96−3	93
July 11	3525	90+0	90
21	3535	90+0	90
1868	240+		
July 25	3539	90+0	90
27	3541	90+3, 96−3	93
Sept. 6	3582	90+3, 96−3	93
Oct. 17	3623	90+3, 96−3	93
Nov. 6	3643	90−0	90
8	3645	90−0	90
15	3652	90−1	89
1878			
June 29	7165	90+5, 96−4	94
1880			
Sept. 6	7965	90+0, 85+2	88
7	7966	85+4, 90−1	89
Oct. 10	7999	90+4, 96−3	94
20	8009	90+2, 96−4	92
Nov. 3	8023	90+3, 96−3	93
1884			
Aug. 24	9413	90+3, 96−3	93
25	9414	90+3, 96−3	93
Sept. 2	9422	90+3, 96−3	93
11	9431	90+4, 96−3	94
17	9437	90+6, 96+0	96
26	9446	90+6, 96−2	95
Oct. 1	9451	90+5, 96−4	94
5	9455	90−2	88
12	9462	90−1	89
Nov. 9	9490	90+5, 96−2	94
13	9494	96−2	94
20	9501	96+2	98
27	9508	96+0	96
Dec. 4	9515	90+6, 96−3	94
20	9531	90+1, 96−6, 85+5	90
24	9535	90+3, 96−7	91

* The number 5 is given in the observation, but it seems almost certainly a slip for 8.

† The number is omitted in the entry, but 8 is supplied conjecturally.

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TABLE V.—*Baxendell's Observations of S (2) Vulpeculæ*—continued.

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1885		240+		
Jan.	8	9550	96 − 2	94
	11	9553	90 + 5, 96 − 3	94
May	11	9673	90 + 1, 96 − 5	91
	20	9682	90 + 3, 96 − 3	93
June	1	9694	90 + 7, 96 − 1	96
	9	9702	90 + 7, 96 − 1	96
July	4	9727	90 + 2, 96 − 5	92
	29	9752	90 + 4, 96 − 3	94
Aug.	14	9768	96 − 1	95
	19	9773	96 − 0	96
Sept.	5	9790	90 + 1	91
	25	9810	90 + 1	91
Oct.	29	9844	96 + 0	96
Nov.	8	9854	96 − 1	95
	15	9861	90 + 3, 96 − 5	92
	19	9865	90 − 2	88
1885		240+		
Dec.	1	9877	90 + 0	90
	26	9902	90 + 4, 96 − 3	94
1886				
Jan.	6	9913	96 + 1	97
	11	9918	96 + 1	97
		241+		
June	14	0072	90 + 0	90
Sept.	18	0168	90 + 1, 96 − 6	90
Oct.	23	0203	90 − 2	88
Nov.	2	0213	90 − 1	89
	4	0215	90 + 0	90
	17	0228	90 + 2, 96 − 6	91
Dec.	4	0245	90 + 5, 96 − 3	94
	20	0261	96 − 2	94
	29	0270	90 + 2, 96 − 7	90
1887				
Jan.	12	0284	90 + 3, 96 − 4	92

The observations were arranged in the usual manner, with an assumed period of 67·5 days, Chandler's formula being 2402239·0 + 67·5 E; but they were found to be too scanty to give good results by the usual methods. They were therefore plotted on squared paper, and it was found that a period of 67·5 days fitted them very well on the whole, both for maximum and minimum. It is never possible to fix these epochs *exactly* from the observations; but when the dates in the following table are compared with the observations, it is seen that in very few cases is there any serious discrepancy. The minima of E = − 1, 113, 116, and 118 seem to fall later than the mean date, and those of E = 18 and − 6 earlier; and the maxima of E = 85 and 107 seem to fall earlier, and that of E = 15 later; but in most of these cases the discrepancy rests upon a single observation—sometimes even upon *half* an observation only!

In the rest of the epochs the mean date, while not always quite the most "probable" one, is not far from being so, and is reasonably consistent with the observations. Chandler's epochs for maximum are 5 days earlier throughout, which, as a rule, makes no appreciable difference to the agreement; but the dates here given seem to be slightly better in more instances than those in which they are slightly worse. The minimum is put at 36 days after the maximum, but there are some indications of a progressive increase in the distance from maximum to minimum, as though the period from minimum to minimum were a little longer than from maximum to maximum.

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TABLE VI.

Mean Epochs observed by Baxendell of S Vulpeculæ.

E.	Max.	Min.
	240 +	
− 17	1096·5	...
− 15	...	1267·5
− 14	(1299·0)	1335·0
− 13	(1366·5)	1402·5
− 12	(1434·0)	1470·0
− 7	1771·5	1807·5
− 6	1839·0	1875·0
− 4	...	2010·0
− 3	2041·5	2077·5
− 2	...	2145·0
− 1	2176·5	2212·5
0	(2244·0)	...
+ 1	...	2347·5
+ 2	2379·0	2415·0
+ 3	2446·5	2482·5
+ 7	2716·5	2752·5
+ 8	2784·0	(2820·0)
+ 9	2851·5	(2887·5)
+ 10	2919·0	2955·0
+ 11	2986·5	...

E.	Max.	Min.
	240 +	
+ 13	3121·5	...
+ 14	...	3225·0
+ 15	3256·5	3292·5
+ 16	(3324·0)	...
+ 17	...	(3427·5)
+ 18	(3459·0)	3495·0
+ 19	3526·5	3562·5
+ 20	(3594·0)	3630·0
+ 21	(3661·5)	...
+ 85	7981·5	8017·5
+ 106	...	9435·0
+ 107	9466·5	9502·5
+ 108	9534·0	(9570·0)
+ 110	9669·0	...
+ 111	9736·5	9772·5
+ 112	9804·0	9840·0
+ 113	9871·5	9907·5
+ 116	10074·0	...
+ 117	...	10177·5
+ 118	10209·0	10245·0
+ 119	10276·5	...

There is in a good many cases some suggestion of a secondary maximum and minimum, not (as with R Sagittæ) about 180° from the primary ones, but at about 8 or 10 days before or after these. But this appearance may be illusory, and be really due to considerable flatness in maximum and minimum, combined with variable atmospheric conditions.

On further consideration, and at the suggestion of Mr. J. Baxendell, junior, it has been decided to add to the 23 stars already discussed, a few others, beginning with S Aquilæ, of which Baxendell took many observations. After the first four years of observation he called it T instead of S.

No. 24. S Aquilæ (Chandler 6726).

$20^h\ 4^m\ 57^s$ $+15°\ 11'·5$ (1855·0)
20 7 1 +15 19·4 (1900·0)

The Comparison Stars.—Baxendell made two charts and one small sketch for this variable, and the identifications do not present

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much difficulty. The *b* of the first chart was clearly shown to be variable. Each *b* was very seldom compared with the variable.

TABLE VII.

Comparison Stars.

First Chart.	Second Chart.	Sketch, 1863 Aug. 24.	A.S.V. No.	A.S.V. Grades.	H.P.	Adopted Mags.	Bax.'s Est.
a	*a*	*a*	27	90, 78	9·36	94	9·2 to 9·7
m	*b*	...	30	96, 86	9·64	96	9·4
b	...	...	33	(98), 87	9·34	99	9·9 ?
c	*c*	...	41	111, 93	...	102	10·1
d	*d*	*d*	50	121, 101	10·69	106	10·4
e	*e*	*e*	53	129, 109	...	112	11·0
...	*f*	*f*	54	146, 123	...	120	12·0
r	...	...	49	121, 101	10·60	...	...
...	...	*g*	...	...	...	123	...
...	...	*h*	...	...	...	...	...
(A)	...	...	?13	5, 43	8·49	85	8·0 ?

The *g* and *h* on the sketch are not given in Hagen's chart.

The *b* of the first chart was discarded after 1863 Oct. 11, from which date the name *b* was transferred to the star previously called *m*; but the latter star was also discarded almost entirely after the 28th of the same month until 1878. A is not named on any of the charts, and its identity is not certain.

TABLE VIII.

Baxendell's Differences between his Comparison Stars.

Years.	102–96.	102–99.	106–102.	112–106.	120–112.	102–94.	112–102.	94–85.
1863–4	$7{\cdot}0_{1}$	$0{\cdot}5_{9}$	$2{\cdot}3_{14}$	$6{\cdot}7_{27}$	$5{\cdot}4_{12}$	$9{\cdot}0_{13}$	$7{\cdot}0_{4}$	$12{\cdot}1_{9}$
1865–7	...	...	$2{\cdot}6_{13}$	$8{\cdot}1_{24}$	$12{\cdot}0_{1}$	$6{\cdot}5_{36}$	$8{\cdot}0_{2}$	$13{\cdot}1_{4}$
1868–9	$1{\cdot}0_{1}$	...	$2{\cdot}5_{4}$	$6{\cdot}2_{16}$	...	$6{\cdot}7_{3}$	...	...
1877–9	$6{\cdot}2_{8}$	...	$2{\cdot}6_{15}$	$7{\cdot}8_{20}$	$13{\cdot}0_{11}$	$7{\cdot}0_{21}$	$9{\cdot}0_{3}$	...
1880–3	...	...	$4{\cdot}0_{16}$	$7{\cdot}1_{14}$	$14{\cdot}5_{5}$	$7{\cdot}7_{41}$	...	...
1884–6	...	...	$4{\cdot}8_{28}$	$8{\cdot}0_{30}$	...	$7{\cdot}3_{17}$	$9{\cdot}5_{2}$	...
Mean.	5·8	0·5	3·5	7·4	10·1	7·4	8·2	12·4
Reduced Mean.	5·4	0·4	3·1	6·4	8·4	6·4	7·1	10·8
No. of Obs.	10	9	90	131	29	121	11	13

Besides these, 5 observations of 96–94 in the years 1863–4 give a mean difference 2·4, or 2·2 when reduced; and 4 observa-

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tions of 99–94 in the same years give a mean difference 6·4, or 5·4 when reduced.

The instruments used were A 68 until 1869 Oct. 26, and E 80 afterwards, with the following exceptions:—R (7) 130 was used 1863 July 28 to Aug. 13; C 39, 1863 Aug. 15 and Sept. 19, 1866 Aug. 3; R (13) 81, 1868 June 18; A 39, 1877 Sept. 4; E 110, 1877 Dec. 31. The intervals obtained between some of the comparison stars vary very much, and Baxendell considered *d* to be variable, but he continued to make use of it.

TABLE IX.

Baxendell's Observations of S Aquilæ.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1859	240+		
Aug. 19	0276	(About) 94+0	94 ?
1860			
June 1	0563	Invisible.	
1863			
June 25	1682	94+0	94
July 28	1715	94+11, <99	*105
29	1716	94+12, 102+3, 106+3	*107
31	1718	102+4, 106+3, 112+0	109☽☽
Aug. 1	1719	112+0, 102+6, 106+5	110
4	1722	112+0, 106+5	112
13	1731	112+2	114
15	1733	112+1	113
20	1738	112+2, 120+1	118
24	1742	112+3, 120−1, 123−3	118
29	1747	112+3	115
Sept. 3	1752	120−2	118
5	1754	112+1, 120−4	114
7	1756	112+0, 120−5	114
10	1759	112+0	112
19	1768	112−3, 106+2	108
20	1769	106+3, 112−3	109
22	1771	106+1, 112−4	108
Oct. 9	1788	94+4, 106−2	101
1863	240+		
Oct. 11	1790	94+6, 102−2	100
,,	,,	94+4, 102−2	99
13	1792	94+4, 102−2	99
24	1803	94−3	91
28	1807	94−2	92
29	1808	94−2	92
30	1809	94−1	93
Nov. 1	1811	94−2	92
2	1812	94−2	92
6	1816	94−1	93
9	1819	94+0	94
10	1820	94+0	94
11	1821	94+0	94
19	1829	102+1, 94+8	102
28	1838	106+3, 112−3	109
29	1839	106+3, 112−3	109
Dec. 5	1845	106+3, 112−3	109
17	1857	106+2, 112−3	108
22	1862	106+1, 112−4	108
27	1867	102+1, 112−5, 106−1	105
1864			
Jan. 1	1872	102+3, 112−3, 106+2	107
May 16	2008	94+4, 102−2	99
26	2018	102+2, 106−1	104
June 6	2029	106+3, 112−3	109
16	2039	106+6, 112−1	112

* "Strong ☽ and hazy."

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1864	240+		
July 4	2057	112+1	113
8	2061	112−1	111
13	2066	106+4, 112−2	110
16	2069	106+3, 112−3	109
19	2072	106+3, 112−3	109
26	2079	106+3, 112−3	109
29	2082	106+3, 112−3	109
Aug. 21	2105	85+2, 94−7	87
23	2107	85+3, 94−7	88
26	2110	85+3, 94−7	88
31	2115	85+4, 94−7	88
Sept. 5	2120	85+3, 94−6	88
9	2124	85+5, 94−6	89
11	2126	85+6, 94−5	90
14	2129	85+6, 94−6	90
19	2134	85+7, 94−5	90
24	2139	94−1	93
26	2141	94+0	94
Oct. 1	2146	94+0	94
3	2148	94+1	95
5	2150	94+3	97
8	2153	94+6, 102−1	100
17	2162	106+4, 112−3	110
24	2169	106+6, 112−3	110
31	2176	112+0	112
Nov. 5	2181	112+0	112
8	2184	112+0	112
23	2199	112+4	116
1865			
May 18	2375	112+2	114
20	2377	112+1	113
24	2381	112+2	114
June 8	2396	106+1, 112−4	108
9	2397	106+0	106
11	2399	106+0	106
13	2401	106+0	106
1865	240+		
June 17	2405	102+1, 106−1	104
20	2408	102+2, 106−1	104
21	2409	102+1, 106−2	104
28	2416	102+0	102
July 1	2419	102+0	102
13	2431	102+1, 106−2	104
14	2432	102+1, 106−2	104
21	2439	102+2, 106+1	106
Aug. 3	2452	106+6, 112−1	112
12	2461	112+0	112
25	2474	112+2	114
31	2480	112+4, 120−6	115
Sept. 7	2487	112+2	114
23	2503	112−1	111
27	2507	106+4, 112−3	110
Oct. 2	2512	106+4, 112−3	110
7	2517	106+2, 112−5	108
10	2520	106+4, 112−3	110
19	2529	106+4, 112−3	110
29	2539	106+2, 112−6	107
Nov. 1	2542	106+0	*106
5	2546	102+0, 106−2	103
7	2548	94+4, 102−2	99
12	2553	94+4, 102−2	99
15	2556	94+3, 102−3	98
26	2567	94+1, 102−3	97
Dec. 22	2593	102+2, 106+0 (about)	†105?
31	2602	102+3, 112−3	‡107
1866			
May 17	2739	106+2, 112−6	107
20	2742	106+2, 112−4	108
22	2744	106+2, 112−4	108
24	2746	106+2, 112−4	108
June 8	2761	106+4, 112−2	110
25	2778	102+1, 106−2	104

* "☽☽ and hazy." † "Sky thick." ‡ "☽☽ and cloud."

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1866	240+		
July 11	2794	94+0, 102−2	97
16	2799	94+2, 102−3	98
20	2803	94+3, 102−3	98
22	2805	94+3, 102−3	98
30	2813	94+3, 102−3	98
31	2814	94+4, 102−2	99
Aug. 3	2817	94+6, 102−1	100
20	2834	94+1, 102−3	*97
27	2841	94−3, 85+8	92
31	2845	85+6, 94−4	90
Sept. 8	2853	85+6, 94−5	90
15	2860	85+10, 94−2	94
17	2862	94+0	94
24	2869	94+4, 102−3	98
Oct. 1	2876	94+3, 102−3	98
6	2881	94+3, 102−2	98
9	2884	94+5, 102−1	100
10	2885	94+0, 102−5	96
14	2889	94+8, 102+2	103
16	2891	102+3, 106+1	106
19	2894	106+2, 112−8	106
28	2903	106+3, 112−4	108
Nov. 3	2909	106+6, 112−3	110
6	2912	112−3	109
9	2915	112−1	111
19	2925	112−1	111
25	2931	106+6, 112−3	110
30	2936	106+3, 112−5	108
Dec. 16	2952	106+1, 112−7	106
30	2966	106+0, 112−6	106
1867			
Jan. 11	†2978	94+9, 102−3	101
		?94−3, 102−9	92
May 21	3108	102+2	104
June 29	3147	94+4, 102−3	98
July 24	3172	102+1, 106−1	104
1867	240+		
July 27	3175	102+2, 106+0, 112−5	106
Aug. 1	3180	106+1, 112−5	107
6	3185	106+2, 112−3	108
23	3202	112+0	112
Sept. 2	3212	112+0	112
10	3220	112−1	111
16	3226	112−1	111
23	3233	112+0	112
24	3234	112−1	111
25	3235	112−1	111
27	3237	112−2	110
Oct. 3	3243	112−1	111
8	3248	112−1	111
17	3257	94+3, 102−3	98
23	3263	94−2	92
Nov. 1	3272	94−2	92
17	3288	94−4	90
19	3290	94−2	92
Dec. 6	3307	94+4, 102−3	98
12	3313	102+1, 106−2	104
18	3319	106+2, 112−4	108
1868			
May 20	3473	106+2, 112−3	108
25	3478	106+3, 112−2	110
June 2	3486	106+3, 112−3	109
4	3488	106+3, 112−2	110
7	3491	106+3, 112−3	109
12	3496	106+3, 112−2	110
15	3499	106+4, 112−2	110
18	3502	106+4, 112−1	110
19	3503	106+4, 112−1	110
25	3509	106+5, 112+0	112
26	3510	106+5, 112−1	111
28	3512	106+4, 112−2	110

* "Clouds—doubtful."
† The two lines are both Baxendell's.

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1868	240+		
June 29	3513	112−2	110
30	3514	112−2	110
July 6	3520	106+3, 112−3	109
10	3524	102+1, 106−2	104
11	3525	102+0	102
12	3526	102−1	101
13	3527	94+4, 102−2	99
21	3535	102+1, 106−2	104
23	3537	102+0	102
25	3539	102+0	102
29	3543	102−1	101
Aug. 3	3548	96−2, 102−3	96
Sept. 4	3580	94−2	92
5	3581	94−1	93
6	3582	94−1	93
11	3587	94+0	94
Oct. 17	3623	106+1	107
19	3625	106+0, 112−5	106
27	3633	102+0, 106−2	103
Nov. 6	3643	About 102+0	* 102 ?
8	3645	102+1, 106−1	104
15	3652	106+3, 112−3	109
1869			
June 8	3857	94−6	88
Oct. 26	3997	106+2, 112−3	108
Nov. 28	4030	94+2, 102−4	97
1877			
Sept. 4	6867	94+3	†97
Nov. 25	6949	94−1	93
Dec. 2	6956	94−1	93
13	6967	94+2	96
14	6968	94+3, 102−3	98
17	6971	94+2	96
18	6972	94+2	96

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1877	240+		
Dec. 23	6977	94+4	‡98
31	6985	106+3, 112−2	110
1878			
June 25	7161	106+4, 112−1	110
28	7164	112−1	111
July 7	7173	112−1	111
12	7178	112+0	112
19	7185	106+2	108
20	7186	106+3, 112−6	108
21	7187	106−1, 112−6	§106
27	7193	102+4, 106−2	105
Aug. 4	7201	102+3, 106−1	105
10	7207	106−3, 102−1	102
18	7215	96+3, 102−1	100
21	7218	102−1	101
25	7222	96+5, 102−1	101
29	7226	96+4, 102−1	100
Sept. 2	7230	94+3, 102−2	98
5	7233	96+4, 102−2	100
11	7239	96+0, 102−3	98
15	7243	94−3, 102−6	94
24	7252	94+5, 102+0	100
Oct. 11	7269	102+1, 94+9	103
17	7275	106+2, ‖102−1	104
26	7284	106+3, 112−2	110
Nov. 1	7290	106+6, 112−2	111
4	7293	106+4, 112−3	110
8	7297	106+6, 112−2	111
17	7306	112+0	112
19	7308	112+0	112
28	7317	112−1	111
Dec. 5	7324	112−2	110
7	7326	112−2	110
8	7327	106+6, 112−2	111

* "Cloud." † "Cloud."
‡ "Hasty observation." § "Does *d* change ?"
‖ Baxendell writes, "*d* is certainly brighter than *c*."

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1878		240+		
Dec.	23	7342	102+4, 112−5, 106+3	107
	30	7349	106+3	109
1879				
Jan.	13	7363	102+1, 106−3	103
	15	7365	102−1	101
	19	7369	94+3, 102−1, 106−6	99
	22	7372	94+7, 102+1	102
Aug.	1	7563	112−2	110
	8	7570	106+0, 112−6	106
	11	7573	96+3, *102−1?	100?
	14	7576	102+3, 106+0	106
	17	7579	106+0, 102+3	106
	21	7583	106+3, 112−5, 102+3	107
	23	7585	112−2	110
	26	7588	106+3	109
	29	7591	106+4, 112−3	110
	31	7593	106+6, 112−1	112
Sept.	2*	7595	106+7, 112+0	112
	7	7600	106+8, 120−6, 112+3	114
	9	7602	112+2, 120−8	113
	12	7605	112+2, 120−7	114
	13	7606	112+1, 120−9	112
	19	7612	112+3, 120−9	113
	20	7613	112+3	115
	21	7614	112+2	114
	23	7616	112+3	115
	25	7618	112+4, 120−6	115☽
	26	7619	112+5	117☽☽
	28	7621	112+7	119☽
	29	7622	112+6	118☽☽
Oct.	2	7625	112+6, 120−4	117
	4	7627	112+6, 120−6	116
1879		240+		
Oct.	5	7628	112+6, 120−6	116
	6	7629	112+6, 120−6	116
	14	7637	112+1, †120−12	110?
	15	7638	112+0	112
	16	7639	112−1	111
	17	7640	112+0	112
	25	7648	106−2, 112−10	103
	26	7649	102+0, 106−4	102
	28	7651	>112	
Nov.	1	7655	96+3, 102−3	99
	2	7656	96+4, 102−2	100
	5	7659	94+1, 102−3	97
	7	7661	94+2, 102−3	98
	10	7664	94+1, 102−3	97
	12	7666	94+1, 102−3	97
	13	7667	94+1, 102−3	97
	22	7676	94−1	93
	23	7677	94+1	95
	24	7678	94+2, 102−3	98
	25	7679	94+0	94
	27	7681	94+1	95
	29	7683	94+3, 102−4	98
	30	7684	94+3, 102−3	98
Dec.	3	7687	94+4, 102−3	98
	6	7690	94+4, 102−5	98
	22	7706	94+6, 102−1	100
	23	7707	94+6, 102−1	100
	24	7708	94+7, 102−2	100
	26	7710	94+8, 102+0	102
	28	7712	102+1	103
1880				
Jan.	2	7717	102+2, 106−2	104
Mar.	17	7792	102+1, 106−3	103
Apr.	17	7823	94+0	94
	24	7830	94+1, 102−4	96
	29	7835	94+3, 102−2	98

* The query is Baxendell's.

† "*f* seems less than usual."

TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.	J.D.	Corrected Comparisons.	Deduced Mag.
1880	240+		
Apr. 30	7836	94+3, 102−2	98
May 5	7841	94+4, 102−1	100
7	7843	94+7, 102−1	101
12	7848	102+3, 106−1	105
13	7849	106+1, 112−4	108
15	7851	102+2, 106−2	104
27	7863	106+5, 112−2	110
June 4	7871	112−1	111
7	7874	112+0	112
17	7884	112+1	113
26	7893	112+4, 120−8	114
July 3	7900	112+4 (or more)	*116?
4	7901	112+4, 120−8	114
9	7906	112+6, 120−6	116
20	7917	112+9, 120−4	118
30	7927	112+0	112
31	7928	112+0	112
Aug. 6	7934	102+0, 106−1	104
9	7937	106−1, 112−4	106
11	7939	102+0, 106−3	102
14	7942	102−1	101
16	7944	94+4, 102−3	98
17	7945	94+2, 102−6	96
22	7950	94+3, 102−3	98
24	7952	94+5, 102−3	99
27	7955	94+6, 102−1	100
31	7959	94+4, 102−3	98
Sept. 2	7961	94+3, 102−5	97
3	7962	94+3, 102−5	97
4	7963	94+3, 102−3	98
7	7966	94+2, 102−4	97
8	7967	94+2, 102−4	97
13	7972	94+3, 102−3	98
16	7975	94+2, 102−4	97
18	7977	94+2, 102−6	96
25	7984	94+0, 102−7	94
1880	240+		
Sept. 26	7985	94+0, 102−7	94
Oct. 2	7991	94+1, 102−7	95
3	7992	94+1, 102−5	96
9	7998	94+3, 102−6	96
17	8006	94+6, 102−2	100
20	8009	94+6, 102−1	100
23	8012	94+7, 102−1	101
29	8018	102+2, 106−2	104
31	8020	102−1, 106−4	102
Nov. 3	8023	102+3, 106−2	104
8	8028	106+2, 112−6	107
19	8039	106+3, 112−3	109
20	8040	106+4, 112−3	110
26	8046	†106+6, 112−2	111
Dec. 1	8051	106+5, 112−1	111
4	8054	112+1	113
18	8068	112−3	109
24	8074	106+3, 112−3	109
1881			
Jan. 12	8093	94+0	94
May 4	8205	106+5, 112−3	110
7	8208	112−1	111
9	8210	112−3	109
12	8213	106+1, 112−4	108
16	8217	106−1, 112−7	105
20	8221	102+0	102
21	8222	102+0	102
22	8223	102−1	101
23	8224	102−2	100
29	8230	94+0	94
30	8231	94+0	94
June 8	8240	94+1	95
12	8244	94+2	96
18	8250	94+7, 102−1	101
27	8259	94+3, 102−3	98
July 8	8270	94+0	94

* "Cloud."

† "$d > c$, ∴ one var."

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1881		240+		
July	11	8273	94−1	93
	14	8276	94+0	94
	17	8279	94+0	94
	19	8281	94+0	94
	27	8289	94+0	94
Aug.	1	8294	94+1	95
	5	8298	94+1	95
	6	8299	94+1	95
	24	8317	102+3, 106−2	104
	31	8324	112+0	112
Sept.	1	8325	112+0	112
	14	8338	112+2	114
	19	8343	112+1	113
	25	8349	112+1	113
Oct.	1	8355	112−1	111
	2	8356	112−2	110
	15	8369	112−2	110
	16	8370	112−3	109
	26	8380	102+3, 106+1	106
	29	8383	106+0, 112−4	107
Nov.	5	8390	102−1	101
	14	8399	9+43, *102−8 ?(or 112−8) or	96? 100
	17	8402	94+3, 102−3	98
	20	8405	94+1, 102−3	97
	22	8407	94+1, 102−4	96
	28	8413	94−1	93
Dec.	1	8416	94+0	94
	10	8425	94−1	93
	19	8434	94+0	94
1882				
May	12	8578	94+0	94
	14	8580	94+0	94
	16	8582	94+2	96
	18	8584	94+3, 102−2	98
	19	8585	94+4, 102−1	100
	20	8586	94+4, 102−2	99

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1882		240+		
May	25	8591	102+3, 106+0	106
	28	8594	102+6, 106+0	107
	29	8595	102+6, 106−1	106
July	9	8636	112+5, 120−6	116
	24	8651	112+2	114
	26	8653	112+1	113
	29	8656	112−1	111
Aug.	19	8677	94+6, 102−3	100
Sept.	4	8693	94+3, 102−2	98
	9	8698	94+4, 102−3	98
	11	8700	94+3, 102−2	98
	13	8702	94+2, 102−4	97
	15	8704	94+0	94
Oct.	5	8724	94+3, 102−3	98
	28	8747	112+0	112
1883				
July	28	9020	102+4, 106−1	106
Aug.	25	9048	106+3, 112−1	110
Oct.	28	9112	102+0	102
1884				
May	18	9315	102+2, 106−3	104
	24	9321	102+3, 106−3	104
	26	9323	102+2, 106−3	104
	27	9324	102+4, 106−1	106
	30	9327	102+3, 106−3	104
June	11	9339	106−1	105
	20	9348	102+3, 106+0	106
	25	9353	102+0, 106−3	102
July	2	9360	102+0, 106−3	102
	6	9364	102−2	100
	12	9370	94+4, 102−2	99
	16	9374	102+0, 106−3	102
	25	9383	94+4, 102+0	100
	27	9385	102+0, 106−3, 94+5	101
Aug.	1	9390	102+0, 106−4	102

* The letter is *c*, but Baxendell adds "?*c*."

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1884		240+		
Aug.	4	9393	102+1, 106−4	102
	6	9395	102+1	103
	11	9400	102+1, 106−4	102
	14	9403	102+3, 106−2	104
	15	9404	102+3, 106−2	104
	16	9405	102+3, 106+2	*106 ?
	18	9407	102+2, 106−2	104
	19	9408	102+4, 106+1	106
	23	9412	102+6, 106+1	108
	25	9414	102+7, 106+3	109
Sept.	1	9421	102+5, 106+3	108
	4	9424	106+3, 112−3	†109
	11	9431	112+0	112
	16	9436	112−1	111
	17	9437	112−2	110
	19	9439	106+3, 112−3	109
	20	9440	106+3, 112−3	109
	24	9444	102−1, 112−8, 106−3	‡103
Oct.	4	9454	94+1, 102−6	96
	5	9455	94+3, 102−3	98
	9	9459	94+3, 102−4	98
	12	9462	94+0, 102−7	94
Nov.	7	9488	94+1	95
	13	9494	94−2	92
	17	9498	94−2	92
	19	9500	94−2	92
	21	9502	94−2	92
	27	9508	94−3	‡91
	28	9509	94−3	91
	29	9510	94−2	92
Dec.	3	9514	94+2	96
	4	9515	94+1	95
	6	9517	94+3	97
	8	9519	94+4	98
	9	9520	94+5, 102−1	100
	14	9525	106+4, 112−3, 94+9	107
	15	9526	106+3, 112−3	109
1884		240+		
Dec.	20	9531	106+6, 112−3	110
	21	9532	106+6, 112−2	111
	24	9535	112−2	110
1885				
Jan.	5	9547	106+2, 112−4	108
	8	9550	106+4, 112−4	109
	11	9553	112−2	110
	14	9556	106+3, 112−5	108
Apr.	29	9661	106+0	106
May	7	9669	106+2, 112−3	108
	11	9673	106+0, 112−6	106
	20	9682	106+6, 112−2	111
June	1	9694	106+5, 112−2	110
	9	9702	106+5, 112−1	111
	12	9705	106+6, 112+0	112
July	4	9727	102+1, 106−5	102
	9	9732	102−1	101
	25	9748	94+2	96
	28	9751	94+0	94
Aug.	1	9755	94+1	95
	3	9757	94+2	96
	14	9768	94+1	95
	16	9770	94+1	95
	18	9772	94+0	94
	19	9773	94+0	94
	20	9774	94+1	95
	22	9776	94+1	95
	28	9782	94+1	95
Sept.	3	9788	94+3	97
	5	9790	94+3, 102−3	98
	6	9791	94+5, 102−2	100
	8	9793	94+7, 102+0	102
	9	9794	94+6, 102+0	101
	12	9797	102+1, 94+8	102
	14	9799	106+0	106
	18	9803	112−2	110
	25	9810	112+1	113
	26	9811	112+1	113

* "*d* evidently changes." † "Clouds and ☽."
‡ "Observation repeated."

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TABLE IX.—*Baxendell's Observations of S Aquilæ*—continued.

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1885		240+		
Sept.	27	9812	112+1	113
Oct.	29	9844	112−3	109
Nov.	8	9854	106+0, 112−4	107
	15	9861	106+0	106
	19	9865	106+2, 112−6	107
	28	9874	106+0	106
Dec.	1	9877	106+0	106
	26	9902	94+2, 102−5	96
1886				
Jan.	5	9912	94+4, 102−2	99
	11	9918	94+4, 102−2	99
		241+		
May	21	0048	94+1	95
	29	0056	94+0	94
June	5	0063	94+1	95
	14	0072	94+2	96
	26	0084	106+0	106
	30	0088	106+6, 112+0	112
Sept.	14	0164	102+2, 106−2	104
	17	0167	102+0	102
Oct.	23	0203	94+4, 102−2	99

Date.		J.D.	Corrected Comparisons.	Deduced Mag.
1886		241+		
Oct.	25	0205	94+4, 102+0	100
	30	0210	102+2, 106−2	104
Nov.	2	0213	102+3, 106−3	104
	4	0215	102+5, 106−2	106
	7	0218	106+2	108
	9	0220	106+3, 112−5	108
	10	0221	106+3, 112−4	108
	13	0224	106+4, 112−2	110
	15	0226	106+5, 112−2	110
	16	0227	106+6	112
	17	0228	106+7, 112−2	112
	18	0229	106+6, 112−3	110
	30	0241	106+7, 112−1	112
Dec.	4	0245	106+7, 112−1	112
	16	0257	102+6, 112−3, 106+4	109
	20	0261	112−4	108
	25	0266	106+1, 112−5	107
	29	0270	106−1?	105
1887				
Jan.	12	0284	102+2	104
June	10	0433	102−1	101

Later Observations of S Aquilæ by J. Baxendell, junior.

Date.		J.D.	Deduced Mag.
1887			
May	16	0408	111
June	7	0430	103½
	10	0433	101
	20	0443	94
	24	0447	93
July	14	0467	90½
	19	0472	90½
	31	0484	94
Aug.	18	0502	108½
	21	0505	110
Sept.	11	0526	111

Date.		J.D.	Deduced Mag.
1888			
May	11	0769	94
	30	0788	94
June	18	0807	93½
July	26	0845	101
Aug.	2	0852	104
	14	0864	100
	27	0877	100½
Sept.	9	0890	100
	17	0898	98
	26	0907	99
Oct.	1	0912	98
	15	0926	97
	23	0934	*95
Nov.	14	0956	106½

Date.		J.D.	Deduced Mag.
1889			
May	20	1143	109½
	25	1148	104
	31	1154	98
June	16	1170	95
July	7	1191	95
	27	1211	99
Aug.	15	1230	†112
Sept.	1	1247	111
	30	1276	114
1890			
Sept.	10	1621	105

* Uncertain; fog and cloud. † "Hazy."

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At first sight these observations present an appearance of great irregularity; but a closer inspection to a great extent removes this impression, and they have been tabulated in the usual manner with an assumed period of 147 days. The results, as far as Period LII., are as follows:—

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
A (I. to XVI.) . .	97	101	104	108	110	112	108	106·5	103·5	100	97	94·5
B (XXXVI. to XLVIII.)	102	107	111	112	113	112	105	100	99	96	96	98
Weighted Mean (B shifted). .	98	102	105·5	109	111	112	110	106	102	99	96	95

There is an evident retrograde movement of about one "portion" ($12\frac{1}{4}$ days) between A and B, so the mean light curve has been obtained by shifting the B figures one place forward. This gives a good, smooth curve with minimum at 67 days and maximum at 136 days, after the arbitrary starting-point; but, before comparing the observations with it in the usual way, a table was drawn up to ascertain how far an amended regular period of 146·6 days would really fit the observed epochs of maximum and minimum, taking for these the widest reasonable limits suggested by the observations. The object of this was to ascertain whether the apparent anomalies were sufficiently numerous and irregular to make the work of obtaining the "corrections to epoch" not worth while. The result (given in Table X.) shows that, with a few startling exceptions, a regular period of 146·6 days is not far from accordance with the observations, especially when we take into account that the "limits" might sometimes be considerably extended by assuming a possible error or abnormal conditions in regard to one or two observations, and also when we notice how very much the magnitude *appears* to fluctuate very often near both maximum and minimum, and how much these vary in magnitude in different periods.

The individual observations have therefore been compared with the mean curve in all cases where they were available on both rise and fall, even when they were very few in number. They were so arranged as to bring the epoch of maximum as nearly as might be into the middle of each set, and *general* variations of brightness were allowed for. The results are given in the last two columns of Table X. In the cases of the last three epochs (from Mr. Baxendell, junior's, observations) the data are so scanty that the results must be considered very doubtful, and this is also the case with Periods X., XLVIII., LVI., and LVIII.

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TABLE X.

Epochs of Minimum and Maximum of S Aquilæ.

Period.	Limits for Min.	Mean Min.	Least Diff.	Limits for Max.	Mean Max.	Least Diff.	Calc. Max.	C - M.
I.	1736–1750	1750	0	1800–1812	1819	−7	1807	−12
III.	2040–2059	2043	0	2100–2118	2112	0	2115	+ 3
V.	? −2385	2336	?+	2413–2424	2405	+8	2420	+15
VI.	2477–2483	2483	0	?2557–?2580	2552	?+5	2562	+10
VIII.	2754–2767	2776	−9	2843–2855	2845	0	2846	+ 1
IX.	2914–2928	2923	0	...	·2992	...	...	...
X.	...	3069	...	...	3138	...	?3140	?+ 2
XI.	3195–3233	3216	0	3280–3289	3285	0	3285	0
XIII.	3506–3510	3509	0	?3569–3580	3578	0	3578	0
XV.	...	...	...	?3843–?3866	3871	?−5	..	...
XXXVI.	...	6881	...	?6925–6960	6950	0	...	...
XXXVIII.	7176–7180	7174	+2	7241–7246	7243	0	7241	− 2
XXXIX.	7302–7312	7321	−9	?7368–?7370	7390	?−20	...	...
XLI.	7620–7626	7614	+6	7672–7680	7683	− 3	7680	− 3
XLII.	...	7760	...	?7814–7826	7829	− 3	7824	− 5
XLIII.	7912–7920	7907	+5	?7945–7987	7976	?0	7981	+ 5
XLIV.	8053–8059	8054	0	...	8123	...	...	...
XLV.	? −8208	8200	?0	?8230–8290	8269	0	8271	+ 2
XLVI.	8333–8351	8347	0	8411–?8440	8416	0	...	...
XLVIII.	?8629–8643	8640	0	8704–8712	8709	0 ?	?8706	?− 3
LII.	...	9226	...	?9353–?9390	9296	?+57	9361	+65
LIII.	9428–9433	9373	+55	?9496–9509	9442	+54	9493	+51
LV.	?9680–9711	9666	+14	9750–9772	9735	+15	9765	+30
LVI.	9808–?9820	9813	?0	9890–9907	9882	+ 8	?9904	?+22
LVII.	...	9960	...	? −0067	0029	?+	...	...
LVIII.	...	0106	...	?0185–0203	0175	?+10	?0185	?+10
LIX.	0227–0250	0253	−3	...	0322	...	...	...
LX.	...	0400	...	0458–0476	0468	0	?0467	?− 1
LXIII.	?0849–?0857	0839	?+10	?0920–?0940	0908	?+12	?0913	?+ 5
LXV.	...	1132	...	1163–?1199	1201	?−2	?1183	?−18
LXVI.	?1262– ?	1279	?0	...	1348	...	...	...

In two cases the epochs obtained by calculation are outside the so-called "limits." Taking the last column as it stands, we see that the calculated epochs are in agreement with the "mean" ones, except for Periods I., V., VI., LII. to LVIII., and LXV.; but the last is quite doubtful.

These results suggest a *general* period of about 146·6 days,

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with two (observed) fluctuations very similar in character. Each consists of a few periods abnormally long, followed by a few much shorter than the average, in both cases bringing the epoch back to its mean position. The variation is greater on the second occasion than on the first, and appears to extend over a longer time, though we may not have the beginning of the first one. There is, of course, ample time for one or two similar fluctuations during the gap between Periods XIII. and XXXVII., and there is, in fact, a distinct suggestion of a shortening of the period between XIII. and XV. made by the isolated observation of J.D. 2403857. The following discontinuity solution gives fairly good results, but it is possible that the fluctuations are really of a wave-like character.

Periods I. to V., 153·5 days; V. to VIII., 141 days; VIII. to XLVIII., 146·6 days; XLVIII. to LII., 161·5 days; LII. to LX., 139 days.

TABLE XI.

Period.	Epoch by Disc.	D−C.	Period.	Disc.	D−C.	Period.	Disc.	D−C.	Period.	Disc.	D−C.
I.	1807	0	X.	3137	?−3	XLII.	7828	+4	LIII.	9493	0
III.	2114	−1	XI.	3284	−1	XLIII.	7975	−6	LV.	9771	+6
V.	2421	+1	XIII.	3577	−1	XLV.	8268	−3	LVI.	9910	?+6
VI.	2562	0	XXXVIII.	7242	+1	XLVIII.	8708	?+2	LVIII.	0188	?+3
VIII.	2844	−2	XLI.	7682	+2	LII.	9354	−7	LX.	0466	?+1

The maximum magnitude varies, in different periods, from 8·7 to 9·9 or fainter; and the minimum from 11·0 (?) to 11·8.

Analysis of the mean light curve gives the expression

$$8{\cdot}08 \cos(\theta - 138^\circ) + 0{\cdot}22 \cos(2\theta - 90^\circ) + 0{\cdot}5 \cos(3\theta - 90^\circ),$$

which becomes, when divided by the range and multiplied by 6, and when, further, the phase of the first harmonic is reduced to 180°, which the Rev. T. E. R. Phillips adopted as standard (*B.A.A. Journal*, October 1916, p. 9),

$$2{\cdot}85 \cos(\theta - 180^\circ) + 0{\cdot}08 \cos(2\theta - 174^\circ) + 0{\cdot}18 \cos(3\theta - 216^\circ).$$

The figures for A (Periods I. to XVI.) and for B (XXXVI. to XLVIII.) give:

$$\text{(A)}\quad 2{\cdot}52 \cos(\theta - 180^\circ) + 0{\cdot}19 \cos(2\theta - 230^\circ) + 0{\cdot}24 \cos(3\theta - 174^\circ)$$
$$\text{(B)}\quad 2{\cdot}85 \cos(\theta - 180^\circ) + 0{\cdot}14 \cos(2\theta - 14^\circ) + 0{\cdot}29 \cos(3\theta - 299^\circ).$$

Now, Mr. Phillips' classification depends on the relation between the phases ϕ_2 and ϕ_3 for the second and third harmonics. But it will be seen that the values of ϕ_2 for (A) and (B) differ by nearly 180°; and, when they are combined into the mean, not only is the coefficient 0·08 very small, but the phase is very uncertain. A change of a tenth of a magnitude in one reading might alter it by 90° or even more. Hence it seems unprofitable to attempt to assign the "group" of S Aquilæ from Baxendell's

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observations alone, though the smallness of the coefficient of the second harmonic suggests Group I. rather than Group II.; but the point on Mr. Phillips's diagram falls near the junction of the two groups.

Notes on T Herculis and R Serpentis.

Since the publication of the sixth and seventh instalments of this series, Mr. C. L. Brook has very kindly allowed us to see the B.A.A. observations of these stars for J.D. 2418730 to 2420500 (about) in MS., and corrections to epoch for T Herculis have been calculated from these observations on the same plan which has been used throughout this series, so that they might be compared with the epochs given in *M.N.*, lxxvi. pp. 478–481 and 492. [There is a misprint on p. 478. The date for Period XIX., in Table VI., should be 3238, not 3288.] In the following Table the "assumed epochs," at 165 days' regular interval, are taken four days later than in the old Tables, as this suits most of the observed maxima better :—

Table XII.

Epochs of Maxima of T Herculis.

Period.	Assumed Epoch.	Obsd. Epoch.	O – A.	Period.	Assumed Epoch.	Obsd. Epoch.	O – A.	Period.	Assumed Epoch.	Obsd. Epoch.	O – A.
I.	0272	0274	+ 2	LVIII.	9677	9658	– 19	CX.	8257	8259	+ 2
X.	1757	1751	– 6	LIX.	9842	9830	– 12	CXI.	8422	8426	+ 4
XII.	2087	2082	– 5	LXI.	0172	?0158	? – 14	CXII.	8587	8584	– 3
XIV.	2417	2417	0	XCII.	5287	5287	0	CXIII.	8752	?8750	? – 2
XVI.	2747	2743	– 4	XCIV.	5617	5616	– 1	CXIV.	8917	8913	– 4
XVII.	2912	2910	– 2	XCVI.	5947	5944	– 3	CXV.	9082	9078	– 4
XIX.	3242	3247	+ 5	CI.	6772	6771	– 1	CXVI.	9247	9245	– 2
XXI.	3572	3578	+ 6	CIII.	7102	7104	+ 2	CXVII.	9412	9406	– 6
XLIII.	7202	7190	– 12	CIV.	7267	7265	– 2	CXVIII.	9577	9577	0
XLVII.	7862	?7842	? – 20	CV.	7432	7431	– 1	CXIX.	9742	9737	– 5
XLVIII.	8027	8004	– 23	CVI.	7597	?7605	? + 8	CXX.	9907	9906	– 1
L.	8357	8335	– 22	CVII.	7762	7763	+ 1	CXXI.	0072	0070	– 2
LII.	8687	8667	– 20	CVIII.	7927	7929	+ 2	CXXII.	0237	0234	– 3
LVI.	9347	9326	– 21	CIX.	8092	8086	– 6	CXXIII.	0402	0407	+ 5
LVII.	9512	9490	– 22								

The "observed" epochs of Periods XCII. and CXIII. are not calculated like the rest, but are estimated from the appearance of the observations.

It must be admitted that this Table gives no support to the theory of sudden periodical changes of phase which was suggested by Baxendell's observations. It is also clear that the theory of an unchanging period of 165 days suits all the observed epochs except those of Baxendell's later series, viz. Periods XLIII. to LXI.

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But the failure in agreement of these epochs is so decided and so consistent that we must suppose either a temporary change in the star's variation or some very strange and persistent error in the observations throughout that whole series. The epochs in the Table from XCII. are those of the B.A.A. observations; but those of Harvard (see p. 492) never differ from them by more than a very few days.

The new B.A.A. observations of R Serpentis have been dealt with in the same manner, and five new epochs (8828, 9188, 9544, 9894, 0248) have been obtained. Compared with the uniform period of 356·7 days suggested in *M.N.*, lxxvi. p. 689, these give residuals of +9, +12, +12, +5, and +2, which are not satisfactory; while the residuals from the "new formula" there proposed are still less so, being −1, −4, −9, −19, and −23.

In the following Table the whole set of epochs is compared with a period of 356·8 days:—

TABLE XIII.

Comparison of Epochs of R Serpentis.

E.	Observed Epoch.	Uniform.	O−U.
31	9582	9556	+26
	240+		
35	0990	0983	+ 7
37	1695	1697	− 2
38	2046	2054	− 8
39	2401	2410	− 9
40	2756	2767	−11
41	3119	3124	− 5
42	3482	3481	+ 1
	241+		
75	5253	5255	− 2
76	5619	5612	+ 7
77	5961	5969	− 8
79	6674	6682	− 8
80	7036	7039	− 3
81	7393	7396	− 3
82	7752	7753	− 1
83	8104	8110	− 6
84	8463	8466	− 3
85	8828	8823	+ 5
86	9188	9180	+ 8
87	9544	9537	+ 7
88	9894	9894	0
	242+		
89	0248	0250	− 2

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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVII. JUNE 8, 1917. No. 8

Major P. A. MacMahon, D.Sc., Sc.D., LL.D., F.R.S., President, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Cyril Heath Stephenson, Bickford House, Fallings Park, Wolverhampton (proposed by Arthur Hands); and
William Wilson, M.B., C.M., Medical Missionary of the China Inland Mission, 43 Fellows Road, South Hampstead, N.W. 3 (proposed by A. M. Newbegin).

Professor C. G. Abbot, Director of the Astrophysical Observatory of the Smithsonian Institution, Washington, D.C., U.S.A.;
Professor Giovanni Celoria, Director, Reale Osservatorio di Brera, Milan, Italy;
M. Pierre Henri Puiseux, Observatoire, Paris; and
Professor Charles Edward St. John, F.R.A.S., Solar Observatory, Mount Wilson, California, U.S.A.,

were balloted for and duly elected Associates of the Society.

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Forty-eight presents were announced as having been received since the last meeting, including, amongst others:—

Enlarged photograph of parts of two letters from Lalande to Sir W. Herschel, suggesting a symbol for the planet Uranus, presented by Miss Francisca Herschel.

SPECIAL GENERAL MEETING.

A Special General Meeting, of which due notice had been given to the Fellows, was held the same day, at 6.45 o'clock. Sir F. W. Dyson, M.A., F.R.S., Vice-President, in the Chair.

The following new By-law, proposed by the Council, was put to the meeting and carried:—

BY-LAW SPECIAL TO THE PRESENT WAR.

26A. Notwithstanding the preceding By-laws, any Fellow of the Society who is on Active Naval or Military Service may, if he so desires, be excused from payment of his Annual Contribution until the end of the "emergency period" (*i.e.* the end of the year following that in which peace is declared). He shall not during that period forfeit his Fellowship, and his name shall not be suspended as a defaulter by reason of non-payment of such Annual Contribution, as provided in By-law 26; but he shall not receive the publications of the Society until he resumes the payment of his Annual Contribution.

At the end of the emergency period he shall have the option of resuming his Annual Subscription from that date, or any previous date, receiving the publications of the Society from the date of resumption; or of resigning his Fellowship without any liability for arrears of Subscription.

Note on the Paper "On Frederick de Houtman's Catalogue of Stars." By E. B. Knobel.

I am indebted to Dr. F. C. Wieder of Wageningen, and to Dr. J. Stein, S.J., of Amsterdam, for calling my attention to the fact that the map of Plancius, dated 1594, mentioned on p. 414, was preceded by a map published by Plancius in 1592, of which only one copy is known to exist, and that at Valencia in Spain. This map of 1592 is referred to by Blundeville in his *Description of Petrus Plancius, his universall Mappe*, and is thus clearly the origin of Blundeville's remarks quoted on p. 416.

I also learn that two further copies of Houtman's Catalogue are known; one in the Royal Library at the Hague, and the other in a private library in Holland.

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The Systematic Motions of the Stars between Dec. + 24° *and Dec.* + 32°. By Sir F. W. Dyson and W. G. Thackeray. (Plates 8 and 9.)

The proper motions discussed in this paper are those which have been determined for the Greenwich Catalogue for 1910·0, and are about 12,000 in number. They were obtained by comparison of the Greenwich positions with those given in the catalogues of the Astronomische Gesellschaft, supplemented by determinations from the earlier catalogues of Bessel and Lalande. In the reduction of Bessel's and Lalande's observations corrections were found for each separate night's observations. The probable error of the proper motions varies considerably. It may be put at ± 1″·25 a century, except for the Right Ascensions of about 2500 stars which are derived from a comparison of Greenwich and Cambridge alone. For these it is larger, say ± 1″·50 a century.

Each hour of Right Ascension has been considered separately, the zone being thus divided into 24 areas of approximately 13° × 8°. The proper motion of every star in each of these areas has been plotted just as if the star were situated at the centre of the area in question. The plots were made from the proper motions in Right Ascension (converted into arc) and Declination, but by making them on paper ruled with circles and radiating lines the composition into total proper motions and position-angles was made graphically. The zero of angle is the direction of increasing Right Ascension, and the direction of increasing angle is clockwise, so that 90° indicates a proper motion towards the North Pole.

From these plots can be read off by inspection either the number of stars between given limits of direction or the number of stars whose total proper motions lie between certain limits. For convenience, the stars in each hour were divided into two groups, those from 0″ to 5″ being plotted on one sheet of paper and those above 5″ on another.

Part I.

Star Streaming.

The plots bring out in a striking manner the want of symmetry in the direction of proper motions due to star streaming. For the larger proper motions there is a considerable sector, amounting sometimes to 180°, in which no stars appear, while in the opposite sector there is no direction of maximum concentration. With smaller proper motions these characteristics are less marked, while with the very small proper motions the effect of accidental error has produced still greater uniformity of distribution. This will be clearly seen in Table I.

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TABLE I.

Number of Stars within certain Limits of Proper Motion for every 10° of Direction.

Angle of Direction.	0^h–1^h.				1^h–2^h.				2^h–3^h.			
	0″–2″.9.	3″.0–4″.9.	5″.0–7″.9.	8″.0+	0″–2″.9.	3″.0–4″.9.	5″.0–7″.9.	8″.0+	0″–2″.9.	3″.0–4″.9.	5″.0–7″.9.	8″.0+
0°–10°	9	6	5	7	7	5	0	6	2	6	3	1
10–20	6	8	3	5	5	3	2	1	5	4	2	3
20–30	4	4	3	3	2	3	0	1	3	2	1	0
30–40	8	2	0	2	2	2	1	0	4	3	0	0
40–50	1	1	1	1	2	0	1	2	2	0	1	0
50–60	3	0	0	0	6	2	1	0	2	0	1	0
60–70	4	1	1	0	3	0	0	0	5	1	3	0
70–80	9	2	1	0	3	1	1	0	1	2	0	0
80–90	2	0	0	0	4	3	0	0	2	0	0	0
90–100	6	0	0	0	4	2	1	0	2	1	0	0
100–110	1	3	0	0	2	0	0	0	1	0	0	2
110–120	5	1	0	0	1	1	0	0	4	1	1	1
120–130	2	1	0	0	1	1	0	0	6	3	0	0
130–140	5	2	2	0	2	2	4	0	1	1	0	1
140–150	3	1	1	0	9	1	0	0	3	2	2	1
150–160	3	2	2	2	3	2	1	0	4	2	0	0
160–170	3	7	0	0	4	0	2	0	5	3	1	0
170–180	5	9	2	0	3	4	4	1	5	4	0	1
180–190	4	3	1	0	4	7	3	0	7	0	2	0
190–200	6	2	2	1	8	3	1	3	4	2	3	1
200–210	13	8	4	2	5	2	1	0	7	0	3	1
210–220	7	6	4	1	8	2	2	1	6	3	5	0
220–230	5	7	3	2	7	12	3	3	3	5	3	5
230–240	3	2	1	0	6	4	2	4	5	3	1	0
240–250	7	3	4	2	10	5	1	2	7	4	6	0
250–260	7	4	1	1	6	2	3	1	7	3	2	2
260–270	7	4	6	2	6	6	1	0	4	4	2	0
270–280	6	3	3	0	4	2	3	3	7	2	2	1
280–290	3	7	2	3	8	2	3	1	4	4	1	0
290–300	6	6	0	1	4	2	1	2	7	5	6	0
300–310	4	3	2	3	6	7	2	5	1	3	7	4
310–320	9	4	0	2	4	8	4	4	7	5	1	3
320–330	5	4	4	5	7	4	3	3	8	7	7	8
330–340	12	2	4	9	14	7	1	9	2	3	4	6
340–350	5	5	5	14	6	7	5	8	7	8	4	7
350–360	6	10	5	11	4	5	7	3	2	9	4	4

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TABLE I.—*Number of Stars within certain Limits of Proper Motion for every* 10° *of Direction*—continued.

Angle of Direction.	3h–4h.				4h–5h.				5h–6h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
0°–10°	4	4	3	0	3	3	0	0	11	7	2	0
10–20	5	2	0	0	3	2	0	2	8	7	1	0
20–30	1	2	0	0	3	2	0	0	10	4	0	0
30–40	4	3	1	0	5	2	1	0	1	5	0	0
40–50	4	3	0	0	1	1	1	1	7	3	0	0
50–60	4	1	0	0	5	1	0	1	2	2	0	0
60–70	3	1	0	0	4	2	0	0	3	0	0	0
70–80	3	1	0	0	1	1	0	0	1	1	0	0
80–90	3	0	0	1	3	1	0	0	6	1	0	0
90–100	6	1	0	0	2	1	0	1	5	0	0	0
100–110	1	0	0	0	0	1	1	0	2	1	1	0
110–120	2	2	0	0	4	2	0	0	9	1	1	0
120–130	3	0	0	0	1	1	0	0	5	3	0	0
130–140	5	0	0	0	1	0	0	0	6	0	1	0
140–150	1	2	0	0	1	0	1	0	8	4	0	0
150–160	4	2	0	0	1	2	0	0	6	2	2	0
160–170	1	3	0	1	1	0	1	0	4	2	0	0
170–180	4	4	0	0	5	2	1	1	9	4	1	1
180–190	3	3	4	0	2	3	2	0	6	4	2	0
190–200	3	5	1	1	3	5	2	0	8	6	5	0
200–210	5	3	2	0	5	2	0	1	5	2	2	0
210–220	5	3	4	0	3	3	4	0	8	5	1	2
220–230	4	7	0	1	1	1	0	3	13	9	1	1
230–240	8	2	3	2	5	4	1	1	13	7	2	1
240–250	9	4	4	2	4	3	1	0	15	6	1	1
250–260	4	10	3	7	4	7	1	4	14	7	5	1
260–270	10	4	1	1	5	4	3	3	16	8	4	3
270–280	9	2	3	1	8	6	1	2	11	8	5	3
280–290	12	8	7	7	7	8	4	6	18	5	10	4
290–300	6	5	9	3	4	8	6	2	13	13	6	5
300–310	5	5	6	1	4	9	4	6	10	17	4	4
310–320	8	4	5	4	5	8	4	4	14	5	6	1
320–330	8	5	6	7	5	0	2	3	12	8	9	0
330–340	3	7	3	7	8	7	4	4	9	12	3	0
340–350	7	3	3	1	5	1	2	1	10	3	3	2
350–360	5	2	4	2	3	4	2	2	11	6	3	0

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TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	6h–7h.				7h–8h.				8h–9h.			
	0″–2″.9.	3″.0–4″.9.	5″.0–7″.9.	8″.0+	0″–2″.9.	3″.0–4″.9.	5″.0–7″.9.	8″.0+	0″–2″.9.	3″.0–4″.9.	5″.0–7″.9.	8″.0+
0°–10°	9	5	2	0	5	5	2	1	6	2	2	0
10–20	9	1	2	0	3	4	2	0	2	1	1	0
20–30	7	1	1	0	8	2	0	0	4	1	0	0
30–40	4	5	2	0	2	0	0	0	0	0	0	0
40–50	9	1	0	0	6	0	0	0	1	0	0	1
50–60	5	1	0	0	3	0	0	1	2	0	0	0
60–70	9	0	0	0	4	0	0	0	2	0	0	0
70–80	5	0	1	0	2	0	0	0	3	1	1	0
80–90	1	2	1	0	2	0	0	0	1	0	0	0
90–100	1	0	0	0	4	2	0	1	1	3	0	0
100–110	5	2	0	0	1	0	0	0	0	1	0	0
110–120	4	1	1	0	3	0	0	0	0	0	0	0
120–130	0	1	0	0	1	0	0	0	4	1	0	0
130–140	5	2	0	0	3	1	1	0	2	0	0	0
140–150	7	2	1	0	2	1	0	0	4	0	0	0
150–160	8	2	0	1	1	3	0	0	2	0	1	0
160–170	14	3	0	2	1	2	0	0	1	2	0	1
170–180	7	1	2	0	9	1	1	0	2	3	1	1
180–190	3	4	4	0	6	3	2	2	1	4	0	1
190–200	8	2	4	0	4	6	3	2	5	5	2	2
200–210	4	6	1	3	9	0	4	1	4	8	2	6
210–220	6	0	1	2	9	5	0	2	7	6	4	1
220–230	19	10	2	0	12	8	7	0	8	3	6	0
230–240	8	12	3	0	8	7	1	2	10	6	4	2
240–250	13	6	5	3	18	8	8	6	9	10	5	3
250–260	12	9	6	3	14	7	7	4	12	9	6	5
260–270	22	9	9	4	14	5	6	4	15	8	5	4
270–280	20	13	6	4	6	8	1	4	8	11	3	2
280–290	20	12	3	5	12	13	3	2	6	9	2	4
290–300	19	18	6	1	8	11	4	7	8	7	2	4
300–310	14	20	4	6	8	5	7	2	4	8	3	4
310–320	21	12	8	3	15	6	6	1	5	5	8	0
320–330	19	12	6	0	11	12	1	3	7	4	1	1
330–340	16	13	4	3	11	10	6	0	7	4	3	0
340–350	15	11	4	0	8	4	2	1	4	8	1	0
350–360	16	7	3	0	4	5	3	0	5	4	2	0

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TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	9^h–10^h.				10^h–11^h.				11^h–12^h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
0°–10°	5	2	0	1	3	0	0	0	4	0	1	0
10–20	0	1	0	0	1	0	0	0	1	3	0	0
20–30	3	2	0	0	2	2	0	0	0	1	0	0
30–40	5	1	0	0	0	1	0	0	2	0	0	0
40–50	1	1	0	0	0	0	1	0	3	2	0	0
50–60	0	0	0	0	0	1	0	0	1	1	0	2
60–70	2	0	0	0	1	0	0	0	3	0	1	0
70–80	0	0	0	0	3	0	0	0	1	1	0	0
80–90	6	0	0	0	1	2	0	0	2	1	0	1
90–100	5	0	1	0	3	1	0	0	1	0	0	0
100–110	2	0	0	1	0	0	0	0	0	0	0	0
110–120	2	2	0	0	0	1	0	0	1	1	1	0
120–130	1	3	0	2	1	0	0	0	1	0	0	1
130–140	1	2	0	1	2	0	0	0	3	1	1	0
140–150	5	0	0	0	2	4	0	1	4	2	1	1
150–160	6	1	1	0	4	0	1	1	2	0	4	3
160–170	3	3	0	0	3	1	4	3	2	2	1	4
170–180	5	1	2	0	3	5	6	4	5	6	9	6
180–190	5	4	4	2	4	4	2	7	2	2	6	7
190–200	9	4	5	7	4	6	6	0	4	9	6	6
200–210	5	6	4	2	2	5	8	11	7	6	7	5
210–220	5	4	3	5	2	5	4	6	4	4	2	3
220–230	6	11	2	2	3	2	7	3	2	6	2	1
230–240	5	9	4	2	10	7	3	3	5	7	1	1
240–250	6	4	2	1	6	2	3	1	1	2	3	3
250–260	7	2	1	3	6	2	4	0	4	3	1	2
260–270	11	7	4	3	6	5	4	3	8	2	2	3
270–280	6	9	2	3	6	1	6	3	2	2	0	1
280–290	9	8	2	2	9	2	3	2	4	4	1	3
290–300	5	4	6	4	1	6	1	3	3	3	0	1
300–310	4	4	1	1	9	4	5	2	2	2	0	1
310–320	4	4	2	0	3	1	4	3	4	0	2	2
320–330	5	3	5	0	2	7	3	2	3	3	3	0
330–340	1	3	3	0	3	5	3	3	7	1	0	4
340–350	5	3	2	0	1	4	4	2	2	2	3	1
350–360	6	4	0	0	2	1	0	0	2	0	0	2

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TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	12h–13h.				13h–14h.				14h–15h.			
	″ ″ 0–2·9.	″ ″ 3·0–4·9.	″ ″ 5·0–7·9.	″ 8·0+	″ ″ 0–2·9.	″ ″ 3·0–4·9.	″ ″ 5·0–7·9.	″ 8·0+	″ ″ 0–2·9.	″ ″ 3·0–4·9.	″ ″ 5·0–7·9.	″ 8·0+
° ° 0–10	3	1	1	0	4	1	1	1	5	0	1	1
10–20	3	1	0	0	0	0	0	0	6	3	0	0
20–30	3	1	0	1	2	3	0	0	5	1	1	0
30–40	2	0	0	0	2	1	1	0	2	1	2	2
40–50	2	2	0	0	1	0	1	0	1	2	0	0
50–60	3	1	0	0	1	0	2	0	1	0	2	0
60–70	2	0	0	0	3	2	0	0	1	0	0	0
70–80	1	0	1	0	0	1	1	0	2	0	0	1
80–90	2	0	1	0	4	1	1	0	3	3	1	0
90–100	2	1	1	0	3	0	0	0	5	1	3	0
100–110	0	0	2	0	1	0	0	0	2	1	1	0
110–120	0	1	0	0	2	1	1	1	1	3	0	0
120–130	4	2	0	0	1	1	0	1	5	4	1	0
130–140	2	2	0	1	4	3	1	1	4	2	3	1
140–150	2	1	3	1	3	2	3	4	1	1	4	3
150–160	1	3	0	1	4	1	4	5	4	4	2	6
160–170	6	7	3	4	6	3	2	4	4	1	2	5
170–180	5	3	6	7	6	4	7	7	5	4	10	2
180–190	6	5	4	1	4	6	7	5	4	2	3	2
190–200	10	4	2	4	4	3	4	4	3	4	8	4
200–210	3	3	6		5	5	0	2	2	4	0	2
210–220	9	4	6	5	1	1	1	2	7	1	2	4
220–230	4	5	2	3	3	4	7	3	1	2	3	2
230–240	5	5	3	2	4	2	4	3	1	3	3	2
240–250	7	4	2	2	3	6	3	2	4	1	2	1
250–260	2	3	1	3	3	4	4	3	4	1	1	0
260–270	2	3	3	1	4	2	1	0	9	2	1	2
270–280	4	0	1	3	3	2	0	1	5	3	1	3
280–290	2	4	3	3	3	2	0	3	3	3	1	1
290–300	4	7	2	5	1	2	1	3	3	3	1	1
300–310	1	2	3	4	2	4	2	1	1	4	1	1
310–320	2	5	4	2	3	3	3	0	4	1	4	0
320–330	5	3	2	5	3	2	4	0	4	2	3	1
330–340	3	2	2	1	4	1	3	2	6	1	4	0
340–350	2	2	1	2	2	3	1	0	2	6	1	0
350–360	6	5	2	1	1	2	1	0	7	1	2	1

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Table I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	15^h–16^h.				16^h–17^h.				17^h–18^h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
0°–10°	3	0	1	1	6	5	0	0	8	2	1	0
10–20	2	2	0	0	6	2	1	0	3	2	0	0
20–30	2	1	0	0	6	2	1	0	5	2	0	1
30–40	2	2	0	0	2	0	1	0	1	2	0	0
40–50	3	2	0	2	1	0	0	0	3	3	0	0
50–60	1	1	0	0	3	0	0	0	3	6	1	0
60–70	3	1	0	1	3	0	0	0	3	2	2	1
70–80	2	1	0	0	2	0	0	1	3	1	3	0
80–90	2	0	0	0	3	0	0	1	4	1	1	0
90–100	1	1	0	0	5	1	1	1	4	3	3	3
100–110	6	2	0	0	3	4	0	0	6	1	0	2
110–120	2	2	1	1	2	1	0	2	8	1	2	0
120–130	5	1	0	2	5	3	1	2	4	1	1	2
130–140	3	6	7	1	2	1	2	2	5	5	1	1
140–150	4	3	3	4	5	0	1	1	6	2	1	0
150–160	8	3	3	4	0	3	2	0	11	2	1	1
160–170	6	3	2	3	4	4	0	2	5	0	1	1
170–180	4	3	2	0	3	3	3	3	6	4	0	1
180–190	5	1	1	2	2	3	0	1	7	4	1	2
190–200	6	2	4	2	3	2	1	1	6	2	1	1
200–210	3	3	3	4	5	3	0	1	7	4	3	0
210–220	6	5	2	3	6	0	2	0	9	1	2	0
220–230	5	3	1	3	7	4	2	2	12	8	4	1
230–240	7	2	0	3	6	4	4	2	5	5	6	1
240–250	2	2	2	5	6	6	2	4	11	7	2	4
250–260	8	2	1	0	13	4	4	3	11	4	2	3
260–270	6	8	2	1	6	6	4	2	17	4	8	3
270–280	7	4	4	2	6	10	3	2	8	5	3	2
280–290	6	2	3	2	4	5	5	1	9	5	6	4
290–300	3	6	0	3	6	5	1	0	8	9	7	1
300–310	6	0	1	4	8	1	4	1	5	5	5	3
310–320	4	3	1	3	6	2	5	1	10	8	5	0
320–330	5	1	3	3	3	1	3	1	12	10	1	0
330–340	2	3	1	1	9	2	7	1	8	5	6	0
340–350	4	2	2	0	4	6	2	1	7	10	3	1
350–360	2	4	3	0	8	3	3	0	4	2	3	1

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TABLE I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	18^h–19^h.				19^h–20^h.				20^h–21^h.			
	0–2·9.	3·0–4·9.	5·0–7·9.	8·0+	0–2·9.	3·0–4·9.	5·0–7·9.	8·0+	0–2·9.	3·0–4·9.	5·0–7·9.	8·0+
0°–10°	9	11	2	0	17	15	4	2	21	10	5	1
10–20	12	3	2	1	25	17	5	1	14	11	6	3
20–30	18	12	4	0	24	20	3	1	20	12	5	2
30–40	13	6	3	1	21	15	4	4	11	7	6	0
40–50	12	3	0	2	19	7	7	4	15	10	3	0
50–60	7	4	2	3	18	6	2	1	17	11	3	1
60–70	9	4	3	2	17	3	1	3	12	9	0	5
70–80	5	3	1	2	12	5	2	0	6	3	2	0
80–90	6	4	2	0	18	3	1	1	13	5	0	0
90–100	5	3	1	1	15	4	1	1	12	7	1	0
100–110	8	1	1	1	8	2	0	0	8	2	0	0
110–120	9	5	1	0	11	2	0	0	2	1	0	0
120–130	8	2	0	0	15	2	2	0	13	4	0	0
130–140	8	2	1	0	9	1	0	0	15	4	2	0
140–150	5	4	0	1	9	3	1	0	8	2	0	0
150–160	12	3	1	0	9	3	0	0	7	3	2	0
160–170	7	4	1	1	10	2	0	0	5	2	1	1
170–180	10	3	1	0	10	2	0	1	8	6	2	0
180–190	7	4	0	0	11	3	1	0	10	7	2	0
190–200	8	5	1	0	15	4	0	0	10	5	0	0
200–210	8	4	1	0	10	3	3	1	10	7	1	1
210–220	12	3	0	0	13	4	2	0	20	2	0	2
220–230	10	5	2	2	11	3	2	3	15	9	3	1
230–240	6	6	4	0	14	5	4	2	9	7	3	3
240–250	13	2	0	2	13	4	7	0	13	5	3	2
250–260	6	4	0	4	10	7	2	6	13	4	4	2
260–270	14	4	6	3	26	3	3	3	20	7	7	1
270–280	14	4	2	2	20	6	5	0	18	4	2	1
280–290	10	8	5	3	20	6	1	2	12	7	2	2
290–300	17	10	9	2	23	8	0	2	14	5	2	1
300–310	10	8	3	2	15	9	3	2	15	7	2	0
310–320	18	8	8	0	24	9	4	0	19	8	1	0
320–330	8	6	5	0	24	17	5	4	13	8	0	1
330–340	17	5	0	2	19	11	4	0	19	7	2	1
340–350	15	8	4	3	23	12	6	2	19	7	5	0
350–360	12	11	3	1	23	13	5	0	18	12	2	3

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Table I.—*Number of Stars within certain Limits of Proper Motion for every 10° of Direction*—continued.

Angle of Direction.	21h–22h.				22h–23h.				23h–0h.			
	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9	3″·0–4″·9.	5″·0–7″·9.	8″·0+	0″–2″·9.	3″·0–4″·9.	5″·0–7″·9.	8″·0+
0°–10°	13	23	7	7	10	8	7	9	12	14	6	6
10–20	10	14	5	5	10	7	9	3	4	8	5	11
20–30	11	3	7	3	10	6	3	5	11	3	3	3
30–40	10	12	3	2	11	6	3	0	11	4	2	1
40–50	16	7	0	3	6	6	1	1	7	3	1	0
50–60	14	5	4	0	8	2	0	0	1	1	0	0
60–70	10	0	1	0	9	1	0	0	2	2	1	0
70–80	9	3	1	0	5	0	2	0	6	4	1	0
80–90	8	4	2	0	5	2	0	1	5	1	0	0
90–100	13	2	0	0	3	1	0	0	6	1	0	1
100–110	6	3	0	1	6	1	0	1	5	2	0	0
110–120	3	3	0	0	6	2	1	1	1	1	1	0
120–130	7	3	0	0	8	1	0	0	1	2	0	0
130–140	8	0	0	0	5	3	0	0	7	3	0	0
140–150	4	1	1	1	4	2	2	0	3	2	1	1
150–160	5	1	1	0	7	4	0	0	10	3	0	0
160–170	8	1	1	0	9	4	3	1	3	2	1	0
170–180	12	2	5	0	15	4	2	3	7	3	1	0
180–190	8	5	3	0	9	5	4	1	5	5	2	1
190–200	7	3	1	0	12	2	5	0	7	5	5	0
200–210	11	7	2	0	14	3	2	5	9	5	3	0
210–220	13	3	2	4	11	7	4	2	9	4	3	2
220–230	10	9	3	2	4	3	2	2	4	5	4	1
230–240	13	6	3	4	11	6	3	5	4	4	1	2
240–250	16	5	2	0	9	3	1	6	8	4	3	2
250–260	7	4	1	2	10	5	1	2	6	3	1	2
260–270	10	4	4	1	4	4	5	3	3	5	2	2
270–280	7	3	0	1	5	3	0	1	7	2	2	0
280–290	10	7	2	2	5	5	0	1	5	2	4	1
290–300	8	2	1	0	7	0	3	2	7	1	2	0
300–310	5	1	1	1	7	1	0	1	7	6	2	1
310–320	6	6	3	0	4	5	1	1	10	2	1	2
320–330	10	2	4	0	12	1	1	1	10	5	0	5
330–340	7	9	1	0	16	6	1	0	10	5	3	1
340–350	15	6	2	4	12	6	1	1	8	5	5	4
350–360	13	8	5	7	11	11	7	2	10	9	5	8

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The initial difficulty of deciding what is the best material to work with is caused by this factor of accidental errors. To adopt arbitrary limits is not free from objection, but to take in everything is to swamp the best with inferior material. For this and other reasons the difficulty has been met by breaking up the large amount of material available into four categories. Proper motions within the limits of $0''$ to $3''$ per century are not used at all, and the rest are divided into sections $3''$ to $5''$, $5''$ to $8''$, and $8''$ and upwards, the statistics being exhibited separately.

The figures of Table I. were plotted on various diagrams, of which that given in Plate 8 may be taken as a specimen. This gives the number of proper motions (1) from $3''$ to $5''$, (2) from $5''$ to $8''$, and (3) $>8''$. Similar diagrams were made, using all the proper motions $>5''$. The observations were smoothed by combining three consecutive azimuths, so that the ordinate at 5° is the number of stars between 350° and 20°. Further smoothing was effected by combining three hours together, giving a double weight to the middle one. Various other diagrams were made with less smoothing.

The existence of two star-streams was shown clearly, but no sufficient evidence of Stream III. was obtained.

Stream I. was very marked, especially the high peaks near 0^h and 12^h. The position of the apex was found to be at R.A. 6^h, Dec. -8°, having regard to the directions alone, but the relative numbers of stars about 6^h and 18^h which belonged to the stream indicated a more southerly declination, and no certain correction was indicated to the value -13°, the mean of the values in Professor Eddington's *Stellar Motions and the Structure of the Universe*, p. 125.

The position of the apex of Stream II. cannot be determined so accurately as that of Stream I., and the mean of several different determinations is R.A. 18^h, Dec. -60°.

The ratio of the stream velocities in Streams I. and II. was determined graphically by comparison of the heights of the peaks in the various diagrams. At the distance θ_1 from the apex of Stream I. the stream velocity is $V_1 \sin \theta_1$, and at a distance θ_2 from the apex of Stream II. the stream velocity is $V_2 \sin \theta_2$. For the whole region from 12^h to 0^h $\sin \theta_2$ is very nearly 1 throughout this zone. It was therefore necessary to find in what parts of the zone the peaks due to Stream I. rose to the mean height of those due to Stream II. from 12^h to 0^h. This was found to be when $\sin \theta_1 = \cdot 60$, and thus V_2 was roughly found $= \cdot 6 V_1$.

The numbers of the stars belonging to the two streams appear to be in the ratio of about 4 : 3. The ratio found by Professor Eddington from the stars of Boss's Catalogue is 3 : 2.

The constants of the two star-streams as determined from a narrow zone naturally do not possess any great accuracy. The larger proper motions give the best agreement, the smaller ones showing deviations which may arise from local irregularities in the movements of the stars or from errors in the proper motions arising from small systematic errors in the observations.

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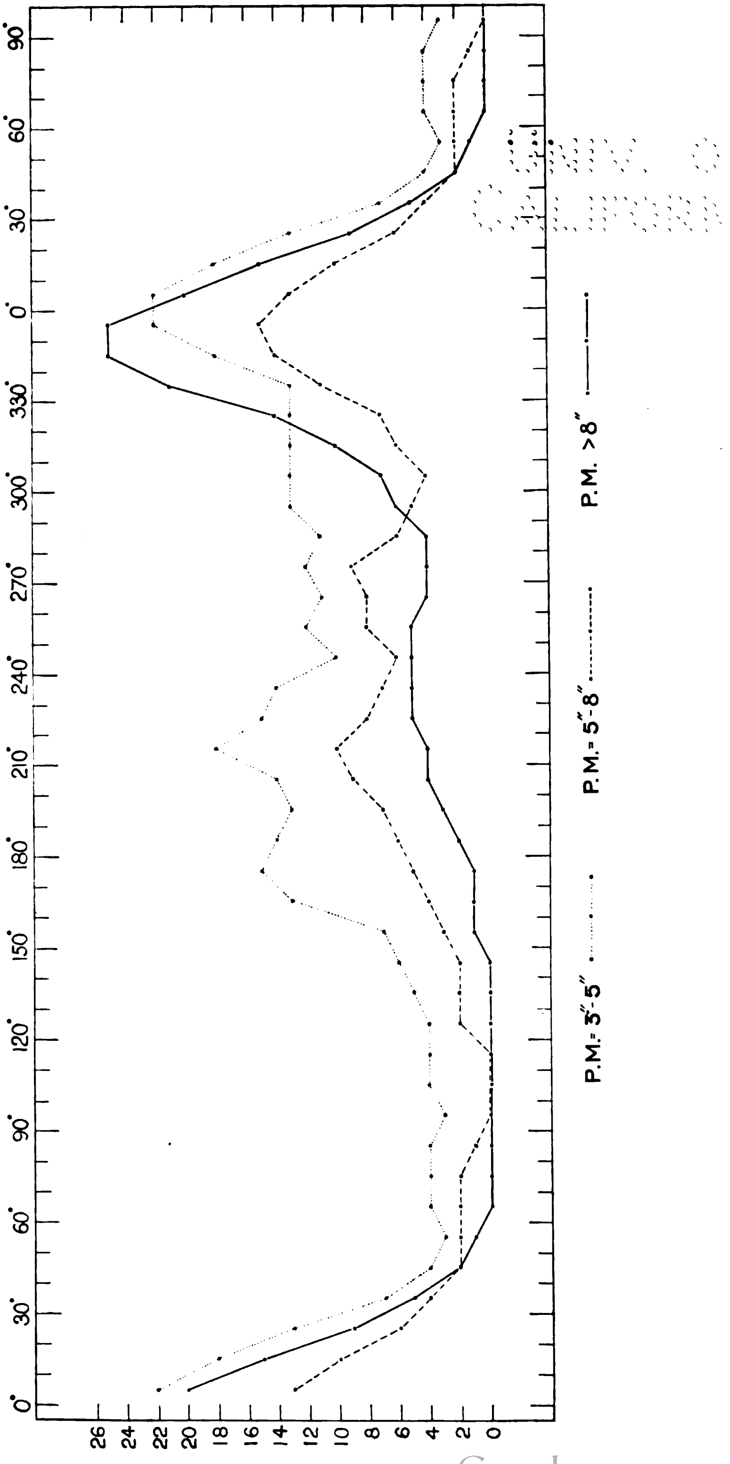
DISTRIBUTION OF PROPER MOTIONS IN DIRECTION
R.A. $0^H - 1^H$



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But in the main, as far as this discussion goes, the general features of the proper motions of the stars in this zone may be said to be well explained by the existence of two star-streams.

Part II.

The Distribution of Proper Motions in Relation to the Galaxy.

The number of stars with proper motions between varying limits were counted in each hour. They are given in detail in Table II. and in a more condensed form in Table III.

Table II.

Number of Stars for varying ranges of Centennial Proper Motion arranged in order of Right Ascension.

Limits of R.A.	Centennial Proper Motion.									
	0″·0–0″·9.	1″·0–1″·9.	2″·0–2″·9.	3″·0–3″·9.	4″·0–4″·9.	5″·0–5″·9.	6″·0–6″·9.	7″·0–7″·9.	8″·0–8″·9.	9″·0–9″·9
h h										
0–1	26	68	107	82	56	31	20	21	12	12
1–2	34	68	81	65	54	34	18	12	11	10
2–3	26	59	61	64	43	30	26	21	10	6
3–4	33	62	73	74	58	40	19	12	10	11
4–5	19	51	56	67	40	17	17	14	9	6
5–6	62	117	136	110	74	41	23	18	6	4
6–7	59	126	186	124	74	49	32	12	12	8
7–8	39	102	91	82	63	36	27	14	7	5
8–9	27	61	75	79	57	33	22	11	7	7
9–10	23	52	78	64	49	25	18	14	6	9
10–11	12	31	52	50	38	35	21	26	10	13
11–12	13	37	43	52	29	24	22	12	13	12
12–13	18	43	60	59	34	25	22	19	12	9
13–14	18	38	42	38	39	25	27	19	9	8
14–15	23	52	57	46	30	31	27	14	16	10
15–16	20	64	60	50	39	19	20	13	10	11
16–17	31	59	79	56	43	34	14	17	5	7
17–18	40	79	125	82	60	49	22	13	7	7
18–19	58	164	145	119	61	31	33	14	14	6
19–20	120	223	228	154	87	43	30	18	13	11
20–21	88	187	194	148	81	37	19	22	10	1
21–22	62	123	158	99	80	41	25	11	5	5
22–23	57	98	132	85	48	39	22	14	13	12
23–24	48	83	100	84	56	31	22	17	8	5
Sums	957	2047	2419	1933	1293	800	548	378	235	195
Means	40	95	101	81	54	33	23	16	10	8

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TABLE II.—*Number of Stars for varying ranges of Centennial Proper Motion arranged in order of Right Ascension*—continued.

Limits of R.A.	Centennial Proper Motion.								
	10″·0–10″·9.	11″·0–11″·9.	12″·0–12″·9.	13″·0–13″·9.	14″·0–14″·9.	15″·0–19″·9.	20″·0–29″·9.	30″·0–49″·9.	50″·0+
h h									
0–1	10	6	4	5	7	9	7	4	0
1–2	8	6	3	5	1	10	7	4	2
2–3	7	4	2	4	3	10	3	4	2
3–4	9	1	0	4	4	4	3	3	1
4–5	6	8	3	2	1	3	5	2	0
5–6	5	4	0	4	0	4	4	1	0
6–7	4	3	1	1	3	4	2	4	1
7–8	7	5	4	2	3	5	4	3	3
8–9	5	5	4	4	1	6	3	4	2
9–10	5	6	6	1	3	5	6	2	1
10–11	12	6	4	2	3	12	9	4	0
11–12	6	6	3	0	7	5	4	3	1
12–13	5	5	7	6	3	8	12	1	0
13–14	5	7	8	2	3	10	4	5	3
14–15	4	5	2	1	2	10	5	2	2
15–16	11	3	4	6	5	9	3	0	2
16–17	11	2	2	3	0	4	3	0	3
17–18	2	3	5	5	2	4	2	4	2
18–19	4	3	1	3	4	3	2	2	1
19–20	3	1	3	3	0	8	7	2	2
20–21	4	4	1	3	2	2	4	2	0
21–22	9	3	1	6	6	6	6	3	1
22–23	5	7	5	3	2	6	8	1	0
23–24	8	3	4	3	6	10	6	3	5
Sums	155	106	77	78	71	157	119	63	34
Means	7	4	3	3	3	7	5	2½	1½

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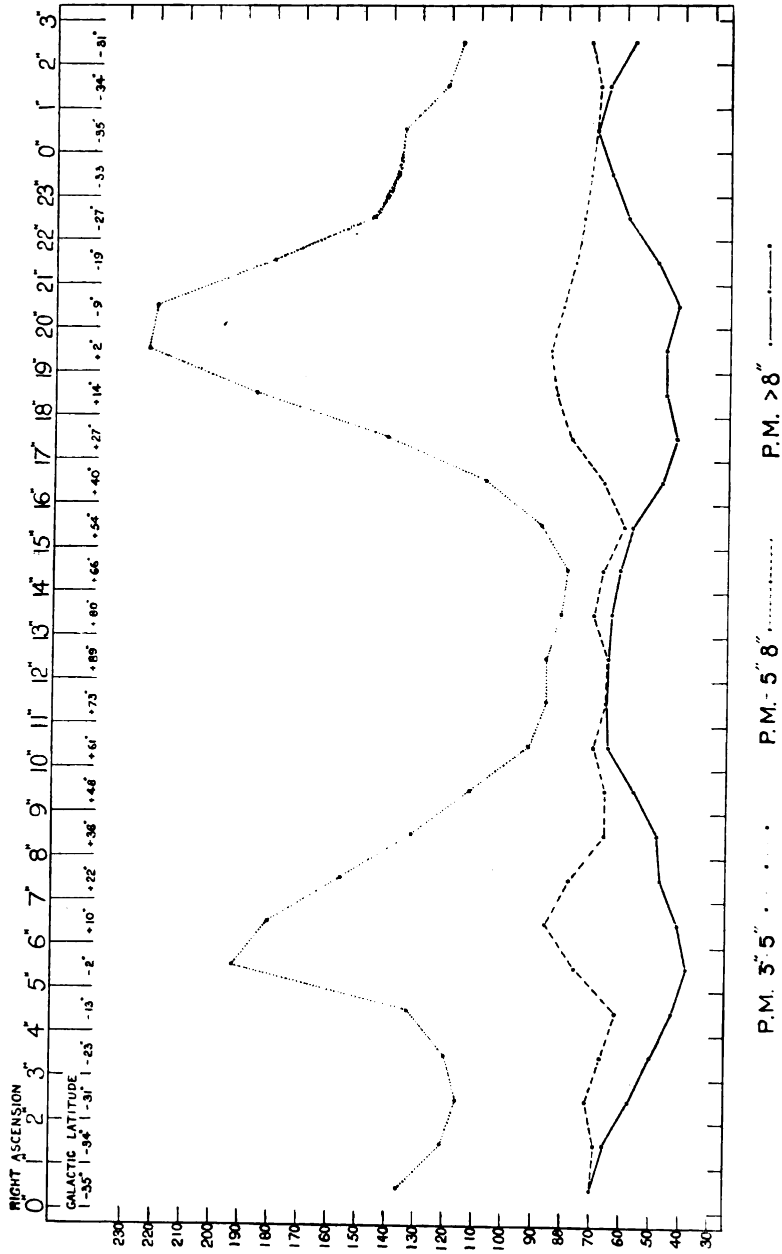

TABLE III.

Numbers and Percentages for varying Groups of Centennial Proper Motion with Galactic Latitude for each hour of R.A.

Limits of R.A.	Numbers				Total No. Stars.	Percentages.				Galactic Latitude.
	″ ″ 0–2·9.	″ ″ 3·0–4·9.	″ ″ 5·0–7·9.	″ 8·0+		″ ″ 0–2·9.	″ ″ 3·0–4·9.	″ ″ 5·0–7·9.	″ 8·0+	
h h										°
0–1	201	135	72	76	487	41	28	15	16	−35
1–2	183	119	64	67	433	42	27	15	16	34
2–3	146	107	77	55	385	38	28	20	14	31
3–4	168	133	71	50	422	40	31	17	12	23
4–5	126	107	48	45	326	39	33	15	14	13
5–6	315	184	82	32	613	51	30	13	6	− 2
6–7	371	198	93	43	705	53	28	13	6	+10
7–8	232	145	77	48	502	46	29	15	10	22
8–9	163	136	66	48	413	39	33	16	12	36
9–10	153	113	57	50	373	41	30	15	14	48
10–11	95	88	82	75	340	28	26	24	22	61
11–12	93	81	58	60	292	32	28	20	20	73
12–13	121	93	76	68	358	34	26	21	19	89
13–14	98	77	71	64	310	32	25	23	20	80
14–15	132	76	72	59	339	39	22	21	18	66
15–16	144	89	52	64	349	41	26	15	18	54
16–17	169	99	65	40	373	45	27	17	11	40
17–18	244	142	84	43	513	48	27	16	9	27
18–19	367	180	78	43	668	55	27	12	6	14
19–20	571	241	91	53	956	60	25	10	5	+ 2
20–21	469	229	78	33	809	58	28	10	4	− 9
21–22	343	179	77	51	650	53	27	12	8	19
22–23	287	133	75	62	557	52	24	13	11	27
23–24	231	140	70	71	512	45	27	14	14	−33

Some of the features of Table III. are shown in the accompanying diagram (Plate 9). It will be seen that the number of stars with proper motions greater than 8″ a century is a maximum about 0^h and 12^h and a minimum about 6^h and 18^h. The minimum at 18^h is not so low as that at 6^h. This distribution seems to be in general satisfactorily explained by reference to the positions of the apices of the two star-streams. Taking the apex of Stream I. at R.A. 6^h Dec. $-13°$, its distance from the zone is nearly 90° at 0^h and 12^h, 40° at 6^h, and 165° at 18^h. This would give maxima at 0^h and 12^h, minima at 6^h and 18^h, that at 6^h being somewhat lower. Taking the apex of Stream II.—which is, however, the slower of the two streams—at R.A. 18^h: Dec. $-60°$, the zone is approximately 90° distant for the whole extent from 12^h to 0^h, and 148°

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at 6^h, leaving only a small stream velocity to be compounded with accidental velocities. The distribution of the stars with proper motions greater than 8″ a century appears therefore to be such as would arise from a uniform distribution of the stars in space.

The next group with proper motions 5″ to 8″ a century shows clearly defined but small maxima in the neighbourhood of the Milky Way, the numbers per hour rising from 68 to 84. Combined with the group 8″+ a century, there would be a nearly uniform distribution of 125 stars per hour.

In the two remaining groups of stars with proper motions 3″ to 5″ and 0″ to 3″, the condensation of stars as the Galaxy is approached is very marked, and the extent to which this is shared by the groups 3″ to 5″ is specially to be noted.

The question whether stars of sensible proper motion crowd towards the Milky Way is discussed by Newcomb,* who makes the following remarks:—"Kapteyn has examined this question in the case of the Bradley stars. His conclusion is that those having a considerable proper motion, say more than 5″ per century, are nearly equally distributed over the sky, but that when we include those having a smaller proper motion, we see a continually increasing tendency to crowd towards the galactic plane. It seems to the writer that the uncertainty as to the smaller proper motions of the Bradley stars renders this result quite unreliable. To reach a more definite conclusion, we must base our work on lists of proper motions which are as nearly complete within their limits as it is possible to make them."

Newcomb then considers the proper motions found in the Zone Catalogues of Auwers, 15° to 20° N. Dec., and Boss 1° to 5° N. Dec., and concludes that no crowding is shown. The proper motions dealt with by Auwers and Boss are in the main large ones, and the present investigation suggests that Newcomb's results are to be attributed to this cause.

The increasing tendency to crowd towards the Milky Way, starting with proper motions between 5″ and 8″ a century, and shown very strongly in stars with proper motions 3″ to 5″ a century, suggested more detailed investigation.

The numbers of Tables II. and III. were accordingly rearranged according to their galactic latitude given in the last column of Table III. The means were then taken for five separate groups, giving the following table:—

Limits of Gal. Lat.	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–11″.	11″–15″.	>15″.
± 2° to ± 10°	82	163	186	134	79	42	26	17	20	8	9
± 13 to ± 27	44	96	111	87	59	36	24	13	23	14	12
± 31 to ± 36	32	68	85	75	53	32	22	17	25	16	20
± 40 to ± 61	22	52	67	55	42	28	18	17	27	12	16
± 66 to ± 89	18	42	50	49	33	26	25	16	27	17	19

* *The Stars: A Study of the Universe*, p. 252.

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Dividing the numbers in each column by the number in the last row, the ratio of the densities in different galactic latitudes to that near the galactic pole is found for stars of proper motion 0″ to 1″, 1″ to 2″, etc., as follows:—

Relative Density of Stars of different Proper Motions at Different Galactic Latitudes compared with the Density near the Galactic Pole.

Galactic Latitude.	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–11″.	11″–15″.	>15″.
±6°	4·6	3·8	3·7	2·7	2·4	1·6	1·1	1·1	0·8	0·5	0·5
±21	2·5	2·3	2·2	1·8	1·8	1·4	1·0	0·8	0·9	0·9	0·6
±34	1·8	1·6	1·7	1·5	1·6	1·3	0·9	1·1	0·9	1·0	1·1
±51	1·3	1·3	1·3	1·1	1·3	1·1	0·7	1·1	1·0	0·7	0·8

As the first group covers the regions where the zone cuts the Milky Way, it has been separated. The first region, extending from R.A. 5^h to 7^h, contains 1318 stars; the second, extending from 19^h to 21^h, 1765 stars, and the densities relative to that near the galactic pole are as follows:—

Relative Densities at Intersection with Galactic Planes.

Limits of R.A.											
5^h–7^h	3·5	2·9	3·2	2·3	2·2	1·7	1·1	0·9	0·7	0·5	0·5
19^h–21^h	5·9	4·9	4·2	3·0	2·5	1·5	1·0	1·2	0·8	0·5	0·4

An inspection of the table shows that in the region of greater density there is a corresponding increase of condensation for stars of smaller proper motion.

An interesting comparison may be made between the area 23^h to 3^h which is at galactic latitude $-35°$ with that of 11^h to 15^h in galactic latitude $+77°$. The effect of star streaming is nearly the same in these two regions. We find—

Relative Density at the Extreme Galactic Latitudes.

	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–11″.	11″–15″.	>15″.
23^h–3^h	48	93	124	88	60	36	22	16	26	18	19
11^h–15^h	18	42	50	49	33	26	25	16	27	17	19
Difference	30	51	74	39	27	10	−3	0	−1	+1	0
Ratio	2·7	2·2	2·5	1·8	1·8	1·4	1	1	1	1	1

Thus the two regions agree closely in the numbers of stars with proper motions >6″ a century, but the ratio continually increases as the Galaxy is approached, more and more as the amount of proper motion diminishes. Although the numbers in this table and previous tables are affected considerably by the accidental errors of the proper motions, the ratios are affected to a smaller extent. The effect has probably been to increase the ratios somewhat between 4″ to 6″ and diminish them between 0″ to 2″. The differences in the last table may be compared with the distribution

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which would arise if these stars had no proper motions at all, and the spreading was due to accidental errors alone.

Limits of P.M.	Assumed Probable Error. 1″·7.	1″·5.	1″·3.	Observed.
″ ″ 0–1	7·5	9·6	12·7	13·2
1–2	19·5	23·7	29·0	22·4
2–3	23·9	26·4	28·6	32·5
3–4	20·8	20·4	18·2	17·1
4–5	14·9	11·9	8·1	11·8
5–6	8·1	5·4	2·6	3·1
6–7	3·7	1·9	0·7	...
>7	2·1	0·7	0·1	...

These figures show that the probable error is between 1″·3 and 1″·5, agreeing with the statement at the beginning of the paper. It is, however, difficult to free the actual numbers of these tables from the effect of accidental error in the determination of proper motion.

The most natural explanation of these variations is to be found in the way in which star density varies in space. In the region nearest to us the density is a maximum, and diminishes as we proceed outwards, but much more rapidly in the direction of the galactic pole, so that the surfaces of equal density are spheroids of continually increasing oblateness. Stars with proper motions greater than 6″ a century in the main lie in the near region where the surfaces of equal density are spheres. But stars with proper motions of 3″ a century are in the mean at distances where the density is much greater in the direction of the plane than in that of the pole of the Galaxy.

Further Notes on the Radiative Equilibrium of the Stars. By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor. (Plate 10.)

13. *Ionisation in the Stars.*—In the first paper * calculations were made on the assumption of an average molecular weight 54 for the material of a star. In the present paper the average molecular weight will be taken to be 2.

The first value fairly represents the hypothesis that the ultimate particles, which move independently, are atoms. It will be shown that the value 2 represents the hypothesis that the atoms are highly ionised, so that most or all of the electrons outside the nucleus have been broken off and move as independent particles. The suggestion that at these high temperatures we are concerned

* P. 16. The paragraphs and formulæ of the present paper are numbered in continuation of those in the first paper.

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with particles smaller than the atom was made to me independently by Newall, Jeans, and Lindemann. By an argument which now appears insufficient, I had supposed that atomic disintegration, though undoubtedly occurring, could not have proceeded very far; but Jeans has convinced me that a rather extreme state of disintegration is possible, and indeed seems more plausible. In any case, the prevalence of this view of the interior state of a star makes it desirable to present the theory of stellar temperatures and densities on this alternative basis.

The principal result of the first paper—that the effective temperature of a giant star varies as the sixth root of the density—is not changed by taking account of ionisation. The main difference in the results is that the influence of the star's *mass* on the temperature is considerably increased by ionisation. The change seems on the whole to improve the agreement with observation in minor details; but since the theory is necessarily only a rough approximation, it is not desirable to lay too much stress on this removal of small discrepancies. A more important confirmation is that the theory, when thus modified by introducing ionisation, provides an explanation of why stellar masses are of the order of magnitude actually found.

It will be remembered that the temperatures within the stars are chiefly from 10^6 to 10^7 degrees. The radiation at these temperatures consists mainly of waves a little longer than X-rays, having strong ionising power. But since we do not know how fast recombination of the ions takes place, it is difficult to predict what proportion of the atoms are ionised at any moment. For many elements the first electron is removed by an ionisation potential of about 4 volts; and, in accordance with the theory of the photoelectric effect, this separation can be effected by radiation of wave-length 3200 Å or less (since a quantum of this frequency carries sufficient energy). At 9000° the maximum radiation is of the required wave-length. Although the theory gives no certain guidance, it seems reasonable to suppose that when the material is in equilibrium with radiation at temperature 9000° a fair proportion of the atoms must be in the ionised state, and that the proportion will increase rapidly as the temperature rises. We should expect then the loss of one electron to take place at temperatures of the order 10,000°; succeeding electrons will be lost at higher temperatures which can scarcely be predicted; but we can estimate on the same basis the temperature at which the last electron is removed, since the nucleus attended by a single electron forms the simple system considered by Bohr. The necessary frequency is given by the formula $2\pi^2 N^2 me^4/h^3$, where N is the atomic number, m and e the mass and charge of a negative electron, and h Planck's constant. The wave-length is, in fact, $\frac{3}{4}$ that of one of the characteristic X-rays of the element. For aluminium it is 5·4 Å; for copper 1·1 Å. The maximum radiation at 10^7 degrees is of wave-length 2·9 Å. It seems probable therefore that, though disintegration is nearing completion, it will

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scarcely be complete even in the hottest part of the stars, and that all except the lightest elements will retain a small remnant of their electrons.

If N electrons break away from each atom, they will act like independent particles in causing pressure, so that for the same temperature and density the pressure is increased $(N+1)$fold. Using the term molecules to denote the ultimate particles, whether atoms or electrons, the average molecular weight will be $A/(N+1)$, where A is the atomic weight; and it is this average which occurs as the molecular weight in the pressure-temperature-density equation of a gas. (I assume that the electric charges of the particles do not affect the calculation of the pressure.) Now the atomic number, or number of electrons outside the nucleus, is very near to half the atomic weight for all elements except hydrogen. Hence, if disintegration is complete, $A/(N+1)$ is approximately equal to 2, whatever may be the composition of the star. Although 2 is thus the extreme lower limit for the molecular weight, the retention of the last few electrons would make comparatively little difference; thus iron (atomic number 26) might retain 8 electrons and gold (79) might retain 14 without the molecular weight increasing above 3.

It may be suggested that the nucleus could also be broken up, leading to a further diminution of the average molecular weight. But it seems probable that this would require much greater energy; and it could scarcely be accomplished by radiation of less frequency than the characteristic X-rays.

14. *Summary of the Theoretical Formulæ.*—The theory given in the first paper will here be collected into a form more convenient for reference and development. I wish especially to show which formulæ are general, and which depend on particular assumptions.

The notation may be summarised as follows:—

ξ is the distance from the centre of the star.
p, ρ, T, the pressure, density, and temperature of the material.
k is the mass-coefficient of absorption.
$\frac{4}{3}\pi B$ is the net outward flow of radiation per unit area.
a is the constant $7{\cdot}06 \times 10^{-15}$ such that aT^4 is the density of the radiant energy.
μ is a constant equal to $ac/4\pi$ where c is the velocity of light.*

The two fundamental equations are

$$d(p+\tfrac{4}{3}aT^4) = -g\rho d\xi \quad . \quad . \quad . \quad (34)$$

$$d(T^4) = -\frac{B}{\mu} \cdot k\rho d\xi \quad . \quad . \quad . \quad . \quad (35)$$

Equation (34) is the usual hydrostatic equation with the correction for radiation-pressure, $d(\frac{4}{3}aT^4)$, as found in (24). Equation (35) is the equation of radiative equilibrium, expressing that the flow of energy, B, is proportional to the gradient of the energy-density,

* This value of μ was not given previously, but can easily be verified.

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$dT^4/d\xi$, and inversely proportional to the opacity of the medium, $k\rho$; it is equivalent to (6). Both equations are perfectly general, and hold for any kind of material; they could only fail if some additional forces or effects were present, *e.g.* electric forces or convection currents.

Eliminating $d\xi$, we have,

$$d(p+\tfrac{4}{3}aT^4)=\frac{g\mu}{B}\cdot\frac{1}{k}d(T^4) \quad . \quad . \quad . \quad (36)$$

The surface-value of B is given by the condition that the net outward flow of radiation near the boundary is $\pi\mu T_1^4$ per unit area, T_1 being the effective temperature.* It follows that the surface-value of $\frac{g\mu}{B}$ is $\frac{4}{3}\frac{g_1}{T_1^4}$ where g_1 is the value of g at the surface. Let the ratio B/g at any interior point be η times the surface-value, then

$$d(p+\tfrac{4}{3}aT^4)=\frac{4}{3}\frac{g_1}{T_1^4}\cdot\frac{1}{\eta k}d(T^4) \quad . \quad . \quad . \quad (37)$$

Now g is proportional to the mass within a sphere of radius ξ, divided by ξ^2; B is proportional to the energy liberated per unit time in the same sphere, divided by ξ^2. Hence η varies as the energy liberated per unit mass, averaged through the part of the star interior to the point considered. Owing to this averaging, η varies very little even when the energy is liberated at very different rates in different parts of the star. For example, if the radiated energy comes entirely from contraction, and the star passes through a series of homologous states of gradually increasing density, the energy liberated in any volume per unit time will be proportional to the energy present in that volume. In that case, with the law of temperature considered in the first paper ($p \propto T^4$), I find that η rises slowly from 1 at the surface to 1·7 at the centre.

We shall henceforth introduce an approximation, setting $\eta=1$. From (37) it appears that the effect of this will be equivalent to altering the value of k, so that the error introduced may be amalgamated with the uncertainties of k. η will almost certainly rise a little above its surface-value unity, since the greatest secular change of temperature and density is likely to take place near the centre. Hence the true coefficient of absorption will be a little less than the k used in the formulæ.

Let

$$\frac{1}{k_0}=\frac{1}{T^4}\int_0^T\frac{1}{k}d(T^4) \quad . \quad . \quad . \quad . \quad (38)$$

so that k_0 is an average value of k taken in such a way as to give enormously greater weight to the higher temperatures. If k is any slowly varying function of T, k_0 will differ very little from k, because of this mode of weighting. It will be understood that k and k_0 may be functions of the temperature, since the constitution of the radiation to be absorbed varies with the temperature.

* I take as definition of effective temperature the temperature of a black-body, which gives the same total radiation per unit area.

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Integrating (37), we have

$$p = \tfrac{4}{3}\left(\frac{g_1}{k_0 T_1^4} - a\right)T^4 . \qquad (39)$$

One useful result follows immediately from this formula. Since p cannot be negative,

$$k_0 < \frac{g_1}{a T_1^4} \qquad (40)$$

If ξ_1 is the radius of the star, $g_1 \xi_1^2$ gives the star's mass, and $a T_1^4 \xi_1^2$ gives the total radiation emitted, *i.e.* the bolometric magnitude. Hence the right-hand side of (40) can be estimated from observation. With the observational data previously used,

$$k_0 < 31.$$

This has to hold for all temperatures up to the highest interior temperature in the star. It follows that k cannot be greater than 31, except possibly for a very limited range of temperature.* We ought to allow some margin for the uncertainty of the observational data; but it seems safe to conclude that k cannot be greater than 100. This limit is interesting, since the observed absorption-coefficients of many elements for soft X-rays considerably exceed 100. The lower value found in the stars is probably due to ionisation, which renders the affected atoms non-absorbing. Laboratory experiments refer to weak streams of radiation; for very intense streams the absorption must diminish, since there will be a saturation effect.

Introduce a quantity β defined by

$$\frac{p}{\tfrac{4}{3} a T^4} = \frac{\beta}{1 - \beta} \qquad (41)$$

where β is in general variable.

We then obtain from (39)

$$T_1^4 = \frac{1}{k_0 a} g_1 (1 - \beta) \qquad (42)$$

Equations (41) and (42) are formally equivalent to (26*a*) and (29).

If k_0 *is constant throughout the star*, β is constant by (42), and hence $p \propto T^4$. Thus (for a perfect gas) the temperature and density distribution is of a well-known type studied by Emden. We find also from (41) that $1 - \beta$ is the ratio of the radiation-pressure to gravitation, since dp represents the excess of gravitation over radiation-pressure on an element, and $d(\tfrac{4}{3} a T^4)$ represents the radiation-pressure. Radiation-pressure can thus be allowed for by replacing the constant of gravitation G by βG.

For a gaseous star in which $p \propto T^4$, the central temperature and

* If k is greater than 31 at any point, the pressure there decreases inwards. If η is taken into account, the true absorption coefficient is, as we have seen, still lower.

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pressure are related to the mass M, mean density ρ_m, molecular weight m, and effective constant of gravitation βG, in the ratios *

$$\left.\begin{aligned} T_0 &\propto \rho_m^{\frac{1}{3}} M^{\frac{2}{3}} \beta m \\ p_0 &\propto \rho_m^{\frac{4}{3}} M^{\frac{2}{3}} \beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (43)$$

Hence (41) leads to an equation for β of the form

$$\frac{\beta}{1-\beta} = \frac{1}{m^4 M^2 \beta^3} \times \text{constant}$$

or

$$1 - \beta = y\beta^4 \quad . \quad . \quad . \quad . \quad (44)$$

where y varies as $M^2 m^4$.

This equation enables us to find β for any giant star, provided k is constant.

In default of information as to how k varies with the temperature, I make this assumption of the constancy of k in what follows. It has been seen that it leads to the law that the bolometric magnitude of a giant star is independent of the density, which seems to be verified by observation; and it is difficult to see how this could happen unless the assumption were approximately true. The assumption appears in two aspects, (1) that k may be taken as constant throughout a star, (2) that it may be taken constant for stars of different spectral types. With regard to (1), it seems natural that we should obtain an approximate representation by replacing the varying k_0 by some kind of average value. If this is admitted, we can by observation compare the values of k_0 (*i.e.* the aforesaid average values) for different stars by (42). Take two stars of the same mass with very different densities, say type A and type M; their total radiations will be inversely proportional to their values of k_0.† Thus if their k_0's were in the ratio 6·3 : 1, the stars would differ bolometrically by 2 magnitudes. I think this is an upper limit to the variations which the observations will allow. The observations, therefore, seem to show that k cannot vary very much.

One apparent alternative might be considered. Suppose that k_0 has a maximum value (about 30) at some rather low temperature. It follows by (39) that aT_1^4/g_1 cannot be greater than $\frac{1}{30}$. Is it possible that a star settles into such a condition that it radiates the maximum possible amount consistent with this limit? This would give $aT_1^4/g_1 = \frac{1}{30}$ for all stars. I scarcely think this could lead to a satisfactory theory; it involves zero pressure and density at some point in the interior of the star.

15. *Radiation-pressure for low Molecular Weight.*—The ratio of radiation-pressure to gravitation, $1 - \beta$, is obtained from the quartic equation (44)

$$1 - \beta = y\beta^4,$$

* Emden, *Gaskugeln*, pp. 69, 97.

† Total radiation $\propto \xi_1^2 T_1^4 \propto \frac{M}{k_0}(1-\beta)$, and by (44) β is independent of ρ and k_0.

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where y varies as M^2m^4. We have given (p. 28) the value of y for $M = 1{\cdot}5 \times \odot$, $m = 54$, viz. $\log_{10} y = 5{\cdot}2980$. Hence we can find y and β for any other values.

In Table I. the results for various masses for molecular weight 2 are given, those for molecular weight 54 being added orcomparison.

TABLE I.

Mass. ($\odot$ = 1)	$1-\beta$. Molecular Weight 2.	$1-\beta$. Molecular Weight 54.
0·5	·036	·920
1·0	·106	·943
1·5	·174	·953
3·0	·320	·967
4·5	·409	·973

It will be seen that $1-\beta$ is now much more sensitive to a change in mass than when the molecular weight was high. Also radiation-pressure no longer counterbalances the greater part of gravitation;* its place in sustaining the equilibrium of the star has been largely taken by the pressure of the free electrons. I am not sure that there is much fundamental difference between the two ideas; we may say that the pressure is now sustained by a mixture of cathode rays and X-rays instead of by X-rays alone.

The coefficient of absorption can now be found from (42). I assumed in the first paper that a star of mass 1·5 and density 0·002 would have an effective temperature 6500°; on reviewing the evidence I believe that this is the best value to adopt. Another way of expressing the same conclusion is that a star of mass 1·5 and type G has an absolute magnitude $-0^{m}{\cdot}3$. The average absolute magnitudes of giant stars are known with considerable accuracy, and the main cause of uncertainty is the doubt as to the masses to which they refer.

On this basis the value of k is 5·4 C.G.S. units. It is interesting to note that the extreme change of molecular weight has not altered the order of magnitude of k; the value previously found was 29·5.

The conclusion that atomic disintegration is not complete is important in connection with this value of k. When X-rays are absorbed, the energy is expended in ionisation; but, if the material is fully ionised, a ray traversing it can find nothing to expend itself on. In that case k should approximate to the coefficient of scattering, 0·2 (see § 12); but k is certainly higher than that.

By (43) the central temperature is modified from the value previously given proportionately to βm, or by (44) proportionately to $\sqrt[4]{(1-\beta)}$. The change is comparatively trifling.

* The importance of radiation-pressure, however, increases rapidly if the ionisation is not quite complete; *e.g.* for molecular weight 2·8 we should take from Table I. the value of $1-\beta$, corresponding to double the actual mass of the star (since $1-\beta$ depends on M^2m^4).

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16. *Limit to the Masses of Stars.*—The equation (44) for determining β involves no constants derived from astronomical observation, and can be applied to theoretical gas-spheres of any size. It is therefore a striking coincidence that the actual masses of stars correspond to the most critical range of values of β, for which radiation-pressure increases from negligible to predominant influence.

To show the character of this coincidence, consider a series of gas-spheres of all sizes—from bubbles to universes.

As the mass increases from 0 to $1·8 . 10^{33}$ gm., the ratio of radiation-pressure to gravitation in the gas-sphere increases from 0 to 0·1.

As the mass increases from $1·8 . 10^{33}$ to $3·7 . 10^{34}$ gm., the ratio of radiation-pressure to gravitation increases from 0·1 to 0·8.

As the mass increases from $3·7 . 10^{34}$ gm. to infinity, the ratio of radiation-pressure to gravitation increases from 0·8 to 1·0.

The range $1·8 . 10^{33}$ to $3·7 . 10^{34}$ gm. corresponds to about 1 to 20 times the sun's mass, and we see that at the beginning of this interval radiation-pressure is only one-tenth of gravitation, and at the end it counterbalances four-fifths of gravitation. These figures correspond to complete ionisation (molecular weight 2); if we had taken only half the electrons to be broken off (molecular weight 4) the range would have been from $\frac{1}{4}$ to 5 times the sun's mass. The truth probably lies between these two assumptions.

I think it is probable that this result is something more than a coincidence, and that it points to the cause which determines why the stars are so nearly of one standard size. A gas-sphere under the influence of two opposed and nearly balanced forces would probably be on the verge of instability; we need only refer to the somewhat analogous case of rotating masses in which centrifugal force balances a considerable part of gravitation.* We should expect, therefore, that masses in which radiation-pressure counterbalances the greater part of gravitation would very readily divide under the influence of comparatively small disturbances—rotation, or perhaps imperfect homogeneity. Hence the primitive material would continually divide and subdivide until the masses were below the dangerous limit; after that there would be no further cause for division.

The suggestion is that though high radiation-pressure does not actually break up the mass, it produces a state in which the mass will, under ordinary circumstances, be broken up by rotation or other causes. Occasional large masses with exceptionally low angular momentum may survive, as in the case of Canopus. But when the mass is reduced to, say, half that of the sun, radiation-pressure lends practically no aid in breaking up, and smaller masses could only be produced in exceptional circumstances.

* *E.g.*, the Maclaurin spheroids become unstable when the centrifugal force is 0·36 of gravity at the equator; the Jacobi ellipsoids become unstable when the ratio varies from 0·19 to 0·55 at different parts of the equator.

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17. *The Temperatures of Giant Stars.*—Although $(1-\beta)$ is now small, the equation

$$T_1^4 \propto g(1-\beta)$$

is not thereby rendered illusory.

Since β is independent of the density, the two principal results in which we previously found agreement with observation are obtained as before—the effective temperature varies as the sixth-root of the density, and the bolometric magnitude is independent of the stage of evolution so long as the star behaves as a perfect gas.

The total radiation varies as $M(1-\beta)$, and we can no longer consider this as practically proportional to the mass. By Table I., for a 9-fold increase of mass (from 0·5 to 4·5) $1-\beta$ increases 11-fold, so that in the range usually considered the total radiation varies more nearly as the square of the mass. This is the principal difference made by assuming ionisation. We see that the masses of stars must be even more uniform than was stated on p. 30.

This greater sensitivity to mass-changes enables us to account more readily for the occasional occurrence of exceptionally brilliant stars. Canopus is probably of magnitude −5·5 or brighter, and certain evidence (such as Shapley's estimate of the parallax of the Hercules cluster) points to the existence of other stars of this brightness. With molecular weight 54 Canopus would have to be 180 times as massive as the sun; but with molecular weight 2 it need not be more than 40 times as massive—a much less violent assumption.

Taking the sun's absolute magnitude to be $5^{m}\cdot1$, the absolute magnitude of a giant star of the same type can be calculated. We obtain the following results *:—

Table II.

Mass.	Abs. Mag. (m)
0·5	+2·6
1·0	+0·7
1·5	−0·3
3·0	−1·7
4·5	−2·4

For any other type the absolute bolometric magnitude will be the same, but a small correction must be applied to obtain the absolute visual magnitude. In Table III. I give the necessary correction for objects which radiate like a black body; since the corrections are small, it may be hoped that the deviations of an actual star from a black body will not make any important difference.

The table is calculated from Nutting's † measures of the visual

* The middle result is practically adopted from observation, as already explained. The others follow from the values of $M(1-\beta)$.

† *Phil. Mag.*, 1915 Feb., p. 304.

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intensity of light of different wave-lengths, together with Planck's radiation formula.

TABLE III.

Effective Temperature.	Ratio of Luminosity to Total Radiation.	Visual—Bolometric Magnitude.
2540°	0·092	+2·59
3000	0·206	+1·71
3600	0·417	+0·95
4500	0·723	+0·35
6000	1·000	0·00
7500	0·985	+0·02
9000	0·893	+0·12
10500	0·749	+0·31
12000	0·616	+0·53

The numbers in the last column are to be added to the absolute magnitude (for the mass in question) given in Table II.

18. *The Temperatures of Dwarf Stars.*—With a high molecular weight it was found that the whole dwarf series must be comprised within densities 0·7 to 1·5; but with molecular weight 2 the range is extended in both directions. Maximum temperature occurs at densities 0·2 to 0·4 according to mass, and it is no longer necessary or possible to assume that the sun is compressed to the maximum possible density. Thus we can have dwarf stars with densities up to 3 or 4. The temperature-density curve becomes more open, and it is possible to follow the relations some way down the dwarf series.

I have thought it worth while to work out in detail the temperatures near the maximum, and as far as practicable down the dwarf series. It is true that in this (as in some other places) I am pressing the theory further than it could be expected to go. But it may be interesting and suggestive to follow out fully the behaviour of a theoretical model; and since a comparison with observation is often possible, we may seek to find out whether the model is defective.

I have represented the deviations from a perfect gas by van der Waal's equation

$$p = R\rho T(1 - \rho/\rho_0)^{-1},$$

the term $a\rho^2$ usually added to p being negligible at the high pressures here considered. The equation is, of course, not accurately satisfied by any gas, but it seems likely to represent sufficiently well for our purpose the kind of deviation from Boyle's law which most concerns us. In order to obtain the correct temperature for the sun, it is found necessary to take ρ_0 to be 3·9. The constants have thus been adjusted to agree with observation at two points, viz. to give a temperature of about 6000° for the sun, and 6500° for a star of mass 1·5 and density 0·002.

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The method of calculation was explained in § 10, but I have now found it much simpler to proceed from the centre of the star outwards; this avoids a series of adjustments by trial and error. Starting with an arbitrary central density and an arbitrary value of β, we can always build up a possible star; its mass and mean density are found at the end of the calculation. The density is unalterable; but we can afterwards change the mass to whatever value we please, changing β so as to keep $\frac{1-\beta}{\beta^4 M^2}$ constant (*cf.* equation (44)). Thus β can be found for stars of that particular density and of any assigned mass. The temperature is then given by (42). In this way I have obtained Table IV.

TABLE IV.

Effective Temperatures of Stars.

Mass / Density.	0·2.	0·5.	1·0.	1·5.	3·0.	4·5.
2·11	...	2590°	3880°	4920°	7350°	9300°
1·53	...	3580	5360	6790	10080	12640
·97	2720	4650	6960	8770	12810	15690
·65	3140	5350	7950	9960	14210	17000
·33	3520	5980	8820	10900	14930	17320
·13	3710	6250	9060	10980	14370	16230
·035	3540	5910	8350	9900	12460	13830
·002	2410	4000	5550	6500	8020	8820

The masses are measured in terms of the sun as unit; the densities in terms of water.

The results are plotted in the diagram, Plate 10, in which the logarithms of the densities and temperatures are taken as abscissæ and ordinates.

The table illustrates H. N. Russell's contention that the small stars cannot rise to the high temperatures characteristic of the so-called earlier types. The maximum temperature attained varies nearly as the square-root of the mass. The following table (derived by interpolation) shows the mass necessary to attain a specified temperature:—

TABLE V.

Effective Temperature.	Minimum Mass.
14000°	2·5
12000	1·8
10000	1·2
8000	0·8
6000	0·46
4500	0·29
3000	0·14

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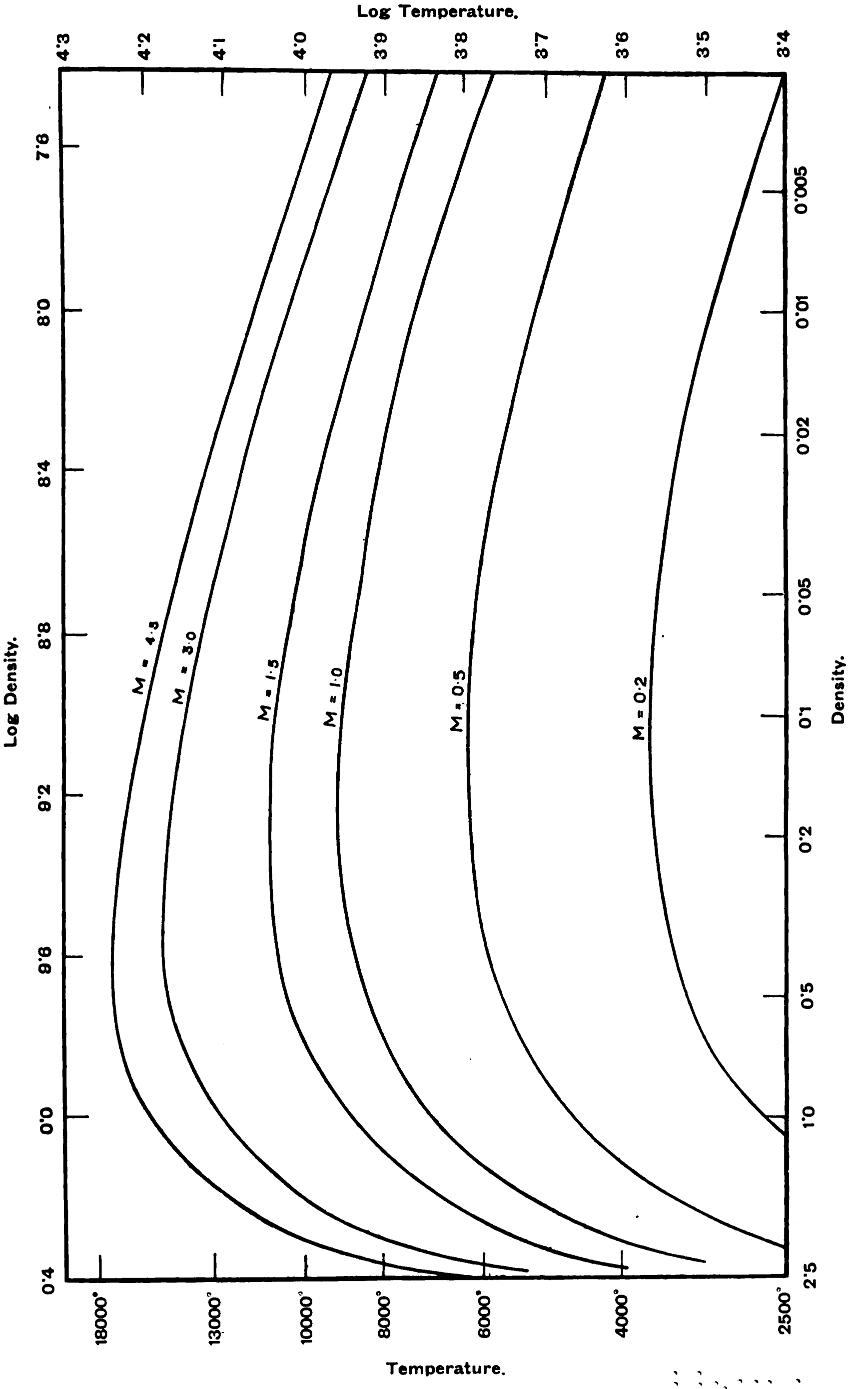

Relation between Effective Temperature and Mass and Mean Density (illustrating Table IV.).

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I had not anticipated so rapid an increase of maximum temperature with mass; but I believe that the observational evidence on the whole favours it.

It is easy to calculate from the curves the absolute luminosity of a star of mass M and effective temperature T. Take the density ρ from the curve; then the volume of the star is to the volume of the sun as $\frac{M}{\rho} : \frac{1}{1{\cdot}38}$; their surfaces are in the ratio $\left(\frac{1{\cdot}38M}{\rho}\right)^{\frac{2}{3}}$ and their total radiations in the ratio $\left(\frac{1{\cdot}38M}{\rho}\right)^{\frac{2}{3}}\left(\frac{T}{6000}\right)^4$. Converting this into magnitudes and applying the correction from Table III., we obtain the difference between the absolute visual magnitude of the star and that of the sun $(+5^{m}{\cdot}1)$.

It may be noted that whereas the luminosities of the giants of given type vary approximately as M^2, the luminosities of the dwarfs cannot vary more widely than $M^{\frac{2}{3}}$, and even this variation is partly compensated by the corresponding changes of density. In the same spectral class the range of luminosity of the dwarfs ought therefore to be much less than that of the giants; but unless we use very fine divisions of spectral classification this uniformity of the dwarfs is likely to be masked by the great change in luminosity from type to type.

19. *Discussion of the Temperature-Density Curves.*—We proceed to deduce tentative conclusions from the curves. Where possible these are compared with observations; in other cases we must judge by the general reasonableness of the results.

(*a*) We calculate by this theory the difference of absolute magnitude of the dwarfs and giants of type M (temperature 3000°). I find the visual magnitudes to be—

Dwarfs + 10·6, 10·6, 10·2, 9·9, 9·5
for masses 0·2, 0·5, 1·0, 1·5, 3·0.
Giants + 4·3, + 2·4, + 1·4, 0·0, − 0·7
for masses 0·5, 1·0, 1·5, 3·0, 4·5.

Adams,* who has determined by the spectroscopic method absolute magnitudes of 10 dwarfs and 48 giants, gives an average magnitude + 10·3 for the dwarfs and + 1·4 for the giants. The greater range of the giants is also apparent in his results, viz.—

Dwarfs + 10·7 to + 9·8; giants + 3·4 to − 1·0.

41 out of his 48 giants were, however, between $+2^{m}{\cdot}0$ and $0^{m}{\cdot}0$, which on our theory would correspond to masses from 1·2 to 3·0.

Russell gives the average observed absolute magnitudes of the dwarfs and giants as + 9·9 and + 0·6 respectively.

(*b*) A star of mass less than $\frac{1}{7}$ of the sun could not, on this theory, attain an effective temperature 3000°, and hence all stars (or, at least, those falling into the usual spectral series) must have masses greater than $\frac{1}{7}$. The smallest masses of which we have

* *Nat. Acad. Sci. Wash.*, 1916, p. 157.

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trustworthy knowledge are the fainter components of ζ Herculis and Procyon, from $\frac{1}{3}$ to $\frac{1}{4}$ the sun's mass; there is no evidence that any visible star has a mass so small as $\frac{1}{10}$ of the sun.*

Stars which reach a maximum temperature only a little greater than that of type M would tend to bridge the gap in luminosity between giants and dwarfs. The clear separation of the two groups seems to show that such stars must be rare (unless we can suppose that, owing to their rapid evolution, they have all reached a dense state). The giants and dwarfs become clearly separated at about type G, for which the minimum mass is 0·46; it is therefore probable that stars of mass less than half the sun's mass are comparatively uncommon. A theoretical reason for this was explained in § 16.

(*c*) The effective temperature for type B is rather uncertain, but perhaps we may take 13000° to correspond to B5. Hence by Table V. a star of type B – B5 must have mass 2·2 at least. It is well known that B stars are more massive than other types. Ludendorff † showed that the spectroscopic binaries of type B are on the average three times as massive as the other types. If the lower limit of mass for type B is 2·2 and for all stars 0·5, the proportion seems about right.

A star of mass 2·2 and type B5 will have absolute magnitude $+0^{m}\cdot 9$, according to our curves. Since this is the least massive star of the type possible, it might perhaps be thought that $+0^{m}\cdot 9$ would be the minimum absolute brightness of a B5 star; but this is not so. A B5 star of mass 3·0 will have a magnitude + 1·4 or – 0·7 according as it is in the later or earlier stage; for mass 4·5 the magnitudes are + 1·4 or – 1·6. There is a true minimum about $+1^{m}\cdot 4$, and evidently there must be a large number of stars with magnitudes near to the minimum, since stars within quite a large range of mass have magnitudes differing from it very little. We may venture to predict that the frequency of luminosity for B5 will not be an error-curve, but will be more like half an error-curve, the frequency falling from the maximum to zero almost abruptly. Kapteyn ‡ has traced the frequency up to a maximum at absolute magnitude + 0·9, but he found it impossible to determine what happened on the other side of the maximum.

(*d*) For a binary star a "hypothetical parallax" can be derived on the assumption that the mass of the brighter component is equal to the sun's mass. The luminosity deduced with this parallax may be called the "hypothetical luminosity." If H is the hypothetical luminosity, L the true luminosity, it is easy to show that $L = HM^{\frac{2}{3}}$.

For a giant star $L \propto M(1-\beta)$, so that $H \propto M^{\frac{1}{3}}(1-\beta)$. We

* Russell, *Nature*, vol. xciii. p. 256. Russell has just announced (*A.J.*, No. 711) for the companion of Krueger 60 a mass "between $\frac{1}{8}$ and $\frac{1}{6}$ of the sun's mass."

† *Ast. Nach.*, No. 4520. According to J. Halm (*Monthly Notices*, vol. lxxi. p. 636) the B stars are seven times as massive as the others, which would not fit our theory so well.

‡ *Ap. J.*, vol. xl. p. 82.

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have seen that for high molecular weight the variations of $1 - \beta$ are practically negligible, and in that case $H \propto M^{\frac{1}{3}}$. A range of mass from 0·5 to 4·0 would give a range of H from 1 to 2, or $0^{m}\cdot75$, in any one spectral class. The diagram given by Russell* shows that H is by no means so restricted. Some allowance must be made for accidental errors of various kinds, but the evidence seems to be irreconcilable with so small a range. With molecular weight 2, however, $(1 - \beta)$ varies approximately as M, and H varies as $M^{\frac{4}{3}}$. The mass range 0·5 to 4·0 would then give a range of 3 magnitudes in H, which is about the amount indicated in Russell's diagram.

For the giants H increases with M, but for the dwarfs the relation is inverted and H diminishes as M increases. In the same spectral class $L \propto M^{\frac{2}{3}}\rho^{-\frac{2}{3}}$, and hence $H \propto \rho^{-\frac{2}{3}}$, and we see from the curves that for dwarf stars of given temperature the density increases with the mass. This enables us to make a rough calculation of the effective temperatures of the different types by assuming that the lowest H in each case (except for evident stragglers) corresponds to a certain maximum mass, say 4·5. If the mass is higher it will make very little difference except in type A, because the density is approaching its limiting value. From the diagram given by Russell I deduce in this way—

Type.	Faintest Hyp. Mag.	Effective Temperature.
A	3·2	10600°
F	4·7	7300
F5	5·0	6800
G	5·7	5800
G5	6·2	5300
K	6·9	4700
K5	7·5	4200

The table is only given in order to show that the progression of temperature is roughly of the amount expected.

(*e*) For double stars with accurate orbits and large parallaxes, whose masses and luminosities can be computed, it is possible to use the curves to determine the temperatures. In this way I find—

Star.	Adopted. Mass.	Adopted Abs. Magnitude. m	Calculated Temperature.	Observed Type.
Sirius	2·45	1·30	12800°	A0
Procyon	1·0	3·03	8300	F5
α Centauri	1·0	4·73	6400	G
	0·85	6·10	5100	K5

The parallaxes were taken from Kapteyn and Weersma's list, and the ratios of the masses of the two components from Boss's Catalogue.

* *Nature*, vol. xciii. p. 257.

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The agreement of the last two columns is scarcely so good as might have been hoped (all the temperatures seem to be too high for the types).

A small uncertainty of the mass makes a rather large difference in the resulting temperature; hence for double stars of smaller parallax it seems better to reverse the procedure, and, adopting a temperature which is believed to correspond to the observed type, calculate the mass, and hence from the orbital elements deduce the parallax. The following results were obtained:—

Star.	Type.	Adopted Temperature.	Calculated M.	Calculated π.	Observed π.
		°		″	
γ Virginis	F	8000	1·05	·093	·058
δ Equulei	F5	7000	2·47	·046	·067
42 Comæ	F5	7000	1·04	·058	·062
η Coronæ	G	5800	0·54	·072	·076
ζ Herculis	G	5800	0·43	·151	·142
ξ Ursæ Majoris	G	5800	0·62	·152	·179
70 Ophiuchi	K	4700	0·36	·264	·212
85 Pegasi	G	5800	1·50	·067	·088

The orbital data and magnitudes were taken from a paper by E. Öpik (*Ap. J.*, vol. xliv. pp. 298–9). Four of the observed parallaxes are by Slocum (*Ap. J.*, vol. xli. p. 237), and the others are from Kapteyn and Weersma's list. The calculated masses refer to the brighter component.

The agreement of the last two columns is satisfactory; but it must be remembered that the ordinary "hypothetical parallax" is usually very close to the truth. The masses of all these stars are too near unity to give a satisfactory test of the theory.

20. *Energy of a Star.*—It is well known that the hypothesis that the sun's energy is derived from contraction leads to a value of the sun's age which is much too small to be reconciled with estimates of the earth's age based on studies of radioactive minerals and other geological evidence. Perhaps it has not been realised that the contraction hypothesis involves a rapidity of evolution of the giant stars which is from some points of view even more startling. I give a numerical instance worked out on the present theory, but on any other theory the order of magnitude of the results could scarcely be altered.

Consider the time taken for a giant star of mass 1·5 to rise from effective temperature 3000° to 6000°, *i.e.* from type M to G. At 6500° the energy of contraction Ω was found (p. 23) to be $1{\cdot}18 \times 10^{48}$ ergs, and since $\Omega \propto T^2$ we find $\Omega_{6000} - \Omega_{3000} = 7{\cdot}54 \times 10^{47}$ ergs. Of this, the fraction $\frac{1+\beta}{4}$ is required for ætherial and translational molecular energy in the star, and therefore, taking the new value of β (0·826), we have at most $4{\cdot}1 \times 10^{47}$ ergs available for radiation. The actual amount is less than this,

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since some energy must be used in increasing the ionisation and internal atomic energy.

Taking the solar constant to be 1·93 gram-calories per sq. cm. per minute, we find that the sun radiates $1{\cdot}20 \,.\, 10^{41}$ ergs per year. The giant star in question is 5·4 magnitudes brighter (bolometrically), and therefore radiates 145 times as fast. Hence the time taken to rise from 3000° to 6000° is not greater than $4{\cdot}1 \,.\, 10^{47} \div (145 \times 1{\cdot}20 \,.\, 10^{41})$, or 24,000 years!

We can only avoid a short time-scale by supposing that the star has some unknown supply of energy. Probably the simplest hypothesis of this kind is that there may be a slow process of annihilation of matter (through positive and negative electrons occasionally annulling one another). This would set free an almost inexhaustible store of energy. But the defect of all such processes is that they supply energy at some fixed rate—unlike the energy of contraction, which is drawn on as required and automatically replaces the amount radiated. A star would soon expand or contract until the rate of radiation balanced the fixed rate of supply. But in that case it is difficult to understand why the rate should be so very different in different stars of similar mass.

The usual view that evolution takes place in the direction of continually increasing density has practically no foundation, unless we believe that the energy comes mainly from contraction.

On the contraction hypothesis the question whether in the giant stage evolution proceeds more rapidly in heavy stars or light stars depends (according to the present theory) on the molecular weight. If r is the radius of the star—

Energy of contraction . . $\propto \dfrac{M^2}{r}$.

Rate of radiation . . . $\propto M(1-\beta)$.

Effective temperature . . $\propto M^{\frac{1}{4}} r^{-\frac{1}{2}}$.

Hence, assuming that a constant fraction of the energy of contraction is available for radiation,*

Time for a given rise of effective temperature $\propto M^{\frac{1}{2}}/(1-\beta)$.

This increases with M when the molecular weight is high, but decreases with increasing M (up to a certain limit) for molecular weight 2.

There is thus a possibility that during the giant stage heavy stars may change type more rapidly than light ones; but, nevertheless, the light stars have, as it were, not so far to go, and may still reach the dwarf stage first.

Summary of Conclusions.

1. If the mass-coefficient of absorption is the same for all stars, the effective temperature varies as the sixth-root of the

* The fraction available for radiation is $(3-\beta)/4$, *less* the unknown internal atomic energy. Our ignorance of the latter renders the conclusion somewhat uncertain.

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density, and the total radiation is independent of the stage of evolution, so long as the star is in a perfect gaseous condition. Conversely, the approximate confirmation of these laws by observation seems to show that the absorption is nearly the same for all stars.

2. Owing to the compensatory effect of radiation-pressure the distribution of temperature in the interior depends only to a very small degree on the molecular weight of the material or the amount of ionisation. For a giant star of type G the central temperature is of the order 5,000,000°.

3. Assuming the highest possible degree of ionisation in the interior, the mass-coefficient of absorption is 5·4 C.G.S. units. If the coefficient depends on the temperature, this value refers to approximately the highest temperature in the interior. The absorption cannot anywhere exceed 30 C.G.S. units, except possibly for a very limited range of temperature.

4. The actual masses of stars correspond to the range in which radiation-pressure increases from trifling to predominant influence. Assuming that a gas-sphere in which gravitation is largely counterbalanced by radiation-pressure would be in a state approaching instability, and would readily break up under small rotational velocity or other causes, it is evident that the matter of the universe must aggregate into bodies of mass agreeing quantitatively with the observed masses of stars.

5. With low ionisation the total radiation of a giant star varies approximately as the mass, with high ionisation as the square of the mass. In either case the observed range of luminosity of the giants indicates a very close uniformity of mass of the majority of the stars.

6. Determinations of the temperatures of stars too dense to behave as a perfect gas involve more doubtful assumptions. Assuming a high degree of ionisation, a theoretical model has been found which appears to represent the chief results of observation satisfactorily. According to this, a body must have at least one-seventh the sun's mass in order to attain an effective temperature of 3000°; stars with mass less than 2·2 times the sun's mass will not reach the B stage. The maximum temperature is attained when the mean density is between 0·2 and 0·5. The distribution of the luminosities of the B stars may be expected to be strongly asymmetrical, the maximum frequency corresponding to minimum luminosity.

7. If contraction is the only source of energy, the giant stage of a star's existence can scarcely exceed 100,000 years.

Errata in the First Paper.

P. 19, formula (8*a*), *read* "amount flowing *inwards* is $\pi A - \frac{2}{3}\pi B$."

P. 21, line 3 from bottom, *for* $\frac{4}{3}$ *read* $\frac{16}{3}$.

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Note on the Southern Magnitude Distribution, with special reference to the Perth Astrographic Zone. By Professor R. A. Sampson, D.Sc., F.R.S. (Plates 11, 12.)

In a recent publication of the National Academy of Sciences,* Mr. Seares concludes that the galactic condensation of small stars arrived at by Chapman and Melotte from a study of the Franklin-Adams charts † must be considered altogether erroneous, and that the truth is to be found much nearer to the older estimates which the counts of Chapman and Melotte displaced. The divergence is extreme; at mag. 16·3 the respective measures are:—Kapteyn, 32; Chapman and Melotte, 4·2; Seares, 21·4. With these Seares compares also Herschel's estimates for mag. 14, viz. Herschel, 12·9, where Kapteyn gives 11 and Chapman and Melotte 3·9. Seares' publication is, so far, preliminary only, but it is based upon counts of some 40,000 stars, in eight galactic zones, down to mag. 17·5 on the Mount Wilson photographic scale, equated to 16·3 of the Groningen visual scale. The totals only are available, not the counts of the separate magnitudes. Chapman and Melotte's counts, if I gather correctly the method described, depend upon some 50,000 or 60,000 stars in specimen areas, and are grouped according to magnitudes. Until a contradiction of this nature is cleared up, we know nothing at all of the galactic condensation. Its importance is my excuse for offering a few remarks which do not contribute directly to its solution, but touch it collaterally through the allied question of using the Chapman and Melotte scale as representative of any chance zone upon the sky, say one of the astrographic zones. I suppose that only by comparing together all possible collateral evidence shall we at length reach a reliable magnitude scale. My attention was turned to the matter by the valuable work lately done by Professor Turner in insisting that the magnitude scales of the various astrographic zones should be collated, and showing that, even from the material at hand, remarkable and consistent evidence was forthcoming of systematic "obscuration" of a large area of the sky, in the relative deficiency of faint stars compared to bright ones.‡ Among the zones which he employs is that of $-32°$, taken and measured at Perth, W.A.§ Mr. Cooke's magnitude scale is professedly provisional only,‖ but the following remarks seem to me to show that it deserves careful attention. His method was to expose upon a 7-mag. or 8-mag. star for various periods, viz. 1^s, $1^s{\cdot}4$, 2^s, . . . 64^s, each period rising with the ratio $\sqrt{2}$ to the last, first at the centre of the plate, then at the middle of one side, and then at one of the corners. These images were then brought side by side, named A, B, . . . N, and

* F. H. Seares, *Proc. Nat. Acad. Sci. U.S.A.*, vol. iii. (1917), p. 217.
† *R.A.S. Mem.*, vol. lx. (1914), p. 145.
‡ *M.N.*, vol. lxxv. (1915), p. 480.
§ *Ibid.*, p. 144.
‖ *Perth Astrographic Catalogue* (1911), vol. i. Introduction.

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turned into an eye-piece scale, with which the appearance of any star in any part of the astrographic plate might be compared with assurance of uniformity. The estimates of magnitude are published accordingly under the letters A, . . . M,—N being invisible; only a provisional attempt is made to translate these into numbers, but by identification with stars in the C.P.D., Mr. Cooke gives the value of 1 step as 0·3 mag., and the magnitude of the star D as 9·3.

We may note that the total number of stars in this zone is upwards of 60,000; therefore, as a piece of classification, it is upon the same scale as the counts of Seares, and Chapman and Melotte, and more satisfactory in some respects, because complete and continuous within its range; but being confined to a single zone of declination, it is not intentionally representative of the whole sky. But suppose for the moment that it is representative of the whole sky, then the values of the symbols A, B, . . . may be derived from comparison with Chapman and Melotte's numbers of the average numbers of stars of each magnitude per square degree. Professor Turner has made this calculation, and shows that it would follow that the Perth star D = 10·08 mag., while 1 step = 0·52 mag.* I wish to point to some considerations that make it almost certain that Mr. Cooke's provisional estimates are much nearer to the truth. First, with regard to the value of the step. Stars of unknown intensity, A, B, . . ., exposed for uniform time T, are equated to images of a standard star Ω, exposed for various times t_A, t_B, . . . If we take Schwarzschild's actinic law as applicable, we have

$$A^q T = \Omega^q t_A,$$

in which experiment gives q pretty consistently the value 1·25. Hence if m_A, m_B, . . . are the associated magnitudes,

$$m_A - m_B = -\frac{2 \cdot 5}{q} \log_{10} (t_A/t_B),$$

so that, if $t_A/t_B = 1/\sqrt{2}$,

$$m_A - m_B = -\frac{2 \cdot 5}{q} \times 0 \cdot 15,$$

$$= -\cdot 30, \text{ for } q = 1 \cdot 25.$$

In order to make $m_A - m_B$ different from this, say equal to $-0 \cdot 5$, we should require $q = 0 \cdot 75$, which is quite at variance with experience.

Next, with regard to the zero point of the scale. Mr. Cooke's comparison with the C.P.D., though derived from "a few plates, taken at random," is very fairly consistent, giving the following equations:—

A = 8·4 mag.	E = 9·7 mag.
B = 8·7 ,,	F = 10·0 ,,
C = 8·8 ,,	G = 10·2 ,,
D = 9·3 ,,	H = 10·3 ,,

* *Loc. cit.*, p. 144.

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One may conclude that to take D = 9·3 mag., with interval 0·3 mag., makes Cooke's scale identical with that of the C.P.D. Now the C.P.D. was adjusted generally to Gould's Zone Catalogue. Hence, if Cooke's D should really be considered as 10·1 mag., we should conclude that the limit of the Cordoba observers, which they give as 9½ mag., with a few of 10 mag., should really be 10½ mag. or more. But as they used only a 5-inch transit circle, I find it very hard to suppose that they habitually observed 10½-mag. stars, with occasional 11 mag.

Nor again can one suppose that the second Perth exposure, which was 2 min. only, and was intended to secure everything of 11 mag., but not more, with a second image, really reached beyond 13 mag., as the comparison given by Professor Turner (*M.N.*, vol. lxxv. p. 144) would require.

I conclude that the Chapman and Melotte numbers do not in the least represent the actual distribution in the Perth zone −32°. But I would interpolate here that this conclusion does not detract in any way from the use subsequently made by Professor Turner of the Perth counts.

But it is permissible to go a little further and ask how many steps must the Perth counts be extrapolated before they give the totals per square degree which Chapman and Melotte find as an average for the whole sky down to 17 mag. It is easily seen that this is reached some 7 or 8 steps beyond Cooke's K, or, according to Cooke's scale, between 13 mag. and 14 mag. No doubt there is little certainty about such an extrapolation, but it seems to point to an irreconcilable divergence between the two sequences. But the zone −32° is not especially rich in stars. We may consider the following numbers, representing the logarithms of the average number of stars per square degree recorded for various zones; they are quoted from Professor Turner's papers referred to, adding only Perth −38°, counted at Edinburgh, and Greenwich.

Zone.		log N.
Greenwich,	+87° − 63°	1·541
Vatican,	+64°, 62°, 60°	1·966
Oxford,	+28°	2·040
Bordeaux,	+17°	1·853
Algiers,	+ 1°	1·689
Perth,	−32°	1·981
,,	−38°	2·013
Cape,	−41°	2·094

Therefore it seems to me that, generally, the material of the *Astrographic Catalogue* makes it difficult to accept the Chapman and Melotte averages as representing the actual mean distribution over the whole sky.

In making the foregoing remarks upon the substantial justice of Cooke's scale, as derived from the C.P.D., I would not overlook

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the paper recently published by Dr. Halm,* of which one section deals with the corrections to the C.P.D. scale. These he finds to be in the same sense and of even greater amount than would bring Cooke's magnitudes into agreement with Chapman and Melotte's, running, namely, as follows:—

C.P.D.	Standard.	Standard *minus* C.P.D.
m	m	m
7·6	7·40	−0·15
8·0	8·50	+ ·49
8·4	9·29	·86
8·7	9·78	1·04
9·0	10·22	1·22
9·3	10·66	1·37
9·5	11·06	1·53
9·7	11·46	1·73
9·9	11·93	+2·00

It is unnecessary to dwell upon the complete rearrangement of scale that such corrections would imply; they are brought out sufficiently above, in the consequences of comparing the Perth zone with Chapman and Melotte. And I would hesitate to criticise a work which is evidently the outcome of a great deal of thought and labour, and is at present only published in a more condensed form than belongs to such fundamental work. But I must draw attention to one feature of it, to which Dr. Halm does not expressly devote any remarks, though in other systems of fundamental photographic magnitudes it has occupied the cardinal place—I mean, care that the quantities measured represent the whole actinic effect of each star, and do not, for example, progressively fall off as the images grow fainter. This would seem to be particularly required where the scale reaches out beyond the point where any other standard investigations have been made.

In the astrographic work it is usual to "collect" the diffused edges of an image and add them to the centre in forming an estimate of diameter. Whether this leads to uniformity may be doubted, especially where the images are not round. Dr. Halm follows another plan: he measures the diameter of the completely blackened nucleus. This, deliberately, is not the whole actinic effect. Its advantages are a matter for experience. The standards are obtained by exposures upon a chosen star (σ *Octantis*, taken at photographic magnitude $m_0 = 5{\cdot}73$) exposed for a series of periods from about 1^s to 1500^s, giving a curve relating diameter and time (t_n); then the equation is employed (see p. 614 above),

$$m - m_0 = \frac{2{\cdot}5}{q} \log (t_m/t_n),$$

* "A System of Photographic Magnitudes for Southern Stars," *M.N.*, vol. lxxiv. (1914), p. 600.

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where t_m is the exposure of the unknown star m, and the unknown magnitude m is derived from it, by equating its measured diameter with one of these shown by the standard star m_0. Thus, for example, with 1000ˢ exposure of the plate (m) we should have the following data *:—

d.	$\log t_n$.	$\log (t_m/t_n)$.	$m - m_0$.	m.	$\sqrt{d}$.
0·92	2·87	0·13	0·27	6	0·96
·80	2·38	0·62	1·27	7	·89
·67	1·89	1·11	2·27	8	·82
·57	1·40	1·60	3·27	9	·76
·48	·92	2·08	4·27	10	·69
·39	·42	2·58	5·27	11	·62
·32	− ·06	3·06	6·27	12	·56

I have added the last column to show that the result is, apparently exactly, a linear relation between m and $\sqrt{d}$. My criticism of this method enters where the ratio of the exposure times t_m/t_n is large. We have seen above that $m = 12$ (p. 616) is required in order to reach 10 mag. on the C.P.D. scale. For this end the standard star *σ Octantis* is exposed for about 1ˢ, and the star investigated for about 1000ˢ. One would suspect that in this time, owing to atmospheric unsteadiness, the image of the fainter star would wander a sensible amount, and its completely blackened nucleus appear systematically smaller in proportion to its whole actinic effect than that of the standard star which was only exposed for a thousandth part of the time.

As against this, a control of the results is given by comparison with Schwarzschild's *Aktinometrie* † down to mag. $m = 9·4$, or with a ratio of exposure times 1 : 30. In the same way Cooke's Perth scale, considered above, which is similar in principle, shows no stretching of magnitudes over a similar ratio of exposure times. It is in the extension from 30 times to 1000 times required to pass over the next three magnitudes that I would suggest a doubt. For example, if we accept Dr. Halm's corrections to the C.P.D., it would follow that the Cordoba observers with a 5-inch instrument habitually took 11-mag. stars, with an occasional 12-mag., and I find this difficult to believe.

I come now to another point. The plates for zones − 38°, − 39°, − 40°, taken at Perth, were measured at Edinburgh. I have recently had counts made of these, with special reference to their bearing on Professor Turner's "obscured patch," or region in which small stars are relatively deficient. The demonstration of this region is essentially cumulative, and gains very much strength from the variety of material and treatment through which it

* Derived from Table II., Halm, *loc. cit.*, p. 604.
† Halm, *loc. cit.*, p. 606.

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appears. Our magnitudes were not taken with Cooke's scale, but by estimating the collected diameters in the method indicated above, in all by six different groups of measures, viz.: 1, S.A.F. and I.C.F; 2, E.W. and L.C.; 3, R.W.W.; 4, E.K. and J.M.; 5, G.C.; 6, J.S.

The counts were made separately over the centre of the plates, that is, between réseau divisions 5 and 23, and the margins beyond this, having areas respectively $\left(\frac{18}{26}\right)^2 = \frac{81}{169}$ and $\frac{88}{169}$ of the whole, in order to examine any systematic effect there might be due to inferior definition at the edges.

When the results were examined, it appeared that only for zone $-38°$ were they so regular in estimates of diameters as to allow one to treat them with confidence hour by hour as parts of one whole. This zone was measured throughout, with trifling exceptions, by 1, S.A.F. and I.C.F. The other zones were much broken up among the six measurers, and the estimates of diameter are occasionally strikingly discordant. In considering the results, I am so much impressed with the superiority of Cooke's method, and with the importance of good magnitude estimates of the plates, as well as good measures of position, that I have decided to re-estimate the whole three zones with his focal scale, provided some test studies now being made seem to justify it. Meanwhile, what is given below depends chiefly upon zone $-38°$. There remained, however, another test besides the diameters, namely, the relative frequency of stars with a third image. This would seem to be much less exposed to personality than the estimates of diameter, depending as it does only on detection of the third image. The results were easily collected, and I give, in fig. 1, Plate 11, the charts of the logarithms of the ratio of the whole number of stars to stars showing a third image, for each of the three zones for each hour of R.A. The numbers 1 to 6, attached, indicate the measurers.

This chart speaks for itself; it may be said, generally, that the most promounced features or breaks are those which are confirmed by all three, and that they do not seem to be associated with change of measurer, or present any obvious relation to an annual term. Take, for instance, the remarkable oscillations between 13^h and 24^h. The best measured of the zones ($-38°$) is also the smoothest. Professor Turner's patch is shown by it quite clearly, running from 23^h to 6^h. To recognise its occurrence here in confirmation of proofs given elsewhere is, of course, a very different thing from claiming its reality upon the evidence of this chart alone.

The following chart (fig. 2, Plate 12) relates to zone $-38°$ alone. The logarithms of the ratios are given separately for whole plate and central portion—(1) for third-image stars compared to the whole; (2) diameter 24 compared to diameter 12; and (3), diameter 16 compared to the whole. The magnitudes associated with these are approximately as below, upon the Perth scale, and upon what

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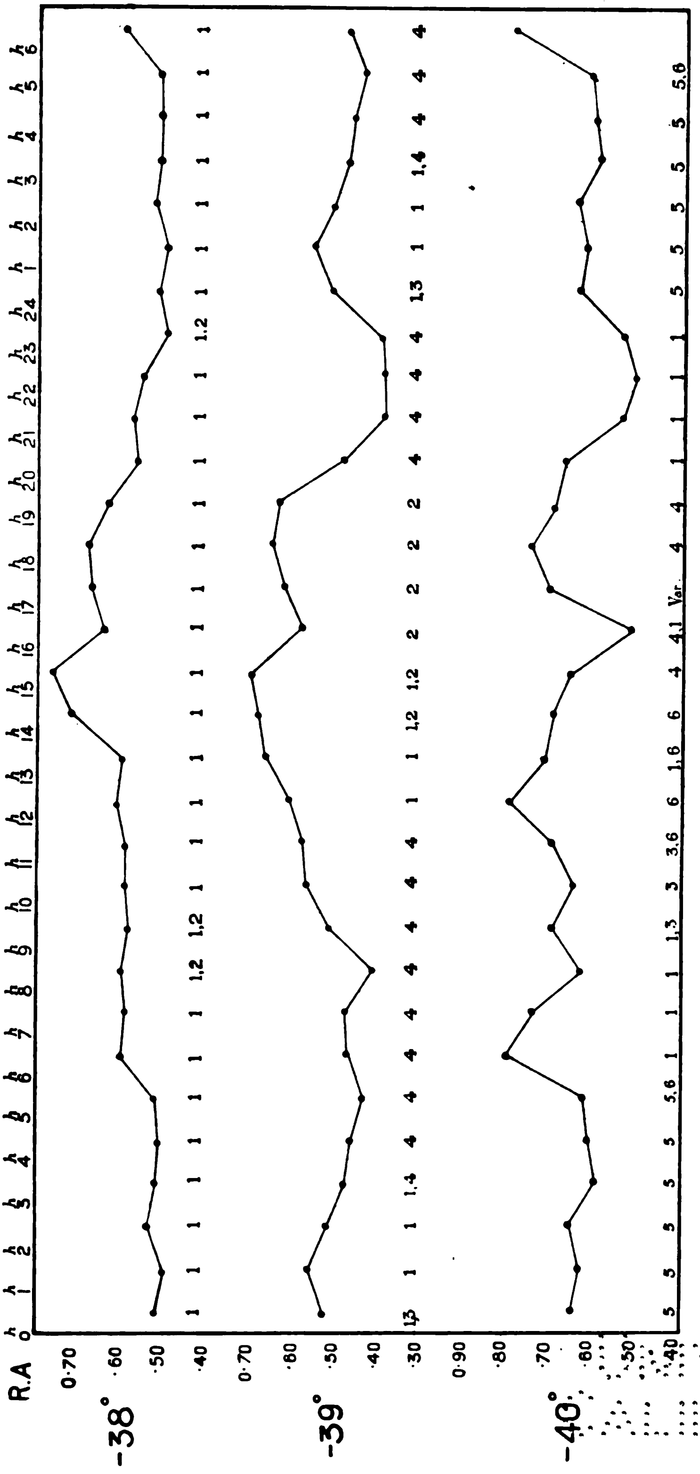

Fig. 1.—Log Ratios of Numbers of Stars with third Image to whole

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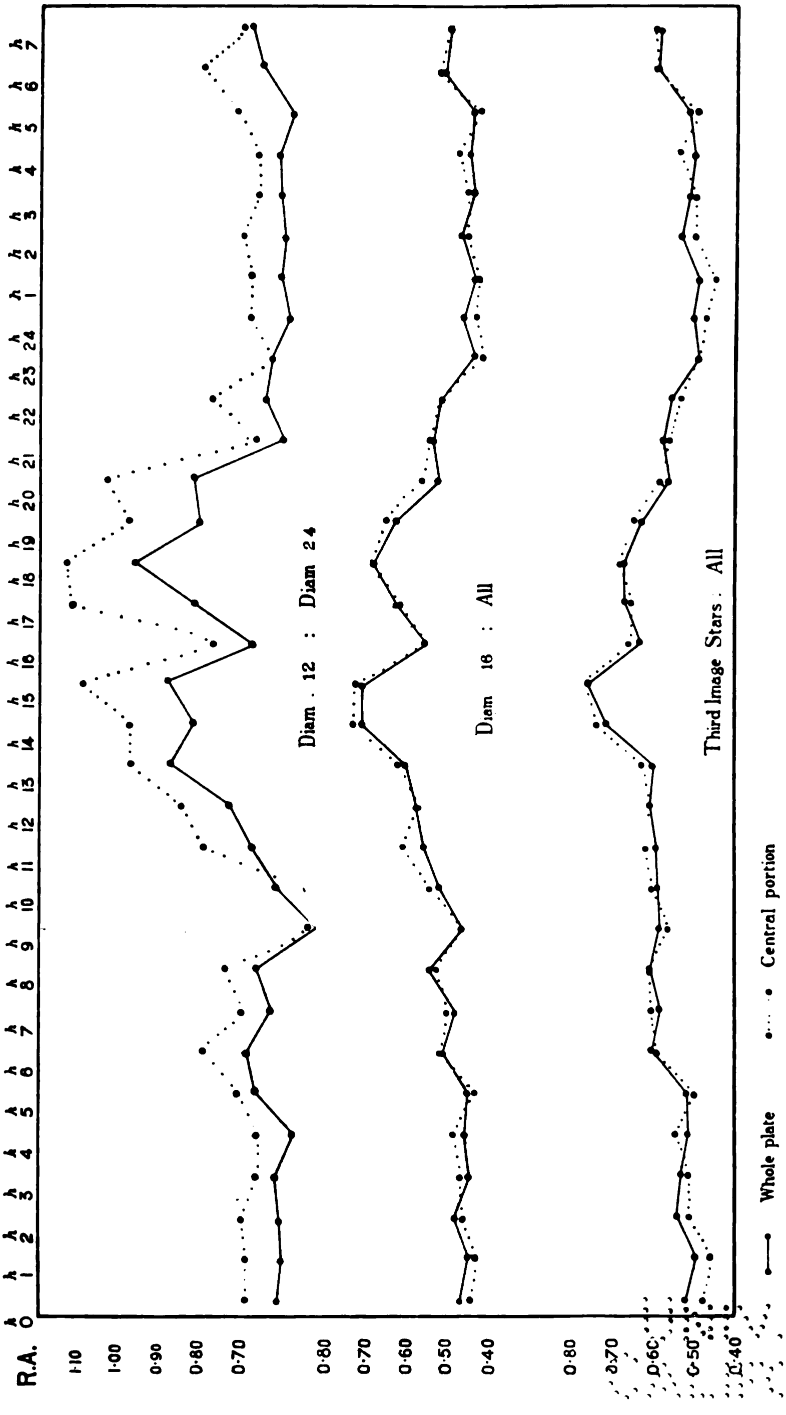

Fig. 2.—Log Ratios of Numbers of Small and Large Stars.

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for convenience I have referred to as the Chapman and Melotte scale respectively :—

	Perth.	C. and M.
	m	m
Third image	10·0	11·4
Diam. 24	9·6	10·7
,, 16	10·1	11·5
,, 12	10·7	12·0
Whole = Diam. 6	11·5	13·1

The following are the numbers represented upon the chart, with notes as to the region of the sky with respect to the galaxy, and the dates when the plates in question were taken :—

R.A.	Whole Plate. log (all /3rd image).	Whole Plate. log (12 diam. /24 diam.).	Whole Plate. log (all /16 diam.).	Central Portion. log (all /3rd image).	Central Portion. log (12 diam. /24 diam.).	Central Portion. log (all /16 diam.).	Region.	Dates of Exposures.
0^h	·52	·61	·47	·48	·68	·44	A	1911 Dec. 9, 12, 18.
1	·50	·60	·44	·46	·68	·43	,,	1911 Dec. 9, 12.
2	·54	·61	·48	·51	·70	·46	,,	1911 Dec. 12, 18.
3	·52	·62	·45	·51	·66	·46	,,	1911 Dec. 18, 1912 Jan. 14.
4	·51	·58	·46	·54	·66	·48	,,	1912 Jan. 14, 21.
5	·52	·66	·44	·50	·72	·43	,,	1912 Jan. 21, Feb. 22.
6	·60	·68	·51	·59	·79	·52	,,	1912 Feb. 22, Mar. 20, Apr. 10.
7	·59	·63	·49	·60	·70	·50	B	1912 Apr. 10, 14.
8	·60	·67	·54	·61	·74	·53	,,	1912 Apr. 10, 12, May 10.
9	·58	·53	·47	·56	·54	·46	,,	1912 Apr. 10, 14.
10	·59	·62	·53	·60	·55	·55	,,	1912 Apr. 10, May 8, 10, 1913 June 8.
11	·59	·68	·56	·62	·79	·61	C	1912 May 5, 8, 10.
12	·61	·73	·58	·61	·85	·58	,,	1912 May 5, 10.
13	·60	·88	·61	·63	·97	·63	,,	1912 May 5, July 21, Aug. 20.
14	·72	·82	·72	·74	·97	·74	,,	1912 July 26, 1913 July 22.
15	·76	·89	·72	·77	1·09	·73	D	1912 July 26, 1913 Aug. 7.
16	·64	·67	·56	·67	·77	·57	E	1911 Aug. 20, Sept. 14, 1912 July 18, 26.
17	·67	·81	·63	·66	1·12	·62	F	1911 Aug. 20, 1912 July 24, Aug. 12.
18	·68	·97	·69	·68	1·13	·68	,,	1912 Aug. 12, 1913 Aug. 20, Sept. 1.
19	·64	·80	·63	·65	·97	·66	G	1912 Oct. 12, 1913 Sept. 1.
20	·57	·82	·53	·59	1·03	·56	,,	1912 Oct. 12, Sept. 19.
21	·58	·60	·54	·57	·67	·55	A	1911 Oct. 25.
22	·56	·65	·52	·54	·77	·52	,,	1911 Oct. 25, Nov. 15.
23	0·50	·59	·43	·50	·63	·42	,,	1911 Nov. 12, 15, Dec. 9.

A, non-galactic, poor in stars ; B, a faint part of galaxy ; C, non-galactic, but fairly rich ; D, bright galactic region ; E, a dark region in galaxy ; F, bright galactic region ; G, fairly bright, bordering galaxy.

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The first comment upon these charts is to the effect that restriction of counts to the central region, in which the star images are perfect and fainter stars are not lost by diffusion of their light, makes, generally speaking, no difference. The next most striking feature is the galactic condensation; it is perhaps emphasised by the remarkable way the dark rift in the middle of it, about 16^h, shows itself by an indentation of the curve. For the rest, the only definite feature is Professor Turner's obscured patch, extending from 23^h to 6^h in this zone. It is unmistakably present in just the position which he derived from analysis of the zones on each side, Perth $-32°$ and Cape $-41°$.

In conclusion, I give the actual totals of stars N_m, counted down to and including each diameter upon zone $-38°$.

Diameter.	N_m.	m.	Residual.
50	536	8·10	−0·07
40	1462	8·94	− ·13
30	2971	9·91	+ ·14
28	3359	10·12	+ ·23
26	3950	10·34	+ ·28
24	5298	10·57	+ ·21
22	6802	10·80	+ ·19
20	9409	11·05	+ ·11
18	13389	11·32	− ·03
16	18586	11·58	− ·14
14	22305	11·88	− ·06
12	26753	12·20	+ ·05
10	39331	12·53	− ·09
8	51683	12·92	− ·01
6	68527	13·35	+ ·01

To test the smoothness of these the corresponding magnitudes are calculated from Professor Turner's tables of Chapman and Melotte's scale, and reduced to the formula

$$m = 16^{m}\cdot 12 - 1^{m}\cdot 14\sqrt{d}.$$

The corresponding magnitudes found are shown in the column m, and the residual excesses over the Chapman and Melotte numbers corresponding to the counted stars in the adjoining columns. A distinct undulation is shown in the curve, due perhaps to a form of "decimal equation" or similar personality in the measurers I, S.A.F. and I.C.F., and comparing somewhat unfavourably, *e.g.* with the numbers shown by Professor Turner,* in putting Cooke's

* *Loc. cit.*, p. 145.

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Perth estimates for zone $-32°$ from comparison images alongside the same standard, when the mean discrepancy of the first nine values is ± 0.04 mag.

I am indebted to Mr. E. A. Baker and Miss I. C. Falconer for the actual counts and statistical collections of zones $-38°$, $-39°$, $-40°$, together with a great deal of associated work that does not appear in this paper.

Sunspots in a High Southern Latitude. By Mrs. Walter Maunder.

(*Communicated by the Astronomer Royal.*)

Since 1910 the Greenwich heliographic register has been very largely supplemented from the Cape Observatory. For the earlier years, as the slight activity of the sun made it possible to measure the solar photographs closely to date, the Greenwich photographs were used whenever available, and the gaps were filled from those sent later from the Cape. Since the war, however, the reduction of the staff, combined with the increase in the solar activity, caused the measuring to fall somewhat into arrear, and the Cape photographs arrived before the Greenwich photographs were selected, and Mr. Newton, who was in charge of the department, had frequently four photographs to choose from for one day instead of two.

One consequence of this was that, in examining the Cape negatives, Mr. Newton detected a large number of very small faint markings which for distinctness I will call "flecks,"—markings that are certainly more than "pores," if not always quite of the order of "spots," though no rigid division can be drawn between them and the more obvious formations. None but an observer as keen and careful as Mr. Newton would have detected them in the first place, but also there seem to be two reasons for the fact that these "flecks" were to be discovered upon the Cape negatives rather than upon those taken at Greenwich. First, when possible, a considerable interval of time is allowed to elapse between the taking of the two daily solar photographs at Greenwich—it may be twenty minutes or an hour or more; at the Cape the photographs are taken immediately following each other, with usually but a minute's interval. No "fleck" was admitted as a "real" marking unless it was found on two photographs, but as it is evident that they are very short-lived as a class, it is clear that though they may survive for a minute or two, they may have vanished in a quarter of an hour. The second reason is more obscure, for there is no doubt that this order of marking does appear more clearly on the Cape photographs than on the Greenwich, even where the definition on the latter is as good or better than on the former. It may be a question of exposure and development as between the two observatories; it may be a question of

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photographic level and focus which makes these faint markings show a stronger contrast to their surroundings on the Cape negatives.

So long as these "flecks" were found only within the usual sunspot latitudes Mr. Newton paid them no particular attention; they were simply examples of very faint and short-lived spots. But in the spring of 1916, when examining the photographs of 1915 December, he found a similar faint marking on the Cape negative of 1915 December 26 which was evidently quite beyond the received limits of sunspot latitudes; it was evidently a genuine solar marking, as it was clearly visible on two solar photographs, but as these were taken at an interval of about a minute, it may have lasted for but a short time; it was not to be detected either on December 25 or 27. When measured and reduced it gave for its heliographic longitude and latitude 63°·9, − 59°·5, a latitude which constituted a record beyond all doubt: the highest registered hitherto was 50°·4 by Professor Peters in 1846. The number of Mr. Newton's spot in the Greenwich register is 7578, and another similar high latitude spot is No. 7486 of 1915 October 21, in hel. long. 143°·6 + 59°·7.

But both of these were evanescent, and until such a "fleck" was detected which persisted for a day or longer, it remained doubtful whether these high latitude markings belonged really to the order of spots or merely to that of the "pores" that characterise the sun's surface everywhere. It is the unfairness of the fortune of war that it should fall to my lot and not to Mr. Newton's, to whom I might say they rightly belong, to bring before the Society an instance which gives us the right to assign to them the rank of sunspots.

A few days ago, when examining the reductions of the photographs for 1916 March, I found that one "fleck" was in south latitude 63°·0, longitude 125°·6, on March 10, and on March 14 I found again a "fleck" in south latitude 63°·3 and longitude 119°·6. These were so close in position that it is evident that if they were not the same actual spot they belonged to the same disturbed area, and I looked up at once not only these two photographs, but also the intermediate days and the duplicate photographs for the whole period. The sun's south pole was then most fully presented to us, which probably accounts for their more easy detection. I have remeasured this region of the sun on all the photographs available for the period 1916 March 8 to March 15 inclusive, rejecting all markings that did not seem to be registered on at least two photographs. The following table gives the positions in three different latitudes of areas that are distinctly disturbed, *i.e.* in latitudes − 63°, − 65°, and − 72°. It will be noted that there is considerable uncertainty in the positions measured on photographs which are quite close together in time, and this may be due partly to the difficulty of measuring such faint indefinite markings on photographs which differ greatly in definition and contrast; partly to the fact that these "flecks" are so close to the

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limb that they are seen strongly foreshortened and may easily be covered in part by faculæ at different times. In these regions, too, a very small difference in the position of a spot as measured, necessarily means a considerable difference in the latitude and longitude as computed. But it is evident that markings, not easily to be distinguished from spots, can persist in very high latitudes; but the evidence is not yet sufficient to find whether the individual spots lie in polar "rings" or "streams" or whether they have a steady drift, or again whether they are subject to abnormal proper motions.

Sun Flecks in High Southern Latitudes, 1916 March.

Date 1916.	No. and Place of Photograph.	G. Civil T.	Heliographic Co-ordinates of Fleck. Long.	Lat.	No. and Place of Photograph.	G. Civil T.	Heliographic Co-ordinates of Fleck. Long.	Lat.
		h m s	°	°		h m s	°	°
March 8	18616 G.	11 59 20	143·2	−72·2	18617 G.	12 2 14	143·7	−72·2
9	3832 C.	7 50 42	136·4	−72·2	3833 C.	7 51 32	136·7	−72·4
10	3834 C.	8 10 28	120·2	−73·3	3835 C.	8 11 28	120·8	−73·3
11	3836 C.	8 19 47			3837 C.	8 20 47		
12	18620 G.	12 18 5	134·4	−71·1	18621 G.	12 28 10	134·4	−71·3
13	3840 C.	7 20 42	135·3	−71·1	3841 C.	7 21 43	135·6	−69·9
14	3842 C.	7 56 7	118·3	−74·6	3843 C.	7 57 7	119·3	−74·6
March 9	3832 C.	7 50 42	137·3	−62·5	3833 C.	7 51 32	136·9	−62·6
10	3834 C.	8 10 28	125·6	−63·1	3835 C.	8 11 28	125·6	−63·0
11	3836 C.	8 19 47	126·8	−62·3	3837 C.	8 20 47	126·5	−62·5
12	3838 C.	10 55 1	130·5	−63·8	3839 C.	10 57 53	131·2	−63·6
12	18620 G.	12 18 5	130·5	−63·4	18621 G.	12 28 10	130·3	−63·0
13	3840 C.	7 20 42	123·0	−63·5	3841 C.	7 21 43	122·3	−63·1
14	3842 C.	7 56 7	119·9	−63·5	3843 C.	7 57 7	120·0	−63·3
March 13	3840 C.	7 20 42	138·6	−65·4	3841 C.	7 21 43	137·7	−65·1
14	3842 C.	7 56 7	133·3	−65·9	3843 C.	7 57 7	133·0	−66·6
March 13	3840 C.	7 20 42	124·3	−64·8	3841 C.	7 21 43	124·6	−64·7
14	3842 C.	7 56 7	108·3	−64·4	3843 C.	7 57 7	107·9	−64·0

Note.—The title "veiled spots," given by M. Trouvelet to certain faint solar markings in high latitudes, may have been applied by him to markings of the same kind as these "flecks," but if so was not, I think, an appropriate one. The "flecks" as shown on the photographs do not, to my eyes, suggest "veiling"; some are faint, others dark, but none appear as if seen through an obscuring medium.

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Observations of Schaumasse's Comet (b 1917) made at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The observations were made with the guiding telescope (aperture 10 inches) of the Astrographic equatorial. The driving clock was running, and the comet and comparison star were bisected by means of the slides, which have motions in the directions of Right Ascension and Declination.

G.M.T.		Comet – *. R.A.	Log Factor of Parallax.	Comet – *. Decl.	Log Factor of Parallax.
1917	h m s	s		′ ″	
May 28	10 24 36	(+ 3·45)	9·7140	+3 0·4	0·7681
	10 32 10	+ 7·68	9·7093	+2 5·0	0·7753
	10 36 7	+10·43	9·7067	+1 35·3	0·7800
	10 40 29	+16·19	9·7036	+0 48·1	0·7852
	10 48 3	+22·19	9·6977	–0 8·0	0·7941
	10 55 19	+29·06	9·6915	–1 3·4	0·8022

The star is B.D. +47° 1481.

R.A. 1917·0.	Red. to App. Place.	N. Dec. 1917·0.	Red. to App. Place.
h m s	s	° ′ ″	″
7 39 6·64	+1·84	47 26 25·66	+4·07

Authority, *Bonn A.G. Catalogue*, 1875.
The observations were made by Mr. Hollis.

Royal Observatory, Greenwich:
1917 *June* 7.

Micrometrical Measures of Double Stars.

By Rev. T. E. R. Phillips, M.A.

The following measures were made at Headley, Surrey, during 1916, with the 8-inch Coleman telescope, lent by the Society. Special attention was given to (1) ξ Böotis, the *comes* of which has shown very rapid motion during recent years, and (2) to 70 Ophiuchi; in the latter case by special request.

Star.	1910. R.A.	1910. Dec.	Mags.	Position-Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 60 (η Cassiop.)	0 44	+57 20	4·0, 7·6	252·2	6·78	3	1916·983
205 (γ² Androm.)	1 58	41 54	5·5, 6·5	111·9	0·52	3	·978
1110 (Castor)	7 29	32 6	2·7, 3·7	216·6	5·02	3	·975
1196 (ζ Cancri) AB	8 7	17 55	5·0, 5·5	301·6	0·95	3	·296
AC	...	..	...	113·2	4·85	3	·296

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Star.	1910. R.A.	1910. Dec.	Mags.	Position-Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
1563 (ω Leonis)	9 24	+ 9 26	6·2, 6·7	125·1	0·94	3	1916 ·265
1424 (γ Leonis)	10 15	20 18	2·0, 3·5	118·0	3·89	4	·240
1536 (ι Leonis)	11 19	+11 2	3·9, 7·1	41·8	1·87	4	·288
1670 (γ Virginis)	12 37	− 0 57	3·3, 3·3	324·6	5·76	4	·359
1687 (35 Comæ Ber.)	12 49	+21 44	5·2, 8·2	91·7	0·98	3	·344
1865 (ζ Böotis)	14 37	14 6	3·5, 3·9	138·4	0·82	3	·249
1877 (ε Böotis)	14 41	27 27	3·0, 6·3	332·1	2·97	3	·249
1888 (ξ Böotis)	14 47	19 29	4·7, 6·6	94·2	2·25	4	·233
	...	...	...	92·5	2·15	6	·587
	Mean of all measures :—			93·4	2·19	10	·445
1937 (η Coronæ)	15 19	30 37	5·5, 6·0	59·2	0·88	3	·252
1938 (μ² Böotis)	15 21	37 40	6·7, 7·3	50·7	1·28	3	·310
2032 (σ Coronæ)	16 11	34 4	5·0, 6·1	218·3	4·82	3	·820
2055 (λ Ophiuchi)	16 26	2 11	4·0, 6·1	70·3	1·05	3	·295
2130 (μ Draconis)	17 3	54 36	5·0, 5·1	130·4	2·16	3	·383
Krueger 46	17 18	58 37	8·8, 9·0	64·8	1·50	3	·383
Σ 2199	17 37	55 48	7·2, 7·8	82·1	1·58	3	·383
2207	17 37	67 10	8·0, 8·5	124·8	0·77	3	·387
2218	17 39	63 43	6·5, 7·7	338·6	2·00	3	·387
2272 (70 Ophiuchi)	18 1	2 32	4·5, 6·0	139·2	4·78	5	·384
				138·6	4·75	6	·559
				138·3	4·70	5	·632
				136·9	4·61	5	·780
	Mean of all measures :—			138·3	4·71	21	·585
2278 BC	18 1	56 26	6·8, 7·3	146·9	5·93	3	·407
2284	18 1	65 57	7·6, 9·2	193·2	3·51	3	·387
Hussey 926	18 9	64 13	9·1, 9·3	267·8	4·03	3	·430
Σ 2307	18 12	69 13	8·5, 8·5	204·4	4·39	3	·408
2368	18 36	52 16	7·2, 7·4	326·8	2·00	3	·422
2384	18 38	67 2	8·0, 8·5	306·8	0·73	3	·400
2410	18 45	59 14	8·2, 8·7	92·5	1·44	2	·433
Hussey 938	18 54	64 25	9·1, 9·5	217·3	1·19	2	·518
Σ 2438	18 56	58 6	7·0, 7·6	17·5	0·58	3	·518
2478	19 2	69 19	8·8, 8·8	295·2	1·01	3	·590
OΣ 369	19 8	71 56	7·0, 7·3	31·1	0·72	2	·561
Σ 2509	19 16	63 2	7·0, 8·1	335·6	1·14	3	·568
β Cygni	19 27	27 46	3·0, 5·3	54·5	34·52	3	·781
Σ 2550	19 29	73 10	8·2, 8·2	249·1	1·95	2	·588

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Notes.

Σ 1110 (Castor). This pair appears to be now closing.

Σ 1888 (ξ Boötis). The *comes* continues its rapid motion. The angle diminishes about a degree in six weeks, but the distance has shown little change during the last few years.

Σ 2272 (70 Ophiuchi). The four groups of measures show a small progressive diminution both in angle and distance. The former is doubtless real in part, but the distance should be slowly increasing. Unless there is a disturbing dark companion, as has been suggested (see vol. lvi., *R.A.S. Memoirs*, p. 519), it would seem that there is some evidence here of a systematic error perhaps depending on the changing slant of the line joining the two stars before and after "opposition." The possibility of such an error seems worth further investigation.

Σ 2478. Stars seem fainter than magnitudes assigned.

Errata in the Rev. T. E. Espin's Recent List of New Double Stars. By Eric Doolittle.

This list, comprising the numbers Espin 1480 to Espin 1600, has just been published in the *Monthly Notices*, vol. lxxvii. pp. 239 to 242. In reducing the positions to the standard epoch 1880·0, and comparing the measures with the manuscript extension of Burnham's General Catalogue, the following notes and corrections are suggested :—

All of the columns are headed with the epoch 1920·0; this should be 1900·0.

Espin 1496. The R.A. is 15 sec. too small.
1499*a*. The R.A. is 20 sec. too large.
1505. The Decl. is 6 min. too small.
1509. The R.A. is 12 sec. too small.
1514. The Decl. is 2 min. too great.
1515. Jonckheere 890 must be near, but the descriptions of the two pairs differ.
1519. The R.A. is 25 sec. too small.
1537. The Decl. is 1 min. too small.
1548. The Decl. is 1·3 min. too large.
1552. Identified by Espin with B.D. +44°, 2450, but if this is correct the R.A. given is 8 min. too great and the Decl. is 9 min. too great. The place given practically agrees with that of B.D. +44° 2470, and it is assumed that this is the pair observed.
1565*a*. This pair is Roe 40. Measured by Roe in 1910 (*A.N.*, 4467).

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Espin 1574. Called B.D. + 41°, 3922 by Espin, but this is a 6·5 magnitude star, 6s following and 45′ north of the place given. This place, however, practically agrees with that of B.D. + 41°, 3921, and it is here assumed that this was the star observed.

1575. The Decl. is 1·5 min. too small.

1576. The Decl. is 1·5 min. too small.

1582*a*. Measured by Fox in 1912 (*Annals, Dearborn Obs.*, vol. i.).

1586. The Decl. is 3 min. too small.

1592. The Decl. is 9 min. too small.

Page 236. B.D. + 44°, 4550. This measure was published in *M.N.*, lxxv. p. 203, where a nearer companion was also given.

Page 238. B.D. + 44°, 3552. Measures made on three nights in 1908 will be found in *M.N.*, lxix. p. 224.

The Flower Observatory:
1917 *May* 5.

Observations of SX Aurigæ. By C. Martin and H. C. Plummer.
(Plate 13.)

1. The variability of SX Aurigæ was discovered in 1907 by Miss Leavitt from Harvard photographs.* Other designations are 79·1907, 050442 on the Harvard system, B.D. + 41°·1101 and A.G. (Bonn) 4192. The range found by Miss Leavitt is 7·4–8·0.

2. The star has been observed by S. Enebo.† His results, based on very numerous observations, are singularly conflicting. Thus for the period he has mentioned 1·533, 1·53165, 1·53172, 1·5323, 1·53186 days. There is no doubt that the determination of the period presents a problem which has not yet found a satisfactory solution. From the usual point of view it may be said that the period is variable. But the material does not exist by which the character of the changes can be decided. It is equally possible that the changes should be attributed to more or less irregular fluctuations superposed upon a regular variation within a fairly definite period. That is a question which cannot be easily answered. The present photographic observations have been reduced with the period 1·53234 days. This serves to reconcile the observations of 1914–15 and 1915–16. It has no other foundation, though it resembles one of the periods mentioned by Enebo, and it leaves anomalies still outstanding.

3. The photographic range of magnitude, based on Schwarzschild's scale, is about 7·8–8·6. According to Miss Leavitt, the

* *H.C.O. Circular*, No. 130; *A.N.*, 4196.
† *A.N.*, 4238; *Beob. auf Dombaas*, iii., iv., vi.

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star is still brighter photographically. For the visual range Enebo has given 8·0–8·7, but, after a revision of the magnitudes of the comparison stars, 8·4–9·3. The basis of this revision does not seem to be clearly indicated. But everything points to the inference that the colour index of the star is negative, perhaps to the extent of half a magnitude. This implies that the type is B–A, and with the period is consistent with the supposition that the star belongs to the same class as β Lyræ. One is then tempted, in view of the abnormal character of the light curve and the palpable irregularities to which it is subject, to see here evidence of a binary system in which the fission has occurred comparatively recently and perhaps even incompletely as yet. In any case, SX Aurigæ seems to present features of exceptional interest, and to deserve accurate photometric study. That method is obviously better suited to a case like this than the photographic method pursued in the (at present) exceptionally unfavourable climate of Dunsink. It has taken three seasons to secure 94 exposures on this one object, and the only outcome is to suggest some features of the problem without offering any solution.

TABLE I.

B.D.	Pl. 1401. m_1.	Pl. 1434. m_2.	Mean. m.	Corr.	Final. m'.	P.E.	Range.
+42°1197	7·70	7·67	7·69	+·001	7·69	±·014	·11
+42·1193	7·73	7·76	7·74	−·006	7·73	±·014	·11
+42·1180	8·27	8·32	8·29	+·006	8·30	±·021	·16
+41·1088	8·29	8·32	8·31	·000	8·31	±·018	·19
+41·1077	8·68	8·76	8·72	−·005	8·72	±·021	·18

4. The details relating to the comparison stars are given in Table I. They are derived from two plates exposed both on the region of the variable and on the χ Persei cluster. The magnitude scale of these plates is based on the magnitudes given by Schwarzschild for the stars in the latter region. The mean magnitudes m have been used in the reductions, though the residuals obtained from all the plates furnish mean corrections, and hence a final magnitude m' for each star on a system which should be slightly more consistent. In the last two columns of the table are also given the probable errors for a single plate, roughly estimated, and the extreme range of the largest (positive and negative) residuals. The method of reduction is that which has been explained in previous papers.

5. Table II. presents the observations on the variable. The date with G.M.T. for the middle of each exposure is given in the second column. The third column contains the corresponding Julian day and decimal fraction. The phase in the fourth column is reckoned from the epoch J.D. 2420 447·420, which is approximately half way between two principal minima, and based on the assumed period 1·53234 days. The fifth column shows the time

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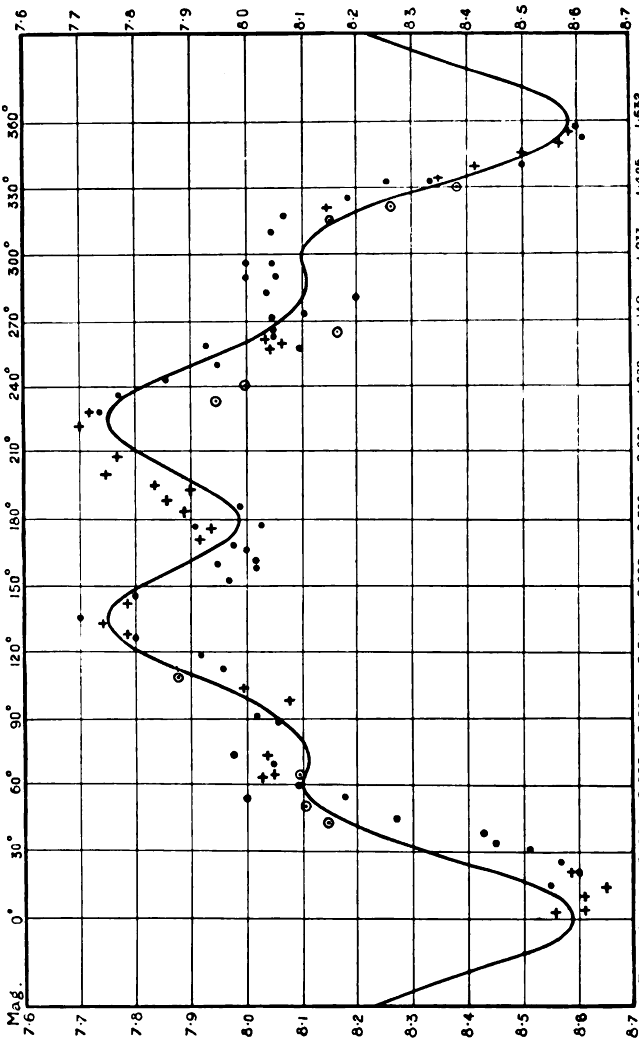
Photographic Light Curve of SX Aurigæ



Observations of 1914–15 denoted by •; of 1915–16 by +; and of 1916–17 by ⊙

Epoch of figure J.D. 2420446·654; period 1·53234 days.

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of exposure, and the sixth the deduced photographic magnitude of SX Aurigæ. In the last column are recorded the measurer's estimates of the quality of the images.

6. In the seventh column of Table II. are given the deviations of the observed magnitudes from the curve shown in the diagram which illustrates this paper (Plate 13). But it is not pretended that any value attaches to them. The curve represents the formula

$$m = 8{\cdot}07 - 0{\cdot}30 \cos \theta + 0{\cdot}11 \cos 2\theta + 0{\cdot}11 \cos 4\theta,$$

where, P being the assumed period, $P\theta/2\pi$ is the phase as defined in § 5. This formula was obtained as the simplest way of interpolating a symmetrical curve among the observations. The latter do not show this symmetry, as the figure indicates, but as they do not define any precise curve, the representation may as well be simple, since it cannot be accurate. The deviations are, therefore, due partly to ignoring the asymmetry in the curve, but there are other irregularities which cannot be explained in this way. Nor can they be attributed to errors of observation. The reduction to one common period is probably responsible to some extent. but the present material is too meagre for any other treatment.

7. Beyond directing further attention to a star which seems to be peculiarly interesting, this note should serve to suggest the necessary conditions for a successful attack on the problem. The means of observation must be accurate and fairly rapid. The kind ot accuracy which depends on multiplying observations of low individual precision is useless here. A good light curve must be determined within the limits of as few consecutive periods as possible. This presupposes a good climate. It will then be necessary to repeat the work in successive intervals, and a comparison of the results may throw some light on the true behaviour of the star.

TABLE II.

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1914	h	m	2420		min.			
1	Nov. 10	9	45	447·407	1·519	30	7·91	−·07	Fair.
2	11	7	51	448·328	0·908	30	8·45	+·15	Fair.
3	11	9	58	448·416	0·996	30	8·18	+·06	Fair.
4	11	10	30	448·438	1·018	30	8·10	·00	Fair.
5	11	11	24	448·475	1·055	30	8·05	−·06	Good.
6	11	11	55	448·496	1·076	30	7·98	−·13	Good.
7	16	9	38	453·402	1·385	30	7·80	+·02	Fair.
8	16	11	1	453·459	1·442	30	7·95	+·07	Fair.
9	16	11	43	453·489	1·472	30	8·00	+·06	Fair.
10	18	10	35	455·449	0·367	30	8·05	+·01	Fair.
11	18	11	16	455·469	0·387	30	8·05	−·02	Fair.
12	18	13	13	455·551	0·469	30	8·06	−·05	Fair.

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TABLE II.—*continued.*

Serial No.	Date G.M.T.				Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1914		h	m	2420		min.			
13	Nov.	18	13	44	455·574	0·492	30	8·05	− ·06	Fair.
14		26	11	25	463·476	0·733	30	8·61	+ ·04	Good.
15		26	11	56	463·497	0·754	19	8·60	+ ·01	Fair.
16	Dec.	7	7	6	474·296	0·826	30	8·55	+ ·03	Good.
17		7	7	38	474·319	0·849	30	8·60	+ ·15	Good.
18		7	8	17	474·345	0·875	30	8·57	+ ·19	Good.
19		7	8	49	474·367	0·897	30	8·51	+ ·19	Good.
20		7	9	33	474·398	0·928	30	8·43	+ ·18	Fair.
21		7	10	14	474·428	0·958	30	8·27	+ ·10	Good.
22		8	7	56	475·331	0·329	40	7·93	− ·04	Good.
23		8	8	39	475·361	0·359	40	8·05	+ ·02	Fair.
24		8	9	32	475·397	0·395	40	8·11	+ ·04	Fair.
25		8	10	11	475·424	0·422	30	8·20	+ ·10	Fair.
26		9	7	19	476·306	1·304	33	7·80	+ ·03	Good.
27		9	8	18	476·346	1·344	22	7·70	− ·05	Fair.
28		9	9	58	476·416	1·414	35	7·97	+ ·14	Fair.
29		9	10	35	476·442	1·440	35	8·02	+ ·14	Fair.
30		18	7	0	485·292	1·096	40	8·00	− ·12	Fair.
31		18	8	25	485·351	1·155	35	8·02	− ·04	Fair.
32		18	10	35	485·442	1·246	35	7·96	+ ·08	Fair.
33		18	11	12	485·466	1·270	35	7·92	+ ·10	Fair.
	1915									
34	Jan.	8	7	4	506·295	0·646	40	8·26	− ·09	Fair.
35		11	6	30	509·268	0·554	40	8·05	− ·09	Good.
36		11	7	12	509·300	0·586	40	8·07	− ·13	Fair.
37		11	8	1	509·334	0·620	40	8·19	− ·09	Fair.
38		11	8	42	509·363	0·649	40	8·34	− ·02	Fair.
39		11	9	32	509·397	0·683	40	8·50	+ ·05	Good.
40		17	6	37	515·276	0·433	40	8·04	− ·06	Fair.
41		17	7	19	515·306	0·463	40	8·00	− ·11	Fair.
42		17	8	6	515·337	0·494	40	8·00	− ·11	Fair.
43		21	8	30	519·354	1·446	38	8·02	+ ·13	Good.
44		21	9	20	519·389	1·481	40	7·98	+ ·03	Fair.
45		21	10	16	519·428	1·520	40	8·05	+ ·06	Fair.
46		21	11	5	519·462	0·022	40	7·99	+ ·01	Fair.
47	Feb.	2	7	29	531·313	1·147	40	8·06	− ·01	Fair.
48	Mar.	19	8	17	576·345	0·208	36	7·74	− ·02	Fair.

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TABLE II.—*continued.*

Serial No.	Date G.M.T.				Julian Day.	Phase in Days.	Ex-posure.	Magn.	Obs. —Curve.	Images.
	1915		h	m	2420		min.			
49	Mar.	19	8	58	576·374	0·237	36	7·78	·00	Fair.
50		19	9	42	576·407	0·270	36	7·76	− ·07	Fair.
51		19	10	25	576·435	0·298	36	7·95	+ ·04	Fair.
52		19	11	11	576·465	0·328	34	8·10	+ ·13	Fair.
53	Nov.	2	10	3	804·419	1·496	30	7·92	− ·04	Fair.
54		2	10	37	804·443	1·520	30	7·94	− ·04	Fair.
55		2	11	19	804·472	0·017	30	7·89	− ·09	Fair.
56		2	11	50	804·493	0·038	30	7·86	− ·10	Fair.
57		2	12	29	804·521	0·066	30	7·84	− ·09	Fair.
58		2	13	0	804·542	0·087	30	7·75	− ·14	Fair.
59		2	13	47	804·575	0·120	30	7·77	− ·06	Good.
60		29	12	57	831·540	1·035	25	8·03	− ·07	Fair.
61		29	13	28	831·562	1·057	25	8·05	− ·06	Fair.
62		29	13	59	831·583	1·078	25	8·04	− ·07	Fair.
63		30	8	48	832·367	0·330	25	8·05	+ ·07	Fair.
64		30	9	13	832·384	0·347	25	8·04	+ ·03	Fair.
65	Dec.	6	12	12	838·508	0·341	17	8·07	+ ·07	Good.
66		7	8	26	839·351	1·184	30	8·08	+ ·07	Fair.
67		7	8	57	839·373	1·206	30	8·00	+ ·03	Good.
68		8	7	11	840·299	0·600	30	8·15	− ·08	Good.
69		8	8	25	840·351	0·652	30	8·35	− ·01	Good.
70		8	9	1	840·376	0·677	30	8·42	− ·02	Fair.
71		8	9	32	840·397	0·698	30	8·50	·00	Good.
72		8	10	10	840·424	0·725	30	8·57	+ ·02	Good.
73		8	10	46	840·449	0·750	30	8·59	+ ·01	Good.
74		8	11	24	840·475	0·776	30	8·56	− ·02	Bad.
	1916									
75	Jan.	27	10	43	890·448	0·182	30	7·70	− ·05	Fair.
76		27	11	23	890·474	0·208	30	7·72	− ·04	Fair.
77	Feb.	8	7	16	902·303	1·310	30	7·79	+ ·02	Fair,
78		8	7	53	902·329	1·336	30	7·75	·00	Bad.
79		8	8	45	902·365	1·372	30	7·79	+ ·02	Bad.
80		9	8	26	903·351	0·826	30	8·65	+ ·13	Fair.
81		9	9	4	903·378	0·853	30	8·59	+ ·15	Fair.
82		22	9	1	916·376	0·060	30	7·90	− ·04	Fair.
83	Mar.	6	8	35	929·358	0·783	30	8·61	+ ·03	Good.
84		6	9	13	929·384	0·809	30	8·61	+ ·06	Fair.

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TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1916	h	m	2421		min.			
85	Nov. 24	8	44	192·365	0·228	40	7·95	+·18	Fair.
86	24	9	34	192·399	0·262	40	8·00	+·18	Good.
87	24	11	58	192·499	0·362	40	8·17	+·14	Fair.
88	25	8	46	193·365	1·228	27	7·88	-·04	Fair.
89	Dec. 22	6	58	220·291	0·572	36	8·16	-·01	Fair.
90	22	7	41	220·321	0·602	39	8·27	+·03	Fair.
91	22	8	27	220·352	0·633	36	8·39	+·08	Fair.
92	27	8	21	225·347	1·038	31	8·10	-·01	Fair.
	1917.								
93	Mar. 12	8	24	300·350	0·949	36	8·15	-·04	Good.
94	12	9	9	300·381	0·980	36	8·11	-·03	Good.

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The Observations by Fabricius and Heller of the Comet of 1556. Being the substance of a paper by Karl v. Littrow in *Sitzungsberichte Wiener Akad. der Wissenschaften*, vol. xx. p. 301. Translated by the late A. S. Herschel. (Plate 14.)

(*Communicated by Professor G. Forbes, and edited by A. C. D. Crommelin.*)

Prefatory Note.—In the middle of the last century there was general expectation of the return of the comet of 1556, which was assumed to be identical with the comets of 975 and 1264. Accordingly, active search was made for possible further records of its path, which might have escaped notice. Littrow succeeded in finding further particulars concerning the observations of Fabricius, and an important series of observations by Joachim Heller. This new material was utilised by Martin Hoek, a few years later, in his definitive orbit of the comet. In 1908 Professor Forbes published a paper in the *Monthly Notices*, vol. lxix. p. 152, in which he suggested that the comet had encountered his hypothetical trans-Neptunian planet when near its last aphelion passage, and returned in four portions, whose orbits had been changed from direct to retrograde, in the years 1843, 1880, 1882, 1887. He had previously consulted A. S. Herschel, who had prepared for him the translation of Littrow's pamphlet, and also made some calculations of his own on the positions of the comet.

Herschel was evidently unacquainted with Hoek's work, and thought no one had yet utilised Heller's observations. Herschel's manuscript was lately submitted to the Society by Professor Forbes. It has been decided to print it in an abridged form, in view of the interesting questions connected with this comet, and the care with which Heller observed it. I have shortened A. S.

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Herschel's paper by the omission of some details of no astronomical importance, and the suppression of many of Heller's circumlocutory descriptions of stars, the modern designation of the star presumably intended being substituted. I have also added some tables at the end, giving the positions of the comet and stars. A. C. D. C.

K. von Littrow, on Observations by P. Fabricius and M. Joachim Heller of the Comet of 1556. Translated from *Sitzungsberichte of Vienna Academy*, vol. xx., by A. S. Herschel.

In deducing the elements of the great comet of 1556, presumably expected to return between 1856 and 1860, all the recent calculators have overlooked the original observations of the Court mathematician of that time, Paul Fabricius. Even Pingré was only acquainted with them from a little chart contained in Conrad Lycosthenes' *Prodigiorum ac ostentorum Chronicon*, and in other works on which were inscribed the comet's places, said to have been observed by Paul Fabricius in Vienna; on these he, as also later calculators, were obliged, through lack of other data, to base their calculations. I do not know whether Halley used any original publications by Fabricius, as I have not access to his *Synopsis of Comets*. Mr. Hind regretted this imperfect knowledge of a source from which useful help in the accurate determination of this body's motion might be expected; he begged me, some two years ago, to search for the original publication of Fabricius. After numerous inquiries among my friends, I resorted to advertisement in the papers of Vienna. I obtained in this manner, through the kindness of Mr. C. Denhart, the first knowledge that modern astronomers have acquired of an original description by Fabricius. In a volume of Royal Patents, preserved in the State archives, there occurs a leaflet with the title "The Comet Seen in March, in the year LVI. in Vienna"; it contains a rough woodcut map of the comet's course, and a dedication to Vito Hütter. This is evidently the map used by Lycosthenes; its scale is three times as large as that of the copy. As the maker of the woodcut did not allow for reversal, the heavens appear reversed right and left in the map. (Littrow corrects this in the copy he gives.) There are seventy-four lines of text, beginning thus: "In my *Pratika*, which I had to make and publish ten months ago for official purposes, it was noted, among other matters, that this year would not pass away without a comet of such magnitude as all may see who have my *Pratika*. Now a comet has just appeared in March in this land of Austria, whose course may be seen in this map, along with the chief stars. It began on the fourth of March, and rose upwards towards midnight, night after night. For further particulars, see my Latin *Judicium*, in which the hours of observation are noted": this is dated 14th March 1556. Following up this clue, with the help of F. Roeth of Augsburg, I found the *Judicium*. It begins, like the pamphlet, with his prediction, and the discovery of the comet: "It was first seen on the ecliptic,

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in longitude 8½° of Libra; it gradually rose, and after . . . nights it passed over Bootes and Ursa Major. The position on March 4 I did not observe, and inserted by guesswork. On March 5, 10^h night, it was as yet small, Longitude 8½° Libra, Decln. about $+3°$, lat. $+7°$; the 3rd, 4th, 5th, 6th positions were on Mar. 6, 7, 8, 9, the last at 9^h night, the others at 10^h. I observed at the residence of Archduke Charles II. of Austria; 7th position, Mar. 10, 9^h; 8th, Mar. 11, 4^h morning; 9th, Mar. 11, 9^h night; 10th, 3 hours later, when the comet occulted a small star in left arm of Bootes (λ Bootis according to Littrow, Piazzi XIII. 316 according to Hoek); 11th position, Mar. 12, 4^h morning; 12th position, Mar. 12, 9^h night; Mar. 13, no observations, cloudy; Mar. 14, 4^h morning, I could with difficulty, through clouds, see the comet near the two chief stars of Ursa Minor (β, γ). . . . The comet itself is not so very large, about equal to a first magnitude star; it is dull in light, in motion not so swift as Mars or Mercury, with only a moderate tail . . . it turned its tail, when setting, somewhat southwards, having hitherto been along parallels. While this is being printed I have added two further observations, useful for the ninth and tenth problem of Regiomontanus on comets; the first near the pole of the zodiac, Mar. 14, 9^h night; the other near the shoulder of Cepheus, Mar. 15, 9^h; the comet was then smaller than before; on Mar. 16 it had advanced a little, and was so small that it was barely visible."

Some of this account is lost through injury to the page, some is omitted here, as being of no astronomical importance. We now have access to all that we can hope to learn about the work of Fabricius on this comet. First, he did not pursue his observations later than March 15, and his position for March 4 was merely a guess, so that his observations extended over only eleven days. Secondly, we gain no more exact positions of the comet than what we can read from the map, of which the inexactitude is very obvious. Lastly, the hours of the observations, which are the chief new matter supplied by the *Judicium*, are evidently only given in round figures. Still, these two new documents are a welcome completion of an astronomical observation of the sixteenth century, and the occultation of the star in Bootes may prove useful.

A third source of information, which I have nowhere seen mentioned, appears to me to provide the best materials for calculation, not excepting Fabricius. Dr. Bethmann, librarian at Wolfsbüttel, drew my attention to this work: *Pratika for* 1557, *with an exposition of what the appearance and motion of the recently observed comet* (which was previously predicted) *in the year* '56 *resembled and signified, by M. Joachim Heller, Astronomical Preceptor in Nürnberg.* In the fourth chapter of this work Heller speaks of having predicted the comet's appearance, and then proceeds as follows: ". . . which actually happened soon after the Full Moon in February, for having watched for a favourable sky for many evenings in my journey,

and perceived nothing but a brightly clear sky on the first two days and nights, on the third evening after, which followed the Thursday after S. Matthias, having ridden after 6 p.m. to the Gefress (according to Stieler's map, 9° 24′ E. of Paris, 50° 5′ N. Lat.), finding myself belated at night on the hills behind Berneck (4′ W. 2′ S. of last station), I saw the Moon rise 2^h 20^m after sunset, Spica Virginis 24^m later; near the latter star, but nearly 3° nearer the south, under Spica, towards the tail of Corvus, a great fiery unusual-looking star, following the moon in its rising, though I had never before seen any such star at that place, & though I well knew that all other planets had set near the sun, and that Jupiter was very far below the horizon. For a time I was somewhat doubtful, as I could so far see no tail to the comet. But on arriving, still doubtful, at Nürnberg, I learnt after Mar. 3 that people there had seen a comet with a little tail on Mar. 3; while respectable, trustworthy people assured me that, with many others, they had seen the unusual star in that shape from the first, on the above-mentioned Thursday after St. Matthias [this Thursday would be Feb. 27]. Hence I am no longer now in doubt about the place where the comet first broke out, or kindled; just as in that kind of body I have before prognosticated. When the 4th and 5th nights became dark, I betook myself with instruments to observation, and found it on the next following night, which followed Mar. 6, a little over 2 hours after sunset, risen several degrees above the horizon; it extended its tail between west and south, and its head, or beginning in front, was still red, but the tail was of mingled pale and dingy colour, such as the flame of burning sulphur usually presents. [Some matter is omitted here.]

"As regards this comet's course, I was careful to note it, either at its rising, or on the meridian, or by known stars, as certain reference-points of its visible position in the sky. And at its first appearance, on the night of Feb. 27, it rose exactly with Spica, though in my judgment it was 3°·4 to the south of it in latitude. Then as the comet in rising was on the same level with Spica, so I also saw over the comet the small star (ψ Virginis) which is the last of the 3 under Virgo's left shoulder, and I judged the comet to be nearly midway between (ψ) and (Piazzi XIII. 9) the first of the 3 small stars (Herschel says x, P, K Virginis, using Bode's nomenclature) that lie below Spica, behind the tail of Corvus; from this I infer that the comet first appeared in Longitude Libra 15°·83, S. Lat. 5°·4; it rose at the same time with R.A. 205°·9, Decl. 0°, and with Long. Libra 18°·67; then on Mar. 6, when it appeared with its tail as above described, I found it had moved nearly 15° higher in Lat., & about 9° in Long.; and again when I noted it in the Observatory, 52^m after midnight, I compared it with the 8th star of Virgo (k Virginis), which preceded it by 3^m of R.A., and with the 9th star (θ Virginis) which followed it by 10^m of R.A., and it stretched its tail downwards to near the 10th star of Virgo (δ Virginis), which came to the meridian 5^m

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before the comet; comet's Long. then Libra 6°, Lat. 10° N. On the next night, Sat. Mar. 7, comet a little above Vindemiatrix (ϵ Virg.), projecting its tail downwards towards the next star below that on the wing of Virgo (σ or e Virg.); at 10^h its distance from Vind^x was nearly 3°; so I waited till midnight, and then noted Vind^x nearly $2\frac{1}{4}$° in Long. from the comet, and about $\frac{1}{3}$° below it; at $12\frac{3}{4}^h$ the star reached meridian in R.A. 189°; 10^m later the comet reached meridian, R.A. $191\frac{1}{4}$°, $\frac{1}{3}$° higher; afterwards at 13^h 12^m Spica reached meridian, R.A. 195° 25′; & I found the comet $21\frac{3}{4}$° N. of that star in Decl.; Sun's R.A. at midnight $357\frac{1}{4}$°. From this observation I found comet's place at midnight, Long. Libra 5°, N. Lat. $16\frac{1}{3}$°, or R.A. 191°, N. Decl. 12°·7. On night of Mar. 8, at 13^h 3^m, I again observed comet with Spica on meridian, above which star it stood 29°·9 in Decl. Spica followed it by 6^m in R.A. on meridian; thus comet was $8\frac{1}{8}$° higher in Decl. than night before, with R.A. 194°; and $\frac{1}{4}$° above (τ) Bootis in Decl., which reached meridian about 13^h 40^m or 36^m after comet; at midnight on this night comet's Long. Libra 4°, N. Lat. $24\frac{5}{8}$°; on night of Mar. 9 comet crossed meridian at $1\frac{1}{4}^h$ after midnight; Spica reached meridian with $198\frac{1}{4}$° (Herschel notes 'query meaning of this, Spica's true R.A. was 195°·5; probably Heller's instruments were out of adjustment'), Spica was 10^m of R.A. before comet, & $39\frac{1}{3}$° S. of it in Decl., *i.e.* comet's N. Decl. was $30\frac{1}{2}$°. α Coronæ followed the comet by 2^h, and was 2° S. of it in Decl. Arcturus followed comet by $\frac{3}{4}^h$, exactly at 2^h a.m.; it was $8\frac{3}{4}$° south in Decl. On night of Mar. 10 the comet crossed meridian between 1^h & 2^h after midnight, exactly 38° N. of equator, 16° 10′ higher than Arcturus, which followed it by a little less than $\frac{1}{2}^h$ on meridian, at 1^h 56^m. On night of Mar. 11 I saw comet on meridian 1^h 57^m after midnight, and star (λ) Bootis at left elbow of Bootes was 4′ above it and 20′ behind it. It passed nearly through Zenith of Nürnberg, and from its position in front of the shafts of the Wain, it seemed to wish to draw it; (η Urs. Maj. preceded comet by 29^m, $3\frac{1}{2}$° above it in decl.). On night of Mar. 12 it had risen in meridian to $10\frac{1}{2}$° beyond the Zenith of Nürnberg, and lay near the last of the 3 stars in last bend of Draco's tail (ι Draconis); this star was $\frac{1}{2}$° north of the comet, & followed it by 12^m 40^s on meridian. On night of Mar. 13 it had crossed N. Polar Circle, and at 4^h after midnight it formed apex of an isosceles triangle with legs 6° long from γ and η Ursæ Minoris towards Pole of Ecliptic. On following night of Mar. 14 it had moved with rapid speed across star-throng of Draco, from sign of Leo to that of Taurus; and on Mar. 16 to end of sign Aries, near β Cephei, which it in fact occulted. On Mar. 17 comet near ι Cephei. Next night cloudy; on 19th near a star in Cassiopeia's left hand (? Groombridge 4023, Hoek). On 20th it occulted ζ Cassiopeiæ (Hoek says δ Cephei). Next successful observation early on last day of March, near (B Andromedæ) Hand of Andromeda. I found it on Apr. 1 under right hand of Andromeda

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(about π Cassiopeiæ) (*sic*; I suppose π Andromedæ is meant). On Apr. 2 it rose $6\frac{1}{4}^h$ after sunset, 55^m after midnight, being at $6\frac{3}{4}^h$ after sunset (shortly after 1^h a.m.), between the stars ρ and σ Andromedæ. On 4th it rose $6\frac{1}{2}^h$ after sunset, 40^m before α Andromedæ, in 3° higher N. Lat. than that star; since the beginning of Apr. it had moved only 3° in Lat. but more in Long. On 5th (Easter Sunday) it rose $6\frac{5}{8}^h$ after sunset, $\frac{1}{4}^h$ before α Andromedæ, was 7° before that star, & stood just 5° quite straight above δ Andromedæ. On Apr. 6 the comet rose $6\frac{3}{4}^h$ after sunset, $5\frac{1}{2}°$ before α Andromedæ, & was $1\frac{1}{2}°$ in Long., 2° in Lat. from δ Andromedæ. On Apr. 7 it rose 6^h 50^m after sunset, & appeared to have reached stationary point. It is now 2° in Long. & 2° in Lat. from δ Andromedæ."

(Littrow notes that after this Heller's star-descriptions became difficult to follow, and their identification provisional.) "On Apr. 8 comet rose 6^h 55^m after sunset, & was then $\frac{5}{8}°$ from (Hoek) π Andromedæ (Littrow says Pi. 0, 86), & about 2° from δ Andromedæ (Hoek) (Littrow says ε Andromedæ). On Apr. 9 it rose 7^h after sunset, & was 5° in Long. from α Andromedæ, $\frac{1}{2}°$ lower in Lat.: still about 1° above δ Andromedæ. On Apr. 10 it rose 7^h 5^m after sunset, 12′ downwards from δ Andromedæ. Apr. 11 cloudy. On Apr. 12 it made an isosceles triangle with δ Andromedæ & ε Andromedæ (Littrow says Piazzi 0, 122) lying towards α Andromedæ, $\frac{2}{3}°$ from that pair of stars. On Apr. 13 it had moved to 1° 10′ below δ Andromedæ, downwards, & $\frac{1}{2}°$ towards α Andromedæ. On Apr. 14 it lay 25′ under ε Andromedæ in Lat, & about $\frac{1}{4}°$ from that star towards α Andromedæ, being then $2\frac{1}{2}°$ under δ Andromedæ & 8° from α Andromedæ. On Apr. 15 the comet lay as far beyond δ Andromedæ as that is from π Andromedæ, in same straight line with both stars. On Apr. 16 it had moved from last place 13′ S. in Lat. & 32′ in Long. On Apr. 17 it was twice as far from the lower shoulder star of Andromeda, ε Andromedæ, as the upper one, π Andromedæ, is from the middle shoulder star, δ Andromedæ, but laterally 50′ nearer to the tropic of Cancer; it thus lay nearly midway in an isosceles triangle from ε Andromedæ & *i* Piscium. On Apr. 19 it reached the Tropic of Cancer, at $1\frac{1}{2}°$ from η Andromedæ.

"After this date I could no longer observe it, as although I glimpsed it once with difficulty in the nights that followed, until Apr. 22, the view was insufficient, as the stars were also fading out near the approaching sun, to permit a sure observation of it. I trust, however, that the learned will have particulars enough in this account for calculating the comet's real course."

Addendum by A. C. D. Crommelin.

This ends the extract from Heller's account. The modern names of the stars are, of course, later additions. Littrow remarks that Heller's observations have a range of 53 days. He draws attention to the three star-occultations of Mar. 11, 16, 20.

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It was by planetary conjunctions that Fabricius and Heller were led to predict a comet in 1556. He adds that astrological prediction was then the main motive for making careful cometary observations, but suggests that this may be pardoned in view of the useful records which they have handed down to us. All attempts to identify the exact spot at Nürnberg where Heller observed have been unsuccessful.

Herschel ends his paper with a list of cometary places deduced by him from Heller's record. I print this in conjunction with Hoek's places. The latter appear to be more reliable; several of Herschel's star places for 1556 are in error, as the appended table shows, and in some cases Hoek has altered the identification of the stars.

The following is Hoek's final orbit:

$$T = 1556 \text{ April } 22{\cdot}1846 \text{ G.M.T.}$$

$$\left.\begin{array}{l}\omega = 100^\circ\ 52'{\cdot}6 \\ \Omega = 175^\circ\ 13'{\cdot}9 \\ i = 32^\circ\ 25'{\cdot}7\end{array}\right\} 1556{\cdot}0$$

$$\log q = 9{\cdot}69092$$

$$x = [9{\cdot}99956]\, r \sin (v + 6^\circ\ 50'{\cdot}95)$$
$$y = [9{\cdot}99481]\, r \sin (v + 276^\circ\ 27'{\cdot}11)$$
$$z = [9{\cdot}20386]\, r \sin (v + 112^\circ\ 50'{\cdot}29)$$

From these elements I computed the appended ephemeris; I assumed for the G.M.T. of Equinox 1556 March 10·209, which is in almost perfect agreement with Grattan Guinness's value. The obliquity of ecliptic was 23°·496.

Ephemeris of the Comet from Hoek's Final Elements and Comparison with Observed Places.

G.M.T.	Calculated. R.A.	Calculated. Dec.	Log r.	Log Δ.	Calculated − Observed. R.A.	Calculated − Observed. Dec.	
1556 Feb. 27·37	186·77	−9·07	0·1058	9·4746	− 2·97	+ 1·33	H_2
Mar. 5·38	191·01	+4·21	·0632	9·1972	− ·10	+ 1·41	F
6·38	192·17	7·96	·0567	9·1703	− ·71	− ·19	F
6·51	192·36	8·53	·0558	9·1630	+ 2·81	+ 1·33	H_2
7·38	193·70	12·73	·0500	9·1192	− 1·23	− ·78	F
7·51	193·93	13·44	·0491	9·1122	+ 1·98	+ ·09	H_2
8·38	195·62	18·72	·0433	9·0677	− 1·55	− 1·74	F
8·52	195·93	19·70	·0423	9·0590	+ 2·33	− ·97	H_2
9·34	198·20	25·95	·0367	9·0169	− 2·21	− 1·08	F
9·53	198·90	27·50	·0354	9·0087	+ ·95	− 3·02	H_2
10·34	202·00	35·27	·0298	8·9708	− 1·99	− ·80	F
10·54	202·92	37·27	·0284	8·9637	+ ·07	− ·41	H_2
10·63	203·47	38·39	·0277	8·9589	− 1·82	− ·73	F

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Ephemeris of the Comet from Hoek's Final Elements and Comparison with Observed Places—continued.

G.M.T.	Calculated. R.A.	Calculated. Dec.	Log *r*.	Log Δ.	Calculated – Observed. R.A.	Calculated – Observed. Dec.	
1556 Mar. 11·34	208°·11	+46°·30	0·0227	8·9361	+ °·18	+ °·79	F
11·46	209·14	47·69	·0218	8·9330	+ ·81	+ ·12	F
11·55	209·85	48·77	·0211	8·9312	− ·10	+ ·19	H_2
11·63	210·48	49·71	·0206	8·9298	− ·45	−1·05	F
12·34	218·38	58·09	·0155	8·9218	+ ·33	+ ·51	F
12·60	222·31	60·94	·0136	8·9217	− 3·17	+ ·99	H_2
13·63	247·13	70·39	·0059	8·9376	+11·14	+ ·01	F
13·64	247·43	70·40	·0058	8·9378	+ 1·42	−1·20	H_2
14·34	273·48	73·26	0·0006	8·9613	− 4·62	− ·58	F
15·80	317·39	69·47	9·9896	9·0276	− 3·16	+ ·64	H_2
17·51	337·73	60·72	·9760	9·1158	− ·77	−3·18	H_1
19·51	346·42	53·21	·9600	9·2117			
20·51	348·80	50·24	·9516	9·2560			
30·63	357·75	35·60	·8595	9·5725	− 1·95	−4·60	H_1
Apr. 1·51	358·50	34·14	·8411	9·6148	− ·70	−3·06	H_1
2·51	358·89	33·41	·8312	9·6361	− ·44	−1·49	H_2
4·53	359·70	32·04	·8111	9·6770	calc. lat. +29°·22, obs. +29°·0		H_1
5·51	0·04	31·46	·8014	9·6954	− 1·30	− ·89	H_2
6·51	0·47	30·83	·7914	9·7141	− 1·73	+1·38	H_2
7·51	0·89	30·22	·7816	9·7323	− ·96	+ ·83	H_2
8·51	1·36	29·64	·7717	9·7498	− 1·61	−1·01	H_2
9·52	1·77	29·08	·7624	9·7670	+ ·73	+ ·43	H_2
10·53	2·25	28·51	·7530	9·7836	− 1·35	+ ·35	H_2
12·53	3·25	27·43	·7357	9·8156	− ·60	− ·14	H_2
13·54	3·80	26·90	·7277	9·8311	+ ·15	− ·42	H_2
14·55	4·35	26·35	·7202	9·8467	+ 1·13	+ ·32	H_2
15·56	4·96	25·83	·7134	9·8610	+ ·78	+ ·48	H_2
16·56	5·57	25·32	·7074	9·8754	+ 1·10	− ·03	H_2
19·57	7·63	+23·83	9·6946	9·9161	+ ·66	+1·93	H_2

F stands for the observation of Fabricius as interpreted by Hoek.
H_2 ,, ,, Heller ,, ,, Hoek.
H_1 ,, ,, Heller ,, ,, Herschel.

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Positions for 1556·0 *of the Stars used for Comparison or mentioned in the Narrative.*

Star's Name.	Alternative Designations.	Mag.	Positions from Boss P.G.C. R.A.	Positions from Boss P.G.C. Dec.	Positions adopted by Herschel. R.A.	Positions adopted by Herschel. Dec.
28 Andromedæ	Piazzi 0, 86	5·3	1°·74	+27°·29		
ζ Cassiopeiæ		3·8	3·21	+51·44		
Piazzi 0, 122		6·7	3·28	+24·80		
π Andromedæ		4·4	3·37	+31·26		
ε Andromedæ		4·5	3·84	+26·88	3·8	+26·9
δ Andromedæ		3·4	3·97	+28·41		
π Cassiopeiæ		5·2	4·85	+44·58	4·9	+44·6
i Piscium		5·7	6·58	+25·27	6·5	+25·3
η Andromedæ		4·6	8·43	+21·00	8·4	+21·0
q Virginis		5·6	182·75	− 6·99	182·86	− 7·00
χ Virginis		4·8	184·11	− 5·54	184·33	− 5·54
ψ Virginis		5·0	187·85	− 7·10	187·85	− 7·12
δ Virginis		3·6	188·32	+ 5·83	188·2	+ 5·2
k Virginis		6·0	189·21	− 1·39	189·7	− 1·4
ε Virginis	Vindemiatrix	2·8	190·02	+13·37	189·9	+13·3
θ Virginis		4·4	191·77	− 3·14	191·7	− 3·2
53 Virginis	x Virg. = Pi. XIII. 9	5·2	192·15	−13·77	191·3	−13·5
e Virginis		5·4	193·69	+11·77		
σ Virginis		5·0	193·81	+ 7·84		
61 Virginis	K Virginis	4·8	193·84	−15·81	192·95	−15·60
α Virginis	Spica	0·9	195·49	− 8·81	195·50	− 8·83
69 Virginis	P Virginis	5·0	195·99	−13·63	195·16	−13·35
τ Bootis		4·6	201·54	+19·88	201·5	+19·8
η Urs. Maj.		1·8	202·48	+51·56	202·5	+51·5
Piazzi XIII. 316		5·6	207·53	+42·71		
α Bootis	Arcturus	0·0	208·86	+21·54	208·45	+21·4
λ Bootis		4·2	209·86	+48·17	209·8	+48·2
β Urs. Min.		2·0	223·31	+75·97		
ι Draconis		3·4	228·79	+60·56	228·8	+60·5
α Coronæ	Alphecca	2·2	228·98	+28·27	229·0	+28·2
γ Urs. Min.		2·9	230·59	+73·41		
ζ Urs. Min.		4·3	240·26	+79·11		
η Urs. Min.		5·2	247·97	+76·73		
β Cephei		3·3	320·37	+68·63	320·6	+68·5
δ Cephei		3·7–4·9	333·22	+56·17		
ι Cephei		3·6	338·54	+63·88	338·5	+63·9
Bradley 3084	Groombridge 4023	5·8	344·21	+50·84		
α Andromedæ		2·0	356·42	+26·64	356·4	+26·7
22 Andromedæ	B Andromedæ	5·2	356·91	+43·60	356·9	+43·7
σ Andromedæ		4·6	358·78	+34·32	358·8	+34·3
ρ Andromedæ		5·3	359·52	+35·50	359·5	+35·5

The star Piazzi 0, 122 is not in Boss; its place is from Greenwich 1910.

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Herschel's (1) *and Hoek's* (2) *Determinations of the R.A. and Dec. of the Comet, from their Independent Interpretations of Heller's Observations.*

G.M.T.	R.A. (1.)	R.A. (2.)	Dec. (1.)	Dec. (2.)	Stars used by Herschel.
1556	°	°	°	°	
Feb. 27·37	191·8	189·74	−11·1	−10·4	ψ Virg., Pi. XIII. 9, Heller's place.
Mar. 6·51	189·0	189·55	+ 7·2	+ 7·2	δ, ε, k Virg., Heller's place.
7·51	191·45	191·95	+12·8	+13·35	α, ε Virg., Heller's place.
8·52	193·2	193·6	+20·9	+20·67	α Virg., τ Bootis, Heller's place.
9·53	197·8	197·95	+29·8	+30·52	α Virg., α Coronæ, α Bootis.
10·54	201·7	202·85	+37·9	+37·68	α Bootis, Heller's place.
11·55	209·4	209·95	+48·2	+48·58	λ Bootis occulted, η Urs. Maj.
12·60	225·7	225·48	+60·0	+59·95	ι Drac.
13·64	247·2	246·01	+71·6	+71·60	γ, ζ Urs. Min.
15·80	320·6	320·55	+68·5	+68·83	β Cephei occulted. Herschel takes date as March 16.
17	338·5		+63·9		ι Ceph.
19					Near a star in left hand of Cassiopeia. Hoek says perhaps Groom. 4023.
20	4·4		+48·1		(ζ Cass.) occulted. Hoek says δ Cephei.
30·63	359·7		+40·2		B Androm.
Apr. 1	359·2		+37·2		B Androm., π Cassiop.
2·51	359·2	359·33	+34·9	+34·90	ρ, σ Androm.
4·53					Latitude +29°·0.
5·51	1·2	1·40	+32·9	+32·35	δ Androm.
6·51	1·7	2·20	+29·9	+29·45	δ Androm.
7·51	1·3	1·85	+29·7	+29·39	δ Androm.
8·51		2·97		+30·65	
9·52	1·0	1·04	+28·05	+28·65	α, ε Androm.
10·53	3·8	3·60	+26·7	+28·16	ε Androm.
12·53		3·85		+27·57	
13·54	3·3	3·65	+25·8	+27·32	ε Androm.
14·55	3·6	3·22	+25·5	+26·03	α, ε Androm., Pi. 0, 122.
15·56		4·18		+25·35	
16·56		4·47		+25·35	
19·57		6·97		+21·90	

Hoek's Determination of the R.A. and Dec. of the Comet from Fabricius's Diagram and Description.

G.M.T.	R.A.	Dec.	G.M.T.	R.A.	Dec.
1556	°	°	1556	°	°
Mar. 5·38	191·11	+ 2·80	Mar. 10·63	205·29	+39·12
6·38	192·88	+ 8·15	11·34	207·93	+45·51
7·38	194·93	+13·51	11·40	208·33	+47·57
8·38	197·17	+20·46	11·63	210·93	+50·76
9·34	200·41	+27·03	12·34	218·05	+57·58
10·34	203·99	+34·47	13·63	235·99	+70·40
			14·34	278·10	+73·84

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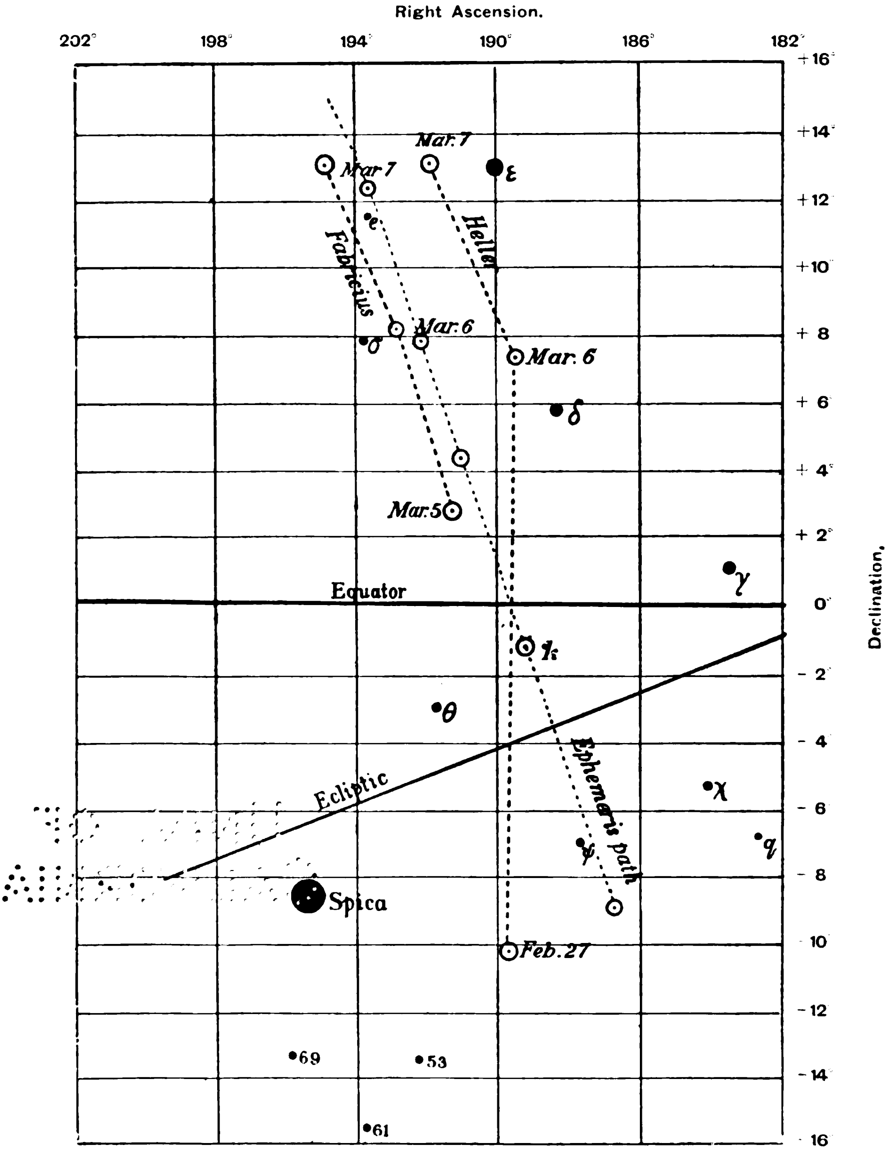

Diagram illustrating the observed and tabular paths of the Comet of 1556 in Virgo.

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Comparison of Heller's and Fabricius's Observations when made on the same day (Hoek's Determination).

(The correction for motion has been applied.)

Day.	Heller – Fabricius. R.A.	Heller – Fabricius. Dec.	Day.	Heller – Fabricius. R.A.	Heller – Fabricius. Dec.
Mar. 6	$-3^\circ.60$	$-1^\circ.48$	Mar. 10	$-2^\circ.09$	$+0^\circ.30$
7	-3.16	-0.77	11	$+0.24$	-0.17
8	-3.85	-0.61	12	$+2.65$	-0.55
9	-3.18	$+2.08$	13	$+9.70$	$+1.13$

The places of Fabricius are simply deduced from the path of the comet drawn on his chart. It is clear that this method is more subject to systematic error than Heller's star comparisons, which were made independently each night. The discordance on March 13 is only 3° when reduced to great circle.

The following are some points calling for note:—

(1) The considerable difference in the paths of Fabricius and Heller in the Virgo region. Heller's path is some three degrees further west; his careful comparisons with stars inspire more confidence than the rough chart of Fabricius, and perhaps Hoek should have given a higher relative weight to his observations. (See Plate 14.)

(2) With reference to the occultation of a star in Cepheus, Hoek says it was probably β Cephei.

(3) On March 19 the comet was "near the outermost star in the left hand of Cassiopeia." It is concluded that the right hand is meant. Hoek adopts (though with doubt) the star Bradley 3084.

(4) March 20: Hoek concludes that the head of Cepheus is meant, not that of Cassiopeia.

(5) There seems to be no doubt that Hoek is right in taking the upper, middle, and lower stars in the shoulder of Andromeda to be π, δ, and ϵ. Littrow (followed by Herschel) took them to be Pi. 0, 86; ϵ Andromedæ, Pi. 0, 122. The last is of magnitude 6·7, and is certainly too faint for Heller to have used. The error in the comet's position due to this wrong assumption is about a degree.

The Orbit Elements of Mellish's Comet (1917*a*).

By C. J. Merfield.

The following preliminary elements were deduced from Melbourne observations, 1917 April 19, April 23, May 1

$$T = 1917 \text{ April } 10{\cdot}66871 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 120^\circ\ 54'\ 4'' \\ \Omega &= 88\ \ 16\ \ 56 \\ \iota &= 32\ \ 44\ \ 38 \end{aligned}\right\} 1917 \qquad \log q = 9{\cdot}2765052.$$

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Residuals.

		cos $\delta d\alpha$.	$d\delta$.
		s	"
March	22·00	−63·1	+149·7
,,	27·00	−46·7	+180·9
April	19·32	− 0·1	− 0·5
,,	23·32	+ 2·5	− 34·4
May	1·31	0·0	+ 0·4
,,	10·32	− 9·6	+ 29·3

The positions for 1917 March 22, March 27, were taken from some positions published in a recent Lick Observatory Bulletin, and are normal places to which I give a weight of four each. The comparison of a position, calculated from the Lick elements, with an observation on May 10, gives the residuals

$$\cos\delta d\alpha = +183^{s}, \qquad d\delta = -552''.$$

With my elements as a basis, I set up a system of condition equations; the solution of these gave the following elliptical elements:—

T = 1917 April 10·683694 G.M.T.

$$\left.\begin{array}{l} \omega = 121^\circ\ 20'\ 55'' \\ \Omega = \ \ 87\ \ 24\ \ 37 \\ \iota = \ \ 32\ \ 28\ \ 16 \end{array}\right\} 1917 \qquad \begin{array}{r} \log q = 9{\cdot}2787499. \\ \log e = 9{\cdot}9974490. \\ e = 0{\cdot}9941434. \end{array}$$

Period 184·8 years.

Residuals.

1917.	cos $\delta d\alpha$.	$d\delta$.
	s	"
March 22	−0·17	+14·4
,, 27	+1·35	− 7·6
April 19	+0·36	−11·7
,, 23	−3·22	−22·1
May 1	−6·20	−35·1
,, 10	+6·23	+28·3

With these elliptical elements I am now setting up a new system of conditions. The solution will, I hope, considerably reduce the residuals, and result in a system of orbit elements that will require very little modification.

The original elements being so far from the truth, the differential coefficients in the equations of condition require a little correction; and further, the second terms, I find, are appreciable. The elliptical elements herewith will, however, require but small corrections to make the calculation and observation agree.

Assuming the eccentricity to be unity, and finding the elements from the condition equations under this assumption, I obtain the following:—

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T = 1917 April 10·675588 G.M.T.

$$\left.\begin{aligned} \omega &= 121^\circ\ 40'\ 53'' \\ \Omega &= \ \ 87\ \ \ 0\ \ 25 \\ \iota &= \ \ 32\ \ 22\ \ 40 \end{aligned}\right\}1917 \qquad \log q = 9{\cdot}2746037.$$

Parabolic Residuals.

1917.	cos δdα.	dδ.
	s	"
March 22	− 2·87	− 99·3
,, 27	− 7·37	+ 74·4
April 19	−14·50	+348·0
,, 23	− 7·20	+228·5
May 1	+11·06	+ 94·7
,, 10	+17·54	− 28·0

Melbourne:
1917 *June* 20.

Addendum, 1917 *July* 1.

A new solution has been made, adopting the above elliptical elements as the elements to be corrected. The results are as follows:—

T = 1917 April 10·688263 G.M.T.

$$\left.\begin{aligned} \omega &= 121^\circ\ 21'\ 47''{\cdot}75 \\ \Omega &= \ \ 87\ \ 23\ \ 10\ {\cdot}36 \\ \iota &= \ \ 32\ \ 37\ \ 57\ {\cdot}24 \end{aligned}\right\}1917 \qquad \begin{aligned} \log q &= 9{\cdot}2786202. \\ \log e &= 9{\cdot}9974886. \\ \log a &= 1{\cdot}5177493. \end{aligned}$$

P = 189^y ±

Co-ordinates.

$$x = [9{\cdot}9255725]r \sin(298^\circ\ 15'\ 37''{\cdot}52 + v).$$
$$y = [9{\cdot}9702722]r \sin(222\ \ 26\ \ 12\ {\cdot}55 + v).$$
$$z = [9{\cdot}8106553]r \sin(159\ \ 17\ \ 57\ {\cdot}62 + v).$$

Observed Errors of Hansen's Tables as modified by Prof. Newcomb and of Delaunay's New Tables, from Observations made at Greenwich with the Transit-Circle and Altazimuth for the Year 1915.

(*Communicated by the Astronomer Royal.*)

The following table is based on the observations of the Moon's limbs and the crater Mösting A made with the transit-circle and altazimuth. The transit-circle was out of use from January to March

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during the mounting of a travelling micrometer. A mean error has been found for each day from the combined results of limb and crater observations with the two instruments.

As is well known, the *Nautical Almanac* gives the Hansen-Newcomb position of the Moon. The places from the new Delaunay theory are taken from pp. 690–692 of the *Connaissance des Temps* for 1915.

The observations at Greenwich were reduced with the *N.A.* value of the parallax and semi-diameter. The former is in the mean, 0″·47 and the latter 0″·49 greater than the values used by Delaunay.

Date 1915.			Errors of R.A.		Errors of N.P.D.	
			N.A.	Delaunay.	*N.A.*	Delaunay.
	d	h	s	s	″	″
Jan.	1	12	-0·99	-0·33	-1·0	-1·1
	2	13	-0·84	-0·22	+0·6	+1·0
	5	16	-0·59	-0·06	-3·7	-2·2
	8	18	-0·70	-0·13	-2·5	-1·4
	9	19	-0·63	-0·03		
	10	20	-0·76	-0·14		
Feb.	2	14	-0·75	-0·17	-3·4	-1·2
	3	15	-0·71	-0·16	-3·8	-1·7
	4	16	-0·72	-0·19	-2·0	-0·3
	7	19	-0·79	-0·21	-0.4	-1·4
Feb.	19	4	-0·73	-0·04	+2·6	-1·1
	21	6	-1·25	-0·49	-0·9	-3·5
	24	8	-1·25	-0·45	-0·8	-1·0
	27	11	-1·11	-0·39		
Mar.	1	12	-0·67	-0·00		
	5	16	-0·67	-0·07	-1·6	-0·8
Mar.	19	3	-0·97	-0·29	+3·5	+0·3
	21	4	-1·18	-0·41	+3·4	+1·6
	23	6	-1·17	-0·35	+1·7	+1·7
	25	8	-0·93	-0·16	-3·1	-1·9
	27	9	-0·88	-0·20	-2·0	-0·2
	29	11	-0·83	-0·19	-5·7	-4·3
	31	13	-0·83	-0·20	-3·6	-2·6
Apr.	5	18	-0·74	-0·20	+1·5	-1·3
	7	20			-2·1	-5·8
Apr.	17	2	-0·73	+0·02	+3·5	+0·9
	18	3	-1·06	-0·27	+0·9	-0·9
	21	6	-1·25	-0·38	-2·5	-1·3
Apr.	23	7	-1·04	-0·23	-5·2	-3·1
	24	8	-0·92	-0·17	-3·3	-1·1
	26	10	-0·90	-0·27	-6·5	-4·7
	27	10	-0·82	-0·22	-5·9	-4·4
	28	11	-0·71	-0·10	-3·5	-2·6
	29	12	-0·75	-0·14	-3·8	-3·6
	30	13	-0·84	-0·23	-2·8	-3·1
May	1	14	-0·87	-0·25	-4·0	-4·8
	2	15	-0·57	+0·03	-1·0	-2·8
	5	18	-0·61	-0·16	+4·7	+1·2
	7	20	-0·97	-0·56	+0·7	-3·0
	8	21	-0·50	-0·07	+4·8	+0·9
May	19	5	-1·17	-0·30	-1·9	-0·3
	22	7	-0·99	-0·14	-5·2	-2·1
	23	7	-0·96	-0·16	-6·2	-3·3
	24	8			-5·7	-3·2
	25	9	-1·01	-0·33	-7·2	-5·0
	26	10	-0·97	-0·32	-3·5	-2·0
	28	12	-0·73	-0·09	-0·9	-1·3
	30	14	-0·83	-0·24	-1·4	-3·1
	31	15	-0·71	-0·17	+5·4	+2·9
June	1	16	-0·66	-0·17	+0·9	-2·2
	4	18	-0·85	-0·40	+4·5	+0·6
	7	21	-0·83	-0·31	+4·4	+0·6
June	15	2	-1·01	-0·22	-1·2	-0·2
	16	3	-0·84	-0·03	-1·7	+0·3
	17	4			-2·1	+0·6
	18	5	-1·02	-0·18	-5·3	-2·1
	19	5	-0·98	-0·13	-4·6	-1·1

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Date 1915.			Errors of R.A.		Errors of N.P.D.	
			N.A.	Delaunay.	*N.A.*	Delaunay.
	d	h	s	s	"	"
June	20	6	−0·89	−0·02	−5·4	−2·2
	21	7	−1·06	−0·19	−5·8	−2·9
	22	8	−1·15	−0·31	−5·5	−3·1
	30	16	−0·66	−0·22	+4·1	+0·9
July	1	16	−0·74	−0·31	+2·3	−1·1
	2	17	−0·82	−0·37	+4·9	+1·5
	3	18	−0·48	−0·02	+4·5	+1·0
	5	19	−1·13	−0·61	+2·1	−0·9
July	19	6	−0·84	+0·06	−4·3	−1·8
	20	6	−1·14	−0·21	−3·5	−1·4
	23	10	−1·10	−0·28	−1·5	−2·7
	24	10	−0·99	−0·27	+0·4	−2·0
	26	13	−0·87	−0·32	+3·1	−0·7
	27	13	−0·88	−0·39	+5·3	+1·5
	28	14	−0·65	−0·20	+3·5	+0·1
	29	15	−0·64	−0·21	+4·5	+1·3
	30	16	−0·58	−0·16	+3·8	+0·8
	31	16	−0·70	−0·25	+2·2	−0·5
Aug.	1	17	−0·74	−0·26	+3·0	+0·5
	3	19	−0·63	−0·10	+3·6	+1·9
	4	20	−0·86	−0·28	−2·7	−4·0
Aug.	18	6	−1·05	−0·04	−2·8	−2·2
	19	7	−1·14	−0·12	−0·7	−1·0
	23	11	−0·86	−0·20	+4·8	+0·9
	25	13	−0·69	−0·16	+3·2	−0·7
	26	14	−0·62	−0·14	+4·2	+0·6
	27	14	−0·65	−0·20	+4·6	+1·5
	29	16	−0·86	−0·39	+2·3	0·0
Sept.	1	18	−0·96	−0·42	+2·6	+1·6
	2	19	−0·95	−0·39	+6·4	+6·0
	3	20	−0·95	−0·35	+2·2	+2·4
	5	22	−1·06	−0·44	−4·4	−3·1
Sept.	16	6	−0·98	−0·11	+1·1	−0·2
	17	7	−0·97	−0·06	+1·3	−0·6
	19	9	−0·94	−0·13	+4·3	+0·5

Date 1915.			Errors of R.A.		Errors of N.P.D.	
			N.A.	Delaunay.	*N.A.*	Delaunay.
	d	h	s	s	"	"
Sept.	20	10	−0·83	−0·11	+6·2	+2·0
	21	11	−0·70	−0·06	+5·4	+1·0
	24	13	−0·67	−0·15	+2·9	−0·8
	25	14	−0·57	−0·07	+2·4	−0·7
	27	15	−0·67	−0·18	+2·1	+0·1
	30	18	−0·87	−0·38	−0·1	−0·5
Oct.	1	19	−0·98	−0·47	−0·6	−0·5
	4	21	−1·09	−0·51	−6·1	−4·0
Oct.	14	5	−0·88	−0·22	−0·4	−2·8
	15	6	−0·99	−0·33		
	17	8	−0·83	−0·15	+4·7	+1·0
	18	9	−0·77	−0·12	+4·4	0·0
	20	10	−0·72	−0·14	+4·8	0·0
	21	11	−0·79	−0·23	+4·1	−0·4
	22	12	−0·76	−0·23	+4·8	+0·7
	25	14	−0·80	−0·27	+1·8	−0·1
	26	15	−0·82	−0·28	+0·5	−0·7
	28	17	−0·82	−0·30	−1·4	−1·5
Nov.	1	20	−0·86	−0·30	−3·9	−1·6
	2	20	−0·87	−0·27	−4·4	−1·6
Nov.	13	6	−0·87	−0·31	+4·5	+0·9
	14	7	−0·82	−0·27	+5·0	+1·3
	15	7	−0·79	−0·24	+3·9	0·0
	16	8	−0·65	−0·09	+5·0	+0·8
	19	10	−0·65	−0·10	+5·8	+2·1
	26	16	−0·79	−0·26	−1·8	−1·2
Dec.	1	20	−0·84	−0·15	−5·8	−2·9
Dec.	10	4			+4·1	+0·3
	11	5	−0·79	−0·18	+5·1	+0·8
	12	5	−0·75	−0·17	+4·9	+0·6
	13	6	−0·75	−0·19	+5·1	+1·1
	21	12	−0·81	−0·22	−0·4	−1·1
	27	17	−0·74	−0·20	−3·7	−2·6
	29	18	−0·74	−0·13	−5·7	−4·2
	30	19	−0·82	−0·15	−5·5	−3·7

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The Relative Density of Stars with Proper Motions between certain limits in reference to their Galactic Latitudes. By Sir F. W. Dyson and W. G. Thackeray.

In the *Monthly Notices* for 1917 June a comparison was made of the relative densities of stars of different proper motions in different galactic latitudes from a consideration of the stars in Zone 24° – 32° N. dec. A similar comparison is now made for the stars of Boss's Preliminary General Catalogue, both for the interest in a comparison of bright stars with much fainter ones, and also because the proper motions of Boss's Catalogue are affected with very much smaller errors than the proper motions dealt with in Zone 24° – 32°.

Dividing the sky into bands of galactic latitude 0° to ± 15°, ± 15° to ± 30°, ± 30° to ± 50°, and ± 50° to ± 90°, which give four regions of approximately equal area, the following table is obtained for the distribution of the proper motions of stars in different galactic latitudes :—

Number of Stars with Proper Motions between certain limits in different Galactic Latitudes.

Gal. Lat.	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–9″.	9″–10″.	>10″.
+ 90° to + 50°	15	43	51	45	39	36	40	41	29	21	199
+ 50° to + 30°	27	53	99	79	74	59	52	37	35	30	197
+ 30° to + 15°	44	82	103	91	100	49	42	24	21	27	165
+ 15° to + 0°	58	175	142	103	83	61	48	30	18	20	152
– 0° to – 15°	128	233	163	131	89	55	30	28	17	24	145
– 15° to – 30°	78	117	127	97	60	59	35	21	24	26	176
– 30° to – 50°	28	85	71	74	60	49	50	36	25	24	204
– 50° to – 90°	21	61	67	64	45	55	42	27	27	22	194

The larger number of stars with proper motions > 10″ a century in high galactic latitudes is attributable to the greater mean effect of star-streaming on these stars owing to the vertices being in the galaxy. The uniformity of the numbers in different galactic latitudes for proper motions > 5″ and the progressive increase in the numbers of stars with proper motions < 5″ a century, as the galaxy is approached, is very clearly shown.

The ratios of the numbers in galactic latitudes 0°–15°, 15°–30°, 30°–50°, compared with those round the galactic poles 50°–90°, are shown in the following table :—

Relative Star Densities.

Gal. Lat.	0″–1″.	1″–2″.	2″–3″.	3″–4″.	4″–5″.	5″–6″.	6″–7″.	7″–8″.	8″–9″.	9″–10″.	>10″.
0° to + 15°	5·8	4·1	2·8	2·3	2·1	1·7	1·2	0·7	0·6	0·9	0·7
+ 15° to + 30°	2·9	1·9	2·0	2·0	2·6	1·4	1·0	0·6	0·7	1·4	0·8
+ 30° to + 50°	1·8	1·2	1·9	1·8	1·9	1·6	1·3	0·9	1·2	1·4	1·0
0° to – 15°	6·1	3·8	2·4	2·1	2·0	1·0	0·7	1·0	0·6	1·1	0·7
– 15° to – 30°	3·7	1·9	1·9	1·7	1·4	1·1	0·9	0·8	0·7	1·2	0·9
– 30° to – 50°	1·4	1·4	1·1	1·2	1·4	0·9	1·2	1·3	0·9	1·1	1·1

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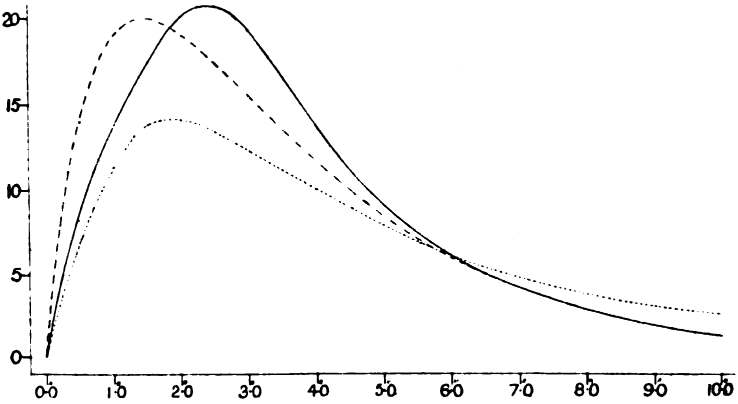
These results are in accordance with those given in *M.N.*, 1917 June, pp. 594–5, in showing that stars of sensible proper motion show a continual increase as the Milky Way is approached. This tendency is much greatest for stars of very small proper motion, but is well marked in the stars of proper motions < 6″ a century.

A comparison of the percentage of stars with different proper motions in Boss's Catalogue with that of the Greenwich Catalogue of Zone 24°–32° is given in the following table and in the accompanying diagram:—

Percentage of Stars with Proper Motions within certain limits.

Proper Motion.	Boss's Catalogue.	Greenwich Dec. 24°–32° Observed.	Greenwich Dec. 24°–32° Corrected.
″ ″			
0 – 1	7·0	8·2	14
1 – 2	13·8	17·5	20
2 – 3	13·4	20·7	16
3 – 4	11·1	16·6	14
4 – 5	9·0	11·0	10
5 – 6	6·9	6·9	
6 – 7	5·5	4·8	
7 – 8	4·0	3·3	
8 – 9	3·2	2·4	
9 – 10	3·2	1·5	
>10	23·4	7·1	



Boss's Catalogue . . . . . .
Greenwich Catalogue ———
Greenwich Catalogue, corrected – – – –

This comparison suggests generally the correction which the numbers obtained from the Greenwich Catalogue require on account of accidental error. The general effect of accidental error

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has been to increase the percentage of stars with proper motions from 2″–5″ a century, and diminish the number from 0″–2″. The curve marked – – – – is a rough attempt to correct the numbers for the effect of accidental error.

The numbers given in *M.N.*, p. 594, are largely affected. But the ratios on p. 595, giving the relative density of stars of different proper motion in different galactic latitudes, are altered very little. There seems no reason to doubt the conclusion that the stars with proper motions less than 5″·0 a century show an increasing tendency to crowd towards the galaxy.

A 13th Magnitude Star in Centaurus, with the same Parallax as α Centauri. By J. Voûte.

In Circular No. 30 of the Union Observatory in Johannesburg, the discovery by Mr. Innes was announced of a faint star in Centaurus with about 5″ proper motion.

Its position is, for

$$1916{\cdot}0 \quad \text{Ep. } 1916{\cdot}11 \quad 14^{h}\ 23^{m}\ 54^{s}{\cdot}28 \quad -62^{\circ}\ 19'\ 10''{\cdot}1,$$

measured from an astrophotographic plate, with reference to the two G.Z. Stars XIV. 1314 and XIV. 1396, near to it. As soon as more recent observations for comparison stars are available, a more accurate place can be given.

The star is near an 11·5 magnitude star. The differences in α and δ between the two stars have been observed as follows:—

$$\begin{array}{lll} 1916{\cdot}11 & \Delta\alpha = -26''{\cdot}06 & \Delta\delta = -5''{\cdot}88 \\ 1917{\cdot}48 & = -31{\cdot}09 & = -4{\cdot}74 \end{array}$$

Assuming that the comparison star has no sensible p.m., we obtain for the star in question one year's p.m.:

$$\text{in } \alpha = -3''{\cdot}66, \qquad \text{in } \delta = +0''{\cdot}83.$$

The difference in magnitude photographically and visually is at least 2·5, viz. visually 11·0 and photographically 13·5. The star therefore must be reddish, and we may suggest it being of the Mb type.

The parallax has been determined by the same method as that described in *M.N.*, lxxvii., No. 2, for three southern binaries, but the full description will be given in the paper in which the whole photographic parallax work will be described.

Every plate gives an equation of condition between the relative shift of the star, its parallax and proper motion.

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From four plates we get:—

$\pi = +0''.755 \pm 0''.028$. Relative mean magnitude comparison stars 11·4.

$\mu_\alpha = -3''.67$. In good agreement with above determination.

The total motion is 3″·76; 282°·7.

Its magnitude, reduced to a distance of 10 parsecs., being 15·4 or 17·9 (visually or photographically), it may be considered at the present moment the faintest star known.

Now we have for α Centauri: p.m. 3″·68, 281°·4
$\pi = +0''.759$ (Gron. 24).

It will be seen that we get a result both as regards parallax and proper motion in close agreement with that of α Centauri, which is certainly interesting for two stars so far separated from each other (viz. 2° 12′).

Are they physically connected or members of the same drift?

Royal Observatory, Cape of Good Hope:
1917 *June* 27.

Photographic Magnitudes of B.D. + 56°.547. By C. Martin and H. C. Plummer.

1. In a former paper* the results of nearly four years' photographic observations of three variable stars in the region of χ Persei, made at Dunsink, have been communicated to the Society. Quite lately, in a letter dated 1917 June 16, Mr. J. Van der Bilt draws attention to the star B.D. + 56°.547, of which he has discovered the variability, with a reference to Harvard Bulletin, No. 625. On his suggestion this fourth variable has now been measured on the Dunsink plates, together with a set of four neighbouring comparison stars.

2. Details relating to the comparison stars are given in Table I. The magnitudes of these four stars for two plates are based on

TABLE I.

B.D. +56°.	Plate 4. m_1.	Plate 6. m_2.	$m_2 - m_1$.	Provisional Phot.-Mag.	Corr.	Final Mag.	P.E.	Range.
534	9·90	9·88	−·02	9·89	+·004	9·89	±·013	·13
557	10·00	10·00	·00	10·00	−·003	10·00	±·011	·11
538	10·21	10·20	−·01	10·20	−·005	10·20	±·015	·16
540	10·39	10·36	−·03	10·38	+·006	10·39	±·014	·16

Schwarzschild's system for the region. The means given in the fifth column have been used in the reduction of the variable. The plate residuals furnish small mean corrections, and hence a set of final magnitudes, which differ so little from those first adopted as

* *M.N.*, lxxiv. p. 686.

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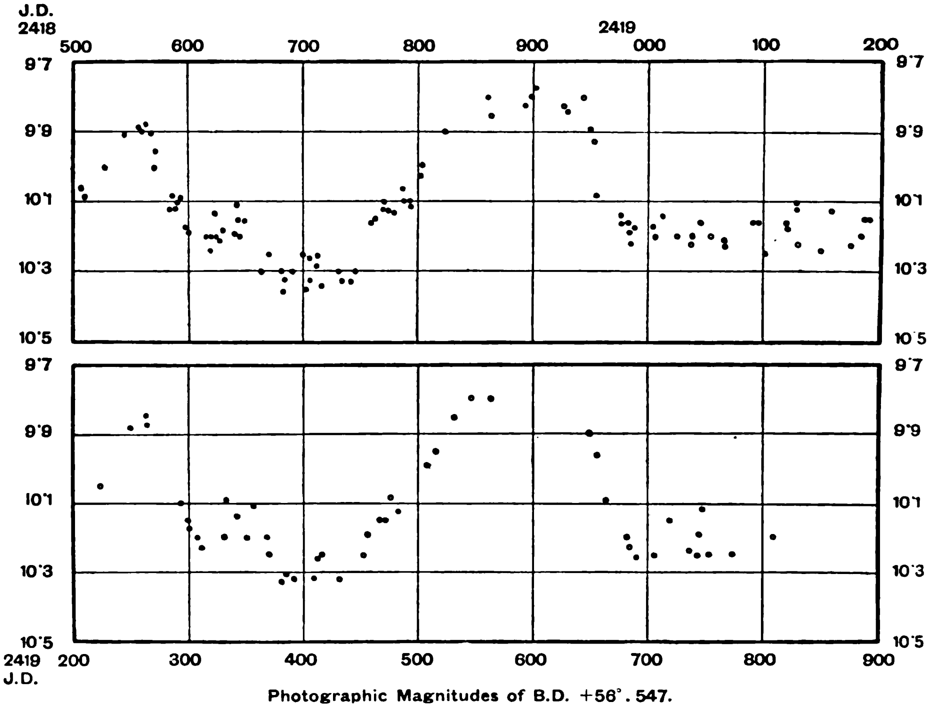
to call for no revision of the work. The probable error refers to a single measure, and the extreme range in the residuals is set down in the last column.

3. The derived photographic magnitudes of B.D. + 56°.547 are given in Table II., together with the Julian date and decimal fraction. The serial number corresponds with that given in Table I. of the paper quoted, from which may be drawn, if desired, the calendar date and G.M.T. for the middle of each exposure and also the length of exposure. It will be noticed that one plate, pre-



Photographic Magnitudes of B.D. +56°. 547.

viously omitted, has been interpolated between Nos. 165 and 166. Also six plates not belonging to the earlier series have been added at the end. These are plates which have been exposed on the regions of other variables, but also on the region of χ Persei, for the purpose of establishing the magnitude scale.

4. With the exception of the results from the six plates last mentioned, all the magnitudes, covering a period of about 1300 days, are plotted in the diagram which illustrates this paper. The total range slightly exceeds half a magnitude (9·8 – 10·3). The three variables of the region, to which reference has been made, appear to be absolutely irregular. At first sight the present star, when the observations are plotted in one continuous strip, appears

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to share this character, though its variations are more definite. The almost stationary phase in the interval J.D. 2419,000 – 200 then suggests a resemblance to SS Cygni, in spite of the relatively small range. But on closer inspection a fact appears which is readily perceived when the record is presented in two consecutive sections of 700 days. In the second interval the star is repeating very faithfully the sequence of changes recorded in the first interval. To all appearance the variations are periodic, and follow a cycle of very approximately 700 days.

TABLE II.

Serial No.	Julian Day.	Magn.	Serial No.	Julian Day.	Magn.	Serial No.	Julian Day.	Magn.
	2418		45	682·35	10·30	87	901·50	9·77
3	507·54	10·06	46	683·35	10·36	88	928·43	9·83
4	510·51	10·08	47	684·36	10·32	89	930·39	9·84
5	528·52	10·00	48	691·37	10·30	90	944·38	9·80
6	544·54	9·91	51	701·34	10·25	91	950·38	9·89
7	557·47	9·88	52	703·44	10·35	92	952·34	9·93
8	560·44	9·89	53	705·35	10·26	93	956·40	10·08
9	563·41	9·87	54	706·35	10·33	94	977·34	10·14
10	567·45	9·90	55	711·31	10·28	95	978·37	10·16
11	570·49	10·00	56	713·30	10·25	97	981·41	10·16
12	572·46	9·95	58	717·51	10·34	98	983·36	10·19
13	585·33	10·12	60	731·46	10·30	99	985·32	10·22
14	586·38	10·08	61	735·40	10·32	100	987·32	10·17
15	588·42	10·12	62	741·46	10·33		2419	
16	591·40	10·10	64	746·46	10·30	101	004·33	10·17
17	593·39	10·09	67	760·38	10·16	102	005·32	10·20
18	598·39	10·17	70	763·40	10·15	103	012·28	10·14
19	600·38	10·18	72	770·42	10·12	104	026·43	10·20
25	617·33	10·20	73	771·42	10·10	105	038·42	10·22
26	618·31	10·20	74	772·43	10·12	106	039·42	10·20
27	620·30	10·24	75	778·45	10·13	107	046·34	10·16
28	621·32	10·20	76	788·42	10·06	108	055·42	10·20
29	623·48	10·13	77	790·42	10·10	109	067·34	10·21
31	627·34	10·21	78	791·46	10·10	110	068·34	10·23
32	631·30	10·18	79	795·54	10·11	111	091·35	10·16
35	641·27	10·19	80	802·59	10·03	112	096·45	10·16
36	642·30	10·11	81	803·57	9·99	113	102·46	10·25
37	644·47	10·15	82	823·53	9·90	114	120·41	10·16
38	645·29	10·20	83	861·54	9·80	115	121·42	10·17
40	649·27	10·15	84	864·54	9·85	116	129·40	10·12
42	664·40	10·30	85	893·50	9·82	117	129·42	10·10
43	672·26	10·25	86	899·45	9·80	118	130·40	10·22

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TABLE II.—*continued.*

Serial No.	Julian Day.	Magn.	Serial No.	Julian Day.	Magn.	Serial No.	Julian Day.	Magn.
119	150·40	10·24	142	371·66	10·25	163	664·43	10·09
121	160·49	10·13	143	382·41	10·33	164	683·42	10·20
123	175·51	10·23	144	386·42	10·31	165	685·41	10·23
124	184·53	10·20	145	393·52	10·32	...	690 52	10·26
125	186·49	10·15	146	410·37	10·32	166	707·43	10·25
126	188·49	10·15	147	413·33	10·26	168	720·40	10·15
127	224·51	10·05	148	416 51	10·25	169	737·49	10·24
128	250·52	9·88	149	432·30	10·32	170	743·34	10·25
129	263·44	9·85	150	453·42	10·25	171	746·56	10·19
130	264·43	9·87	151	456·36	10·19	172	748·55	10·12
131	294·44	10·10	152	468·38	10·15	173	753·43	10·25
132	300·41	10·15	153	470·41	10·15	175	773·52	10·25
133	301·40	10·17	154	477·43	10·08	176	809·32	10·20
134	308·35	10·20	155	483·37	10·13		2420	
135	313·39	10·23	156	509·40	9·99	...	064·39	10·17
136	333·39	10·20	157	516·42	9·95	...	065·37	10·13
137	334·64	10·09	158	532·49	9·85	...	210·38	10·07
138	343·40	10·14	159	547·52	9·80	...	216·38	10·00
139	351·35	10·20	160	565·54	9·80	...	369·51	10·02
140	357·28	10·11	161	651·51	9·90	...	425·37	10·22
141	369·69	10·20	162	657·51	9·96			

5. The star appears then to be one of quite peculiar interest. By identifying the more salient features of the curve in the two successive cycles a period of 702 days was obtained. The material is insufficient for a much closer estimate. In order to extend the period of observation and to confirm the nature of the variation, the six plates at the end of Table II. were brought in. When the six magnitudes are antedated by 1400 days and plotted on the upper section of the diagram, they conform perfectly with the general run of the earlier magnitudes—so well, indeed, that it is difficult to deduce from their positions any decided correction to the above period. If anything, they seem to fall a little late, and suggest that the period is rather more than 700 days. The elements of principal maximum may be taken to be:

J.D. 2418 884 + 704 E.

Some little time must elapse before any considerable improvement on these elements can be expected, but Mr. Van der Bilt is already in possession of later observations.

6. Table III. contains some information relative to the four known variables in the region of χ Persei. None of them in any way resembles the cluster type. The visual magnitudes in the table

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are taken from the A.G. Catalogue, and are compared with the mean photographic magnitude in deriving the colour indices. The latter must therefore be regarded as merely rough estimates. But the inference is tolerably plain that all these stars are very red.

TABLE III.

B.D.	A.G.C. (Helsingfors-Gotha).	Mag. (visual).	Mag. (photographic).	Colour index.
+56°·551	2131	7·8	9·8 – 10·5	+2·3
583	2166	8·5	9·9 – 11·2	+2·1
55 ·597	2160	7·7	9·2 – 10·3	+2·1
56 ·547	2120	7·2	9·8 – 10·3	+2·8

Mr. Van der Bilt suggests in his letter that the fourth star may be found to resemble the three stars already discussed at Dunsink. In physical condition it probably does so. But in type of variation there appears to be a difference. The three stars vary in that irregular way which is probably characteristic to a greater or less extent of all very red stars. If now the regular variation in a periodic cycle (which was entirely unexpected) be confirmed in a star presumably in a similar physical condition—the assumption may be unjustified, of course,—we may see here an interesting link between the regular and irregular types.

7. The curve is evidently complex, and its details cannot easily be represented by a formula. Nevertheless its main outline is given by the very simple equation

$$m = 10·09 - 0·095 \ (\cos \theta + 2 \cos 2\theta),$$

where $\theta = 0$ corresponds to the above principal maximum. To obtain a closer approach to the curve it is necessary to add a number of higher harmonics, none of which exceeds 0·035 in amplitude.

Dunsink Observatory:
1917 *Aug.* 14.

On the Spiral Nebula, M 51, Canum Venaticorum.
By Professor L. Becker, Ph.D.

1. The accompanying diagram is the result of measurements of the photograph of the Spiral Nebula, M 51, as printed on plate xxix. in the publications of the Yerkes Observatory of the University of Chicago, vol. ii., one of the spirals (No. II.) being turned 180° in the diagram.

The measuring apparatus employed permits of the measurement of polar co-ordinates. The central condensation of the photograph was carefully centred with reference to the centre of rotation of the measuring slide, and the distances of the outlines of the two spirals (I. and II.) were measured at every tenth degree in position-angle.

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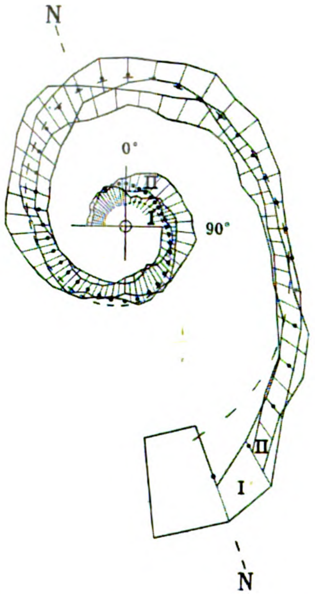
Let p (0° to 520°) designate the position-angle of a point on Spiral I. as counted from an arbitrary zero, and let R_I and R_{II} be the scale readings of two corresponding and diametrically opposite points on the two spirals. I find from these figures that

$$\frac{1}{2}(R_I + R_{II}) = R_0 + v,$$

where R_0 differs from a constant value by 1·2 sin $(2p - 220°)$. This periodic term, whose amplitude is given in unit of 0·01 inch and is about a sixteenth of the diameter of the central condensation, must be due to the measuring machine. The diagram is



drawn with the distances $\rho_I = R_I - R_0$ and $\rho_{II} = R_0 - R_{II}$, Spiral II. being turned 180°. The distances between the two spirals equal $2v$, and these departures are of the order of the irregularities occurring in the outlines of the spirals. It may be said that the two spirals show the same tendency, and may be considered identical apart from accidental differences.

The table contains, under the heading "Observed Middle Curve," the arithmetic mean of the distances belonging to inner and outer outline.

2. I calculate $\frac{1}{2}(\rho_I + \rho_{II})$ for both inner and outer outlines and their means, ρ. Obviously ρ is independent of the adopted values of R_0. The average spiral as represented by ρ is the apparent spiral, and it is the projection of the real spiral on our reference plane drawn at right angles to the line of sight. The distances, ρ, of the apparent spiral generally increase with the position-angle, p,

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though at some places they actually decrease with p. The observed distances may be represented by a power series in p, but evidently a large number of terms will be required so as to make the differential quotient by p to vanish at many values of p. The difficulty may be the effect of projection.

Consider three points of the real spiral, 180° apart in position-angle, and draw lines parallel to each other from these points to the line of sight that passes through the origin of ρ. Such lines can always be drawn even though the spiral is screw-shaped. The lengths of the lines are respectively r_p, r_{p+180}, r_{p+360}. The declinations of r_p and r_{p+360} are δ_p, and that of r_{p+180} is $-\delta_p$, the declinations being counted from the reference plane. Tentatively I then investigate whether the r_p may be so drawn that they are represented by a power series containing no higher power than p^2, while no assumption is made as to the relative position of the r_p at different values of p. Hence

$$r_p = x(1 + yp + zp^2),$$

$$r_{p+180}/r_p = \rho_{p+180} \sec \delta_p/\rho_p \sec \delta_p = \rho_{p+180}/\rho_p = \mathrm{M}_p,$$

and further

$$ay + bz = n,$$

$$a = p + 180 - \mathrm{M}_p p, \quad b = (p + 180)^2 - \mathrm{M}_p p^2, \quad n = \mathrm{M}_p - 1.$$

The 35 equations belonging to the values of p from 0° to 340° are solved by the method of least squares. The resulting values of y and z are then substituted, and the numerical values of $x \cos \delta_p$ are calculated for each value of p.

$$x \cos \delta_p = \rho_p/(1 + yp + zp^2).$$

The 52 values of $x \cos \delta_p$ turn out to be periodical in p, and show that the δ_p are the declinations of the points of a plane which is inclined to the reference plane. The longitude, N, of the node of the plane and its inclination, i, are

$$\mathrm{N} = 160^\circ, \quad i = 42^\circ.$$

3. Though the power series in p satisfactorily represents the measured distances, I introduce an equivalent formula which is independent of the line of sight. Let l designate the longitude in the orbit, $l = \mathrm{N} + u$, then

$$\tan u = \tan (p - \mathrm{N}) \sec i \quad \text{and} \quad \tan \delta_p = \sin (p - \mathrm{N}) \tan i,$$

$$r_p = \rho_p \sec \delta_p, \quad r_p = x'(1 + y'l + z'l^2).$$

The unknowns, x', y', z', are computed from 52 equations by the method of least squares. The result is, in units of 0·01 inch,

$$r_p = 13{\cdot}61 \pm 1{\cdot}15 + \frac{l}{500}(7{\cdot}42 \pm 5{\cdot}2) + \left(\frac{l}{500}\right)^2 (55{\cdot}3 \pm 4{\cdot}8)\,;$$

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or, since 1 mm. on the photograph represents $13''\cdot 2$,

$$r_p = 44''\cdot 9 \pm 4''\cdot 0 + \left(\frac{l + 33^\circ \pm 24^\circ}{500}\right)^2 (185''\cdot 4 \pm 16'').$$

The values of r_p and ρ_p (col. 5 in table) were calculated from these formulæ, and with them the curve is drawn which is marked by dots on the diagram. Of the 53 points, 31 points lie either within the space where the two spirals overlap or between the two spirals, 17 lie on Spiral I., and 5 on Spiral II. On the whole, the trend of the spirals appears to me to be satisfactorily reproduced by the dotted curve.

4. The results derived from the measurements may again be stated.

(1) The two spirals resemble each other, and corresponding points stand diametrically opposite to each other and equally far from a certain point near the middle of the condensation.

(2) Lines of length $r_p = a + bl^2$ may be drawn from the points of the real spiral to the line of sight that passes through the central point, defined under (1), and these lines, when displaced parallel to themselves to a point, all lie in a plane. The plane's longitude of node is 160°, and its inclination to the sphere is 42°.

I infer from the first result that the motion of the particles is outwards, for it is unlikely that particles coming from the outside would arrange themselves in the two spirals in an identical way.

The second result suggests that the longitude, l, may increase proportional to time, and that the formula for r does not only give the positions of the particles at the present time, but also the positions of a particle at any time. On this hypothesis a particle would be accelerated by a constant central force passing through the point to which the line r_p is drawn.

Such a constant force may be produced, as will be shown below, by the particles of a certain spheroid whose equator coincides with the plane of the nebula, and in which each particle acts according to the law of the inverse square of the distance.

The central force exerted by a homogeneous spheroid of density, D, on an exterior point situated on its equator at distance, r, from its centre is

$$R = -c \cos\phi \sin^{-3}\phi(\psi - \tfrac{1}{2}\sin 2\psi)Dr,$$

in which ϕ is the angle of eccentricity, $2a$ the major axis, $\sin\psi = \sin\phi a/r$, and the constant, c, is positive for attraction and negative for repulsion. The force exerted on an interior point is represented by the same formula, provided ψ be changed into ϕ. Let now the density, D, change with the distance from the centre and be constant in each shell enclosed between two spheroids of the same eccentricity and major axes $2a$ and $2(a+da)$. The force exerted by the particles within the spheroidal space

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on one of these particles which is situated on the equator at distance, r, is then found from the formula

$$R = \int_0^r D \frac{\partial(R/D)}{\partial a} da.$$

Assume $D = c'(a+\alpha)^{-k}$ where c', α, k have constant values and α is small compared with the distances, r, that have been measured.

$$R = -2cc' \cos\phi r^{-2} \int_0^r a^2(a+\alpha)^{-k}(1 - \sin\phi^2 a^2/r^2)^{-\frac{1}{2}} da.$$

This has a constant value for $k = 1$, as α is negligible compared with r. Hence, on the hypothesis that the longitude, l, increases proportionally to time, the dotted curve gives the position of the particles if they had been moving in a space filled with particles whose number per unit volume in each spheroidal shell is inversely proportional to $(a+\alpha)$.

5. I shall now investigate whether the spiral may also be satisfactorily represented on the assumption of another law of density of the particles. Taking $k = 3/2$, I have calculated the corresponding curve, and shown it in the diagram by a broken line. The formulæ may be derived in the following way. The formula for R becomes, with $k = 3/2$,

$$R = Cr^{-\frac{1}{2}},$$

where C is a positive constant for repulsion. The equation of motion is

$$\ddot{r} = Cr^{-\frac{1}{2}}.$$

The constant, H, in the integral of energy is positive and, therefore, also $G = 2C/H$ is a positive constant quantity. The integral becomes, if x be put for $Gr^{\frac{1}{2}}$ and c and t_0 designate constants,

$$c(t - t_0) = (x - 2)(1 + x)^{\frac{1}{2}} = X.$$

Let again the longitude, l, increase proportional to time, then

$$X = al + b.$$

With the object of finding the numerical value of G, I choose the numerical values of $r_1 = 10{\cdot}9$, $r_2 = 37{\cdot}1$, $r_3 = 90{\cdot}4$, belonging respectively to $l_1 = 6°$, $l_2 = 256°$, $l_3 = 506°$. The following equation must be satisfied:

$$F = 2(Gr_2^{\frac{1}{2}} - 2)(1 + Gr_2^{\frac{1}{2}}) - ((Gr_1^{\frac{1}{2}} - 2)(1 + Gr_1^{\frac{1}{2}}) \\ - ((Gr_3^{\frac{1}{2}} - 2)(1 + Gr_3^{\frac{1}{2}}) = 0.$$

There is no positive root of this equation for G, because $F = 0$ for $G = 0$, and dF/dG is always positive for positive G. It follows that the three values of r cannot be perfectly represented on the above assumptions. A curve defined by the equations above can, however, be drawn near to the observed

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curve. I choose arbitrarily $G=1$, which value leads to as good a representation of the measurements as any of the other values tried. I then find the values of X for each observed value of r belonging to longitude, l, and hence the constants a and b by means of a graph of X with the abscissæ l. The graph is not a straight line, as it would be if the formula for D gave the actual density, but becomes steeper as l increases. I find $X = l18/520 + 1{\cdot}0$ and the inclination of the plane of the spiral $i = 35^{\circ}$. The points of the broken line are calculated with these values, and the figures for their co-ordinates are given in the last two columns of the table. The broken line obviously does not represent the spiral satisfactorily at its beginning and end, and this might have been expected from the graph of X.

The adopted number per unit volume, $D = c(a+\alpha)^{-1{\cdot}5}$, nearly agrees with that in the spirals if the particles of the spiral move according to the hypothesis stated in section 4, and keep between two parallel planes. For the space occupied by a given number, N, of particles is proportional to $r\dot{r}$, and therefore

$$D = cN/r\dot{r} = Cr^{-1}(r-e)^{-0{\cdot}5} = Cr^{-1{\cdot}5} \text{ nearly,}$$

as $r = e + fl^2$, $\dot{l} = n$, and e is small compared with r.

6. According to the last two sections, the central force which gives to a particle the required acceleration in a radial direction might be supplied by particles arranged in spheroidal shells containing a certain number of particles. This number per unit volume is nearly inversely proportional to the kth power of the major axis of a shell where k has a value near to unity and is certainly less than $1{\cdot}5$.

As to the uniform increase of the longitude with time, the particles may actually move at constant longitudes, and successive particles may start from the centre at regular intervals of time in directions which are turning round the centre at a uniform rate. Such conditions may happen when there are two bodies moving in eccentric orbits round each other, and a third body causing their periastron to move round at a nearly uniform rate. At and near to periastron great changes in the intensity of surface gravity may be conceived to occur, followed by eruptions. Conditions may prevail which are similar to those sometimes happening on the sun, and the deductions made by Arrhenius in the case of the sun* would also hold good here. Negatively charged particles would be emitted by both bodies and at points diametrically opposite to each other. After a stellar year the same process would repeat, but at a longitude slightly larger than at the preceding disturbance, in consequence of the secular perturbation of the periastron. These radially emitted particles would form the spirals. But, apart from them, particles also negatively charged leave the stars in all directions, and it may be these which supply the forces which propel the particles almost proportional to the square of time.

* Arrhenius, *Lehrbuch der kosmischen Physik*, part i., p. 150, etc.

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	Observed Middle Curve.		Calculated.			
			Section 3.		Section 5.	
p.	$\rho_{I.}$	$\rho_{II.}$	*l*.	ρ.	*l*.	ρ.
0	8.4	12.4	6.1	13.1	4.0	6.6
10	6.3	13.7	17.8	12.8	15.2	7.3
20	6.9	14.2	28.5	12.3	25.7	7.9
30	6.7	15.7	38.1	11.9	35.5	8.5
40	6.9	15.8	46.8	11.7	44.7	9.0
50	10.5	15.8	54.9	11.5	53.4	9.5
60	9.9	16.0	62.5	11.5	61.8	10.0
70	11.0	15.3	70.0	11.7	70.0	10.6
80	11.4	16.1	77.5	12.0	78.2	11.3
90	12.1	17.0	85.1	12.6	86.6	12.2
100	13.5	18.4	93.2	13.3	95.3	13.3
110	13.8	18.2	101.9	14.4	104.5	14.5
120	14.9	17.9	111.5	15.7	114.3	15.9
130	16.2	18.0	122.2	17.1	124.8	17.5
140	18.0	18.5	133.9	18.7	136.0	19.3
150	18.8	17.4	146.7	20.4	147.9	20.9
160	18.1	19.6	160.0	21.7	160.0	22.4
170	18.4	20.7	173.3	22.6	172.1	23.5
180	18.0	20.0	186.1	23.0	184.0	24.3
190	20.3	19.7	197.8	22.9	195.2	24.8
200	21.3	20.8	208.5	22.7	205.7	25.1
210	23.2	20.8	218.1	22.6	215.5	25.3
220	24.0	22.7	226.8	22.4	224.7	25.5
230	23.8	24.3	234.9	22.4	233.4	25.7
240	23.6	25.2	242.5	22.6	241.8	26.2
250	24.3	25.5	250.0	23.1	250.0	26.8
260	24.6	28.1	257.5	24.0	258.2	27.9
270	25.7	31.0	265.1	25.3	266.6	29.1
280	27.8	32.6	273.2	26.9	275.3	30.6
290	29.6	33.4	281.9	29.2	284.5	32.6
300	31.6	35.5	291.5	31.8	294.3	34.7
310	33.4	38.7	302.2	34.9	304.8	37.2
320	37.4	41.6	313.9	38.2	316.0	39.7
330	40.0	42.4	326.7	41.5	327.9	42.1
340	44.6	41.3	340.0	44.2	340.0	43.9
350	46.9	38.8	353.3	45.9	352.1	45.1
360	48.1	39.4	366.1	46.5	364.0	45.7
370	48.7	39.2	377.8	46.2	375.2	45.7
380	49.8	38.6	388.5	45.7	385.7	45.4
390	49.8	40.8	398.1	45.0	395.5	45.0
400	47.0	42.3	406.8	44.4	404.7	44.7
410	46.8	42.3	414.9	44.3	413.4	44.7
420	45.8	42.2	422.5	44.5	421.8	45.0
430	46.4	41.7	430.0	45.3	430.0	45.6
440	45.8	43.5	437.5	46.8	438.2	46.8

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	Observed Middle Curve.		Calculated.			
			Section 3.		Section 5.	
p.	ρ_{I}.	ρ_{II}.	l.	ρ.	l.	ρ.
			°		°	
450	46·0	44·7	445·1	49·0	446·6	48·4
460	49·0	47·8	453·2	51·9	455·3	50·6
470	52·9	51·5	461·9	55·7	464·5	53·2
480	57·8	60·7	471·5	60·4	474·3	56·2
490	65·0	66·6	482·2	65·9	484·8	59·6
500	71·8	70·9	493·9	71·4	496·0	63·0
510	83·3	82·5	506·7	77·1	507·9	65·8
520	90·3	...	520·0	81·1	520·0	68·0

Note on the Latitude of Glasgow Observatory.
By Professor L. Becker, Ph.D.

In the years 1897 to 1907 I observed with the Transit Circle the pole star B.D. 89°. 37 whose daily orbit lies in the field of the telescope. The results of these observations are being published in the *Memoirs* of the Society.* I have explained there the method by which, without reference to the almanac positions of pole stars, the position of the true pole can be found from two or more observations of B.D. 89°. 37 taken during a night, and the interval in sidereal time between the observations. The interval was about three hours. These observations give finally the co-ordinates of the reference star, and herewith for each day the pole point, (D) + dD, of the circle, *i.e.* the circle reading when the telescope is directed towards the true pole. (D) stands for $N \pm (90° + c)$ where N denotes the circle reading at the Nadir direction of the telescope, and c has the constant value 55° 52′ 42″·20. The upper sign belongs to the position of the axis when the circle lies towards the west, and the lower sign to the reversed position. The latitude, ϕ, on a certain day satisfies the relation, $\mp(90 + \phi) = N + dN \pm w - (D) - dD$. Hence

$$\phi = c \pm dD \mp dN - w \quad . \quad . \quad . \quad . \quad (1)$$

dN is the unknown division error at the Nadir position, and it has the same value at both positions of the axis. $w = v_0 + v$, v_0 denoting systematic station errors and v the accidental errors.

Let further ϕ_0 be the geographical latitude referred to the "mean pole" on the earth's surface adopted in the publications of the International Latitude Service, and let the instantaneous pole on the earth's surface have the co-ordinate ξ along the axis drawn from the mean pole towards Glasgow. Hence

$$\phi = \phi_0 + \xi \quad . \quad . \quad . \quad . \quad (2)$$

$$\phi_0 = c \pm dD \mp dN - \xi - v_0 - v \quad . \quad . \quad . \quad (3)$$

* Vol. lxii.

The observations utilised here were taken on 79 days when the circle was placed on the west side, and on 74 days with circle east. They give $dN = +0''.14$, and this value of dN was employed in correcting the dD according to formula (1). The accuracy of dD on a certain day depends on the number of points of the orbit of B.D. 89°.37 used in its determination. The adopted weight, p, is 1 for 1 point, 1·5 for 2 points, . . . and 3 for 5 points. The daily values of $\pm(dD - dN)$ are combined in normal places which, together with their weights, p, are shown in the second and third columns of the table below. The values of ξ in the fourth column were obtained from the publications of the Zentralbureau der Internationalen Erdmessung. The fifth column contains the systematic errors v_0, which, if real, are probably not due to erroneous values of ξ, but to periodic changes in the direction of gravity. $v_0 = +0''.14 \sin(L + 217°)$, L being the longitude of the sun.

Year.	p.	$\pm(dD-dN)$.	$-\xi$.	$-v_0$.	$-v$.
		"	"	"	"
1897·61	1·5	−0·45	−0·17	+0·01	−0·50
98·23	7	−1·06	+0·18	+0·09	−0·68
98·58	12·5	−0·17	−0·01	+0·04	−0·03
98·73	10·5	−0·03	−0·18	−0·09	−0·19
98·92	10·5	+0·27	−0·14	−0·13	+0·11
99·20	12·5	−0·03	+0·05	+0·07	+0·20
99·39	11·5	−0·36	+0·08	+0·14	−0·03
99·64	13	−0·03	+0·04	−0·02	+0·10
99·95	10	−0·01	−0·03	−0·12	−0·05
1900·12	12·5	+0·09	−0·06	0·00	+0·14
00·40	9·5	−0·49	+0·05	+0·14	−0·19
00·61	9	−0·04	+0·06	+0·01	+0·14
00·82	12·5	−0·16	+0·07	−0·13	−0·11
01·11	12	+0·23	−0·03	−0·01	+0·30
01·68	12	+0·36	−0·03	−0·05	+0·39
01·83	13·5	−0·04	+0·04	−0·14	−0·03
02·08	14·5	−0·28	+0·07	−0·04	−0·14
02·28	8	+0·08	−0·03	+0·12	+0·28
02·78	19	−0·12	−0·05	−0·12	−0·18
03·25	9	−0·61	+0·10	+0·10	−0·30
04·11	9	−0·45	+0·15	−0·01	−0·20
04·23	9·5	−0·63	+0·17	+0·09	−0·26
05·29	9	−0·24	+0·15	+0·13	+0·15
06·99	5	+0·06	−0·05	−0·10	+0·02

The mean latitude becomes $\phi_0 = 55° \ 52' \ 42''.09 \pm 0''.05$.

The sum of the squares of the residuals with regard to weight, $[pvv]$, is 21·8 for zero values of both ξ and v_0, 14·6 when v_0 is neglected, and 12·2 when both ξ and v_0 are taken into account.

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Acta Mathematica. Zeitschrift, herausgegeben von G. Mittag-Leffler. Band 40, pt. 4.
(*Purchased.*) 4to. Stockholm, 1916

Allegheny Observatory:
Publications. Vol. 3, No. 22, 23.
(*Observatory.*) 4to. Lancaster, Pa., 1916

American Journal of Mathematics. Edited by Frank Morley. Published under the auspices of the Johns Hopkins University. Vol. 38, No. 3, 4; Vol. 39, No. 1, 2.
(*Editor.*) 4to. Baltimore, 1916–1917

American Journal of Science. Editor E. S. Dana [and others]. Fourth series, Vol. 41, 42, 43 (No. 246–257).
(*Editors.*) 8vo. New Haven, 1916–1917

Arcetri, Reale Osservatorio:
Pubblicazioni del R. Istituto di studi superiori pratici e di perfezionamento, fasc. 33, 34.
(*Observatory.*) 4to. Firenze, 1915–1916.
A. Abetti, Osservazioni astronomiche fatte . . . 1914–15.

Astronomical Herald (the). Published by the Astronomical Society of Japan [in Japanese]. Vol. 9, No. 2–12.
(*Society.*) 4to. 1916–1917

Astronomical Journal. Founded by B. A. Gould. Vol. 29, 30 (No. 692–711).
(*Editor.*) 4to. Boston, 1916–1917

Digitized by Google

Astronomie (L'), see **Paris, Société Astronomique de France.**

Astronomische Mitteilungen. Gegründet von Rudolf Wolf: herausgegeben von A. Wolfer. No. 106.
(*Editor.*) 8vo. Zürich, 1917

Astronomische Nachrichten:
——: Begründet von H. C. Schumacher: herausgegeben von H. Kobold. Band 202–204 (No. 4847–4882).
(*Editor.*) 4to. Kiel, 1916–1917
——: Literarisches Beiblatt zu den Astronomischen Nachrichten. No. 27.
(*Editor.*) 4to. Kiel, 1917

Astronomische Nachrichten. Vol. 36–107.
4to. Altona und Kiel, 1853–1884
(*Messrs. J. and W. Baxendell.*)

Astronomischer Jahresbericht. Begründet von W. F. Wislicenus. . . . Bearbeitet im Kgl. Astronomischen Rechen-Institut zu Berlin. Band 17. Die Literatur des Jahres, 1915.
(*Rechen-Institut.*) 8vo. Berlin, 1916

Astrophysical Journal (the). An International Review of spectroscopy and astronomical physics. Edited by G. E. Hale [and others]. Vol. 43, No. 4, 5; Vol. 44, No. 1–5; Vol. 45, No. 1–3.
(*Editors.*) 8vo. Chicago, 1916–1917
(*Another Copy purchased.*)

Athenæum (the). Journal of English and foreign literature, science, the fine arts, music, and the drama, No. 4606–4613.
(*Editor.*) 4to. London, 1916–1917

Banachiewicz (Th.):
*Sur la résolution de l'Equation de Gauss dans la détermination d'une orbite planétaire.
(*Author.*) 4to. Petrograd, 1916

Barcelona, Sociedad Astronomica:
Boletin, Año 7, 8. (No. 61–70.)
(*Society.*) 8vo. Barcelona, 1916–1917

Barnard (Edward Emerson):
——: *A few astronomical events of the past fifty years.
(*Author.*) 8vo. Northfield, Minn., 1916
——: *A small Star with large Proper Motion.
(*Author.*) 8vo. Northfield, Minn., 1916

Digitized by Google

Basel, Naturforschende Gesellschaft:
Verhandlungen; Band 27.
(*Society.*) 8vo. Basel, 1916

Batavia, Royal Magnetical and Meteorological Observatory:
Observations . . . published by order of the Government of Netherlands India. Vol. 35.
(*Observatory.*) 4to. Batavia, 1915–1916

Birmingham, Natural History and Philosophical Society:
——: Annual Report for 1915, and List of Members for 1916.
——: Proceedings, Vol. 14, pt. 1.
(*Society.*) 8vo. Birmingham, 1916

British Association for the Advancement of Science:
Report of the 85th meeting, held in Manchester, 1915.
(*Association.*) 8vo. London, 1916

Brown (Ernest William):
*The relations of Mathematics to the Natural Sciences. Presidential Address delivered at the 23rd Annual Meeting of the American Mathematical Society.
(*Author.*) 8vo. New York, 1917

Brown's Nautical Almanac, Harbour and dock guide, and daily tide tables for 1917.
(*Editor.*) 8vo. Glasgow [1916]

Brunt (David):
The Combination of Observations.
(*Author.*) 8vo. Cambridge, 1917

Bulletin Astronomique. Fondé en 1884 par E. Mouchez et F. Tisserand. Publié par l'Observatoire de Paris. Tome 33, No. 5–12; Tome 34, No. 1, 2.
(*Purchased.*) 8vo. Paris, 1916–1917

Bulletin des Sciences Mathématiques. Rédigé par G. Darboux et E. Picard. Série 2. Tome 40, No. 6–12; Tome 41, No. 1–4.
(*Editors.*) 8vo. Paris, 1916–1917

Calcutta, Astronomical Society of India:
——: Journal, Vol. 6, No. 4–9; Vol. 7, No. 1–3.
(*Society.*) 8vo. Calcutta, 1916
——: Monthly Notices, 1916 June, July, Aug., Oct., Nov.; 1917 Jan.–April.
(*Society.*) 8vo. Calcutta, 1917

Digitized by Google

Cambridge Philosophical Society:
Proceedings, Vol. 18, pt. 5, 6; Vol. 19, pt. 1.
(*Society.*) 8vo. Cambridge, 1916–1917

Cambridge University; Solar Physics Observatory:
Third Annual Report of the Director . . . to the Solar Physics Committee, 1915–1916.
(*Observatory.*) 4to. Cambridge, 1916

Canada, Royal Society:
Transactions, Section III (Mathematical, Physical, and Chemical Sciences). Series 3, Vol. 10, 1916 June, Sept., Dec., 1917 March.
(*Society.*) 8vo. Ottawa, 1916–1917

Cape of Good Hope, Royal Observatory:
——: Annals, Vol. 12, pt. 4.
(*Observatory.*) 4to. London, 1907
Vol. 12, pt. 4. Determination of the orbits of Jupiter's Satellites from photographs by Bryan Cookson.
——: Report of H. M. Astronomer . . . for the year 1915.
(*Observatory.*) 4to. London, 1916

Cape Town, Royal Society of South Africa:
Transactions, Vol. 5, pt. 4, 5.
(*Society.*) 8vo. Cape Town, 1916

Cape Town, South African Association for the Advancement of Science.
The South African Journal of Science, Vol. 12, No. 9–13; Vol. 13, No. 1–6.
(*Association.*) 8vo. Cape Town, 1916–1917

Catania, Società degli Spettroscopisti Italiani:
Memorie . . . Fondata da P. Tacchini: pubblicazione mensile continuata per cura di A. Riccò. Serie 2. Vol. 5, Disp. 5–12; Vol. 6, Disp. 1–4.
(*Society.*) 4to. Catania, 1916–1917

Chatham, Royal Engineers' Institute:
The Royal Engineers' Journal, Vol. 24, 25.
(*Institute.*) 8vo. Chatham, 1916–1917

Chevalier (S.):
*Étude photographique des Taches Solaires.
4to. Chang-hai, 1916

Chicago, Society for Practical Astronomy:
The Monthly Register, Vol. 8, No. 4–7.
(*The Editor.*) 8vo. Chicago, 1916

Digitized by Google

Cincinnati Observatory:
Publications, No. 18, pt. 2.
(*Observatory.*) 4to. Cincinnati, 1916
Catalogue of proper motion Stars.

Comas Solá (Jose):
*La Visión Esterioscópica aplicada a la Astronomia.
4to. Barcelona, 1916

Copenhagen, Kongelige Danske Videnskabernes Selskab:
——: Oversigt over det K. Danske Videnskabernes Selskabs Forhandlinger. 1916, No. 3.
(*Society.*) 8vo. Kjöbenhavn, 1916
——: Skrifter, naturvidensk. og mathematisk Afd.: 8te Række, Bd. 1, No. 3; Bd. 2, No. 2, 3.
(*Society.*) 4to. Kjöbenhavn, 1916

Copenhagen, Observatorium:
*Publikationer og mindre Meddelelser. No. 24–26.
(*Observatory.*) 4to. Kiel, 1916–1917

Cordoba, Observatorio Nacional Argentino:
Resultados. Vol. 21.
(*Observatory.*) 4to. Cordoba, 1914
Catalogo de las Zonas de Exploración, 4: —52° à —62°.

Dublin, Royal Irish Academy:
Proceedings (third series), Section A, Vol. 33, No. 1–6.
(*Academy.*) 8vo. Dublin, 1916

Dublin, Royal Society:
Scientific Proceedings. New Series. Vol. 15, No. 1–14.
(*Society.*) 8vo. Dublin, 1916

Duhem (Pierre):
Le Système du Monde. Histoire des Doctrines cosmologiques de Platon à Copernic. Tome 3, 4.
(*Purchased.*) 8vo. Paris, 1915–1916

Edinburgh, Royal Society:
——: Proceedings. Vol. 36, pt. 1–4; Vol. 37, pt. 1.
(*Society.*) 8vo. Edinburgh, 1916–1917
——: Transactions, Vol. 50, pt. 3, 4; Vol. 51, pt. 1.
(*Society.*) 4to. Edinburgh, 1916

English Mechanic and World of Science (the). Vol. 103–105 (No. 2673–2724).
(*Editor.*) fol. London, 1916–1917

Digitized by Google

Europe, Association Géodésique Internationale:
Rapport sur les Travaux du Bureau Central . . . en 1916.
(*Association.*) 4to. Leide, 1917

Evanston, Illinois, Dearborn Observatory:
Annals. Vol. 1.
(*Observatory.*) 4to. Evanston, Ill., 1915
Measures of double stars.

Fiji, Trigonometrical Survey:
Report of the Trigonometrical Survey of Viti Levu. By G. T. McCaw.
(*Crown Agents for the Colonies.*) 4to. London, 1916

Flint (Albert S.):
*On Evidences of Systematic Variation in recent longer series of Determinations of Stellar Parallax.
4to. Albany, N.Y., 1916

Furness (Caroline E.):
An Introduction to the Study of Variable Stars.
(*Purchased.*) 8vo. Boston and New York, 1915

Furuhjelm (Ragnar):
*Recherches sur les mouvements propres des Étoiles dans la zone photographique de Helsingfors. 1. Clichés de 9^h à 12^h.
4to. Helsingfors, 1916

Garavito (Julio):
La paradoja de la Optica Matematica (Teoria de la aberracion y de la refraccion de la luz). [Spanish and French.]
(*Author.*) 8vo. Bogota, 1916

Gautier (Raoul):
Rapport sur le Concours de réglage de chronomètres, 1915.
(*Author.*) 8vo. Genève, 1916

Geneva, Société de physique et d'histoire naturelle:
——: Mémoires, Tome 38, fasc. 4, 5; Tome 39, fasc. 1.
(*Society.*) 4to. Genève, 1916–1917
——: Comptes rendus des séances, 32, 1915.
(*Society.*) 8vo. Genève, 1916

Greenwich, Royal Observatory:
Report of the Astronomer Royal to the Board of Visitors, 1916.
(*Observatory.*) 4to. London, 1917

Digitized by Google

Halifax, Nova Scotian Institute of Science:
Proceedings and Transactions. Vol. 14, pt. 2.
(*Institute.*) 8vo. Halifax, 1916

Hall (Angelo):
An Astronomer's Wife: The Biography of Angeline Hall.
(*Dr Glaisher.*) 8vo. Baltimore, 1908

Harvard College Astronomical Observatory:
——: Annals, Vol. 73, pt. 3; Vol. 76, No. 11, 12; Vol. 80, No. 4–8; Vol. 84, No. 1–7.
(*Observatory.*) 4to. Cambridge, Mass., 1916–1917
——: Circular, No. 191–200.
(*Observatory.*) 4to. Cambridge, Mass., 1916

Helsingfors, Finska Vetenskaps-Societet:
Acta Societatis Scientiarum Fennicæ. Tomus 48, No. 1.
(*Society.*) 4to. Helsingforsiae, 1916

Hobart, Royal Society of Tasmania:
Papers and Proceedings, 1916.
(*Society.*) 8vo. Hobart, 1917

Horological Journal (the). Vol. 58, 59 (No. 694–705).
(*Brit. Horol. Institute.*) 8vo. London, 1916–1917

Hull (Edward):
*The Tidal Wave on the off side of the Earth from the Moon.
(*Author.*) 8vo. London, 1916

Hyderabad, Nizamiah Observatory:
Report, 1915–1916.
(*Observatory.*) 8vo. 1916

India, United Service Institution:
Journal, Vol. 45, 46, No. 204, 206, 207.
(*Institution.*) 8vo. Simla, 1916–1917

Innes (R. T. A.):
*The Masses of Visual Binary Stars.
(*Author.*) 8vo. Cape Town, 1916

Johannesburg, Union Observatory:
Circular, No. 34–36.
(*Observatory.*) 4to. Pretoria, 1916

Journal des Observateurs:
Vol. 1, No. 8–17.
(*Purchased.*) 4to. Marseille, 1916–1917

Digitized by Google

Kodaikánal and Madras Observatories:
——: Annual Report of the Director for 1916.
(*Observatory.*) fol. Madras, 1917
——: Bulletin, No. 49–54.
(*Observatory.*) 4to. Madras, 1916

La Plata, Observatorio Astronomico:
Resultado de las Observaciones en la Zona $-52°$ a $-57°$ 1913–1915. Tomo 3.
(*Observatory.*) 4to. La Plata, 1916

Le Coultre (Fridtjof):
*Contribution à l'étude physique des Étoiles Filantes.
(*The Laboratory, Geneva.*) 8vo. Lausanne, 1915

Leeds Astronomical Society:
Journal and Transactions, No. 23 (1915).
(*Society.*) 8vo. Leeds, 1916

Lick Observatory, University of California:
Bulletin, No. 281–290.
(*Observatory.*) 4to. Berkeley, 1916-1917

Lisbon, Observatório Astronómico:
Dados astronómicos para os Almanaques de 1917 para Portugal.
(*Observatory.*) 8vo. Lisboa, 1916

Lockyer (Sir J. Norman) and H. E. Goodson:
*On the oxy-hydrogen flame Spectrum of Iron.
(*Sir J. N. Lockyer.*) 8vo. London, 1916

London, British Astronomical Association:
——: Journal, Vol. 26, No. 7, 8, 9; Vol. 27, No. 1–6.
(*Association.*) 8vo. London, 1916–1917
——: Memoirs, Vol. 20, pt. 3, 4.
(*Association.*) 8vo. London, 1916
——: List of members, 1916.
(*Association.*) 8vo. London, 1916

London, Meteorological Office:
The Geophysical Journal, 1915, No. 1–7.
(*Meteorological Office.*) 4to. London, 1916

London, Nautical Almanac and Astronomical Ephemeris for the meridian of the Royal Observatory, Greenwich, 1919.
(*Lords Commissioners of Admiralty.*) 8vo. London, 1916

London, Physical Society:
Proceedings, Vol. 28, pt. 4, 5; Vol. 29, pt. 1–3.
(*Society.*) 8vo. London, 1916–1917

Digitized by Google

London, Royal Astronomical Society:
——: Monthly Notices, Vol. 77. 8vo. London, 1916–1917
——: Memoirs, Vol. 61, 62 pt. 1. 4to. London, 1917

London, Royal Geographical Society:
The Geographical Journal. Vol. 49, No. 6, 1917 June. [Containing article by Dr Ball on modern methods of finding the latitude with a theodolite.]
(*R. Geographical Society.*) 8vo. London, 1917

London, Royal Photographic Society:
The Photographic Journal; including the Transactions of the Royal Photographic Society, Vol. 56, No. 6–8, and Supplement; Vol. 57, No. 1–5.
(*Society.*) 8vo. London, 1916–1917

London, Royal Society:
——: Philosophical Transactions. Series A, Vol. 216, 217 (No. 547–551).
(*Society.*) 4to. London, 1916–1917
——: Proceedings, Series A, Vol. 92, 93 (No. 642–650).
(*Society.*) 8vo. London, 1916–1917
——: Year-book, 1917.
(*Society.*) 8vo. London, 1917
——: Provisional Reports [of meetings], 1916 Nov. 2–1917 June 28.
(*Society.*) 8vo. [London, 1916–1917]
——: Catalogue of Scientific Papers. Fourth Series, 1884–1900. Vol. 15. (Fitting-Hyslop.)
(*Purchased.*) 4to. Cambridge, 1916

London, University College:
Abridged Calendar, session 1916–1917.
(*The College.*) 8vo. London, 1916

Lowell Observatory:
Bulletin, No. 73–75.
(*Observatory.*) 4to. 1916

Lund, Astronomiska Observatorium:
——: *Meddelanden . . . No. 68–77.
(*Observatory.*) 8vo. Uppsala & Stockholm, 1915–1916
——: *Meddelanden . . . Serie 2, No. 14, 15.
(*Observatory*). 4to. Lund, 1916–1917
No. 14. C. V. Charlier. Studies in Stellar Statistics, III: Stars of Spectral Type B.
„ 15. Axel Jönsson. Über die Rotation des Mondes.

Madrid, Observatorio Astronómico:
Anuario, año 1917.
(*Observatory.*) 8vo. Madrid, 1916

Digitized by Google

Madrid, Real Academia de Ciencias exactas, físicas y naturales:
——: Revista . . . Tomo 12, No. 8–12; Tomo 13, No. 1–12; Tomo 14, No. 1–12; Tomo 15, No. 1–5.
(*Academy.*) 8vo. Madrid, 1914–1916
——: Anuario, 1915–1917.
(*Academy.*) 16mo. Madrid

Manchester Astronomical Society:
Journal of the Session 1915–1916. No. 3.
(*Society.*) 8vo. [Manchester, 1916]

Manchester Literary and Philosophical Society:
Memoirs and Proceedings, Vol. 60, pt. 1–3.
(*Society.*) 8vo. Manchester, 1916–1917

Manchester Municipal School of Technology:
Report of the Godlee Observatory for the years 1913 and 1915.
(*Observatory.*) 8vo. Manchester, 1916

Mascart (Jean):
*Introduction à l'Observation des Étoiles Variables.
(*Author.*) 8vo. Torino, 1916

Mauritius, Royal Alfred Observatory:
——: Results of the Magnetical, Meteorological, and Seismological Observations, 1916. New Series, Vol. 2, pt. 8–12.
(*Observatory.*) 4to. Mauritius, 1916
——: Annual Report of the Director . . . for 1915.
(*Observatory.*) fol. Mauritius, 1916

Mee (Arthur):
Welsh astronomical and nature notes. 1915 Jan.–Dec.
(*Editor.*) 4to. [Cardiff]

Melbourne Observatory:
Report of the Board of Visitors, together with the Report of the Government Astronomer. 41st–46th Reports, 1906–1914.
fol. Melbourne, 1908–1915

Milan, Reale Osservatorio di Brera:
Pubblicazioni, No. 53, 54.
(*Observatory.*) 4to. Bologna, 1916
No. 53. F. Vercelli. Oscillazione periodiche e previsione della pressione Atmosferica.
„ 54. L. Volta e G. Silva. Sulla riduzione al vuoto della durati di oscillazioni di Pendoli Gravimetrice.

Missouri University, Laws Observatory:
Bulletin, Astronomical Series. No. 24, 25.
(*Observatory.*) 4to. 1916

Digitized by Google

Monthly Evening Sky Map (the). A Star, constellation and planet finder map for the current month. Vol. 10, No. 7–12; Vol. 11, No. 1–6.
(*Messrs. Wm. Wesley & Son.*) 4to. New York, 1916–1917

Montpellier, Académie des Sciences et Lettres:
Bulletin mensuel, 1916, No. 6–12; 1917, No. 1–4.
(*Academy.*) 8vo. Montpellier, 1916–1917

Mount Wilson, California, Solar Observatory:
——: *Contributions . . . No. 109, 112–126.
(*Carnegie Institution.*) 8vo. Chicago, 1916
——: *Communications to the National Academy of Sciences. No. 27–43.
(*Carnegie Institution.*) 8vo. Boston, 1916–1917
——: Annual Report of the Director . . . 1916.
(*Carnegie Institution.*) 8vo. Washington [1917]

National Physical Laboratory:
Report of the year 1915–1916.
(*Laboratory.*) 8vo. Teddington, 1916

Nature. A weekly illustrated journal of science. Vol. 97–99, No. 2433–2484.
(*Editor.*) 4to. London, 1916–1917

Neuchâtel, Observatoire Cantonal:
Rapport du Directeur . . . pour l'année 1914, 1915.
(*Observatory.*) 8vo. Neuchâtel, 1916

New Zealand, Department of Lands and Survey:
Report of the Survey operations, 1915–1916.
(*Dept. of Lands & Survey.*) fol. Wellington, 1916

Nijland (A. A.):
——: *Over het vermogen van der Utrechtschen kijker en de fotometrische schalen van Harvard en J. A. Parkhurst.
(*Author.*) 8vo. 1916
——: *Het readsel van V.19=SS Cygni.
(*Author.*) 8vo. 1917
——: *Beobachtungen von SS=V.19 Cygni, nebst Ableitung von mittleren Kurven für de letzten 12 Jahre.
(*Author.*) 4to. Kiel, 1917

Observatory (the). A monthly review of astronomy; edited by A. S. Eddington, F. J. M. Stratton, and H. S. Jones. Vol. 39, 40 (No. 502–514).
(*Editors.*) 8vo. London, 1916-1917

Olsen (Ole Theodor):
The Fisherman's Nautical Almanac and Tide Tables. 1917.
(*Author.*) 8vo. Grimsby, 1916

Digitized by Google

Ottawa, Dominion Astronomical Observatory:
Publications. Vol. 3, No. 6, 7.
(*Observatory.*) 4to. Ottawa, 1916

Oxford, Radcliffe Observatory:
Results of meteorological observations, 1911–1915. Vol. 51.
(*Radcliffe Trustees.*) 8vo. Oxford, 1916

Oxford, University Observatory:
Forty-second Annual Report of the Savilian Professor of Astronomy, 1916–1917.
(*Observatory.*) 8vo. Oxford, 1917

Paris, Académie des Sciences:
——: Comptes rendus hebdomadaires des séances. Tome 162, No. 23–26; Tome 163, No. 1–26; Tome 164, No. 1–22.
(*Academy.*) 4to. Paris, 1916–1917
——: Recueil du Fonds Bonaparte. Publication annuelle. No. 1, 1915.
4to. Paris, 1916

Paris, Bureau des Longitudes:
——: Annuaire pour l'an 1917: avec des notices scientifiques.
(*Bureau.*) 16mo. Paris
——: Connaissance des Temps, ou des mouvements célestes pour le méridien de Paris, à l'usage des astronomes et des navigateurs, 1918.
(*Bureau.*) 8vo. Paris, 1916
——: Connaissance des Temps. Extrait à l'usage des marins, 1916.
(*Bureau.*) 8vo. Paris, 1915

Paris, Ministère de l'Instruction Publique:
Enquêtes et documents relatifs à l'enseignement supérieur: Rapport sur les Observatoires astronomiques de province, Année 1915. CXI.
(*Minister of Public Instruction.*) 8vo. Paris, 1916

Paris, Société Astronomique de France:
L'Astronomie: Revue mensuelle d'astronomie, de météorologie et de physique du globe, et Bulletin de la Société Astronomique de France. Année 30, No. 6–12; Année 31, No. 1–5.
(*Society.*) 8vo. Paris, 1916–1917

Paris, Société Mathématique de France:
——: Bulletin. Tome 44, fasc. 1–3.
(*Society.*) 8vo. Paris, 1916–1917
——: Comptes Rendus des Séances de l'Année 1916.
8vo. Paris, 1916

Digitized by Google

Paris, Société Philomathique :
Bulletin. 10me série. Tome 8, 10, No. 1, 2.
(*Society.*) 8vo. Paris, 1916–1917

Philadelphia, American Philosophical Society :
Proceedings. Vol. 54, No. 220 ; Vol. 55, No. 1–8.
(*Society.*) 8vo. Philadelphia, 1916

Philadelphia, Franklin Institute :
Journal. Vol. 181, No. 6; Vol. 182, No. 1–6; Vol. 183, No. 1–5.
(*Institute.*) 8vo. Philadelphia, 1916–1917

Philosophical Magazine :
The London, Edinburgh, and Dublin Philosophical Magazine. Series 6, Vol. 32, 33 (No. 187–198).
(*Purchased.*) 8vo. London, 1916–1917

Pickering (William H.) :
—— : *Evidences of Erosion on the Moon.
(*Author.*) 8vo. Northfield, Minn., 1916
—— : *The Aristillus test for planetary definition.
(*Author.*) 8vo. Northfield, Minn., 1916
—— : Invisible Comets.
8vo. Northfield, Minn., 1916
—— : *Monthly Report on Mars, No. 15, 16.
(*Author.*) 8vo. Northfield, Minn., 1916

Pomona College, Astronomical Society :
Publication, Vol. 5, No. 3.
(*Society.*) 8vo. Claremont, Cal., 1916

Popular Astronomy. Founded by W. W. Payne : edited by H. C. Wilson [and others]. Vol. 24, No. 6–10 ; Vol. 25, No. 1–5.
(*Editors.*) 8vo. Northfield, Minn., 1916–1917

Princeton University Observatory :
Contributions, No. 4.
(*Observatory.*) 4to. Princeton, 1916
R. Smith Dugan. Photometric Researches of Eclipsing Variables.

Pulkovo, Observatoire Central Nicolas :
Bulletin, Band 7, No. 7.
(*Observatory.*) 4to. 1916

Rio de Janeiro, Observatorio :
Annuario . . . para o anno de 1916.
(*Observatory.*) 8vo. Rio de Janeiro, 1916

Digitized by Google

Revue du Ciel (la) :
Revue mensuelle Théorique et Pratique d'Astronomie, de Météorologie, de Physique du Globe et des Sciences qui s'y rattachent. Directeur Abbé Th. Moureux. 1re Année, No. 1–3 ; 2me Année, No. 4–9.
(*M. Moureux.*) 8vo. Bourges, 1916–1917

Rome, Reale Accademia dei Lincei :
——: Atti . . . Anno 313, 314, Serie quinta. Rendiconti, Classe di scienze fisiche, mathematiche e naturali. Vol. 25, semestre 1, fasc. 9–12 ; semestre 2, fasc. 1–12 ; Vol. 26, semestre 1, fasc. 1–8.
(*Academy.*) 4to. Roma, 1916–1917
——: Rendiconti dell' Adunanza Solenne. 1916 12 Nov. Vol. 3.
(*Academy.*) 4to. Roma, 1916

Rome Specola Astronomica Vaticana :
Pubblicazioni. No. 11.
(*Observatory.*) 4to. Roma, 1916
J. G. Hagen. Aggiunti al Catalogo dell' Atlas Stellarum Variabilium.

San Francisco, Astronomical Society of the Pacific :
Publications, Vol. 28, 29 (No. 164–168).
(*Society.*) 8vo. San Francisco, 1916–1917

Salcombe Regis, the Hill Observatory :
Bulletin, Vol. 1, No. 5.
(*Observatory.*) 4to. 1916

Science Abstracts. Section A (Physics). Vol. 19, pt. 6–12 ; Vol. 20, pt. 1–5.
(*Institution of Electrical Engineers.*)
8vo. London, 1916–1917

Shaw (Napier) :
The Weather Map : an introduction to modern Meteorology.
(*Meteorological Office.*) 16mo. London, 1916

Stockholm, Kungliga Svenska Vetenskaps Akademie :
——: Arkiv för mathematik, astronomi och fysik. Bandet 10, Häfte 3, 4.
(*Academy.*) 8vo. Uppsala & Stockholm, 1915
——: Arsbok för år 1916.
(*Academy.*) 8vo. Uppsala & Stockholm, 1916

Stockholm, Observatorium :
Astronomiska Iakttagelser och Undersökningar . . . utgivna af Karl Bohlin. Bandet 10, No. 4.
(*Observatory.*) 4to. Uppsala & Stockholm, 1916

Digitized by Google

Stonyhurst College Observatory :
Results of meteorological, magnetical, and seismological observations, with report and notes of the Director, Rev. W. Sidgreaves, 1916.
(*Observatory.*) 8vo. Blackburn, 1917

Switzerland, Schweizerische Geodätische Kommission :
Astronomisch-geodätische Arbeiten in der Schweiz. Band 15.
(*The Commission.*) 4to. Zürich, 1916

Sydney, Royal Society of New South Wales :
Journal and Proceedings. Vol. 49, pt. 3, 4; Vol. 50, pt. 1, 2.
(*Society.*) 8vo. Sydney, 1916

Tacubaya, Observatorio Astronómico Nacional :
——: Anuario . . . para el año de 1917.
(*Observatory.*) 8vo. México, 1916
——: Boletin, No. 5.
(*Observatory.*) fol. México, 1916
——: Catalogo Astrofotografico, 1900, —9° a —17°, tomo 1.
(*Observatory.*) fol. México, 1916

Terrestrial Magnetism and Atmospheric Electricity :
An International Quarterly Journal. Vol. 21, No. 2–4; Vol. 22, No. 1.
(*Editor.*) 8vo. Baltimore, 1916–1917

Thompson (Silvanus Phillips):
Optical Tables and Data for the use of Opticians. Second edition, revised.
(*Purchased.*) obl. 8vo. London and New York, 1907

Toronto, Royal Astronomical Society of Canada :
——: Journal, Vol. 10, No. 6–10; Vol. 11, No. 1–5.
(*Society.*) 8vo. Toronto, 1916–1917
——: The Observer's Handbook, 1917.
(*Society.*) 8vo. Toronto, 1917

Tortosa, Observatorio del Ebro :
Boletin mensual, Vol. 6, No. 7–12; Vol. 7, No. 1–4.
(*Observatory.*) 4to. Barcelona, 1916–1917

Turin, Reale Accademia della Scienze :
——: Atti, Vol. 51, No. 1–15; Osservazioni Meteorologiche, e Indice Generali dei Volume 41–50.
(*Academy.*) 8vo. Torino, 1916
——: Memorie, serie seconda, Vol. 65. 4to. Torino, 1916

Digitized by Google

Turin, Società Urania:

Saggi di Astronomia Popolare, Anno 6, No. 6–12; Anno 7, No. 1, 2.
(*Society.*) 8vo. Torino, 1916–1917

United States Coast and Geodetic Survey:

——: Centennial Celebration of the United States Coast and Geodetic Survey, April 5 and 6, 1916.
(*Survey Office.*) 8vo. Washington, 1916

——: Results of observations made at the . . . Magnetic Observatory near Honolulu, Hawaii, 1913–1914; near Tucson, Arizona, 1913–1914; at Sitka, Alaska, 1913–1914; at Vieques, Porto Rico, 1913–1914.
(*Survey Office.*) 4to. Washington, 1916

——: Special Publications, No. 29, 36.
(*Survey Office.*) 4to & 8vo. Washington, 1916

No. 29. F. Morse and O. B. French. Difference of longitude, Washington, Cambridge, and Far Rockaway.
„ 36. D. L. Hazard. Results of Magnetic Observations in 1915.

Utrecht, Observatoire Astronomique:

Recherches Astronomiques, No. 6.
(*Observatory.*) 4to. Utrecht, 1916

No. 6. J. van der Bilt. Light-curve of RV Tauri.

Walkey (O. R.):

*Adopted measured parallaxes of 625 Stars, grouped according to the Types of their Spectra.
(*Author.*) 8vo. London, 1917

Washington, National Academy of Sciences:

Proceedings, Vol. 2, No. 6–12; Vol. 3, No. 1–4.
(*Academy.*) 8vo. Baltimore, 1916–1917

Washington, Smithsonian Institution:

——: Smithsonian Miscellaneous Collections. Vol. 64, No. 5; Vol. 65, No. 9; Vol. 66, No. 6, 9–15
(*Institution.*) 8vo. Washington, 1916

Vol. 65, No. 9. C. G. Abbot. Arequipa Pyrheliometry.

——: Annual Report of the Board of Regents . . . . 1915, 1916.
(*Institution.*) 8vo. Washington, 1916

Washington, United States Naval Observatory:

——: Annual Report . . . for the year 1916.
(*Observatory.*) 8vo. Washington, 1916

——: Publications. . . . Second series. Vol. 9, Appendix.
(*Observatory.*) 4to. Washington, 1916

Digitized by Google

Wolf (Max):
——: *Der Ringnebel und der Dumbbellnebel.
(*Author.*) 8vo. Heidelberg, 1915
——: *Über die Spektren der Wolf-Rayet Sterne.
(*Author.*) 8vo. Heidelberg, 1915
——: *Geschichtete Emission im Nebelfleck H iv. 39 Argûs.
(*Author.*) 8vo. Heidelberg, 1916

Zeitschrift für Instrumentenkunde. Organ für Mittheilungen aus dem gesammten Gebiete der wissenschaftlichen Technik. Jahrgang 36, Heft 5–12.
(*Purchased.*) 4to. Berlin, 1916

Zürich, Naturforschende Gesellschaft:
Vierteljahrsschrift, Jahrgang 61, Heft 1, 2.
(*Society.*) 8vo. Zürich, 1916

PHOTOGRAPHS, &c., PRESENTED TO THE SOCIETY.

ffoulkes (Charles): Two photographs of the Turret of the White Tower, used by Flamsteed as an Observatory in 1675.

Helwân Observatory: Photographs of the Variable Nebula in Corona Australis. (24 lantern slides.)

Herschel (Miss F.): Enlarged photographs of parts of letters from Lalande to Sir W. Herschel, suggesting a symbol for the planet Uranus. (Paris, 1782 and 1784.)

McGegan (J. E.): Star Identifier and Diagram for the graphical solution of problems in Nautical Astronomy [with explanation and examples]. 1916.

Reynolds (J. H.): Photographs of Jupiter taken Oct.–Dec. 1916. (7 lantern slides.)

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